AI Assignment 6

Q.2.

$$A \longrightarrow B \longrightarrow C \longrightarrow D$$

$P(A=T)=0.4$

$P(B=T|A=T)=0.1$
$P(B=T|A=F)=0.8$

$P(C=T|B=T)=0.7$
$P(C=T|B=F)=0.4$

$P(D=T|C=T)=0.82$
$P(D=T|C=F)=0.37$

A	P(A)
T	0.4
F	0.6

B	P(B|A=T)
T	0.1
F	0.9

B	P(B|A=F)
T	0.8
F	0.2

C	P(C|B=T)
T	0.7
F	0.3

C	P(C|B=F)
T	0.4
F	0.6

D	P(D|C=T)
T	0.82
F	0.18

D	P(D|C=F)
T	0.37
F	0.63

Joint distribution -

$$P(A,B,C,D) = P(A)\,P(B|A)\,P(C|B)\,P(D|C)$$

(a) $P(B)$.

$$\sum_A \sum_C \sum_D P(A)\,P(B|A)\,P(C|B)\,P(D|C)$$

$$= \sum_A P(A)\,P(B|A) \sum_C P(C|B) \underbrace{\sum_D P(D|C)}_{f_1(C)}$$

$$\sum_{D} P(\overset{T}{\underset{F}{D}} \mid \overset{T}{\underset{F}{C}})$$

C	$f_1(C)$
T	$0.82 + 0.18 = 1$
F	$0.37 + 0.63 = 1$

$\therefore f_1(C)$ is a function of $\langle 1, 1 \rangle$

$$\sum_{A} P(A)\, P(B \mid A) \sum_{C} P(C \mid B)\, f_1(C)$$

$$\sum_{A} P(A)\, P(B \mid A) \underbrace{\sum_{C} P(C \mid B)}_{f_2(B)}$$

B	$f_2(B)$
T	$0.7 + 0.3 = 1$
F	$0.4 + 0.6 = 1$

$$\sum_{C} P(\overset{T}{\underset{F}{C}} \mid \overset{F}{\underset{T}{B}})$$

$f_2(B)$ is a function of $\langle 1, 1 \rangle$

$$\sum_{A} P(A)\, P(B \mid A) \cdot f_2(B)$$

$$\sum_{A} \underbrace{P(A)\, P(B \mid A)}_{f_3(B)}$$

A	B	$P(A) \cdot P(B \mid A)$
T	T	$0.4 \times 0.1 = 0.04$
T	F	$0.4 \times 0.9 = 0.36$
F	T	$0.6 \times 0.8 = 0.48$
F	F	$0.6 \times 0.2 = 0.12$

Summing over A. -

B	$f_4(B)$
T	$0.04 + 0.48 = 0.52$
F	$0.36 + 0.12 = 0.48$

Normalizing - Since Evidence is empty, we do not need to divide.

(b) $P(C \mid A=T)$

~~$P(A,B,C,D) = P(A)\,P(B|A)\,P(C|B)$~~ $P(D|C)$

Evidence $\Rightarrow A=T$

~~P(A,B.~~

Joint Distribution - $P(A=T)\,P(B|A=T)\,P(C|B)\,P(D|C)$

→ Eliminate the variables which are not in query and evidence ∴ B & D.

$$\sum_B \sum_D P(A=T)\,P(B|A=T)\,P(C|B)\,P(D|C)$$

$$P(A=T) \sum_B P(B|A=T)\,P(C|B) \underbrace{\sum_D P(D|C)}_{f_1(C)}$$

C	$f_1(C)$
T	$0.82 + 0.18 = 1$
F	$0.37 + 0.63 = 1$

$$P(A=T) \sum_B P(B|A=T)\ P(C|B) \cdot f_1(C).$$

$$P(A=T) \underbrace{\sum_B P(B|A=T)\ P(C|B)}_{f_2(C,\ A=T)}$$

C	$P(B\|A=T)\ P(C\|B)$
T	$(0.1 \times 0.7) + (0.9 \times 0.4) = 0.07 + 0.36 = 0.43$
F	$(0.1 \times 0.3) + (0.9 \times 0.6) = 0.03 + 0.54 = 0.57$

$$\underbrace{P(A=T) \cdot f_2(C,\ A=T)}_{f_3(C\ |\ A=T)}$$

C	$P(A=T) \cdot f_2(C,\ A=T)$
T	$0.4 \times 0.43 = 0.172$
F	$0.4 \times 0.57 = 0.228$

