Question: 5.(a)

Answer:

Question: 5.(b)

Answer:

Question: 6.(a)

Answer:

Question: 6.(b)

Answer:

Question: 8. A norm $||\cdot||$ on a vector space $(\mathcal{X}, \mathbb{R})$ is said to be strict when $||x+y|| = ||x|| + ||y||$ holds if and only if there exists a non-negative constant α such that either $y = \alpha x$ or $x = \alpha y$. One then says that $(\mathcal{X}, \mathbb{R}, ||\cdot||)$ is strictly normed. Suppose that $(\mathcal{X}, \mathbb{R}, ||\cdot||)$ is strictly normed. Let M be a subspace of $\mathcal{X}$ and suppose that $x \in \mathcal{X}$ is such that $d(x, M) > 0$. Show that there exists $m^* \in M$ such that

$$||x - m^*|| = d(x, M) := \inf_{y \in M} ||x - y||$$

then m^* is unique.

Answer: