Mathematics for Robotics
ROB-GY 6103
Homework 4 Answers

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Question: 1.

Answer: Firstly, consider the inner product $\langle x, y\rangle = x^T\bar{y}$. To show that it is an inner product over $(\mathbb{C}^n, \mathbb{C})$ we need to check if it follow the following properties -

a. $\langle x, y\rangle = \overline{\langle y, x\rangle}$

$$LHS = \langle x, y\rangle = x^T\bar{y} \tag{1}$$

Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$. Substituting in $Eq^n(1) \Rightarrow$

$$LHS = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} \bar{y_1} \\ \bar{y_2} \end{bmatrix} = x_1\bar{y_1} + x_2\bar{y_2} \tag{2}$$

Now consider,

$$RHS = \overline{\langle x, y\rangle} = y^T\bar{x} \tag{3}$$

Substituting the values for x and y in above $Eq^n(3) \Rightarrow$

$$RHS = \overline{\begin{bmatrix} y_1 & y_2 \end{bmatrix} \begin{bmatrix} \bar{x_1} \\ \bar{x_2} \end{bmatrix}} = \overline{y_1\bar{x_1} + y_2\bar{x_2}} = x_1\bar{y_1} + x_2\bar{y_2} = LHS \tag{4}$$

b. $\langle a_1x_1 + a_2x_2, y\rangle = a_1\langle x_1, y\rangle + a_2\langle x_2, y\rangle$ Let, $x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix}$, $x_2 = \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}$ and $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$. So,

$$\begin{aligned}
\langle a_1x_1 + a_2x_2, y\rangle &= (a_1x_1 + a_2x_2)^T\bar{y} && (5)\\
&= \left[a_1 \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix} + a_2 \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}\right]^T \begin{bmatrix} \bar{y_1} \\ \bar{y_2} \end{bmatrix} && (6)\\
&= \begin{bmatrix} a_1x_{11} + a_2x_{21} & a_1x_{12} + a_2x_{22} \end{bmatrix} \begin{bmatrix} \bar{y_1} \\ \bar{y_2} \end{bmatrix} && (7)\\
&= (a_1x_{11} + a_2x_{21})\bar{y_1} + (a_1x_{12} + a_2x_{22})\bar{y_2} && (8)\\
&= a_1x_{11}\bar{y_1} + a_2x_{21}\bar{y_1} + a_1x_{12}\bar{y_2} + a_2x_{22}\bar{y_2} && (9)\\
&= a_1(x_{11}\bar{y_1} + x_{12}\bar{y_2}) + a_2(x_{21}\bar{y_1} + x_{22}\bar{y_2}) && (10)\\
&= a_1 \begin{bmatrix} x_{11} & x_{12} \end{bmatrix} \begin{bmatrix} \bar{y_1} \\ \bar{y_2} \end{bmatrix} + a_2 \begin{bmatrix} x_{21} & x_{22} \end{bmatrix} \begin{bmatrix} \bar{y_1} \\ \bar{y_2} \end{bmatrix} && (11)\\
&= a_1{x_1}^T\bar{y} + a_2{x_2}^T\bar{y} && (12)\\
&= a_1\langle x_1, y\rangle + a_2\langle x_2, y\rangle && (13)
\end{aligned}$$

c. $\langle x, x\rangle\ \geq 0$ and $\langle x, x\rangle = 0 \Leftrightarrow x = 0$ Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ s.t. $x_1 = a_1 + b_1\iota$ and $x_2 = a_2 + b_2\iota$ So,

$$\begin{aligned}
\langle x, x\rangle &= \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} \bar{x_1} \\ \bar{x_2} \end{bmatrix} && (14)\\
&= x_1\bar{x_1} + x_2\bar{x_2} && (15)\\
&= (a_1 + b_1\iota)(a_1 - b_1\iota) + (a_2 + b_2\iota)(a_2 - b_2\iota) && (16)\\
&= ({a_1}^2 + {b_1}^2) + ({a_2}^2 + {b_2}^2) && (17)
\end{aligned}$$

By observing above $Eq^n(17)$ we can see that $\langle x, x\rangle$ will always be ≥ 0 and *iff* $x = 0 \Leftrightarrow \langle x, x\rangle = 0$.