Since, evidence is non-empty, we have to normalize. Divide it by 0.4

$\therefore P(C \mid A=T) =$

C	$P(C \mid A=T)$
T	$0.172/0.4 = 0.43$
F	$0.228/0.4 = 0.57$

(c). $P(A, B \mid C=T, D=F)$

Evidence $\Rightarrow$ $C=T$, $D=F$

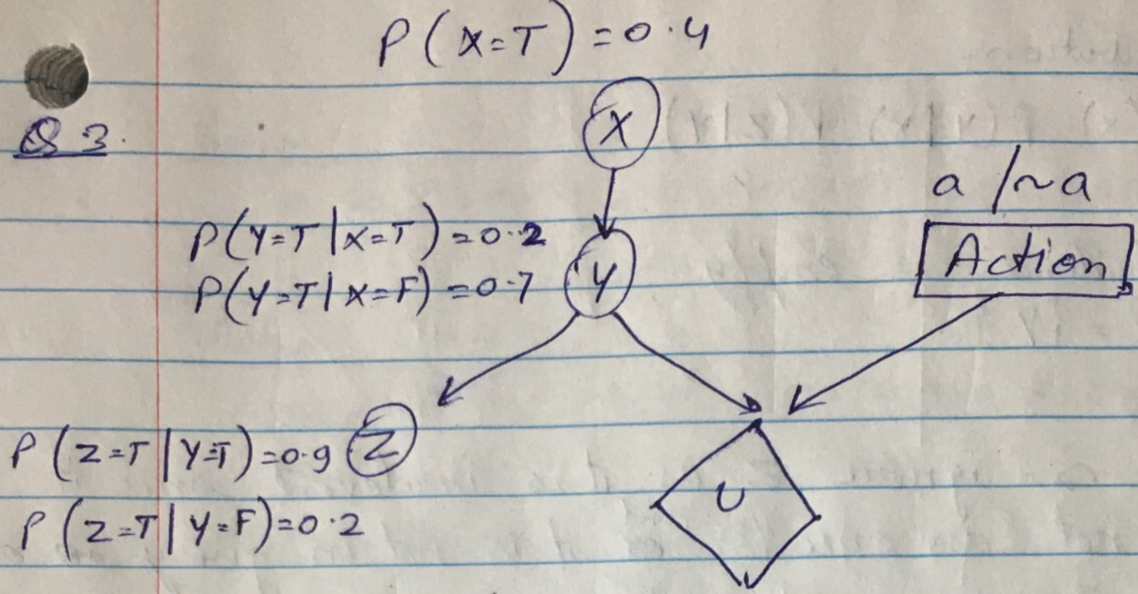
Joint Distribution-

$P(A) \cdot P(B \mid A) \, P(C=T \mid B) \; P(D=F \mid C=T)$.

Eliminate the variables which are not in query and evidence $\Rightarrow$ No elimination.

A	B	C	D	$P(A) \cdot P(B \mid A) \; P(C=T \mid B) \; P(D=F \mid C=T)$
T	T	T	F	$0.4 \times 0.1 \times 0.7 \times 0.18 = 0.00504$
T	F	T	F	$0.4 \times 0.9 \times 0.4 \times 0.18 = 0.02592$
F	T	T	F	$0.6 \times 0.8 \times 0.7 \times 0.18 = 0.06048$
F	F	T	F	$0.6 \times 0.2 \times 0.4 \times 0.18 = 0.00864$

Q 3.



Y	Action	U(Y, Action)
T	a	800
T	~a	400
F	a	200
F	~a	1000

X	P(X)
T	0.4
F	0.6

X, Y	P(Y|X)
T T	0.2
T F	0.8
F T	0.7
F F	0.3

Y Z	P(Z|Y)
T T	0.9
T F	0.1
F T	0.2
F F	0.8

(a) What action should you take?

We need to find the probability of Y from the Bayesian network

Joint Distribution.

$P(X)\ P(Y|X)\ P(Z|Y)$

Evidence is empty.

$Q = \{Y\}$

We can ignore Z as it is irrelevant since it is not an ancestor and it is not in query or evidence.

∴ Joint distribution ⇒ $P(X)\ P(Y|X)$

Query ⇒ $P(Y)$.

∴ To eliminate X –

$$\sum_{X} \underbrace{P(X)\ P(Y|X)}_{f_1(Y)}$$

X	Y	$f_1(Y)$
T	T	0.4 × 0.2 = 0.08
T	F	0.4 × 0.8 = 0.32
F	T	0.6 × 0.7 = 0.42
F	F	0.6 × 0.3 = 0.18

Summing ~~of~~ over X ⇒

Y	$f_2(Y)$
T	0.08 + 0.42 = 0.50
F	0.32 + 0.18 = 0.50

No need to normalize, since there is no evidence.

