Chapter 2

$\boxed{{}^{A}\mathbf{P} = {}^{A}R_B\,{}^{B}\mathbf{P} + {}^{A}\mathbf{P}_{BORG}}$

$${}^{A}T_B = \left[\begin{array}{c:c} {}^{A}R_B & {}^{A}\mathbf{P}_{BORG} \\ \hdashline \mathbf{0}^T & 1 \end{array}\right]$$: **Homogeneous transform**

- Cayley's formula: $R = (I_3 - S)^{-1}(I_3 + S)$ (where S is a skew-symmetric matrix; $S = -S^T$)
- $S = \begin{bmatrix} 0 & -s_z & s_y \\ s_z & 0 & -s_x \\ -s_y & s_x & 0 \end{bmatrix}$ → $\therefore$ R: 3 independent parameters

X-*Y*-*Z* Fixed Angle

$$\boxed{{}^{A}R_{BXYZ}(\gamma,\beta,\alpha) = R_Z(\alpha)R_Y(\beta)R_X(\gamma)}$$

$$= \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix} = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

Z-*Y*-*X* Euler Angle

$$\boxed{{}^{A}R_{BZ'Y'X'}(\alpha,\beta,\gamma) = R_Z(\alpha)R_Y(\beta)R_X(\gamma)}$$

$$= \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix} = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

Angle-Axis

- Equivalent rotation matrix for ${}^{A}\hat{K} = [k_x \;\; k_y \;\; k_z]^T$

$$R_K(\theta) = {}^{A}R_B(\hat{K},\theta) = \begin{bmatrix} k_x k_x v\theta + c\theta & k_x k_y v\theta - k_z s\theta & k_x k_z v\theta + k_y s\theta \\ k_y k_x v\theta + k_z s\theta & k_y k_y v\theta + c\theta & k_y k_z v\theta - k_x s\theta \\ k_z k_x v\theta - k_y s\theta & k_z k_y v\theta + k_x s\theta & k_z k_z v\theta + c\theta \end{bmatrix}$$

(versed sine: $\text{versine}(\theta) = \text{vers}(\theta) = v\theta = 1 - c\theta$)

Rodriques' formula: $Q' = R_K(\theta)Q = Q\cos\theta + \sin\theta\left(\hat{K}\times Q\right) + \left(1-\cos\theta\right)\left(\hat{K}\bullet Q\right)\hat{K}$

Chapter 3

DH Table

Joint i	θ_i	d_i	a_i	α_i	Joint variable $\mathbf{q}$
Revolute	$\theta_i = \tilde{\theta}_i + q_i$	d_i	a_i	α_i	q_i
Prismatic	θ_i	$d_i = \tilde{d}_i + q_i$	a_i	α_i	q_i

$$\boxed{{}^{i-1}T_i = \left[\begin{array}{ccc:c} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ \hdashline 0 & 0 & 0 & 1 \end{array}\right]}$$

Chapter 4

- Let $\boxed{u = \tan\frac{\theta}{2}}$ and substitute $\cos\theta = \frac{1-u^2}{1+u^2}$, $\sin\theta = \frac{2u}{1+u^2}$ (Weierstrass Substitution)

- Two-argument arctangent function $\phi = \operatorname{atan2}(y, x)$
  Defined on all four quadrants ($-\pi \le \phi < \pi$)

Case	Quadrants	$\phi = \operatorname{atan2}(y,x)$
$x > 0$	1, 4	$\phi = \arctan(y/x)$
$x = 0$	1, 4	$\phi = \underbrace{\operatorname{sgn}(y)}_{=\pm 1}(\pi/2)$
$x < 0$	2, 3	$\phi = \arctan(y/x) + \operatorname{sgn}(y) \cdot \pi$

Law of Cosines: $a^2 + b^2 - 2ab\cos C = c^2$

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$