

High level summary

The purpose of this term paper is to highlight the three important elements in any arbitrage opportunity and then provide an illustrative example specific to financial markets. A discussion of (1) *people* is followed by (2) *processes* and within them (3) *probabilities*, followed by the example.

The 3Ps' in arbitrage

As individuals, we are affected by constant and emergent properties (random variables) at the automatic and reflective levels of cognition. In both forms, we perceive signals, process them and make a decision. The key difference is evident in the fact that magnitude or abstractness of axioms affecting the automatic cognitive system is more complex than what we experience consciously. When it comes down to goal-based choices we make, a process that considers both types of "information" can be explained mathematically through the branch of set theory and notion of countability. Although it is said to lead to paradoxical conclusions regarding an economic agent's preference regarding the desired quantity of information in making a particular decision, information modeling is achieved through the use of σ -fields (Dubra & Echenique, 2004). Essentially, signal $f: \Omega \rightarrow X$ generates a partition P_f on Ω and is mapped to f as an inverse function of f defined by $P_f = \{f^{-1}(x): x \in X\}$. Such a signal f_1 belongs to σ -field $\mathcal{F} \subset \Omega$ and said to be measurable, if for (Ω, \mathcal{F}) and (X, \mathcal{G}) , $f^{-1}(A) \in \mathcal{F}, \forall A \in \mathcal{G}$.

Since the collection of events is simultaneously a subset of itself, the power set, or the number of total sets is the entire collection raised by the base of two; $\mathcal{F} = 2^\Omega$. Essentially, as an individual, the older you become, the larger this power set grows. However, sense-making at the reflective level is dominated by composition of more recent events that influence a particular decision and that may lead to an overvaluation of the present in some regards (Langlois, 2014). For instance, a steady decline for a week in your equity portfolio value may lead you to conclude that it will decline perpetually and therefore a change is necessary. While not an exact fit with the definition of a Markov chain, ad-hoc sense-making can be explained as a modification that places more weight to the recent past, as opposed to an all-encompassing historical and neutral perspective utilized in a decision.

In order to demonstrate mathematically, we let a set of states $S = \{s_1, s_2, s_3 \dots s_n\}$ define the historical experience in some particular regard for an individual. In a state described by the Markov chain, the history of states up to s_n is disregarded and s_n along with respective transition probabilities determine s_{n+1} . In the sense-making paradigm, the history up to, say, s_{n-4} may be ignored and events $[s_{n-4}, s_n]$ represent the subset taken into consideration. Since $\mathbb{E}[s_{n+1} | s_n, s_{n-1}, s_{n-2}, s_{n-3}, s_{n-4}]$ the transition probabilities are likely to be tied to whether the discrete $f(x)$ in a continuous setting is monotone increasing or decreasing. Moreover, as optimization is a given at the automatic level of cognition, we naturally reduce costs associated with cognitive processing: i.e., the brain does not want to think (Langlois, 2014). Under such a view, the reference to a relevant variable influencing the decision may just be one reference point. What determines which reference point to use is explained through mental accounting, defined "quite narrowly as 'an outcome frame which specifies (i) the set of elementary outcomes that are evaluated jointly and the manner in which they are combined and (ii) a reference outcome that is considered normal or neutral.'"

Given the fact there is a limit to each of our cognitive processing ability, we are subject to blind variation and selective retention (McLennan, 2007), influenced by the sequence of sets that we have been exposed to. This limitation may not always allow us to process all of the complete and axiomatic choices we have. Hence, together with our propensity to avoid energy expenditure (principle of least effort) and procrastinate (i.e., a pattern seeking brain), we are disinclined to lose something of value (mental accounting) as a result of the filtration that takes place on the subsets of events in continuous time based cognitive processes. Disinclination and filtration associated with something of value, is what we call risk- or loss aversion. Thaler (1999) explains it clearly identifying loss aversion as one of three elements making up mental accounting; *“Losing \$100 hurts more than gaining \$100 yields pleasure: $v(x) < -v(-x)$. The influence of loss aversion in mental accounting is enormous.”*

Risks are of two categories (risk⁺ and risk⁻ as discussed above) resulting from complex adaptive systems characterized by circular causality and perceived by us through our cognition. Though negative risks have greater bearing on our decisions, there is something to be said about risks that are positive. Positive risks, or the more familiar term, opportunity, do exist. They are however, more difficult to identify and quantify. It is at the intersection of positive and negative risk do we find the discrepancy between the observable and theoretical and identify them as arbitrage opportunities in the economic realm. Though the types of risks vary and are numerous, generally they are grouped as either speculative or pure. For the most part, risks within financial markets are speculative in nature (Wind, 2010).

