Mechatronics

Topic #3

Basic Electricity

Charge

•Symbol: (q)

•Unit: Coulomb (C)

–The fundamental electric quantity is charge.

–Atoms are composed of charge carrying particles: **electrons** and **protons**, and neutral particles, **neutrons**.

–The smallest amount of charge that exists is carried by an electron and a proton.

–Charge in an electron:

$$q_e = -1.602\text{x}10^{-19}\ \text{C}$$

–Charge in a proton:

$$q_p = 1.602\text{x}10^{-19}\ \text{C}$$

Current

- Symbol: I
- Unit: Ampere

– Current is a through variable.

– It moves through circuit elements.

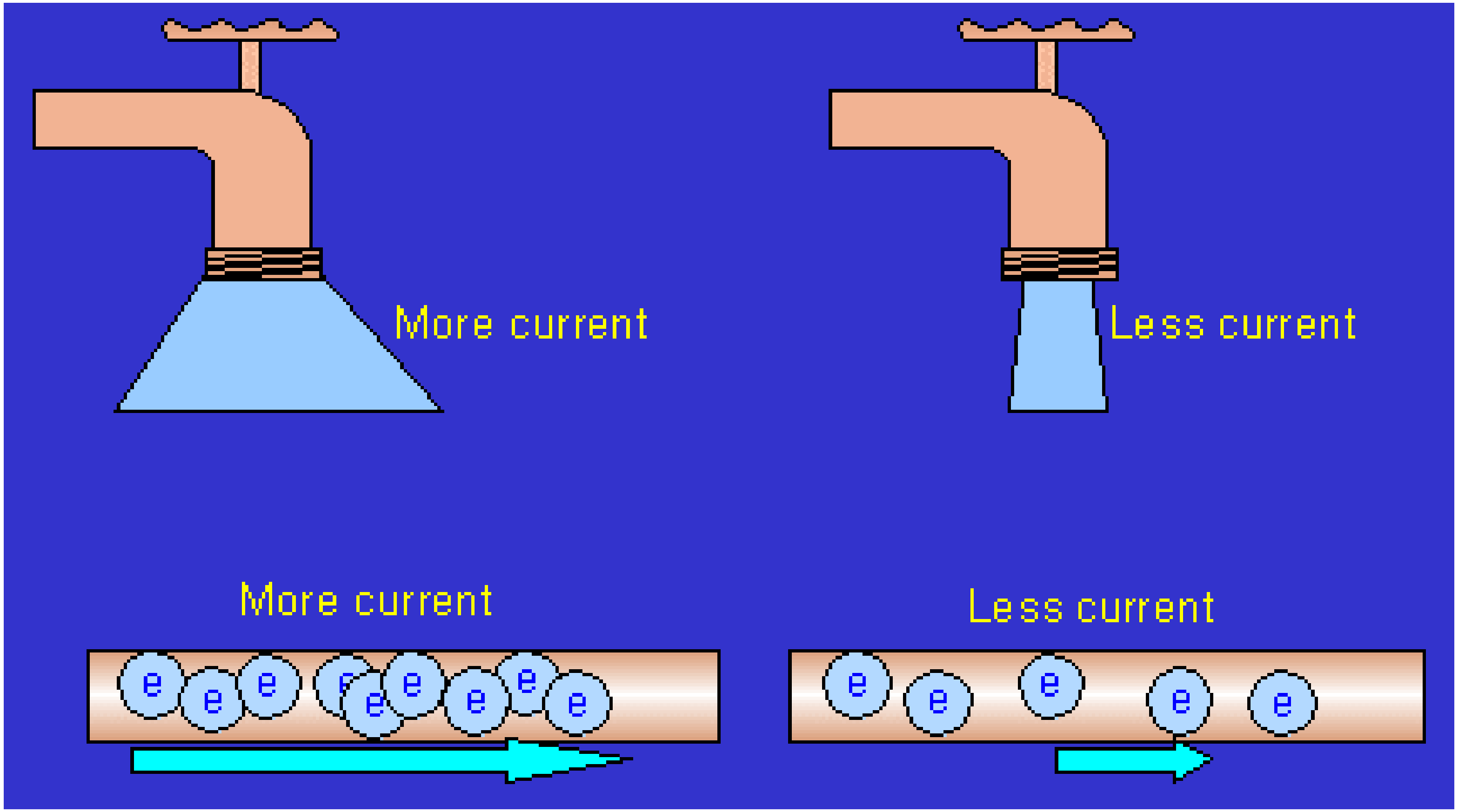
– Current is rate of flow of electric charge through a predetermined cross-sectional area in a conductor.

– Essentially, flow of electrons in an electric circuit leads to the establishment of current.

$$I(t) = \frac{dq}{dt}$$

- q : relatively charged electrons (C)
- Amp = C/sec

Current-Water Analogy



Voltage

- Symbol: V
- Unit: Volt

– Voltage is an across variable.

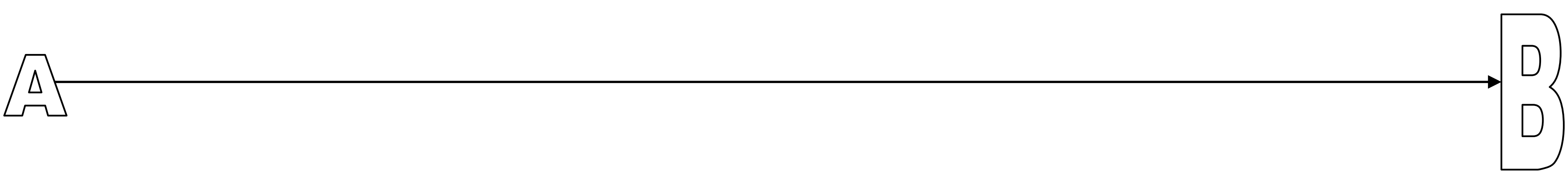
– It is measured across two terminals in a circuit.

– In order to move charge from point A to point B, work needs to be done.

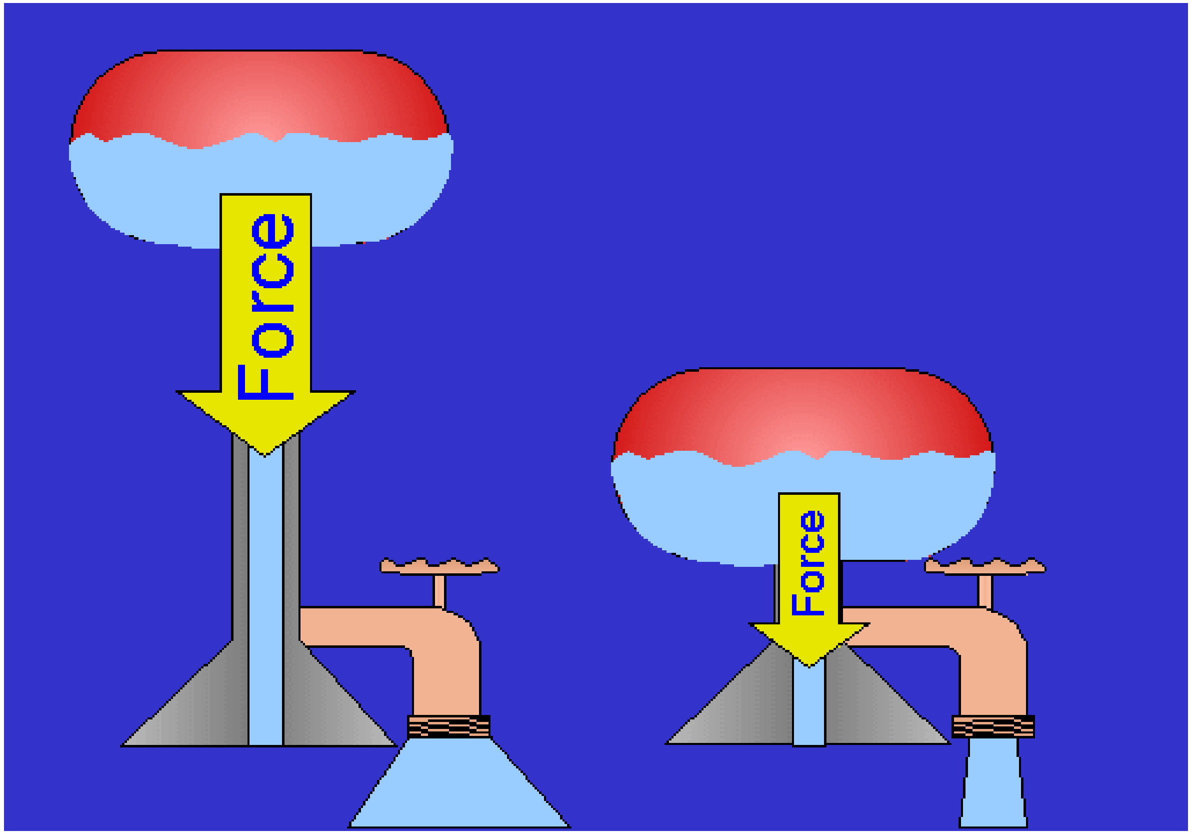
– Let A be the lower potential/voltage terminal

– Let B be the higher potential/voltage terminal

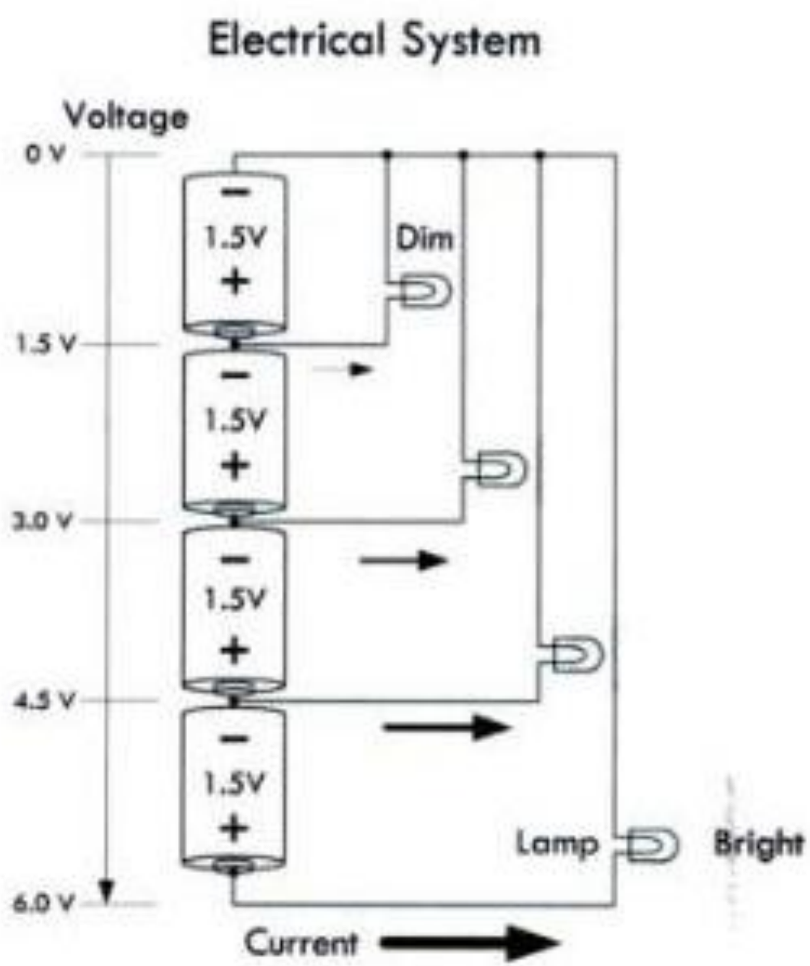
- o Then, voltage across A and B is the cost in energy required to move a unit positive charge from A to B.

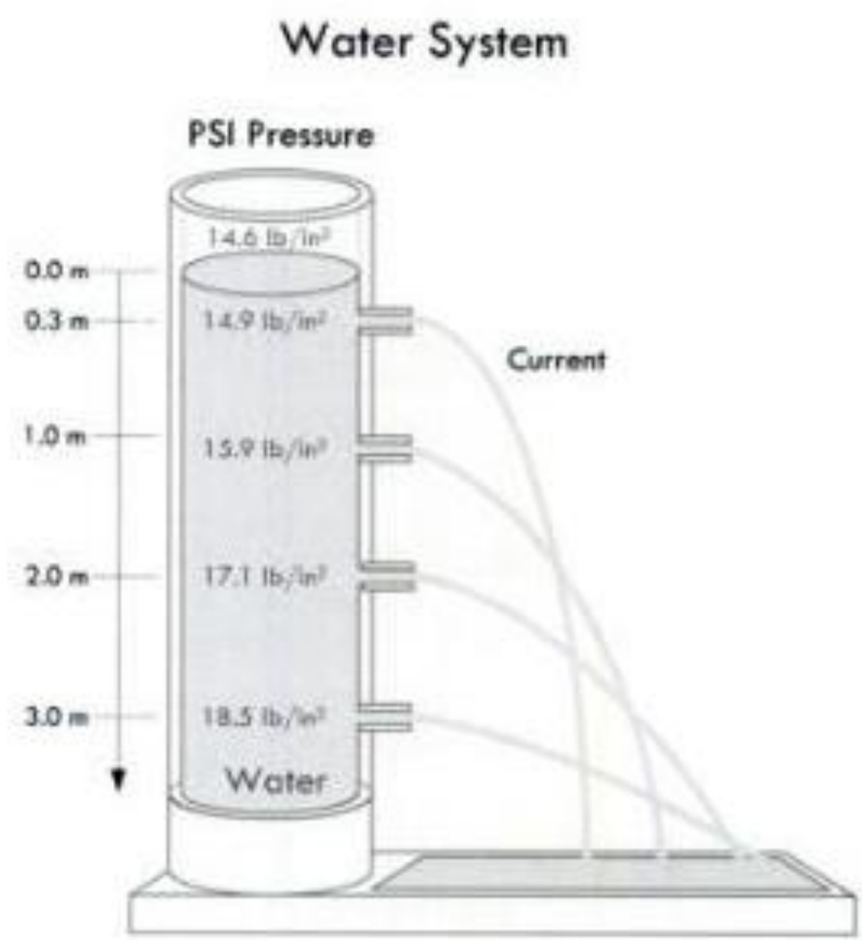


Voltage-Water Analogy

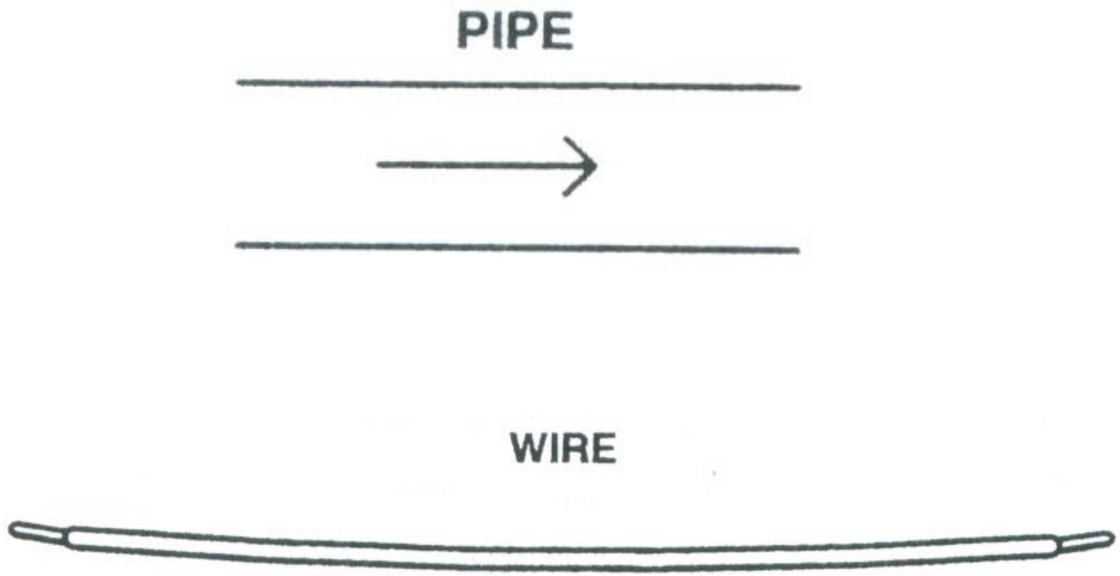


Voltage/Current-Water Analogy





Wire-Water Analogy



Resistor Concept —I

•Flow of electric current through a conductor experiences a certain amount of resistance.

•The resistance, expressed in ohms (Ω, named after George ohm), kilo-ohms (kΩ, 1000Ω), or mega-ohms (MΩ, $10^6\Omega$) is a measure of how much a resistor resists the flow of electricity.

•The magnitude of resistance is dictated by electric properties of the material and material geometry.

•This behavior of materials is often used to control/limit electric current flow in circuits.

•Henceforth, the conductors that exhibit the property of resisting current flow are called resistors.

Resistor Concept —II

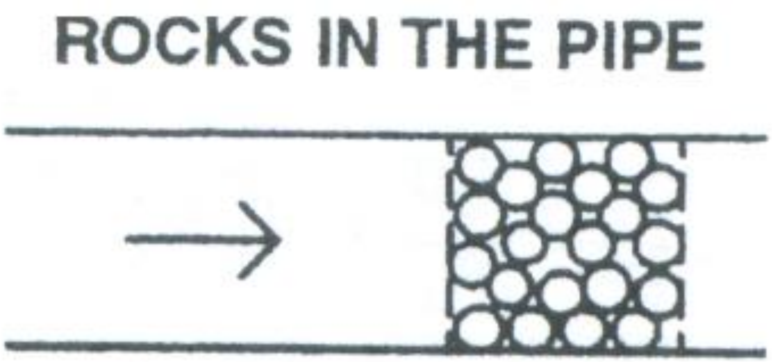
•A resistor is a dissipative element. It converts electrical energy into heat energy. It is analogous to the viscous friction element of mechanical system.

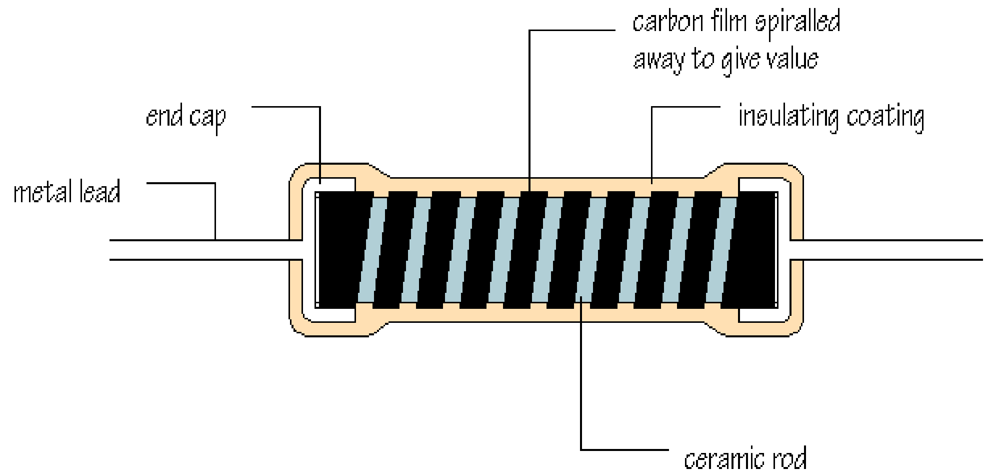
•When electrons enter at one end of a resistor, some of the electrons collide with atoms within the resistor. These atoms start vibrating and transfer their energy to neighboring air molecules. In this way, a resistor dissipates electrical energy into heat energy.

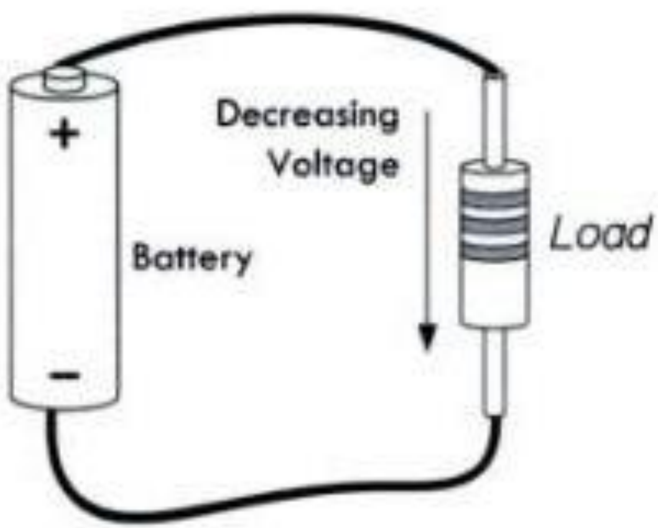
•Resistors can be thought of as analogous to water carrying pipes. Water is supplied to your home in large pipes, however, the pipes get smaller as the water reaches the final user. The pipe size limits the water flow to what you actually need.

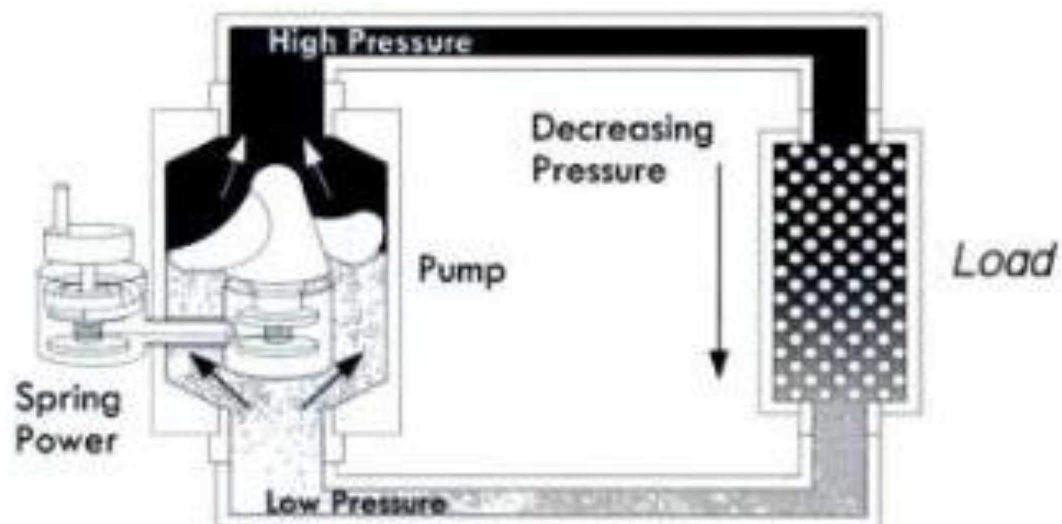
•Electricity works in a similar manner, except that wires have so little resistance that they would have to be very very thin to limit the flow of electricity. Such thin wire would be hard to handle and break easily.

Resistors-Water Analogy









Resistor V-I Characteristic

•In a typical resistor, a conducting element displays linear voltage-current relationship. (i.e., current through a resistor is directly proportional to the voltage across it).

$$I \propto V$$

•Using G as a constant of proportionality, we obtain:

$$I = GV$$

•Equivalently,

$$V = RI \quad (\text{or } V = IR)$$

where $R = 1/G$.

–R is termed as the resistance of conductor (ohm, Ω)

–G is termed as the conductance of conductor (mho, ℧)

Resistor Applications

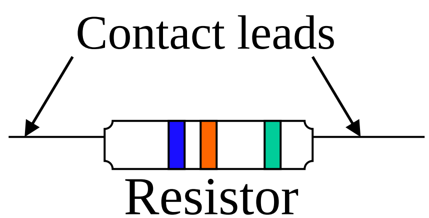
- Resistors are used for:
  - Limiting current in electric circuits.
  - Lowering voltage levels in electric circuits (using voltage divider).
  - As current provider.
  - Photoresistor are used to detect “light” condition.
  - In electronic circuits, resistors are used as pull-up and pull-down elements to avoid floating signal levels.

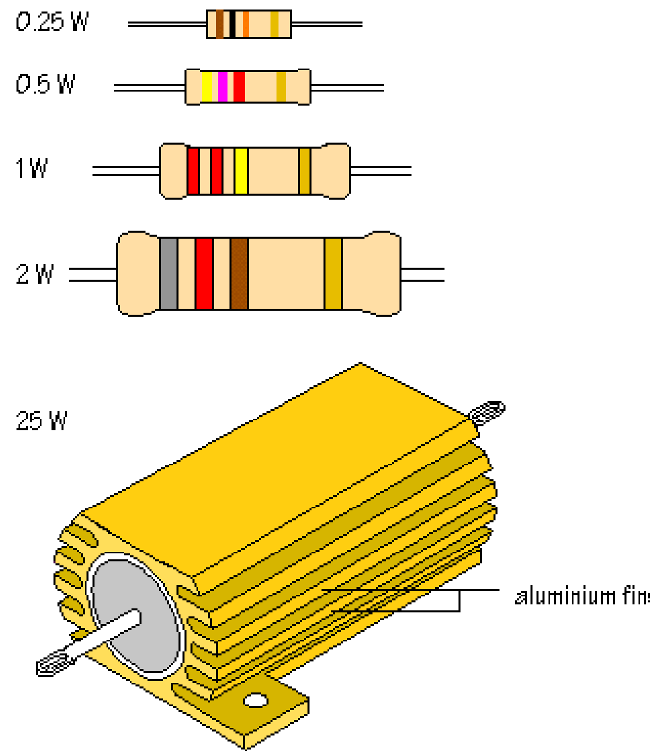
Resistors: Power Rating and Composition

- It is very important to be aware of power rating of resistor used in circuits and to make sure that this limit is not violated. A higher power rating resistor can dissipate more energy than a lower power rating resistor.
- Resistors can be made of**:**
  - Carbon film (decomposition of carbon film on a ceramic core).
  - Carbon composition (carbon powder and glue-like binder).
  - Metal oxide (ceramic core coated with metal oxide).
  - Precision metal film.
  - High power wire wound.

Resistor Examples

Symbol for resistor

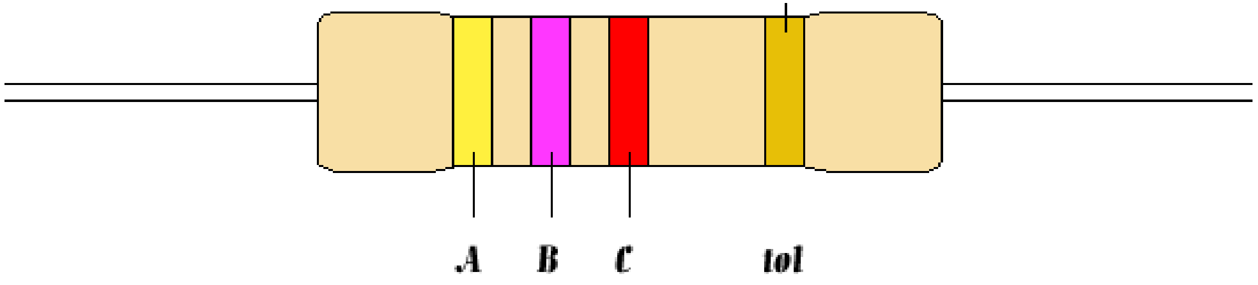




Resistor Labels

- Wire-wound resistors have a label indicating resistance and power ratings.
- A majority of resistors have color bars to indicate their resistance magnitude.
- There are usually 4 to 6 bands of color on a resistor. As shown in the figure below, the right most color bar indicates the resistor reliability, however, some resistor use this bar to indicate the tolerance. The color bar immediately left to the tolerance bar (C), indicates the multipliers (in tens). To the left of the multiplier bar are the digits, starting from the last digit to the first digit.

$$\text{Resistor value} = AB \times 10^{C} \pm tol\%(\Omega)$$

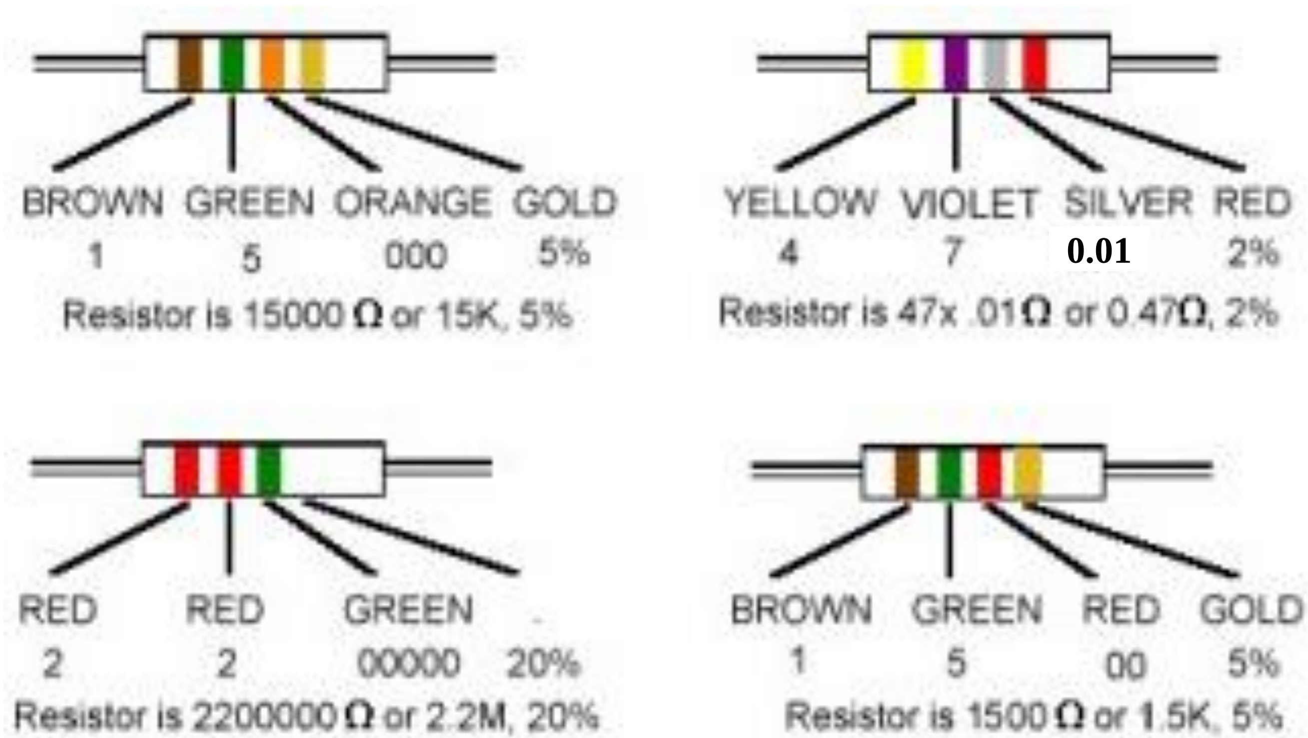


Resistor Color Codes

Color	Tolerance
Brown	±1%
Red	±2%
Gold	±5%
Silver	±10%
None	±20%

Band color	Digit	Multiplier
Black	0	X1
Brown	1	X10
Red	2	X100
Orange	3	X1000
Yellow	4	X10000
Green	5	X100000
Blue	6	X1000000
Purple	7	X10000000
Grey	8	X100000000
White	9	X1000000000
Silver	-	x.01
Gold	-	x.1

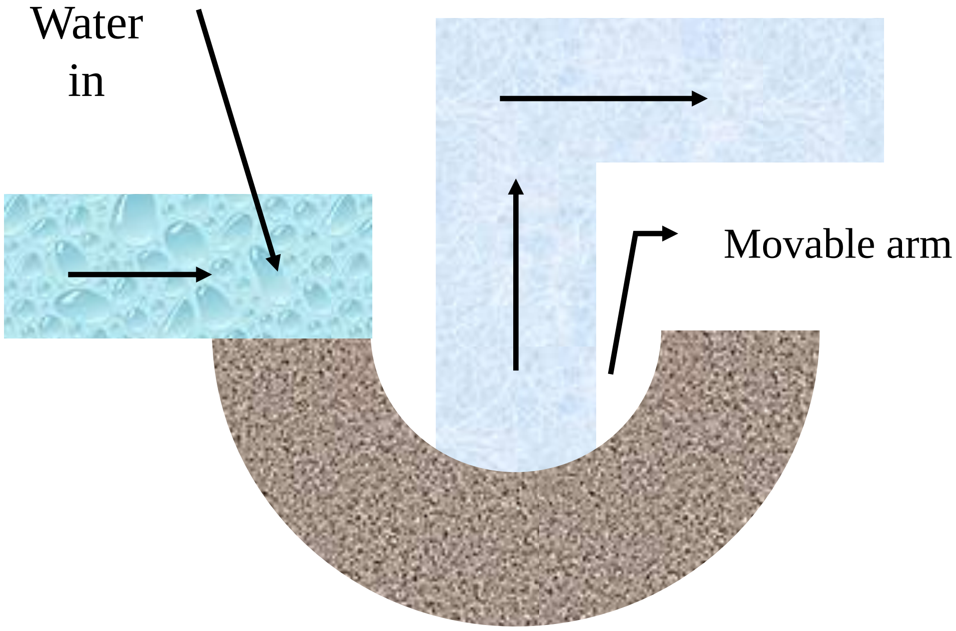
Resistor Labels: Examples



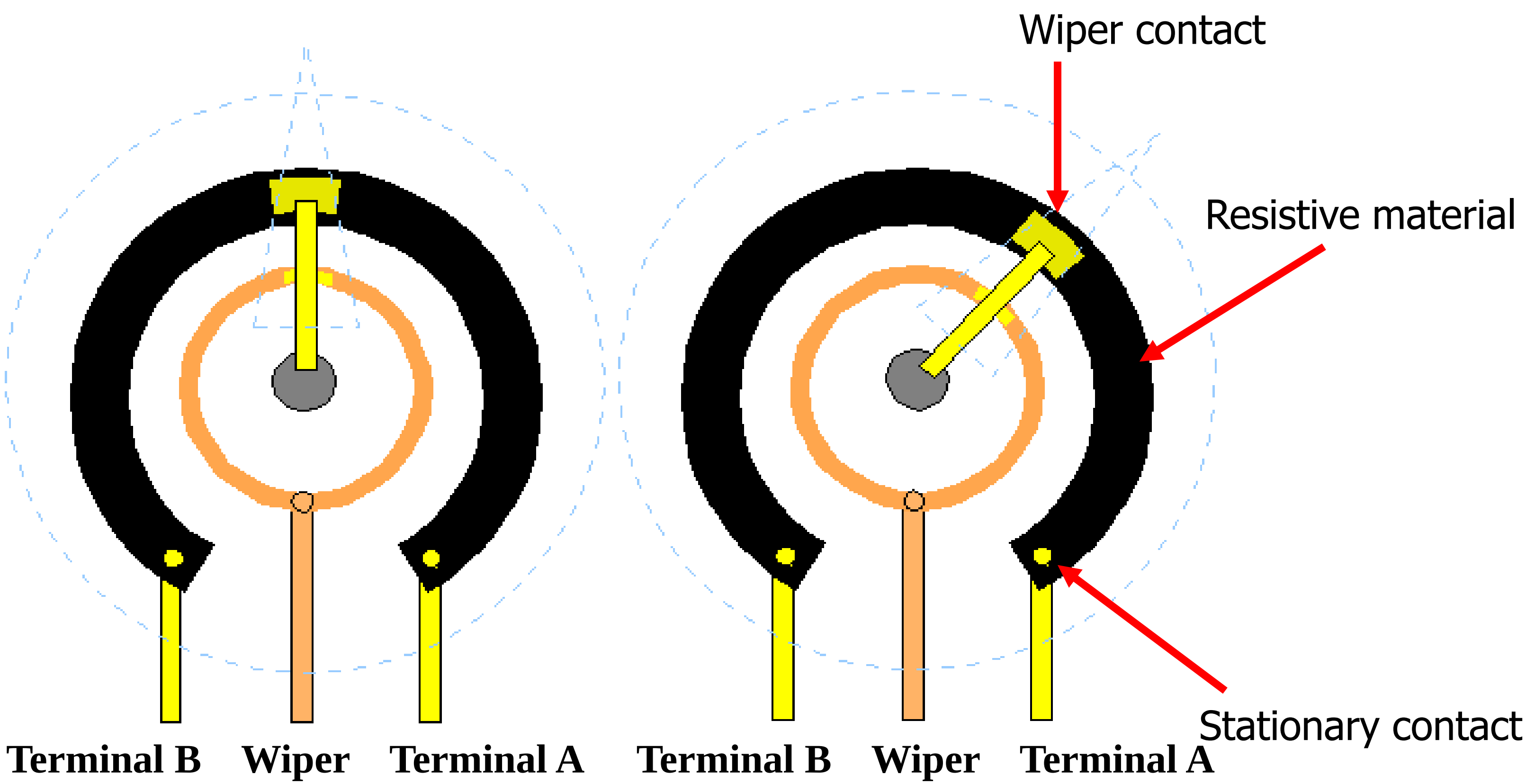
Images taken from http://www.electrician.com/resist_calc/resist_calc.htm

Variable Resistor Concept

- In electrical circuit, a switch is used to turn the electricity on and off just like a valve is used to turn the water on and off.
- There are times when you want some water but don't need all the water that the pipe can deliver, so you control water flow by adjusting the faucet.
- Unfortunately, you can't adjust the thickness of an already thin wire.
- Notice, however, that you can control the water flow by forcing the water through an adjustable length of rocks, as shown to the right.



Variable Resistor Construction



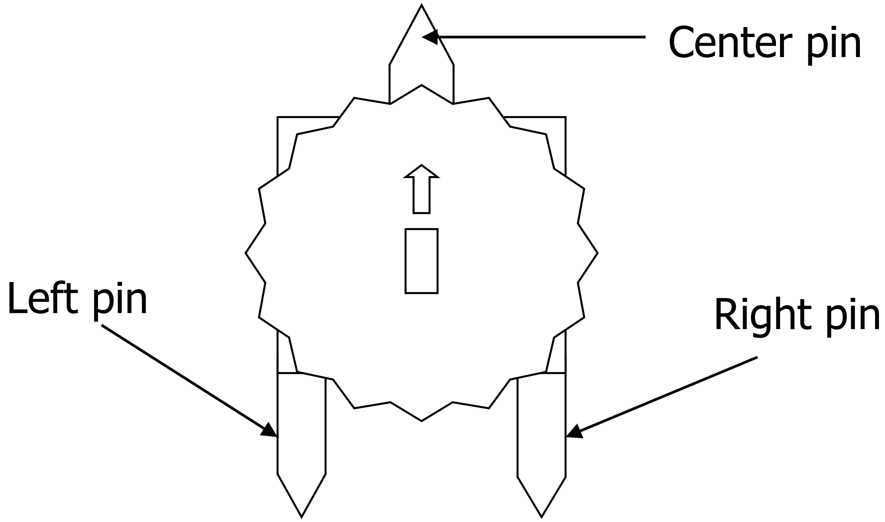
• To vary the resistance in an electrical circuit, we use a variable resistor.

•This is a normal resistor with an additional arm contact that can move along the resistive material and tap off the desired resistance.

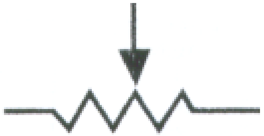
Variable Resistor Operation

•The dial on the variable resistor "pot" moves the arm contact and sets the resistance between the left and center pins. The remaining resistance of the pot is between the center and right pins.

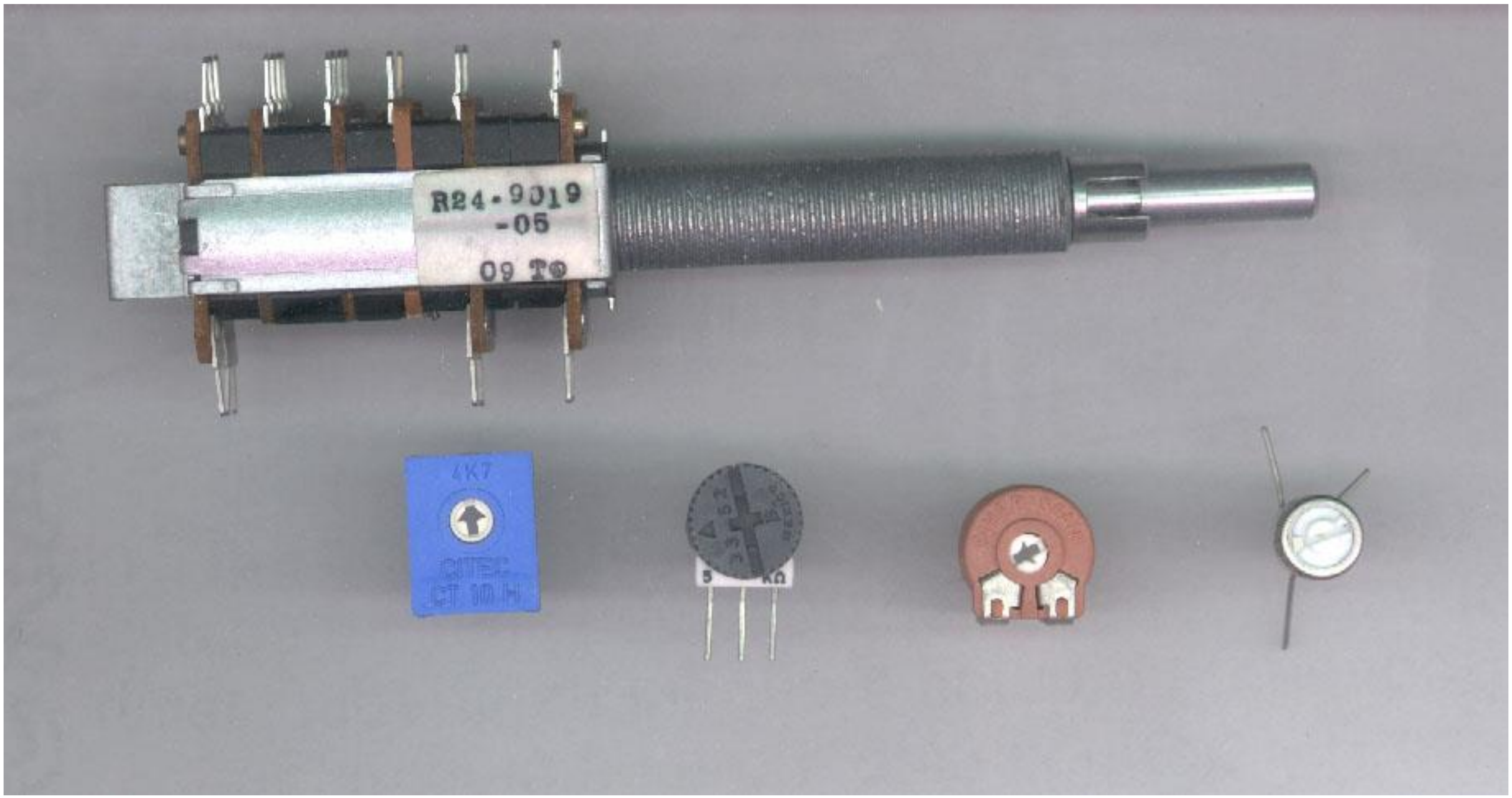
•For example, when the dial is turned fully to the left, there is minimal resistance between the left and center pins (usually 0Ω) and maximum resistance between the center and right pins. The resistance between the left and right pins will always be the total resistance.



Symbol for variable resistor



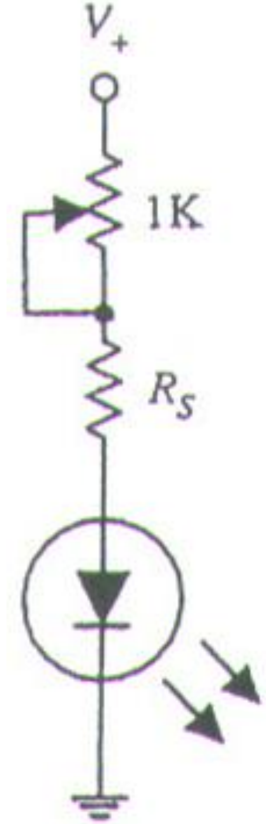
Variable Resistor: Rotary Potentiometers



Variable Resistor: Linear Potentiometer

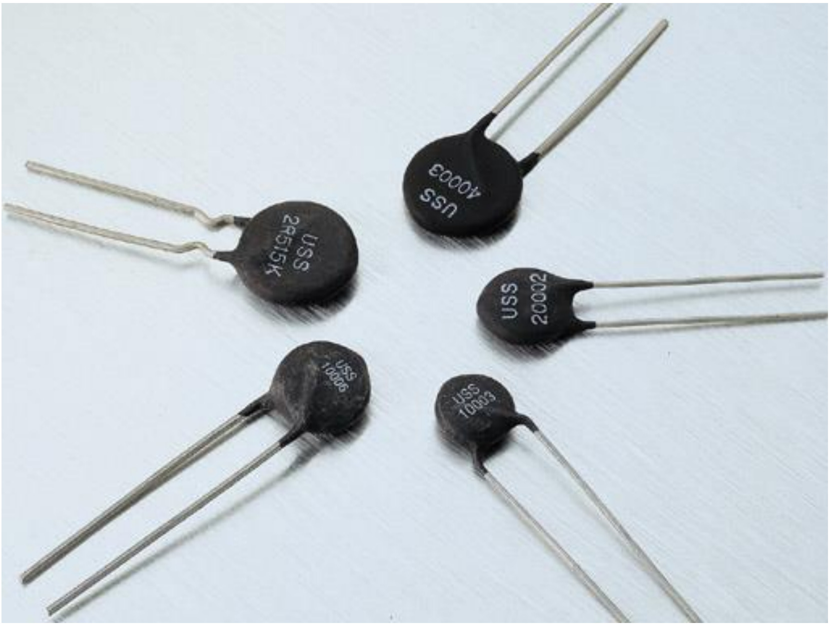
Variable Resistor: Example Application

- A variable resistor is connected to a rotary device
- Limit of travel of the rotary device is to be indicated by an LED
- Suppose rotary device causes variable resistor to effectively offer 0Ω
- Use a safety resistor R_s to protect the LED in this situation
- Suppose rotary device causes variable resistor to effectively offer its full resistance, in this case, effective current in the circuit is very small and the LED is not lit
- As the rotation of the device causes variable resistor to offer lower resistance, current flow will slowly increase, size various components so that at the rotational limit LED turns on and gets brighter if the rotation continues



Variable Resistor: Other Examples

Photoresistor



Thermistor

Resistance Formula

- For a resistor made using a homogenous material

$$R = \frac{\rho L}{A}$$

where

ρ = specific resistance of material (material property)

L = length of conductor used to make the resistor

A = cross-section area of conductor used to make the resistor

Capacitor Concept

- A capacitor is an energy storage element which is analogous to the spring element of mechanical systems.
- It can store electrical pressure (voltage) for periods of time.

