CS- 480 Assignment 5

Q.1. Convert FOL sentences into CNF -

(a) $\forall x \; P(x) \Rightarrow Q(x)$

$\forall x \; [\neg P(x) \vee Q(x)]$ { Eliminate implication ($\Rightarrow$) $A \Rightarrow B = \neg A \vee B$

$\forall x \; [\neg P(x) \vee Q(x)]$ {Drop Universal quantifier}

$\neg P(x) \vee Q(x)$

(b) $\forall x \forall y \; P(x,y) \Rightarrow Q(x)$

$\forall x \forall y \; [P(x,y) \Rightarrow Q(x)]$ { Eliminate implication ($\Rightarrow$) $A \Rightarrow B = \neg A \vee B$

$\forall x \forall y \; [\neg P(x,y) \vee Q(x)]$ {Drop Universal Quantifier}

$\neg P(x,y) \vee Q(x)$

(c) $\exists x \; P(x) \wedge Q(x)$

$\exists x \; [P(x) \wedge Q(x)]$ { Remove $\exists x$ by Skolemize Add constant A }

$P(A) \wedge Q(A)$

(d) $\exists x \exists y \; P(x,y) \wedge Q(y,x)$

$\exists x \exists y \; [P(x,y) \wedge Q(y,x)]$ { Remove $\exists x$ and $\exists y$ by Skolemize Using A for x and B for y }

$P(A,B) \wedge Q(B,A)$

(e) $\exists x \forall y \; P(x,y)$

$\exists x \forall y [P(x,y)]$ {Remove $\exists x$ by Skolemize using A for x.}

$\forall y \; P(A,y)$ {Drop Universal Quantifier}

$P(A,y)$

(f) $\forall x \exists y \; P(x,y)$

$\forall x \exists y [P(x,y)]$ {Remove $\exists y$ by Skolemize using Fx for y.}

$\forall x \; P(x, F(x))$ {Drop Universal Quantifier}

$P(x, F(x))$

(g) $\forall x \forall y \exists z \; P(x,y,z)$

$\forall x \forall y \; P(x,y,F(x,y))$ {Remove $\exists z$ by Skolemize using $F(x,y)$ for z}

$P(x,y,F(x,y))$ {Drop Universal Quantifier}

(h) $\exists x \forall y \forall z \; P(x,y,z)$

$\forall y \forall z \; P(A,y,z)$ {Remove $\exists x$ by Skolemize using A for x.}

$P(A,y,z)$ {Drop Universal Quantifier}

(i) $\forall x (\exists y \; P(x,y) \wedge Q(y)) \Rightarrow R(x)$

$\forall x [(\exists y\ P(x,y) \wedge Q(y)) \Rightarrow R(x)]$ {Eliminate implication ($\Rightarrow$)}

$\forall x [\neg (\exists y\ P(x,y) \wedge Q(y)) \vee R(x)]$

$\forall x [(\forall y \neg P(x,y) \vee \neg Q(y)) \vee R(x)]$ {Remove universal Quantifier}

$(\neg P(x,y) \vee \neg Q(y)) \vee R(x)$

(j) $\forall x\ (\forall y\ P(x,y) \Rightarrow Q(y)) \Rightarrow R(x)$

$\forall x [(\forall y\ P(x,y) \Rightarrow Q(y)) \Rightarrow R(x)]$ {Remove implication ($\Rightarrow$)}

$\forall x [\neg (\forall y\ P(x,y) \Rightarrow Q(y)) \vee R(x)]$ {Remove implication ($\Rightarrow$)}

$\forall x [\neg (\forall y \neg P(x,y) \vee Q(y)) \vee R(x)]$ {Move $\neg$ inward}

$\forall x [\exists y (P(x,y) \wedge \neg Q(y)) \vee R(x)]$ {Remove $\exists y$ by Skolemize using $F(x)$ for y}

$\forall x [(P(x, F(x)) \wedge \neg Q(F(x))) \vee R(x)]$ {Drop Universal Quantifier}

$(P(x, F(x)) \wedge \neg Q(F(x))) \vee R(x)$ {Distribute $\vee$ over $\wedge$}

$[P(x, F(x)) \vee R(x)] \wedge [\neg Q(F(x)) \vee R(x)]$

Q-2 Unify.

a. $P(x)$

b $P(A)$

Unify $(P(x), P(A)) = \{x/A\}$

c $P(x) \vee Q(x, A)$

d $P(B) \vee Q(x, A)$

Unify $((P(x) \vee Q(x, A)), (P(B) \vee Q(x, A))) = \{x/B\}$

e. $P(x) \vee Q(A, x)$

f. $P(x) \vee Q(A, B)$

Unify $((P(x) \vee Q(A, x)), (P(x) \vee Q(A, B))) = \{x/B\}$

g $P(x, A) \vee Q(A, x)$

h. $P(B, y) \vee Q(y, B)$

Unify $(g, h) = \{x/B, y/A\}$

i $P(x) \vee Q(F(x))$

j $P(A) \vee Q(F(A))$

Unify $(i, j) = \{x/A\}$

k. $P(x, A) \vee Q(F(x), x)$

l. $P(B, y) \vee Q(F(B), B)$

Unify $(k, l) = \{x/B, y/A\}$

m. $P(x, A) \vee Q(F(x), x)$

n. $P(B, y) \vee Q(F(A), A)$

Unify (m, n) = Fail. No solution as x can't be both A and B.