Why are individual and collective cognition, the processes of mental accounting, category and types of risks important? Because collectively they run parallel with the notion of “uncertainty,” which is rooted in \mathbb{R} (in a mathematical sense) which then gives rise to the art and science of probabilities by the way of \mathbb{Q} and the gap that exists between them. See appendix exhibit 1 for the conceptual relationships at a high-level.

A comprehensive valuation example through fundamental analysis

As equities financial markets around the world are comprised of a finite set of listed entities, their individual stock price valuation (from either a book or fair value perspective) is ascertained given the figures found on their financial statements. When this perceived (speculative) yet “risk neutral” price exceeds the trading stock price, an arbitrage opportunity exists from the market’s mispricing. Consider the set of financial statements for XYZ Company found in appendix exhibit 2. Suppose there are 100,000 common shares outstanding and the book value associated with XYZ’s shares = Common Stock + Additional Paid In Capital + Retained Earnings = 3,043,000, scaled by (\$000’s) on the balance sheet. The book value per share is = $3,859,000/100,000 = \$38.59$. Let’s also assume the XYZ stock’s trading price is \$43.62. The goal now is to calculate fair value of XYZ’s price per share using fundamental analysis and in the process discuss how the 3Ps’ of arbitrage interrelate with one another.

Several pieces of information are required to arrive at fair value. Categorically, they can be grouped as forecasts from (1) growth & profitability (2) free cash flows and (3) discounted cash flows to account for the time value of money. Using the financial statements found in exhibit 2 of

the appendix along with some basic assumptions (for simplicity's sake) regarding operations found in exhibit 3, the appropriate forecasts will be attained. To recognize that forecasting future numbers in a stochastic (continuous; going concern entity) process is really deterministic in nature is important. The reason for that stems from randomness. No two iterations are likely to be exact in comparison with one another.

Growth & profitability

Suppose XYZ is in a subscription based business. That is, they acquire and then attempt to maintain long-term relationships with their customers. The collective effort within the firm; sales, marketing, product specific departments, etc. will have an impact on how the customers are acquired and how long such relationships are maintained. Mathematically, this relationship is defined by the following notation. In practice, it is referred to as customer lifetime value (CLV).

$$CLV = \sum_{t=0}^T \frac{(R_t - C_t)\mathbb{P}_t}{(1+i)^t} - A$$

R_t = revenue at time t from customer, C_t = cost to retain customer at t ,
 \mathbb{P}_t = probability the customer is active at time t , i = discount rate,
 A = cost to acquire customer, T = cumulative time interval

In arriving at $E(CLV)$, affinities specific to that customer will be identified and noted as dichotomous attributes, which then allows XYZ to improve accuracy in forecasting future values. For instance, assume one of the many attributes associated with any particular customer is defined as: $(x) = \begin{cases} 1, & x = \text{yes} \\ 0, & \text{otherwise} \end{cases}$, $x = \text{more than 2 purchases}$. By utilizing indicator functions such as the one discussed, segmentation (partitioning) of the customers (sample space) enables XYZ to assign probabilities that are specific to a particular subset of customers.

Assume that XYZ has been in business for 5 years and is expecting the launch of a new product to generate added demand for the foreseeable future. As the business activity transitions to a new state, the probabilities are likely to change as well, expectation. The uncertainty surrounding the new product launch is captured within \mathbb{P}_t , modeled as closely to existing product with similar, identifiable affinities. In order to arrive at expectation, a simulated iterative method that conforms to the behavior observed with current customers and the range of values relevant to those customers need be ascertained. Luckily, there are existing methods such as the Monte-Carlo that can be utilized to line up such parameters and evaluate them iteratively.

Now suppose market size for the first year has already been derived and estimated growth rates short and long terms are also available. Then, we will begin with segmenting the current customer list according to some criteria and choose the segment that best represents the new offering. We then drill down to summarize purchase behavior periodically for the relevant group. In our example, we will assume subscriptions follow the pattern seen in a convex graph as it approaches an asymptotic level representative of the segment's population. That is, there is a step-down pattern as time progresses, which is represented by $\frac{K_{t-1} - K_t}{\Delta t}$. At this point set $\{s_1, s_2, s_3 \dots s_M\}$ is representative of the particular segment and we are able to get other pertinent data such as

Though the previous few paragraphs summarized one aspect in evaluation of a “project,” it is usually accompanied by a thorough exercise in order to allocate capital expenditure in practice. Moreover, as this project may be just one of many, XYZ can potentially be evaluating this project as a “real option.” In putting together an analysis for one specific project (let’s say our new product launch), XYZ may develop something like the following utilizing the probabilities attained by the way of the iterative simulation in order to inform investment decisions.