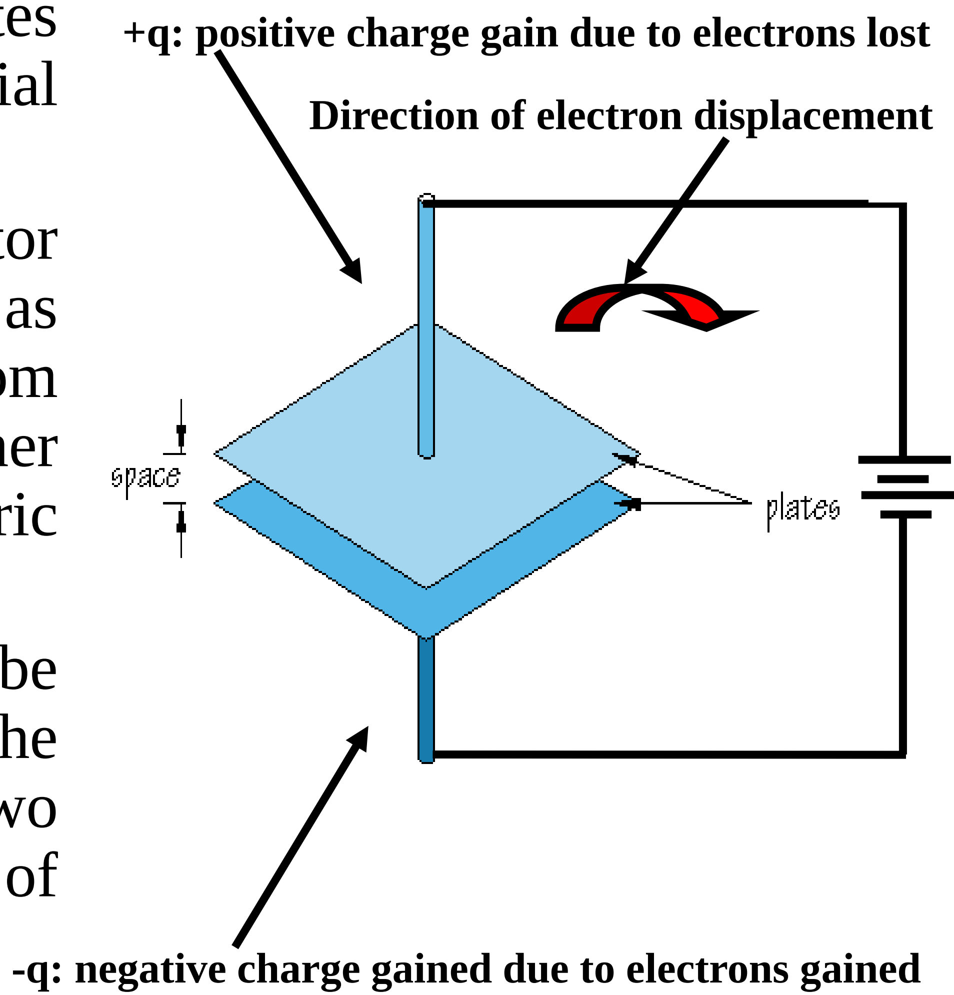
  -When a capacitor has a difference in voltage (electrical pressure) across its plate, it is said to be charged.

  -A capacitor is charged by having a one-way current flow through it for a period of time.

  -It can be discharged by letting a current flow in the opposite direction out of the capacitor.

Capacitor Construction

- A capacitor is constructed using a pair of parallel conducting plates separated by an insulating material (dielectric).
- When the two plates of a capacitor are connected to a voltage source as shown, charges are displaced from one side of the capacitor to the other side, thereby establishing an electric field.
- The charges continue to be displaced in this manner until the potential difference across the two plates is equal to the potential of voltage source.



Capacitor Water Pipe Analogy —I

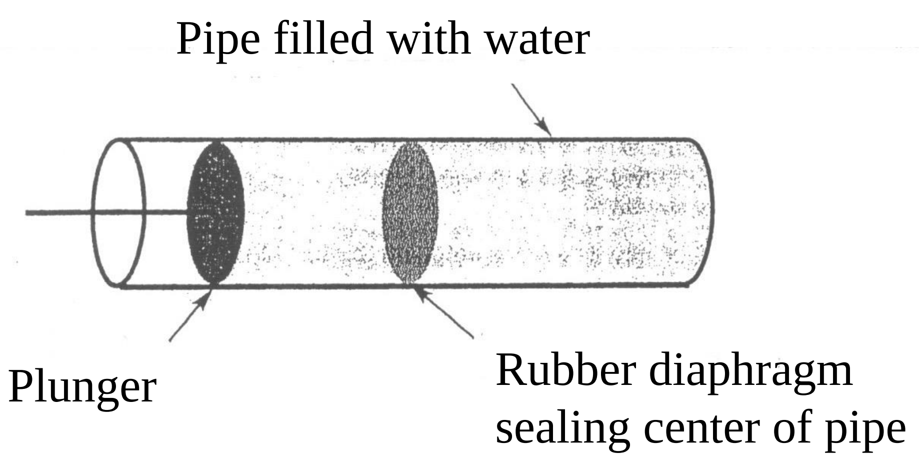
•In the water pipe analogy, a capacitor is thought of as a water pipe:

– with a rubber diaphragm sealing off each side of the pipe and

–a plunger on one end.

•When the plunger pushes toward the diaphragm, the water in the pipe forces the diaphragm to stretch until the force of the diaphragm pushing back on the water equals the force on the plunger→pipe is charged!

•If the plunger is released, the diaphragm will push the plunger back to its original position →pipe is discharged.



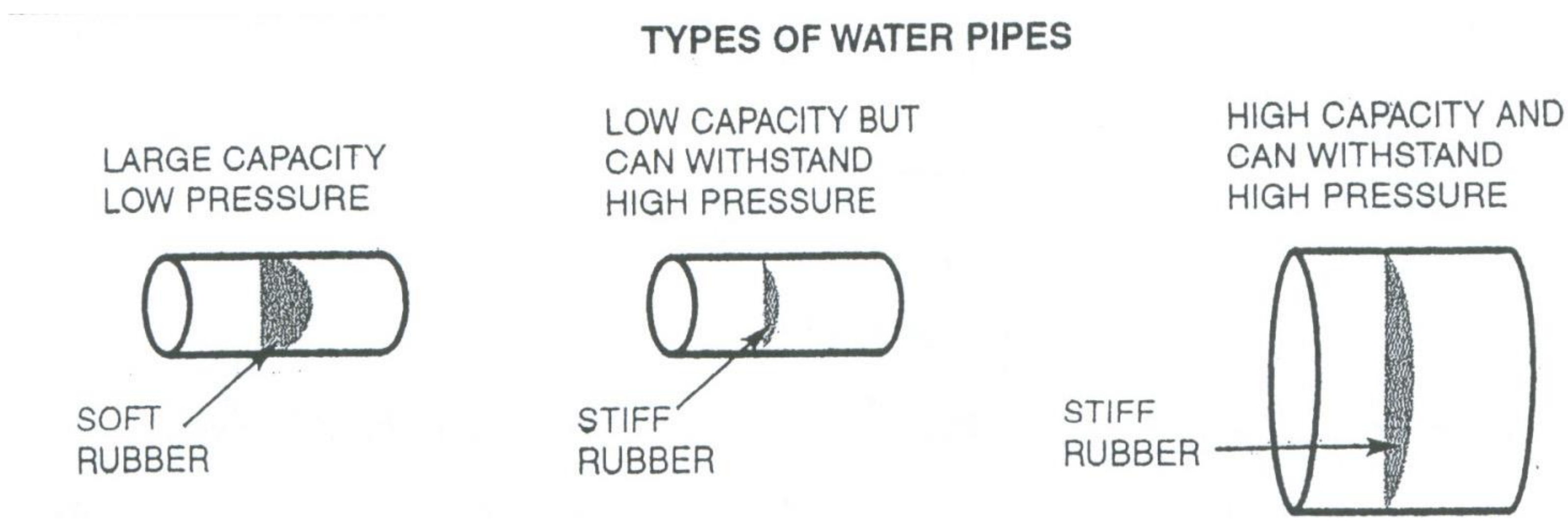
Capacitor Water Pipe Analogy —II

•If the rubber diaphragm is made very soft, it will stretch out and hold a lot of water but will break easily (large capacitance but low working voltage).

•If the rubber diaphragm is made very stiff, it will not stretch far but withstand higher pressure (low capacitance but high working voltage).

•By making the pipe larger and keeping the rubber stiff, we can achieve a device that holds a lot of water and withstand high pressure.

•So the pipe size is determined from the amount of water to be held and the amount of pressure to be handled.



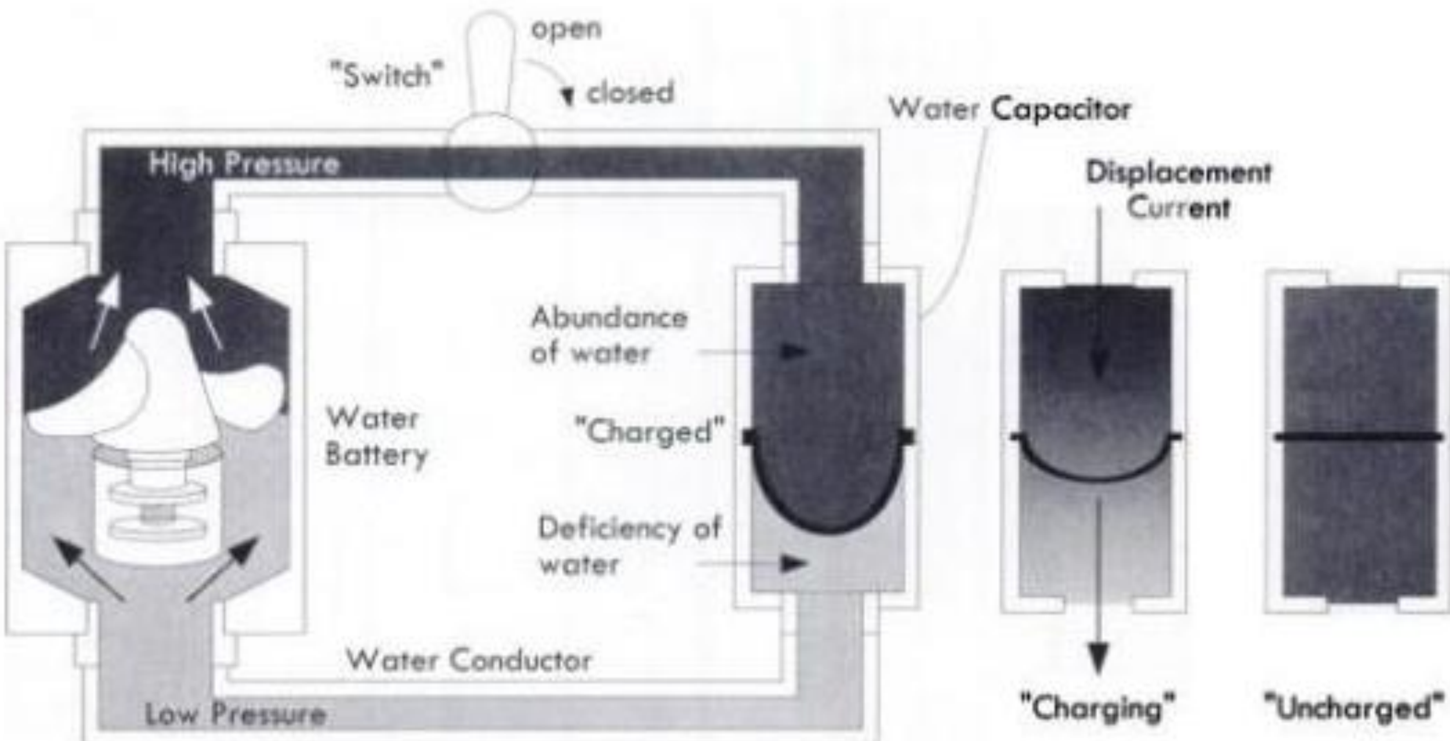
Capacitor Water Pipe Analogy —III

•Water capacitor: a tube with a rubber membranne in the middle

•Rubber membranne analogous to the dielectric, two chambers analogous to two capacitor plates

•When no water pressure is applied on the water capacitor, the two chambers contain same amount of water (uncharged)

•When pressure is applied on the top chamber, the membrane is pushed down causing the water to be displaced from the bottom chamber (appearance of current flow → displacement current)



Capacitor V-I Characteristic

•The charge accumulated on capacitor plates is directly proportional to voltage applied across the plates.

$$q \propto V \longrightarrow q = CV$$

where C is the constant of proportionality and is called capacitance (unit: Farad).

•V-I characteristic of a capacitor is obtained by computing

$$\frac{d}{dt}[q = CV] \longrightarrow \frac{dq}{dt} = C\frac{dv}{dt} \longrightarrow I(t) = C\frac{dv}{dt}$$

•Alternatively, integrating the above equation w.r.t. time, and rearranging terms, we get

$$V(t) = \frac{1}{C}\int_0^t I(\tau)d\tau$$

Capacitance Formula

•For a parallel capacitor:

$$C = \frac{\varepsilon_0 A}{D}$$

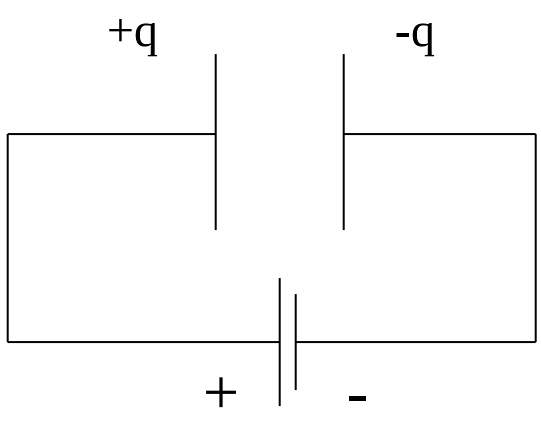
- ε_0 = permittivity of free space

- A = plate area

- D = separation distance of plate.

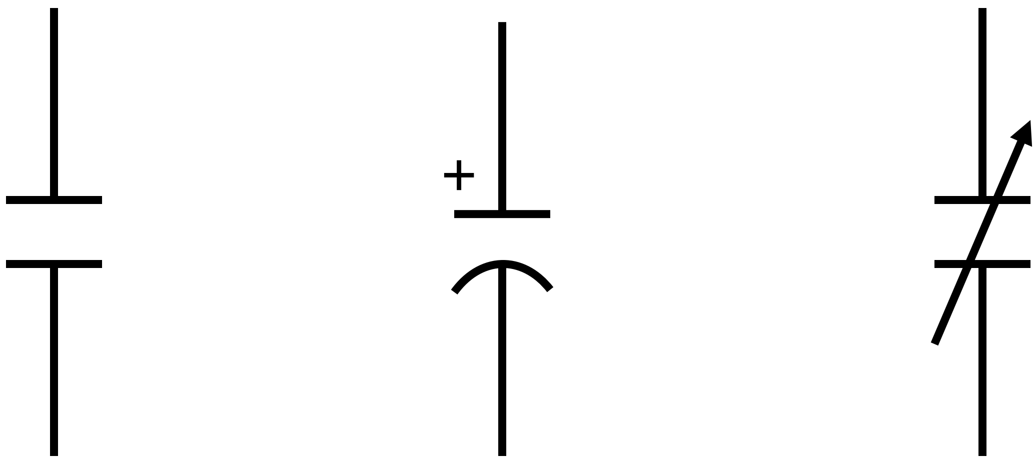
•Often, we use $G = A/D$ as geometry factor (for other types of capacitors as well).

•If a dielectric material with dielectric constant K separates the two plates of the capacitor, then $C = K\varepsilon_0 G$, where K = dielectric constant. Usually $K > 1$.