Action = a

$0.50 \times 800 + 0.50 \times 200 = 400 + 100 = 500$

Action = ~a

$0.50 \times 400 + 0.50 \times 1000 = 200 + 500 = 700$

MEU ⇒ ~a = 700

Therefore action = ~a.

(b) Value of Information of Z-

If. Z = T

Joint distribution -

$P(X)\ P(Y|X)\ P(Z=T|Y)$

Evidence ⇒ Z = T

Query ⇒ P(Y)

We can only eliminate X.

$P(Z=T|Y) \underbrace{\sum_X P(X)\ P(Y|X)}_{f_1(Y)}$

X	Y	P(X) P(Y|X)
T	T	0.4 × 0.2 = 0.08
T	F	0.4 × 0.8 = 0.32
F	T	0.6 × 0.7 = 0.42
F	F	0.6 × 0.3 = 0.18

0.50.

0.50

$P(Z=T \mid Y) \cdot f_1(Y)$

Y	Z	$P(Z=T \mid Y) \cdot f_1(Y)$
T	T	$0.9 \times 0.50 = 0.45 / 0.55 = 0.8182$
F	T	$0.2 \times 0.50 = 0.10 / 0.55 = 0.1818$
		0.55.

Action = a

$0.8182 \times 800 + 0.1818 \times 200 = 654.56 + 36.36$

$\Rightarrow$ 690.92

Action = ~a

$0.8182 \times 400 + 0.1818 \times 1000 = 327.28 + 181.8$

$\Rightarrow$ 509.08

MEU | Z=T $\Rightarrow$ a, 690.92.

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If $Z=F$

Joint Distribution

$P(X)\ P(Y|X)\ P(Z=F|Y)$

Evidence $\Rightarrow$ $Z=T$

Query $\Rightarrow$ Y

~~After~~ For eliminating $X \Rightarrow$ $P(Z=F|Y) \underbrace{\sum_X P(X)P(Y|X)}_{f_1(Y)}$

X	Y	P(X) P(Y|X)	
T	T	$0.4\times0.2 = 0.08$	0.50
T	F	$0.4\times0.8 = 0.32$	
F	T	$0.6\times0.7 = 0.42$	0.50
F	F	$0.6\times0.3 = 0.18$	

$P(Z=F|Y)\cdot f_1(Y)$

Y	Z	$P(Z=F\|Y)\cdot f_1(Y)$
T	F	$0.1\times0.50 = 0.05 / 0.45 = 0.111$
F	F	$0.8\times0.50 = 0.40 / 0.45 = 0.889$
		0.45

Action $= a$

$0.111\times800 + 0.889\times200 = 88.8 + 177.8 = 266.6$

Action $= \sim a$

$0.111\times400 + 0.889\times1000 = 44.4 + 889 = 933.4$

$MEU|Z=F \Rightarrow \sim a,\ 933.4$

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$P(z)$ - We need to find.

$P(x)\, P(y|x)\, P(z|y)$

Eliminate X and Y.

~~$P(z|y)$~~

~~$\sum_{x} P(x) \sum_{y} P(y|x) \cdot P(z|y)$~~

$$\sum_{y} P(z|y) \underbrace{\sum_{x} P(x)\, P(y|x)}_{f_1(y)}$$

X	Y	$f_1(Y)$
T	~~F~~	$0.4 \times 0.2 = 0.08$
T	F	$0.4 \times 0.8 = 0.32$
F	T	$0.6 \times 0.7 = 0.42$
F	F	$0.6 \times 0.3 = 0.18$

0.50

0.50

$$\sum_{y} P(z|y)\, f_1(y) \Rightarrow f_2(z)$$

Y	Z	$P(z|y)\, f_1(y)$
T	T	$0.9 \times 0.50 = 0.45$
T	F	$0.1 \times 0.50 = 0.05$
F	T	$0.2 \times 0.50 = 0.10$
F	F	$0.8 \times 0.50 = 0.40$

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Summation over $Y \Rightarrow$

Z	$f_2(Z)$
T	$0.45 + 0.1 = 0.55$
F	$0.05 + 0.40 = 0.45$

No need to normalize.

~~$EVI(Z) = 0.55x$~~

MEU ~~without~~ before $Z = 700$

$MEU \mid Z = T \quad = 690.92$

$MEU \mid Z = F \quad = 933.4$

$$EVI(Z) = (0.55 \times 690.92) + (0.45 \times 933.4) - 700$$
$$= 380.006 + 420.03 - 700$$
$$= 100.036$$

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(c) Value of information of x.