*Cash flows of potential outcomes discounted at the xx.x% cost of capital:

****Joint probabilities:** Probs 1 and 2 = $0.8 \times 0.6 \times 0.5 = 0.24$; Prob 3 = $0.8 \times 0.4 = 0.32$; Prob 4 = 0.2.

Moving on, assuming assets of XYZ are proportionally tied to revenue generation, and the marginal cost of operating the business in all respect do not change (i.e., no price breaks for increased quantity of inventory orders), we will model growth and profitability, free cash flows and discounted cash flows as follows.

XYZ Growth & Profitability

(In Millions)	FY ENDED	PROJECTIONS FOR FISCAL YEAR ENDING										Stable Capital Structure
	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018		
Total Revenue	20,678	22,332	23,895	25,568	27,358	28,999	30,739	32,276	33,245	34,242	34,584	
% Growth	4.32%	8.00%	7.00%	7.00%	7.00%	6.00%	6.00%	5.00%	3.00%	3.00%	1.00%	
-COGS	8,987	9,706	10,385	11,112	11,890	12,604	13,360	14,028	14,449	14,882	15,031	
% of Revenue	43.46%	43.46%	43.46%	43.46%	43.46%	43.46%	43.46%	43.46%	43.46%	43.46%	43.46%	
Gross Profit	11,691	12,626	13,510	14,456	15,468	16,396	17,380	18,249	18,796	19,360	19,553	
% of Revenue	56.54%	56.54%	56.54%	56.54%	56.54%	56.54%	56.54%	56.54%	56.54%	56.54%	56.54%	
-OPEX	8,494	9,174	9,816	10,503	11,238	11,912	12,627	13,258	13,656	14,066	14,206	
% of Revenue	41.08%	41.08%	41.08%	41.08%	41.08%	41.08%	41.08%	41.08%	41.08%	41.08%	41.08%	
EBIT	3,197	3,453	3,694	3,953	4,230	4,484	4,753	4,990	5,140	5,294	5,347	
% of Revenue	15.46%	15.46%	15.46%	15.46%	15.46%	15.46%	15.46%	15.46%	15.46%	15.46%	15.46%	
-Tax Provision	1,642	941	1,007	1,077	1,152	1,222	1,295	1,360	1,400	1,442	1,457	
% of EBIT	51.36%	27.24%	27.24%	27.24%	27.24%	27.24%	27.24%	27.24%	27.24%	27.24%	27.24%	
NOPAT	1,555	2,512	2,688	2,876	3,077	3,262	3,458	3,631	3,740	3,852	3,890	
% of Revenue	7.52%	11.25%	11.25%	11.25%	11.25%	11.25%	11.25%	11.25%	11.25%	11.25%	11.25%	

Free & Discounted cash flows

XYZ Free Cash Flow

(In Millions)	FY ENDED	PROJECTIONS FOR FISCAL YEAR ENDING										Stable Capital Structure
	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018		
NOPAT	1,555	2,512	2,688	2,876	3,077	3,262	3,458	3,631	3,740	3,852	3,890	
% of Revenue	7.52%	11.25%	11.25%	11.25%	11.25%	11.25%	11.25%	11.25%	11.25%	11.25%	11.25%	
-CAPEX	418	1345	1439	1540	1647	1746	1851	1944	2002	2062	2082	
% of Revenue	2.02%	6.02%	6.02%	6.02%	6.02%	6.02%	6.02%	6.02%	6.02%	6.02%	6.02%	
+ D & A	117	262	425	610	818	1043	1292	1552	1800	2062	2082	
% of Revenue	0.57%	1.17%	1.78%	2.38%	2.99%	3.60%	4.20%	4.81%	5.42%	6.02%	6.02%	
Working Capital	3,496	3,776	4,040	4,323	4,625	4,903	5,197	5,457	5,621	5,789	5,847	
% of Revenue	16.91%	16.91%	16.91%	16.91%	16.91%	16.91%	16.91%	16.91%	16.91%	16.91%	16.91%	
- D in WC	496	280	264	283	303	278	294	260	164	169	58	
% of Revenue	2.40%	1.25%	1.11%	1.11%	1.11%	0.96%	0.96%	0.81%	0.49%	0.49%	0.17%	
Free Cash Flow	758	1,149	1,410	1,663	1,946	2,281	2,605	2,980	3,374	3,683	3,832	
% of Revenue	3.67%	5.15%	5.90%	6.51%	7.11%	7.87%	8.47%	9.23%	10.15%	10.76%	11.08%	