Voltage source

Capacitor Symbols

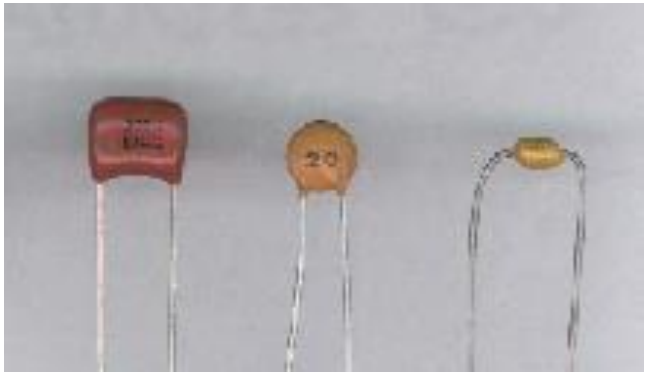


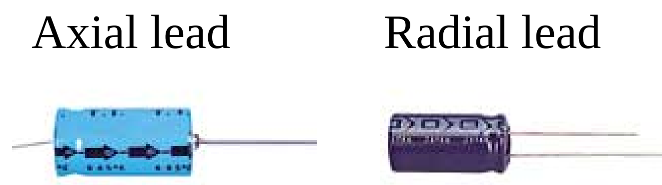
Fixed capacitor

Polarized capacitor

Variable capacitor

Capacitor Variations

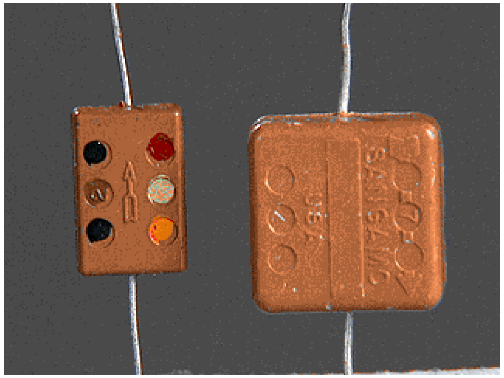




- Ceramic capacitors
  - –very popular nonpolarized capacitor
  - –small, inexpensive, but poor temperature stability and poor accuracy
  - –ceramic dielectric and a phenolic coating
  - –often used for bypass and coupling applications

- Electrolytic
  - –Aluminum, tantalum electrolytic
  - –Tantalum electrolytic capacitor has a larger capacitance when compared to aluminum electrolytic capacitor
  - –Mostly polarized.
  - –Greater capacitance but poor tolerance when compared to nonelectrolytic capacitors.
  - –Bad temperature stability, high leakage, short lives

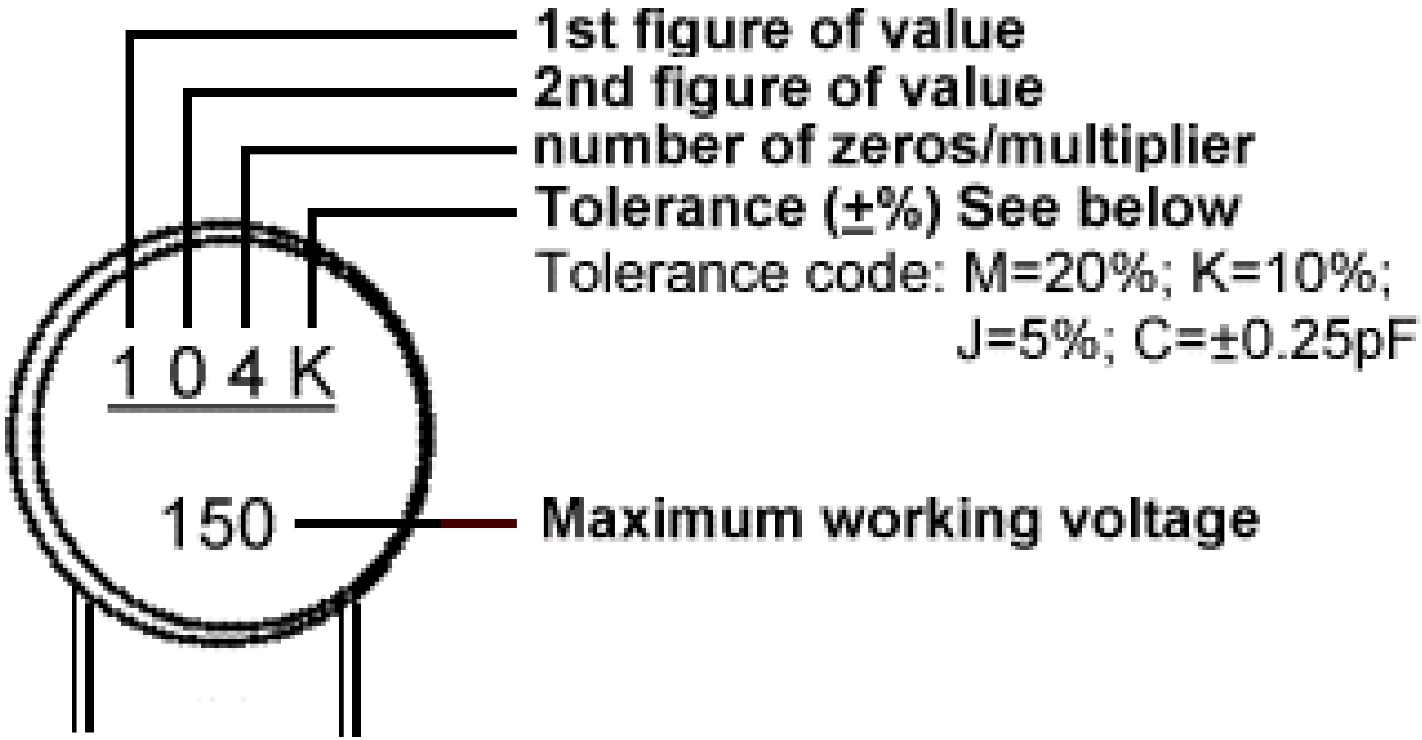
Capacitor Variations



- Mylar
  - –very popular, nonpolarized
  - –reliable, inexpensive, low leakage
  - –poor temperature stability

- Mica
  - –extremely accurate, low leakage current
  - –constructed with alternate layers of metal foil and mica insulation, stacked and encapsulated
  - –small capacitance
  - –often used in high-frequency circuits (i.e. RF circuits)

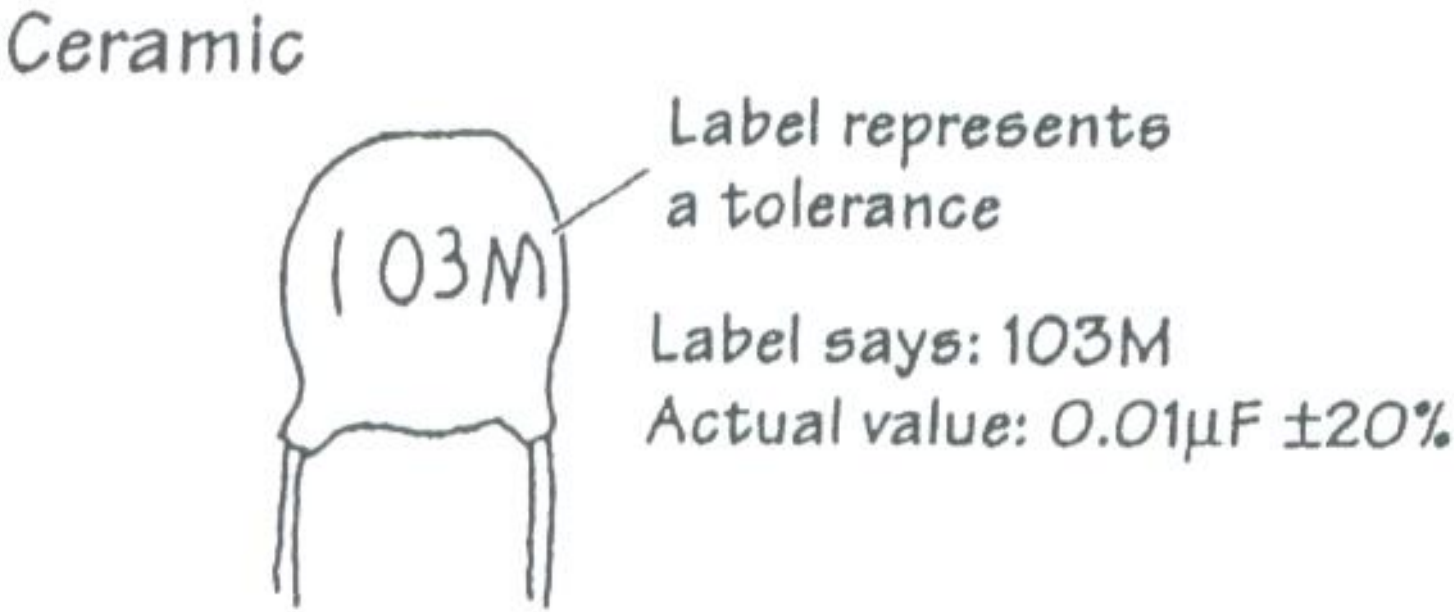
Capacitor Reading Example —I



$$10\times10^{4}\,\text{pF}=10^{5}\times10^{-12}\,\text{F}=10^{-7}\,\text{F}=0.1\times10^{-6}\,\text{F}=0.1\mu\text{F}$$

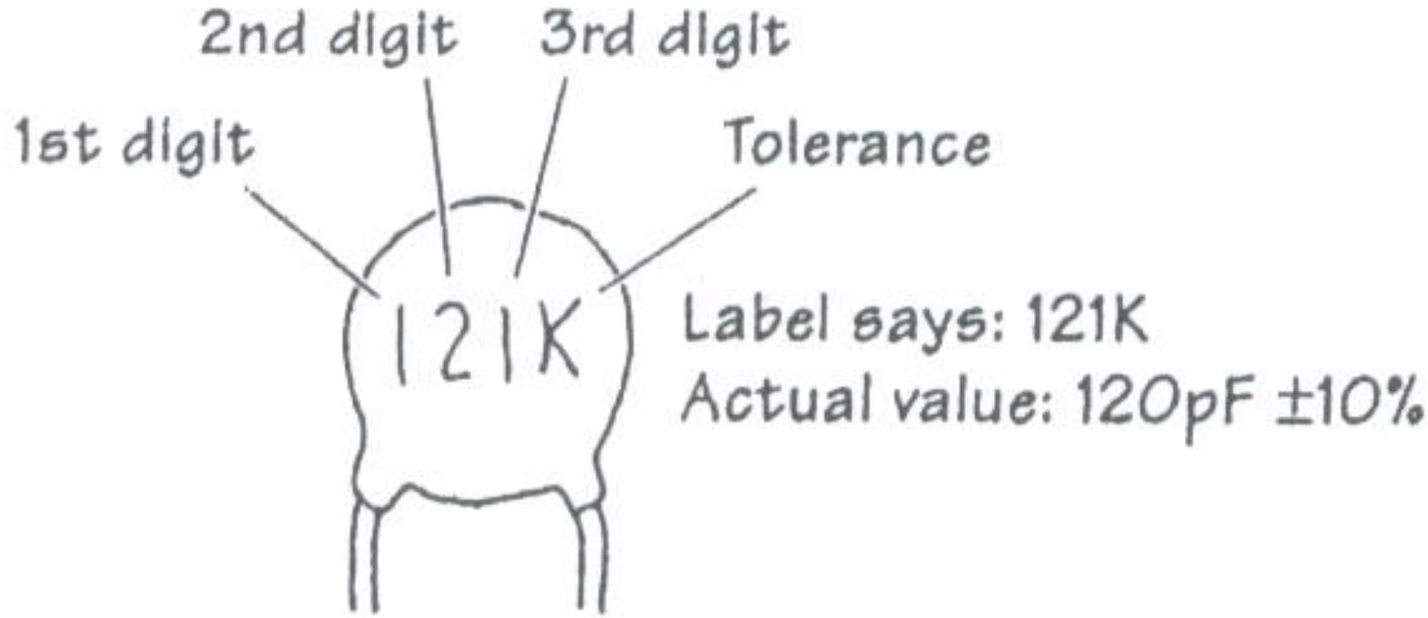
- Thus, we have a 0.1μF capacitor with ±10% tolerance.

Capacitor Reading Example —II



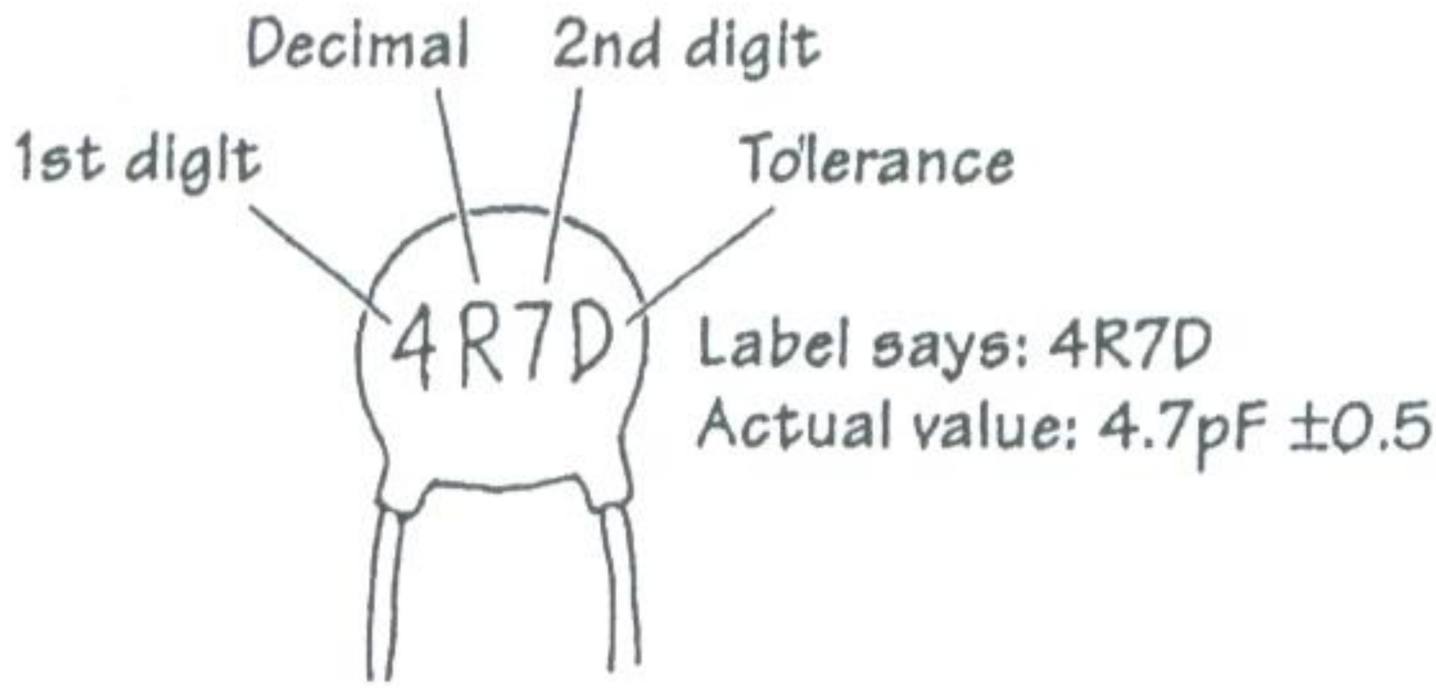
$$10\times10^{3}\,\mathrm{pF}=10^{4}\times10^{-12}\,\mathrm{F}=10^{-8}\,\mathrm{F}=0.01\times10^{-6}\,\mathrm{F}=0.01\,\mu\mathrm{F}$$