If $X = T$
Joint distribution - $P(X=T)\,P(Y|X=T)\,P(Z|Y)$

We can ignore Z
$\therefore P(X=T)\,P(Y|X=T)$

X	Y	$P(X=T)\,P(Y\|X=T)$
T	T	$0.4 \times 0.2 = 0.08/0.4 = 0.2$
T	F	$0.4 \times 0.8 = 0.32/0.4 = 0.8$
		0.40

Action = a
$\cancel{(0.08\times 800) + (0.32\times 200) = 64 + 64 = 128}$
$(0.2\times 800) + (0.8\times 200) = 160 + 160 = 320$

Action = ~a
$\cancel{(0.08\times 400) + (0.32\times 1000) = 32 + 320 = 352}$
$0.2\times 400 + 0.8\times 1000 = 80 + 800 = 880$

MEU | X = T ⇒ ~a, 880

If $X = F$
Joint distribution $\Rightarrow P(X=F)\ P(Y|X=F)\ P(Z|Y)$
We can ignore Z $\Rightarrow P(X=F)\ P(Y|X=F)$.

X	Y	$P(X=F)\ P(Y\|X=F)$
F	T	$0.6 \times 0.7 = 0.42 / 0.60 = 0.7$
F	F	$0.6 \times 0.3 = 0.18 / 0.60 = 0.3$
		0.60

Action = a
~~$(0.42 \times 800) + (0.18 \times 200) = 336 + 36 = 372$~~
$0.7 \times 800 + 0.3 \times 200 = 560 + 60 = 620$

Action = ~a
~~$(0.42 \times 400) + (0.18 \times 1000) = 168 + 180 = 348$~~
$(0.7 \times 400) + (0.3 \times 1000) = 280 + 300 = 580$

$MEU | X=F \Rightarrow a,$ ~~372~~ 620

X	P(X)
T	0.4
F	0.6

~~MEU without~~
$VOI(X) = P(X=T) \cdot (MEU | X=T)$
$+ P(X=F) \cdot (MEU | X=F)$
$-$ MEU before X.
~~$= (0.4 \times 352) + (0.6 \times 372) - 700$~~
~~$= 140 + 223.2 - 700 = -336.8$~~

$= (0.4 \times 880) + (0.6 \times 620) - 700$

$= 352 + 372 - 700$

$= 24.$

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(d) Given $Z=T$, what is value of information of X.

Evidence $\Rightarrow Z=T$

Joint distribution $\Rightarrow P(X)\,P(Y|X)\,P(Z=T|Y)$

Compute $P(Y)$

Eliminate X.

$$P(Z=T|Y)\underbrace{\sum_X P(X)\,P(Y|X)}_{f_1(Y)}$$

X	Y	$f_1(Y)$
T	T	$0.4 \times 0.2 = 0.08$
T	F	$0.4 \times 0.8 = 0.32$
F	T	$0.6 \times 0.7 = 0.42$
F	F	$0.6 \times 0.3 = 0.18$

Summation over $X \Rightarrow$

Y	$f_1(Y)$
T	$0.08 + 0.42 = 0.50$
F	$0.32 + 0.18 = 0.50$

$P(Z=T|Y)\,f_1(Y)$

Y	Z	$P(Z=T|Y)\,f_1(Y)$
T	T	$0.9 \times 0.5 = 0.45/0.55 = 0.8182$
F	T	$0.2 \times 0.5 = 0.10/0.55 = 0.1818$

0.55

Action = a

$(0.8182 \times 800) + (0.1818 \times 200) = 654.56 + 36.36 = 690.92$

Action = ~a

$(0.8182 \times 400) + (0.1818 \times 1000) = 327.28 + 181.8 = 509.08$

$MEU \mid Z=T$, $\underline{690.92}$, a

If X = T

$P(X=T)\, P(Y \mid X=T)\, P(Z=T \mid Y)$

No variable elimination.

X	Y	Z	$P(X=T)\, P(Y \mid X=T)\, P(Z=T \mid Y)$
T	T	T	$0.4 \times 0.2 \times 0.9 = 0.072/0.136$
T	F	T	$0.4 \times 0.8 \times 0.2 = 0.064/0.136$

0.136

0.471

0.529

Action = a

$(0.529 \times 800) + (0.471 \times 200) = 423.2 + 94.2 = 517.4$

Action = ~a

$(0.529 \times 400) + (0.471 \times 1000) = 211.6 + 471 = 682.6$

$MEU \mid_{X=T,\ Z=T} \Rightarrow \sim a,\ \underline{682.6}$

If $X=F$

$P(X=F)\,P(Y\mid X=F)\,P(Z=T\mid Y)$

X	Y	Z	$P(X=F)\,P(Y\mid X=F)\,P(Z=T\mid Y)$
F	T	T	$0.6 \times 0.7 \times 0.9 = 0.378/0.414 = 0.913$
F	F	T	$0.6 \times 0.3 \times 0.2 = 0.036/0.414 = 0.087$