o. $P(x, y) \vee Q(F(A), B)$

p. $P(x, y) \vee Q(x, y)$

Unify (o, p) = $\{x/F(A), y/B\}$

q. $P(x, y) \vee Q(F(A), A)$

r. $P(x, y) \vee Q(x, y)$

Unify (q, r) = $\{x/F(A), y/A\}$

s. $P(x, y) \vee Q(F(x), y)$

t. $P(z, y) \vee Q(z, y)$

Unify (s, t) = Fail. No solution as z can't be both x and F(x).

Q.3. Probability Distribution

Q-3 Probability Distribution

A	B	C	P(A,B,C)
T	T	T	0.014
T	T	F	0.126
T	F	T	0.012
T	F	F	0.048
F	T	T	0.392
F	T	F	0.168
F	F	T	0.144
F	F	F	0.096

(a) $P(A,C)$ $\quad P(A,C) = P(A|C)\,P(C)$

A	C	P(A,C)
T	T	0.014 + 0.012 = 0.026
T	F	0.126 + 0.048 = 0.174
F	T	0.392 + 0.144 = 0.536
F	F	0.168 + 0.096 = 0.264

C	P(C)
T	0.562/1
F	0.438/1

$P(A/C) =$

(b)

C	P(C)
T	$(0.014 + 0.012 + 0.392 + 0.144)/1 = 0.562/1 = 0.562$
F	$(0.126 + 0.048 + 0.168 + 0.096)/1 = 0.438/1 = 0.438$

(c) $P(A|C)$

A	$P(A/C=T)$
T	$0.014 + 0.012 = 0.026/(0.026 + 0.536) = 0.026/0.562 = 0.046$
F	$0.392 + 0.144 = 0.536/(0.026 + 0.536) = 0.536/0.562 = 0.953$

A	$P(A/C=F)$
T	$0.126 + 0.048 = 0.174/(0.174 + 0.264) = 0.174/0.438 = 0.397$
F	$0.168 + 0.096 = 0.264/(0.174 + 0.264) = 0.264/0.438 = 0.602$

(d) $P(A, B|C)$ $P(A, B/C)$

A	B	C	=
T	T	T	$= 0.014/0.562 = 0.562 = 0.024$
T	T	F	$= 0.126/0.438 = 0.438 = 0.287$
T	F	T	$= 0.012/0.562 = 0.562 =$
T	F	F	$= 0.048/0.438 = 0.438 =$
F	T	T	$= 0.392/0.562 = 0.562 =$
F	T	F	$= 0.168/0.438 = 0.438 =$
F	F	T	$= 0.144/0.562 = 0.562 =$
F	F	F	$= 0.096/0.438 = 0.438 =$

(d) $P(A, B|C)$

A	B	C	$P(A, B|C)$
T	T	T	0.014/0.562 = 0.024
T	T	F	0.126/0.438 = 0.287
T	F	T	0.012/0.562 = 0.021
T	F	F	0.048/0.438 = 0.109
F	T	T	0.392/0.562 = 0.697
F	T	f	0.168/0.438 = 0.383
F	F	T	0.144/0.562 = 0.256
F	f	f	0.096/0.438 = 0.219

(e) $P(B|A, C)$

A	B	C	$P(B|A, C)$
T	T	T	0.014/0.026 = 0.538
T	T	F	0.126/0.174 = 0.724
T	F	T	0.012/0.026 = 0.461
T	f	F	0.048/0.174 = 0.275
F	T	T	0.392/0.536 = 0.731
F	T	F	0.168/0.264 = 0.636
F	F	T	0.144/0.536 = 0.268
F	f	f	0.096/0.264 = 0.363

Q4 Random variables $x_2, x_3 \dots x_n$ where $n > 2$

(a) (i) $P(x_2) = 1$ $\rightarrow (2^1 - 1 = 1)$

(ii) $P(x_n) = 1$ $\rightarrow (2^1 - 1 = 1)$

(iii) $P(x_2, x_3 \dots x_n) = 2^{n-1} - 1$

(iv) $P(x_2 | x_3, \dots x_n) = 2^{n-2}$

(v) $P(x_2, x_3, \dots x_{n-1} | x_n) = 2 \times (2^{n-2} - 1) = 2^{n-1} - 2$

(b) (i) $P(x_2) = 1$

(ii) $P(x_n) = n - 1$

(iii) $P(x_2, x_3 \dots x_n) = n! - 1$

(iv) $P(x_2 | x_3 \dots x_n) = n!/2$

(v) $P(x_2, x_3 \dots x_{n-1} | x_n) = n[(n-1)! - 1] = n! - n$