Capacitor Reading Example —III

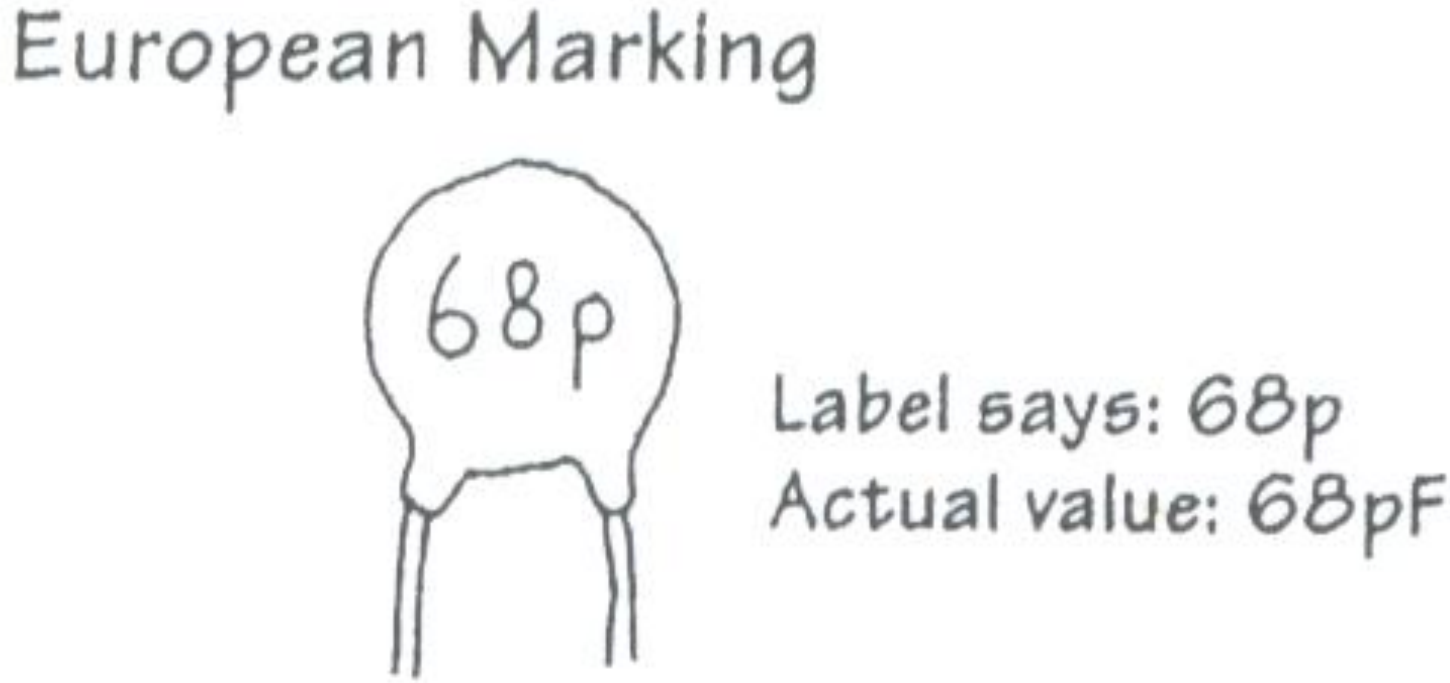


$$12 \times 10^{1}\,\mathrm{pF} = 120\,\mathrm{pF}$$

Capacitor Reading Example —IV



Capacitor Reading Example —V



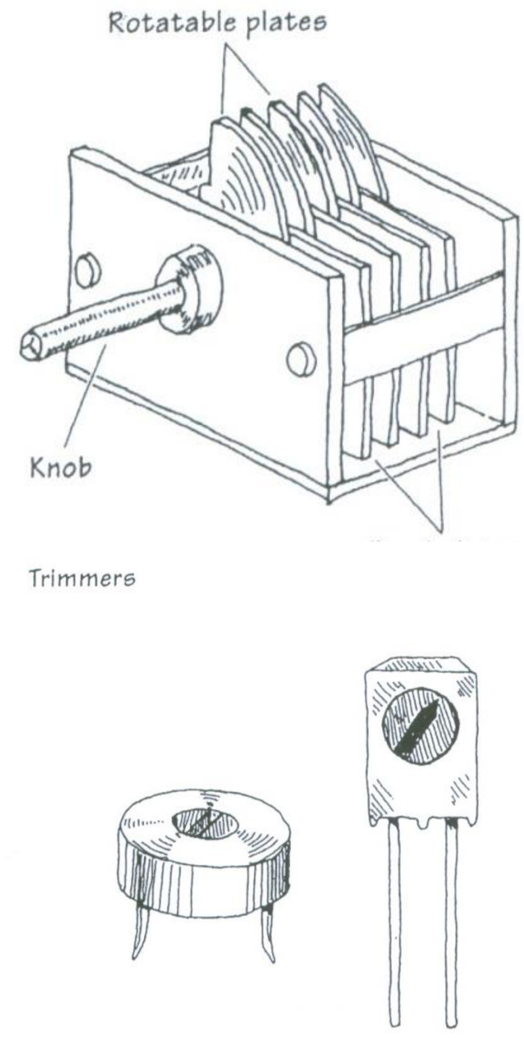
Variable Capacitors

•Devices that can be made to change capacitance values with the twist of a knob.

•Air-variable or trimmer forms

–Air-variable capacitor consists of two sets of aluminum plates (stator and rotor) that mesh together but do not touch. Often used in frequency adjusted tuning applications (i.e., tuning communication receivers over a wide band of frequencies).

–A trimmer capacitor is a smaller unit that is designed for infrequent fine-tuning adjustment (i.e., fine-tuning fixed-frequency communications receivers, crystal frequency adjustments, adjusting filter characteristics)



Inductors

- Inductor is a passive energy storage element that stores energy in the form of magnetic field.
- Inductor characteristic is governed by Faraday's law:

$$V(t) = \frac{d\lambda}{dt}$$

  - V = voltage induced across an inductor
  - λ = magnetic flux (unit: Webers, Wb) through the coil windings (a coil made using resistance-less wires) due to current flowing through inductor.

- For an ideal coil, magnetic flux is proportional to current, so

$$\lambda \propto I \;\; or \;\; \lambda = LI$$

  - L is constant of proportionality, called inductance (unit: Henry, Wb/Amp).

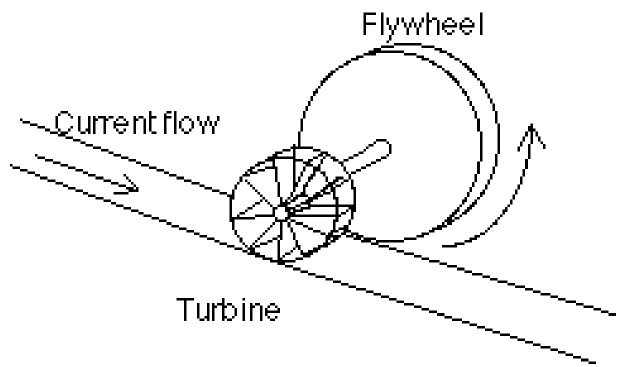
- So, now, the V-I characteristic of an inductor is:

$$V(t) = \frac{d}{dt}(\lambda) = \frac{d}{dt}(LI) = L\frac{dI}{dt}$$

$$I(t) = \frac{1}{L}\int_0^t V(\tau)d\tau$$

- The above V-I characteristics demonstrate that the current through an inductor can not be altered instantaneously.

Inductor-Water Analogy —I

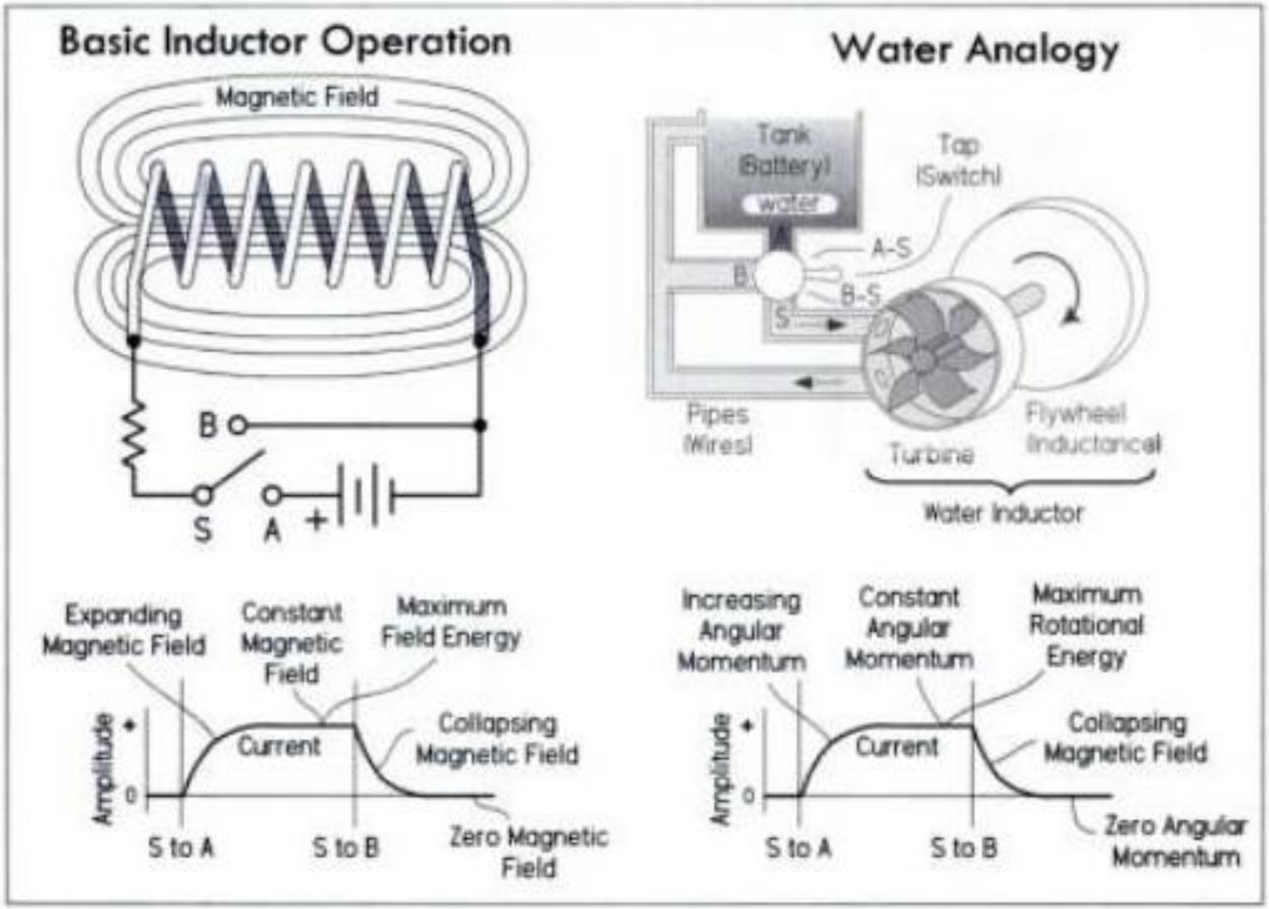


•Suppose a turbine is hooked up to the flywheel and water is supplied to the turbine. The flywheel will start to move slowly. Eventually, the flywheel will move at the same rate as the current.

•If the current alternates back and forth, the flywheel/turbine will take some time to build up to the initial direction that the water wants to flow.

•As the current moves back and forth, the flywheel creates the extra "resistance" to the change in current flow, but eventually the flywheel/turbine will move in the same direction as the current flow.

Inductor-Water Analogy —II



Mechanical inertia and inductor both resist sudden change in their state

•When switch S contacts A, the field generated by the applied positive voltage creates a reverse induced voltage that initially resists current flow

•Based on the value of inductance, as the magnetic field reaches steady-state, the reverse voltage decays

•A collapsing field is generated when applied voltage is removed (switch S contacts B), creating a forward induced voltage that attempts to keep current flowing

•Based on the value of inductance, as the magnetic field reaches zero steady-state, the forward voltage decays

Inductance of a Cylindrical Coil

$$L = \frac{\mu_0 N^2 \pi r^2}{\ell}$$

– μ_0= permeability of free space

– N = number of turns in coil

– ℓ = length of resistance-less wire used in coil

– r = radius of coil cross section.

• If number of turns per unit length is "n", then N=$n\ell$, so:

$$L = \frac{\mu_0 (n^2 \ell^2) \pi r^2}{\ell} = \mu_0 n^2 \ell \pi r^2 = \mu_0 n^2 \ell A$$

– A = cross-sectional area of coil.

– If a magnetizable material forms the core of coil, then permeability μ will be larger than μ_0.

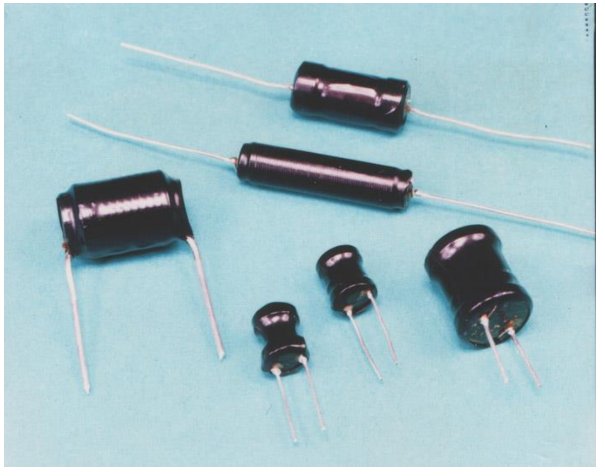
Inductor Variations —I

Inductor Variations —II

- Antenna coil
  - –contains an iron core that magnifies magnetic field effects
  - –used to tune in ultra-high-frequency signals, i.e. RF signals

- Tuning coil
  - –screw-like “magnetic field blocker” that can be adjusted to select the desired inductance value
  - –used in radio receivers to select a desired frequency.

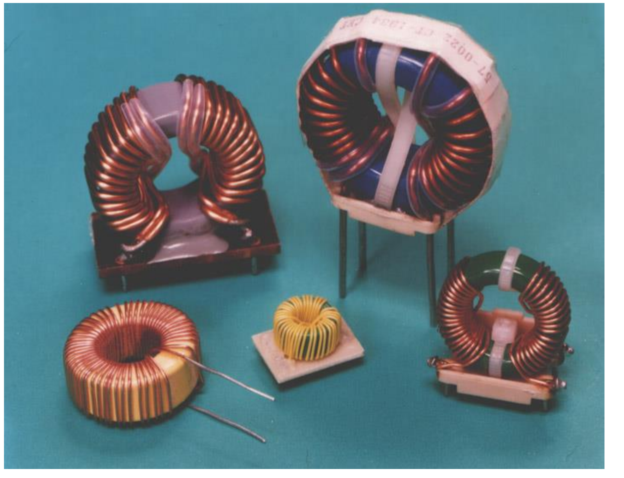
Inductor Variations —III



•Chokes

–general-purpose inductors that act to limit or suppress fluctuating current.

–some use a resistor-like color code to specify inductance values.