XYZ Discounted Cash Flows

(In Millions)	PROJECTIONS FOR FISCAL YEAR ENDING									
	10	11	12	13	14	15	16	17	18	Stable capital structure
Free Cash Flow	1,149	1,410	1,663	1,946	2,281	2,605	2,980	3,374	3,683	3,832
WACC*	23.67%	23.67%	23.67%	23.67%	23.67%	23.67%	23.67%	23.67%	23.67%	23.67%
PV Factor	1.000	0.809	0.654	0.529	0.428	0.346	0.280	0.226	0.183	0.183
PV of FCF	1,149	1,140	1,088	1,029	975	900	833	763	673	3,090

Summing up PV of FCF gives us our total capitalization. From that we deduct claims found on the balance sheet. This gives us with the portion of capital left for the common stock equity shareholders. Dividing that portion by the number of common shares outstanding yields the fair value of XYZ. Before presenting discussed exercise, it is noteworthy to mention the stable capital structure column, especially the terminal value number factors in the fact XYZ is a continuous, going concern entity. That number is derived via Gordon growth model which utilizes the long-term growth rate and weighted average cost of capital (WACC). Further, XYZ's WACC is at almost 24% because the firm is pretty much a startup, with only 5 years of history behind it. Chances are, they are venture capital backed with equity shareholders and mezzanine debt issued that carry with them very high required rates of return, given the commensurate level of risk those creditors and investors have to assume. Now to calculate XYZ stock's fair value:

Fair value per share of XYZ's stock

Total PV of Capitalization (Debt + Equity)	11,639	KEY ASSUMPTIONS	
-Claims		LT-Growth Rate	1.00%
Long Term Debt	2,464	WACC*	23.67%
Preferred Stock	512		
Total External Claims	2,976		
Net Present Value of Equity (Shareholder Claims)	8,663		
Common Shares Outstanding	100		

Fair Value Per Outstanding Share of Common Stock	\$86.63
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Arbitrage opportunity

If one were to agree with the assumptions that were made in putting together the aforementioned analysis, they will note that clearly there is an arbitrage opportunity given XYZ stock's trading price of \$43.62 with its fair value of \$86.63. The stock is underpriced and the gain potential by

investing in XYZ would be to double the investment. However, unless you are an insider who has access to pertinent data such as segmentation based customer life time value, or information from other plans in the works that is not publicly available, there will be a significant amount of subjectivity that will be placed in any valuation exercise. Like you, many others will also use their judgement in arriving at what the fair price for a share of stock should be and those differences determine whether you win or lose with your investment. The uncertainty in people's cognition, the processes of markets and the probabilities within them make things interesting. Whether such a game makes you happy, I suppose, depends on you.