0.414

Action $= a$

$0.913 \times 800 + 0.087 \times 200 = 730.4 + 17.4 = 747.8$

Action $= \sim a$

$0.913 \times 400 + 0.087 \times 1000 = 365.2 + 87 = 452.2$

$MEU \mid \begin{smallmatrix} X=F \\ Z=T \end{smallmatrix} \Rightarrow a,\ \underline{747.8}$

Now, we need to find $P(X \mid Z=T)$

~~Eliminat~~

$P(X)\,P(Y\mid X)\,P(Z=T\mid Y)$

Eliminate Y

$P(X)\sum_{Y} P(Y\mid X)\,P(Z=T\mid Y)$

X	
T	$0.4 * ((0.2 \times 0.9) + (0.8 \times 0.2)) = 0.136/0.55 = 0.247$
F	$0.6 \times ((0.7 \times 0.9) + (0.3 \times 0.2)) = 0.414/0.55 = 0.753$

0.550

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$$VOI(X|Z=T) = P(X=T|Z=T) \times (MEU|X=T, Z=T)$$
$$+ P(X=F|Z=T) \times (MEU|X=F, Z=T)$$
$$- MEU|Z=T \text{ before } X.$$

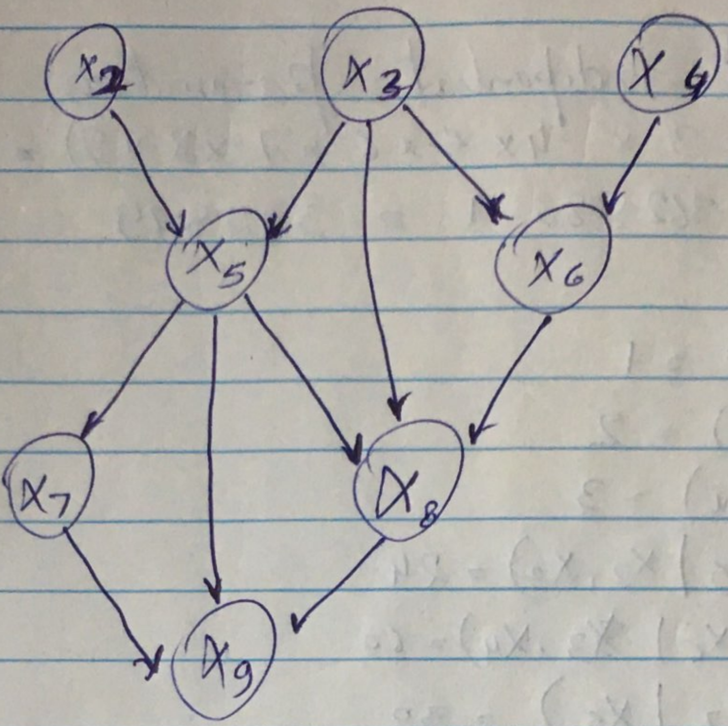
$\Rightarrow$ $0.247 \times 682.6 + 0.753 \times 747.8$

$- \quad 690.92$

$\Rightarrow$ $168.6022 + 563.0934 - 690.92$

$= 40.77.$

Q.1



(a) $P(X_2, X_3 \cdots X_9) = P(X_2) \cdot P(X_3) \cdot P(X_4) \cdot P(X_5 \mid X_2 X_3)$
$P(X_6 \mid X_3, X_4) \cdot P(X_7 \mid X_5) \cdot P(X_8 \mid X_3, X_5, X_6) \cdot$
$P(X_9 \mid X_5, X_7, X_8)$

(c) (i) True
(ii) False
(iii) True
(iv) False
(v) True.

(b) (i) No. of independent parameters

$\Rightarrow (2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9) - 1$

$\Rightarrow 362880 - 1 = 362879$

(ii) $P(X_2) = 1$

$P(X_3) = 2$

$P(X_4) = 3$

$P(X_5 | X_2, X_3) = 24$

$P(X_6 | X_3, X_4) = 60$

$P(X_7 | X_5) = 30$

$P(X_8 | X_3, X_5, X_6) = 630$

$P(X_9 | X_5, X_7, X_8) = 2240$

$\therefore$ Total no. of independent parameters =

$1 + 2 + 3 + 24 + 60 + 30 + 630 + 2240 = 2990$