•Toroidal coil

–resembles a donut with a wire wrapping

–high inductance per volume ratios, high quality factors, self-shielding, can be operated at extremely high frequencies

Inductor Symbols

Air core

Iron core

Powered-iron core

Variable core

Transformer

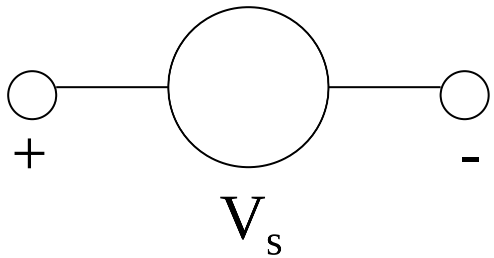
- Isolation
  - –acts exclusively as an isolation device; does not increase or decrease the secondary voltage
  - –usually come with an electrostatic shield between the primary and secondary. Often come with a three-wire plug and receptacle that can be plugged directly into a power outlet

- High Frequency
  - –often come with air or powered-iron cores
  - –used for high frequency applications, i.e. matching RF transmission lines to other devices (transmission line to antenna)

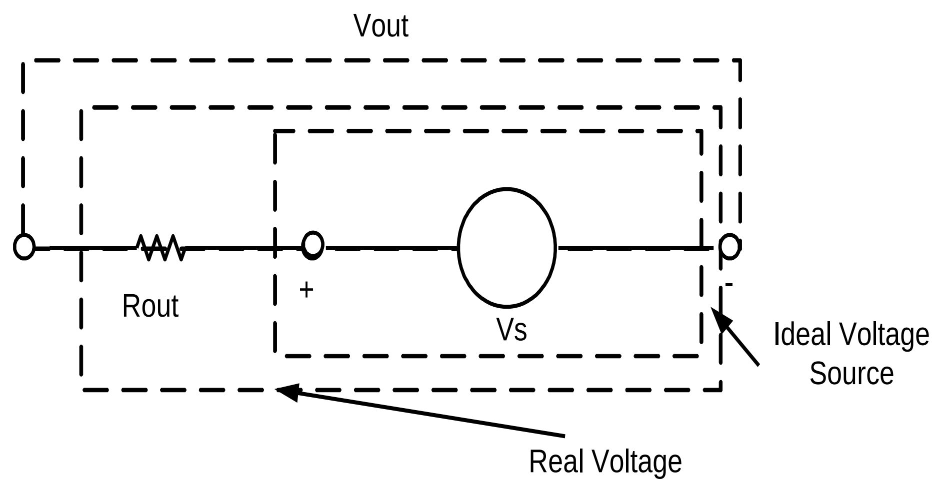
- Audio
  - –used primarily to match impedances between audio devices
  - –work best at audio frequencies from 150Hz to 12kHz
  - –come in a variety of shapes and sizes, typically contain a center tap

Voltage Sources

- Ideal:
  - zero output resistance
  - can supply as much current as needed by the load
  - maintains the prescribed voltage across its terminals



- Real:
  - modeled as an ideal voltage source in series with a small resistor (R_{out})



$$V_{out} = V_s - V_{R_{out}}$$

- If the load across the real voltage source is a resistance R_L, then:

$$V_{out} = \left(\frac{R_L}{R_L + R_{out}}\right) V_s$$

- If $R_L >> R_{out}$, then $V_{out} \cong V_S$

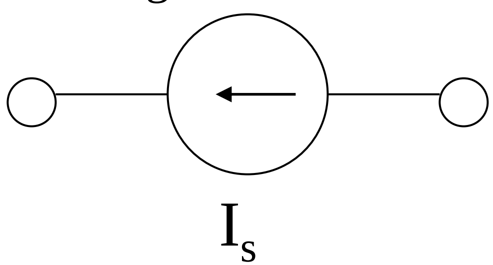
Current Sources

•Ideal:

–infinite output resistance

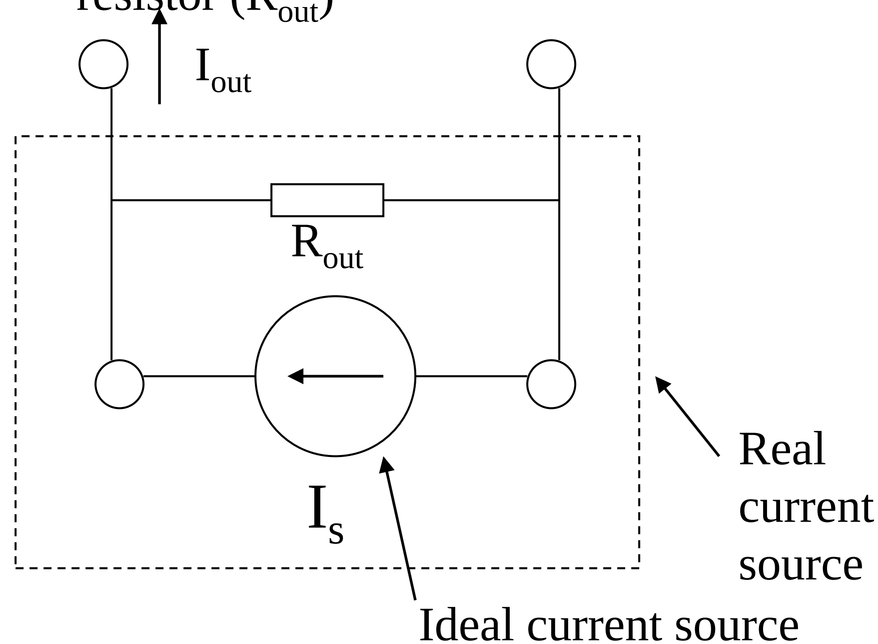
–supply as much voltage as needed by the load

–prescribed current irrespective of the voltage across terminals



•Real:

–modeled as an ideal current source in parallel with a large resistor (R_{out})



$$I_{out} = I_s - I_{R_{out}}$$

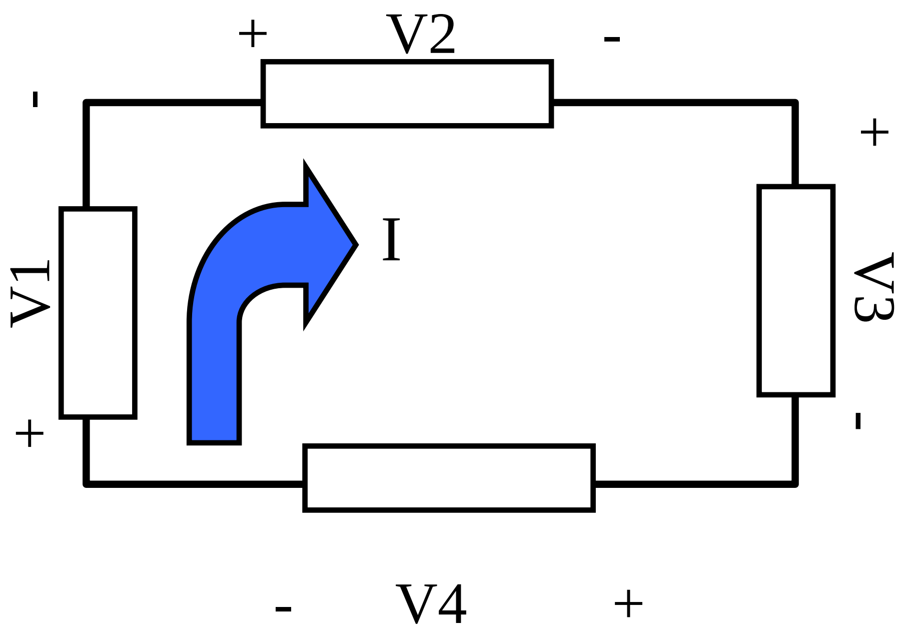
•If the load across the real current source is a resistance R_L, then:

$$I_{out} = \left(\frac{R_{out}}{R_L + R_{out}}\right) I_s$$

Kirchoff's Voltage Law

- The algebraic sum of voltage around a loop is zero.
- Assumption:
  - –Voltage drop across each passive element is in the direction of current flow.

$$V_1 + V_2 + V_3 + V_4 = 0$$

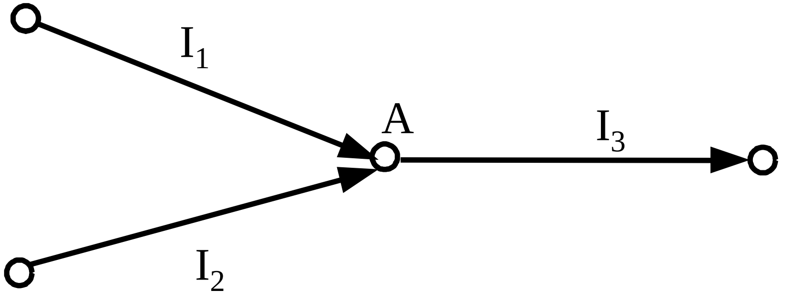


Kirchoff's Current Law

• Algebraic sum of all currents entering and leaving a node is zero.

• At node A:

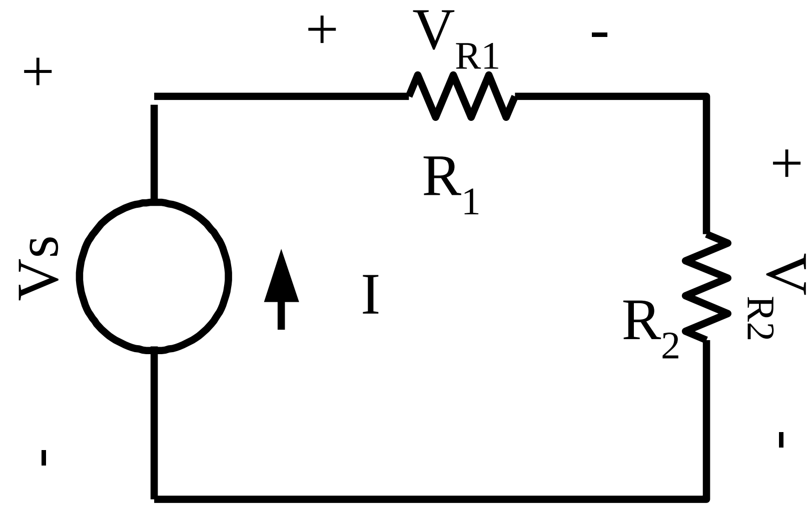
$$I_1 + I_2 - I_3 = 0$$



• Current entering a node is assigned positive sign. Current leaving a node is assigned a negative sign.

Law of Voltage division

$$I = \frac{V_s}{R_1 + R_2}$$



$$\underbrace{V_{R_1} = IR_1 \quad \text{and} \quad V_{R_2} = IR_2}$$

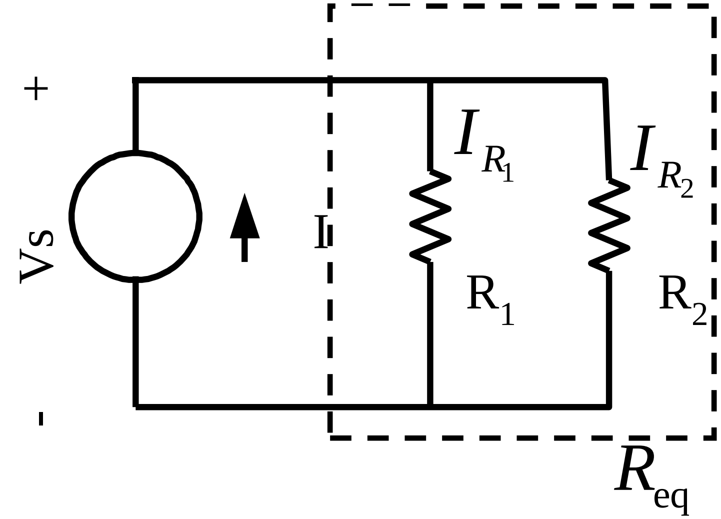
$$\Downarrow$$

$$V_{R_1} = \frac{R_1}{R_1 + R_2} V_s \qquad V_{R_2} = \frac{R_2}{R_1 + R_2} V_s$$

Law of Current division

$$I = \frac{V_s}{R_{eq}}$$

$$V_s = IR_{eq} = I\,\frac{R_1 R_2}{R_1 + R_2}$$



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$I_{R_1} = \frac{V_S}{R_1} \quad \text{and} \quad I_{R_2} = \frac{V_S}{R_2}$$

$$\Downarrow$$

$$I_{R_1} = \frac{R_2}{R_1 + R_2} I \qquad I_{R_2} = \frac{R_1}{R_1 + R_2} I$$

AC Circuit Analysis using Phasors

•Ohm's law which states that the V-I relationship for a resistor is linear can be extended to AC circuit analysis.

•A circuit may have resistor, capacitors, and inductors. In particular, for a circuit containing an element such as resistor, capacitor, or inductor:

$$P[V] = Z\,P[I]$$

–where V and I are alternating voltage and current

–Z: circuit impedance

•The symbol P denotes the phasor operator, it captures information on signal magnitude and phase.

•If we used *Sin ωt* as a reference signal, and have $V(t) = V_0 \sin(\omega t + \phi)$, then:

$$P[V(t)] = V_0 < \phi >= V_0\, e^{j\phi}$$

–V_0=magnitude

–ϕ =phase

AC Circuit Analysis using Phasors—Resistor

•Using last mentioned formula, for a resistor, if we apply $V(t)=V_0\sin(\omega t)$ across it, from Ohm's law:

$$V(t) = RI(t)$$

$$\Rightarrow I(t) = \frac{V(t)}{R} = \frac{V_0}{R}\sin\omega t$$

$$\boxed{I_0 \equiv \frac{V_0}{R}}$$

$$P[V(t)] = V_0 < 0 >, \quad P[I(t)] = I_0 < 0 >$$

•Hence, finally,

$$P[V] = R\,P[I] \quad \& \quad \boxed{Z = R}$$

AC Circuit Analysis using Phasors—Inductor

Consider $I(t)=I_0 \sin\omega t$

$$V(t) = L\frac{dI}{dt}$$

$$= L\frac{d}{dt}(I_0 \sin\omega t)$$

$$= I_0 L\omega \cos\omega t$$

$$\underbrace{= I_0 L\omega \sin(\omega t + \frac{\pi}{2})}_{V_0 \sin(\omega t + \phi)}$$

$$\Rightarrow$$

$$V_0 \Box I_0 L\omega$$

$$\phi = \frac{\pi}{2}$$

•Now, to get Z

$$P[V(t)] = Z\ P[I(t)]$$

$$V_0 < \phi > = Z\ I_0 < 0 >$$

$$\Rightarrow I_0 L\omega < \frac{\pi}{2} > = Z\ I_0 < 0 >$$

$$\Rightarrow Z = L\omega < \frac{\pi}{2} >$$

$$= L\omega\, e^{j\frac{\pi}{2}}$$

$$= L\omega\,[\cos\frac{\pi}{2} + j\sin\frac{\pi}{2}] = jL\omega$$

•So: $\boxed{Z_L = j\omega L}$

AC Circuit Analysis using Phasors—Capacitor

- Consider $V(t)=V_0\sin\omega t$

$$I(t) = C\frac{d}{dt}[V(t)]$$

$$I(t) = C\frac{d}{dt}[V_0 \sin\omega t]$$

$$= C\omega V_0 \cos\omega t$$

$$= C\omega V_0 \sin(\omega t + \frac{\pi}{2})$$

$$\Rightarrow P[I(t)] = C\omega V_0 < \frac{\pi}{2} >$$

- Thus:

$$P[V(t) = Z\ P[I(t)]$$

$$V_0 < 0 >= ZC\omega V_0 < \frac{\pi}{2} >$$

$$\Rightarrow Z = \frac{1}{C\omega} < -\frac{\pi}{2} >$$

$$= \frac{1}{\omega C}[\cos(-\frac{\pi}{2}) + j(\sin(-\frac{\pi}{2}))]$$

$$= \frac{-j}{\omega C} = \frac{1}{j\omega C}$$

$$\boxed{Z_C = \frac{1}{j\omega C}}$$

Laplace Transform Defined

- Time domain function: $f(t)$
- Laplace domain representation: $F(s)$

$$F(s) = \mathcal{L}[f(t)] = \int_0^{\infty} e^{-st} f(t)\,dt$$

  - s: complex variable (called Laplace variable)
  - Require $\mathrm{Re}(s) > 0$ so that the integral exists

- Sufficient condition for $f(t)$ to be Laplace transformable:

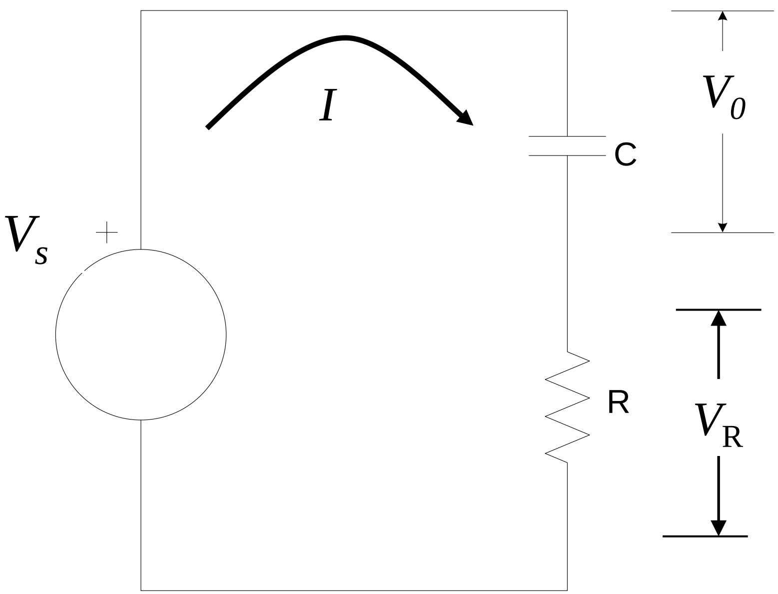
$$\int \left| f(t) \right| e^{-s_1 t}\,dt < \infty \text{ , for } s_1 > 0$$

- If $f(t)$ is bounded by some exponential function, $\mathcal{L}[f(t)]$ exists, i.e., need

$$\left| f(t) \right| < M e^{at}, M > 0, a > 0$$

- Finally, $s \equiv \dfrac{d}{dt}$ and $\dfrac{1}{s} \equiv \displaystyle\int_0^t (\cdot)\,dt$

Math Model: R-C Circuit



- Linear resistor: $V_R = IR$
- Capacitor: $I = C\dot{V}_o$

Use KVL and V-I relations

$$-V_s + V_0 + V_R = 0$$

$$V_0 + IR = V_s$$

$$V_0 + (C\dot{V}_0)R = V_s$$

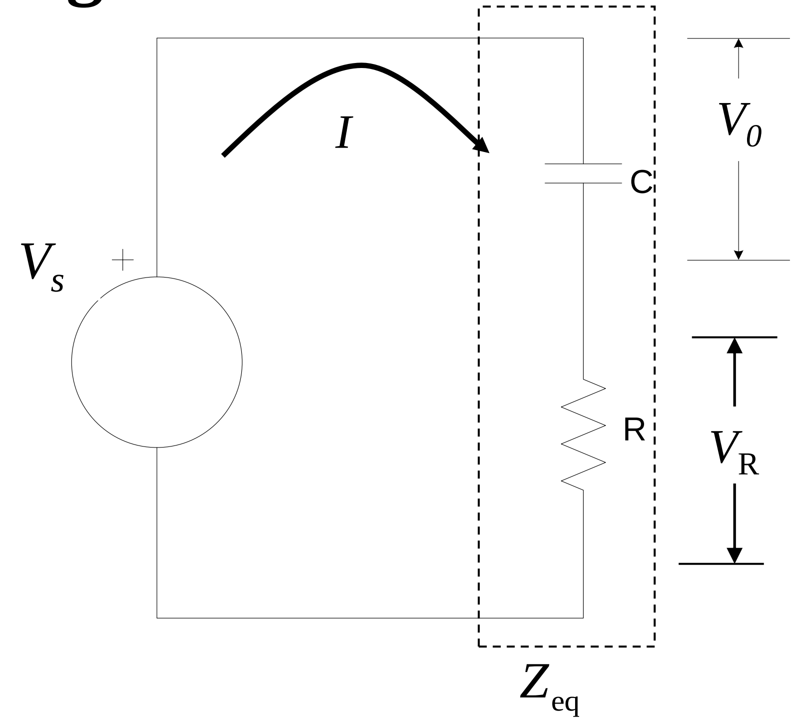
$$\boxed{RC\frac{dV_0}{dt} + V_0 = V_s}$$

- System parameters: R and C
- System output: V_0 and system input: V_s
- Solution: separation of variables/Laplace transforms
- Certain mechanical, fluid, and hydraulic systems have similar model

R-C Circuit Using Phasors

- Linear resistor: $V_R = IR$
- Capacitor: $I = C\dot{V}_o$

$$\left.\begin{array}{l} \\ \\ \end{array}\right\} \quad \begin{array}{l} Z_R = R \\ Z_C = \dfrac{1}{j\omega C} \end{array}$$



Series resistor-capacitor circuit:

$$Z_{eq} = R + \frac{1}{jwC} = \frac{jwRC + 1}{jwC}$$

Voltage divider:

$$\bar{V}_0 = \frac{Z_C}{Z_{eq}}\bar{V}_s = \frac{\dfrac{1}{jwC}}{\dfrac{jwRC+1}{jwC}}\bar{V}_s = \frac{1}{jwRC+1}\bar{V}_s \xrightarrow[\textbf{Domain}]{\textbf{Laplace}} V_0(s) = \frac{1}{sRC+1}V_s(s)$$

$$(sRC + 1)V_0(s) = V_s(s)$$

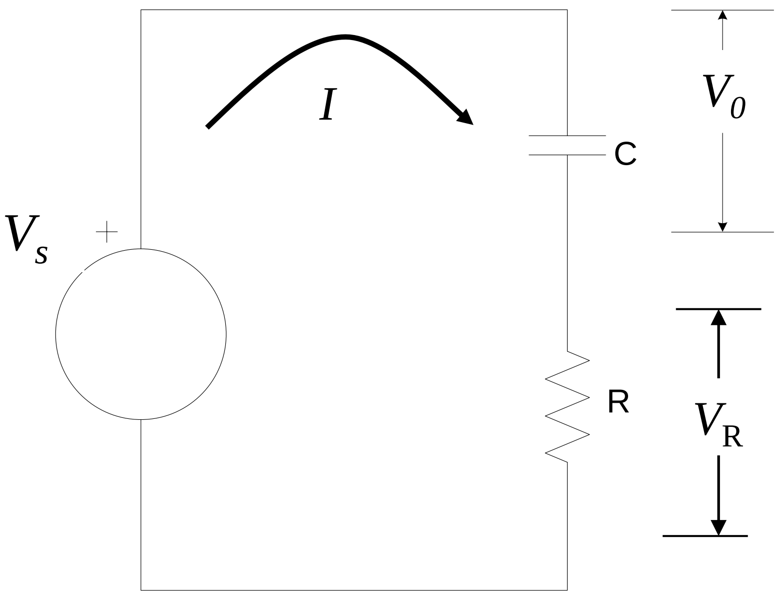
$$\boxed{RC\frac{dV_0}{dt} + V_0 = V_s}$$

Note

$\bar{V}_0 \; @ \, P[V_0(t)]$

$\bar{V}_s \; @ \, P[V_s(t)]$

Transfer Function: R-C Ckt



ODE Model

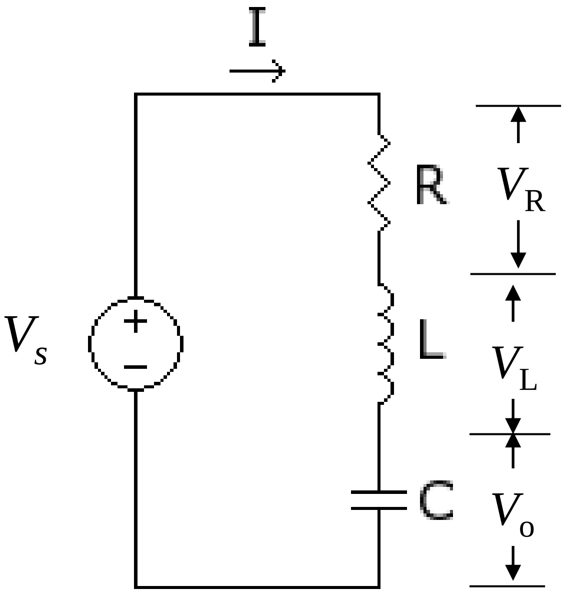
$$RC\frac{dV_0}{dt} + V_0 = V_s$$

$$\mathcal{L}\left(RC\dot{V}_0(t) + V_0(t) = V_s(t)\right)$$

$$(RCs + 1)V_0(s) = V_s(s)$$

$$G(s) = \frac{V_0(s)}{V_s(s)} = \frac{1}{RCs + 1}$$

Math Model: R-L-C Circuit



- Linear resistor: $V_R = IR$
- Capacitor: $I = C\dot{V}_o$
- Inductor: $V_L = L\dot{I}$

Use KVL and V-I relations

$$-V_s + V_R + V_L + V_0 = 0$$

$$IR + L\dot{I} + V_0 = V_s$$

$$(C\dot{V}_0)R + L(C\ddot{V}_0) + V_0 = V_s$$

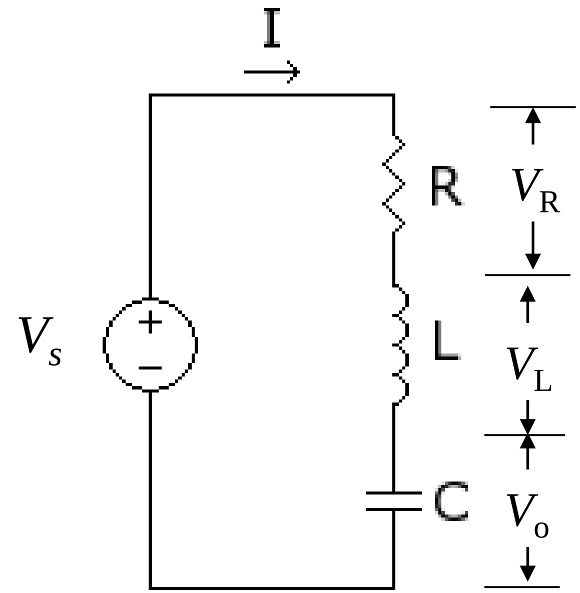
$$\boxed{LC\frac{d^2V_0}{dt^2} + RC\frac{dV_0}{dt} + V_0 = V_s}$$

- System parameters: *R*, *L*, and *C*
- System output: V_0 and system input: V_s
- Solution: method of undetermined coefficients/Laplace transforms
- Certain mechanical, fluid, and hydraulic systems have similar model

R-L-C Circuit Using Phasors

- Linear resistor: $V_R = IR$
- Capacitor: $I = C\dot{V}_o$
- Inductor: $V_L = L\dot{I}$

$$\left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \quad \begin{aligned} Z_R &= R \\ Z_C &= \frac{1}{j\omega C} \\ Z_L &= j\omega L \end{aligned}$$



Series resistor-capacitor-inductor circuit:

$$Z_{eq} = R + jwL + \frac{1}{jwC} = \frac{jwRC + (1 - w^2LC)}{jwC}$$

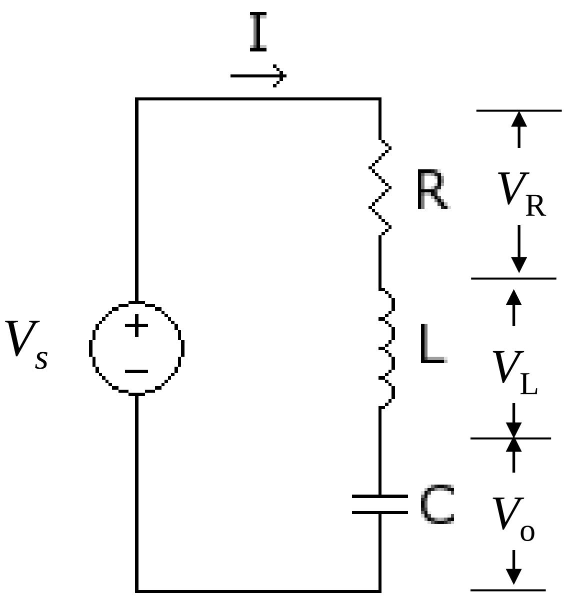
Voltage divider:

$$\bar{V}_0 = \frac{Z_C}{Z_{eq}}\bar{V}_s = \frac{1}{jwRC + (1 - w^2LC)}\bar{V}_s \quad \xrightarrow[\textbf{Domain}]{\textbf{Laplace}} \quad V_0(s) = \frac{1}{s^2LC + sRC + 1}V_s(s)$$

$$(s^2LC + sRC + 1)V_0(s) = V_s(s)$$

$$\boxed{LC\frac{d^2V_0}{dt^2} + RC\frac{dV_0}{dt} + V_0 = V_s}$$

Transfer Function: R-L-C Ckt



ODE Model

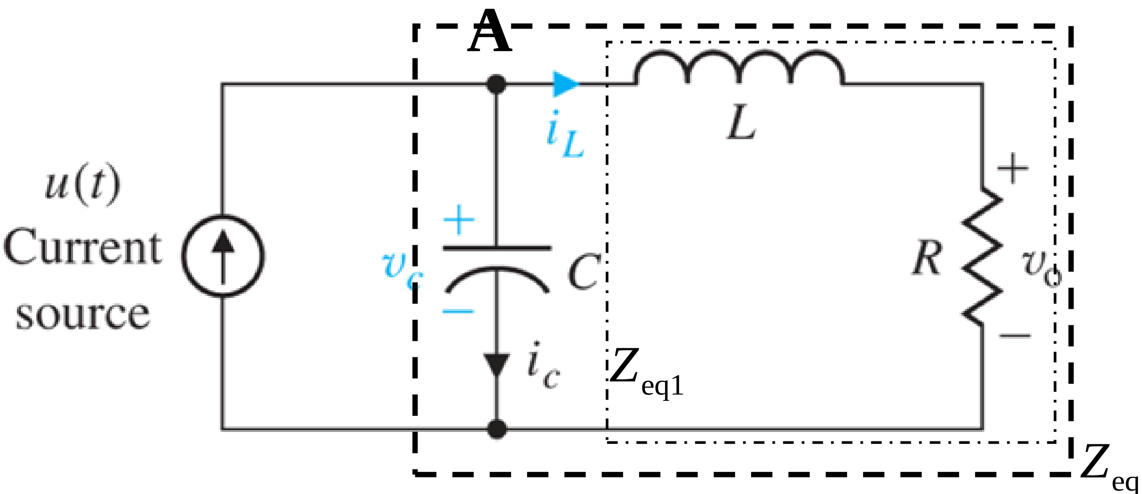
$$LC\frac{d^2V_0}{dt^2} + RC\frac{dV_0}{dt} + V_0 = V_s$$

$$\mathcal{L}\left(LC\ddot{V}_0(t) + RC\dot{V}_0(t) + V_0(t) = V_s\right)$$

$$(LCs^2 + RCs + 1)V_0(s) = V_s(s)$$

$$G(s) = \frac{V_0(s)}{V_s(s)} = \frac{1}{LCs^2 + RCs + 1}$$

Another R-L-C Circuit Using Phasors



$$i_C = C\frac{dv_C}{dt}$$

$$v_L = L\frac{di_L}{dt}$$

$$v_o = Ri_L$$

$$Z_{eq1} = R + jwL \quad \& \quad \frac{1}{Z_{eq}} = \frac{1}{Z_C} + \frac{1}{Z_{eq1}} = jwC + \frac{1}{R + jwL} = \frac{jwRC + (1- \; w^2 LC)}{R + jwL}$$

$$\bar{v}_c = \bar{u} Z_{eq} \quad \& \quad \bar{v}_0 = \frac{R}{Z_{eq1}} \bar{v}_c = \frac{Z_{eq} R}{Z_{eq1}} \bar{u}$$

$$\bar{v}_0 = \frac{R}{jwRC + (1- \; w^2 LC)} \bar{u} \quad \text{Þ} \quad V_0(s) = \frac{R}{s^2 LC + sRC + 1} U(s)$$