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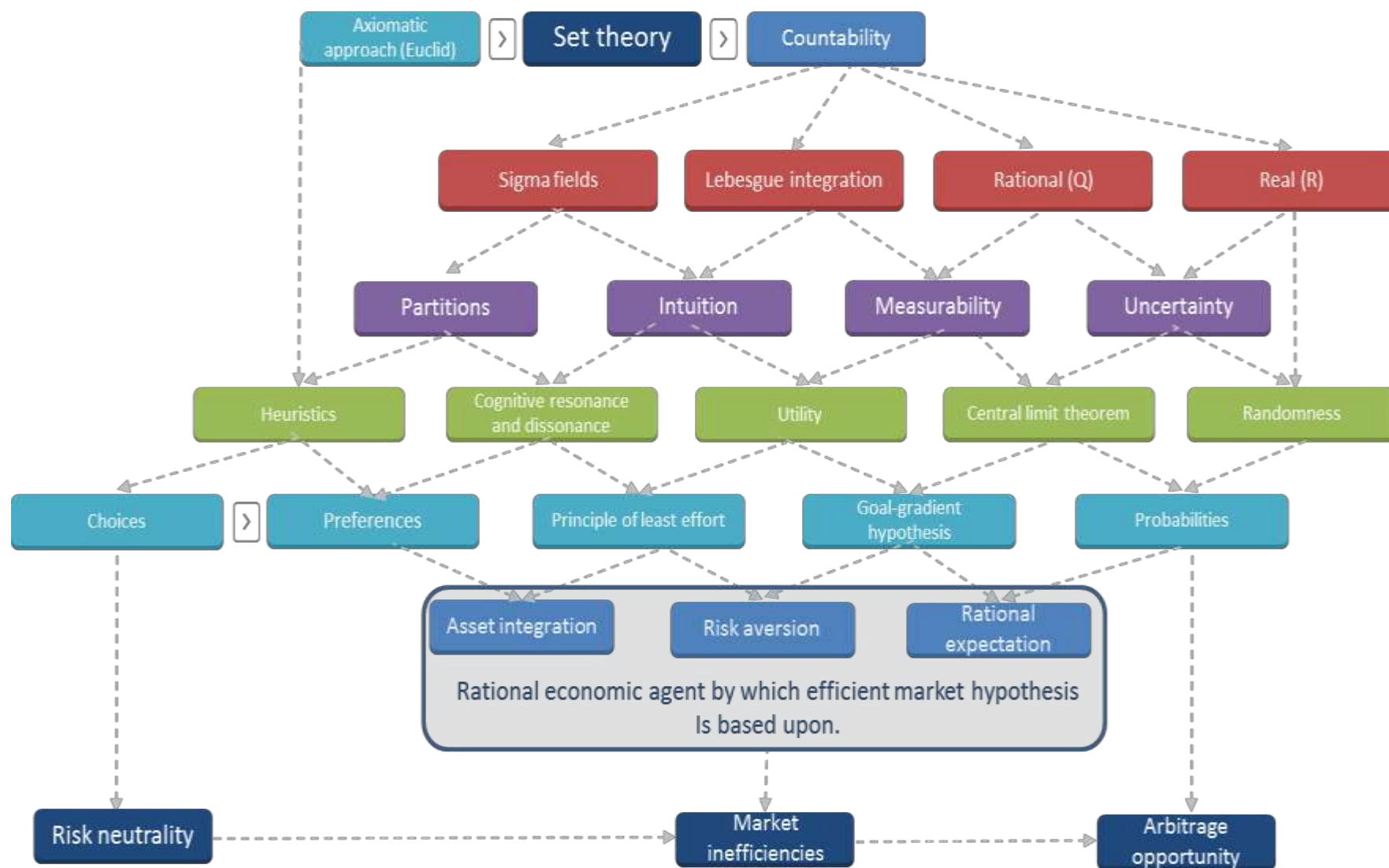
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Appendix

Exhibit 1: Mindmap of variables leading to Arbitrage opportunity



Appendix

Exhibit 3: Assumptions and segmented information including forecast

XYZ Assumptions

Long-term growth rate 1.00% Cannibalization: 20% from segment A and B are cannibalized by new product
WACC 23.67% Retention includes first purchase.

Operating segment

	A		B		C		D		E - New Product	
	Current Yr.	Forecast	Current Yr.	Forecast	Current Yr.	Forecast	Current Yr.	Forecast	Current Yr.	Forecast
# of customers (000's)	78	62	91	73	57	57	15	15	-	55
# of purchases (000's)	497	398	493	395	292	292	115	115	-	327
Retention	6.37	6.37	5.44	5.44	5.12	5.12	7.59	7.59	-	5.91
Segment product price per unit	\$8	\$8	\$15	\$15	\$22	\$22	\$25	\$25	-	\$12
Revenue	\$3,978	\$3,182	\$7,398	\$5,918	\$6,423	\$6,423	\$2,879	\$2,879	-	\$3,929
Product cost per unit	\$4	\$4	\$6	\$6	\$9	\$9	\$12	\$12	-	\$5
Total product cost	\$1,977	\$1,582	\$3,056	\$2,444	\$2,516	\$2,516	\$1,438	\$1,438	-	\$1,665
Cost to acquire 1 cust.	\$9	\$9	\$9	\$9	\$9	\$9	\$9	\$9	-	\$9
Total acquisition cost	\$736	\$589	\$855	\$684	\$538	\$538	\$143	\$143	-	\$522
Total variable cost	\$2,713	\$2,171	\$3,911	\$3,128	\$3,054	\$3,054	\$1,581	\$1,581	-	\$2,188
Contribution Margin	\$1,265	\$1,012	\$3,487	\$2,790	\$3,369	\$3,369	\$1,298	\$1,298	-	\$1,741