Question: 1.

Answer: We are given the matrix,

$$A = \begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 2 & 0 \\ \sqrt{2} & 0 & 0 \end{bmatrix} \tag{1}$$

It gives us the three eigenvalues,

$$\begin{aligned} \lambda_1 &= 2 \\ \lambda_2 &= 2 \\ \lambda_3 &= -1 \end{aligned} \tag{2}$$

$$\Rightarrow \Lambda = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{3}$$

Using MATLAB, we can find the three eigenvectors to be,

$$v^1 = \begin{bmatrix} 0.57 \\ 0 \\ -0.81 \end{bmatrix},\ v^2 = \begin{bmatrix} 0.81 \\ 0 \\ 0.57 \end{bmatrix},\ v^3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{4}$$

$$V = \begin{bmatrix} 0.57 & 0.81 & 0 \\ 0 & 0 & 1 \\ -0.81 & 0.57 & 0 \end{bmatrix} \tag{5}$$

We can see that,

$$V^{-1} = \begin{bmatrix} 0.57 & 0 & -0.81 \\ 0.81 & 0 & 0.57 \\ 0 & 1 & 0 \end{bmatrix} = V^T \Rightarrow V \textit{ is Orthogonal} \tag{6}$$

By multiplying the matrices $V\Lambda V^T \rightarrow$

$$V\Lambda V^T = \begin{bmatrix} 0.57 & 0.81 & 0 \\ 0 & 0 & 1 \\ -0.81 & 0.57 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0.57 & 0 & -0.81 \\ 0.81 & 0 & 0.57 \\ 0 & 1 & 0 \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 2 & 0 \\ \sqrt{2} & 0 & 0 \end{bmatrix} \tag{8}$$

$$= A \tag{9}$$

$\therefore$ we can see that the statement *even with repeated e-values, we can still diagonalize a symmetric matrix using orthogonal matrices* is true using above numerical example.