CS 480 – Assignment 5 solutions

1. Convert each of the following FOL sentences into CNF form.

a. $\forall x\ P(x) \Rightarrow Q(x)$

$\neg P(x) \vee Q(x)$

b. $\forall x \forall y\ P(x,y) \Rightarrow Q(x)$

$\forall x \forall y\ (\neg P(x,y) \vee Q(x))$

$\neg P(x,y) \vee Q(x)$

OR

$\forall x\ \neg(\forall y\ P(x,y) \vee Q(x))$

$\forall x\ \exists y\ \neg P(x,y) \vee Q(x)$

$\neg P(x, F(x)) \vee Q(x)$

c. $\exists x\ P(x) \wedge Q(x)$

$P(A) \wedge Q(A)$

d. $\exists x \exists y\ P(x,y) \wedge Q(y,x)$

$P(A,B) \wedge Q(B,A)$

e. $\exists x \forall y\ P(x,y)$

$P(A,y)$

f. $\forall x \exists y\ P(x,y)$

$P(x, F(x))$

g. $\forall x \forall y \exists z\ P(x,y,z)$

$P(x, y, F(x,y))$

h. $\exists x \forall y \forall z\ P(x,y,z)$

P(A, y ,z)

i. $\forall x(\exists y\ P(x,y) \wedge Q(y)) \Rightarrow R(x)$

$\forall x(\neg \exists y\ (P(x,y) \wedge Q(y))\ V\ R(x)$

$\forall x(\forall y\ \neg(P(x,y) \wedge Q(y))\ V\ R(x)$

¬P(x,y) V ¬Q(y) V R(x)

j. $\forall x(\forall y\ P(x,y) \Rightarrow Q(y)) \Rightarrow R(x)$

$\forall x(\neg \forall y\ P(x,y) \Rightarrow Q(y))\ V\ R(x)$

$\forall x(\neg \forall y\ \neg P(x,y)\ V\ Q(y))\ V\ R(x)$

$\forall x(\neg\neg \exists y\ \neg(\neg P(x,y)\ V\ Q(y)))\ V\ R(x)$

$\forall x(\exists y\ P(x,y)\ \Lambda\ \neg Q(y))\ V\ R(x)$

(P(x, F(x)) Λ ¬Q(F(x)) V R(x)

(P(x, F(x)) V R(x)) Λ (¬Q(F(x)) V R(x))

2. We are given the following pairs of FOL sentences. For each pair of sentences, provide a substitution to unify the sentences. If no such substitution exists, please write so.

a. P(x)
b. P(A)

Sol: {x/A}

c. $P(x) \vee Q(x, A)$
d. $P(B) \vee Q(x, A)$

Sol: {x/B}

e. $P(x) \vee Q(A, x)$
f. $P(x) \vee Q(A, B)$

Sol: {x/B}

g. $P(x, A) \vee Q(A, x)$
h. $P(B, y) \vee Q(y, B)$

Sol: {x/B, y/A}

i. $P(x) \vee Q(F(x))$
j. $P(A) \vee Q(F(A))$

Sol: {x/A}

k. $P(x, A) \vee Q(F(x), x)$
l. $P(B, y) \vee Q(F(B), B)$

Sol: {x/B, y/A}

m. $P(x, A) \vee Q(F(x), x)$
n. $P(B, y) \vee Q(F(A), A)$

Sol: Fail

o. $P(x, y) \vee Q(F(A), B)$
p. $P(x, y) \vee Q(x, y)$

Sol: {x/F(A), y/B}

q. $P(x, y) \vee Q(F(A), A)$
r. $P(x, y) \vee Q(x, y)$

Sol: {x/F(A), y/A}

s. $P(x, y) \vee Q(F(x), y)$
t. $P(z, y) \vee Q(z, y)$

Sol: Fail

3. We are given the following joint distribution for variables A, B, and C. Please compute the requested probabilities. Show each probability distribution as a table/vector. Feel free to use a calculator.

A	B	C	P(A,B,C)
T	T	T	0.014
T	T	F	0.126
T	F	T	0.012
T	F	F	0.048
F	T	T	0.392
F	T	F	0.168
F	F	T	0.144
F	F	F	0.096

a. P(A, C)

A	C	P(A,C)
T	T	0.014 + 0.012 = **0.026**
T	F	0.126 + 0.048 = **0.174**
F	T	0.392 + 0.144 = **0.536**
F	F	0.168 + 0.096 = **0.264**

b. P(C)

C	P(C)
T	0.014 + 0.012 + 0.392 + 0.144 = **0.562**
F	0.126 + 0.048 + 0.168 + 0.096 = **0.438**

c. P(A|C)

A	P(A | C = T)
T	0.026 / 0.562 = **0.046**
F	0.536 / 0.562 = **0.954**

A	P(A | C = F)
T	0.174 / 0.438 = **0.397**
F	0.264 / 0.438 = **0.603**

d. P(A, B | C)

A	B	P(A,B | C =T)
T	T	0.014 / 0.562 = **0.025**
T	F	0.012 / 0.562 = **0.021**
F	T	0.392 / 0.562 = **0.698**
F	F	0.144 / 0.562 = **0.256**

A	B	P(A,B | C =F)
T	T	0.126 / 0.438 = **0.288**
T	F	0.048 / 0.438 = **0.110**
F	T	0.168 / 0.438 = **0.384**
F	F	0.096 / 0.438 = **0.219**

d. P(B | A, C)

B	P(B | A = T, C = T)
T	0.014 / 0.026 = **0.538**
F	0.012 / 0.026 = **0.462**

B	P(B | A = T, C = F)
T	0.126 / 0.174 = **0.724**
F	0.048 / 0.174 = **0.276**

B	P(B | A = F, C = T)
T	0.392 / 0.536 = **0.731**
F	0.144 / 0.536 = **0.269**

B	P(B | A = F, C = F)
T	0.168 / 0.264 = **0.636**
F	0.096 / 0.264 = **0.364**

4. We are given random variables X2, X3, …, Xn, where n>2. (There is no X1). Please answer the following questions.
   a) Assuming all variables are binary, how many independent parameters are needed to represent
      i. $P(X_2)$?

      Sol: 1

      ii. $P(X_n)$?

      Sol: 1

iii. $P(X_2, X_3, ..., X_n)$?

Sol: $2^{n-1} - 1$

iv. $P(X_2 \mid X_3, ..., X_n)$?

Sol: 2^{n-2}

v. $P(X_2, X_3, ..., X_{n-1} \mid X_n)$?

Sol: $2 * (2^{n-2} - 1)$

b) Assuming the size of the domain of Xi is i for all $i \in \{2, 3, \ldots, n\}$, how many independent parameters are needed to represent

i. $P(X_2)$?

Sol: 1

ii. $P(X_n)$?

Sol: n – 1

iii. $P(X_2, X_3, ..., X_n)$?

Sol: n! – 1

iv. $P(X_2 \mid X_3, ..., X_n)$?

Sol: n! / 2

v. $P(X_2, X_3, ..., X_{n-1} \mid X_n)$?

Sol: n * ((n-1)! – 1)