Secondly, consider the inner product $\langle x, y\rangle = \bar{x}^Ty$. To show that it is an inner product over $(\mathbb{C}^n, \mathbb{C})$ we need to check if it follow the following properties -

a. $\langle x, y\rangle = \langle y, x\rangle$ for $\mathbb{F} = \mathbb{R}$.

Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$. So,

$$\begin{aligned}
LHS = \langle x, y\rangle &= \bar{x}^Ty && (18)\\
&= \begin{bmatrix} \bar{x_1} & \bar{x_2} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} && (19)\\
&= \bar{x_1}y_1 + \bar{x_2}y_2 && (20)
\end{aligned}$$

Assuming x_1, x_2, y_1 and $y_2 \in \mathbb{R} \Rightarrow x_1 = \bar{x_1},\ x_2 = \bar{x_2},\ y_1 = \bar{y_1},\ y_2 = \bar{y_2} \Rightarrow$

$$= x_1 y_1 + x_2 y_2 \tag{21}$$
$$= 2\bar{y_1} x_1 + \bar{y_2} x_2 \tag{22}$$
$$= \begin{bmatrix} \bar{y_1} & \bar{y_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{23}$$
$$= \langle y, x \rangle \tag{24}$$

b. $\langle x, y \rangle = \overline{\langle y, x \rangle}$ for $\mathbb{F} = \mathbb{C}$ Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$. So,

$$\langle x, y \rangle = \bar{x}^T y \tag{25}$$
$$= \begin{bmatrix} \bar{x_1} & \bar{x_2} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{26}$$
$$= \bar{x_1} y_1 + \bar{x_2} y_2 \tag{27}$$
$$= \overline{\bar{y_1} x_1 + \bar{y_2} x_2} \tag{28}$$
$$= \overline{\begin{bmatrix} \bar{y_1} & \bar{y_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} \tag{29}$$
$$= \overline{\langle y, x \rangle} \tag{30}$$

c. $\langle a_1 x_1 + a_2 x_2, y \rangle = a_1 \langle x_1, y \rangle + a_2 \langle x_2, y \rangle$

Let, $x_1 = \begin{bmatrix} x_{11} \\ x_{12} \end{bmatrix}$, $x_2 = \begin{bmatrix} x_{21} \\ x_{22} \end{bmatrix}$ and $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$. So,

$$\langle a_1 x_1 + a_2 x_2, y \rangle = \overline{a_1 x_1 + a_2 x_2}^T y \tag{31}$$
$$= \begin{bmatrix} a_1 \overline{x_{11}} + a_2 \overline{x_{21}} & a_1 \overline{x_{12}} + a_2 \overline{x_{22}} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \tag{32}$$
$$= a_1 \overline{x_{11}} y_1 + a_2 \overline{x_{21}} y_1 + a_1 \overline{x_{12}} y_2 + a_2 \overline{x_{22}} y_2 \tag{33}$$
$$= a_1 (\overline{x_{11}} y_1 + \overline{x_{12}} y_2) + a_2 (\overline{x_{21}} y_1 + \overline{x_{22}} y_2) \tag{34}$$
$$= a_1 \bar{x_1}^T y + a_2 \bar{x_2}^T y \tag{35}$$
$$= a_1 \langle x_1, y \rangle + a_2 \langle x_2, y \rangle \tag{36}$$

d. $\langle x, x \rangle \ \geq 0$ and $\langle x, x \rangle = 0 \Leftrightarrow x = 0$ Let $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ s.t. $x_1 = a_1 + b_1 \iota$ and $x_2 = a_2 + b_2 \iota$ So,

$$\langle x, x \rangle = \bar{x}^T x \tag{37}$$
$$= \begin{bmatrix} \bar{x_1} & \bar{x_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{38}$$
$$= \bar{x_1} x_1 + \bar{x_2} x_2 \tag{39}$$
$$= (a_1 + b_1 \iota)(a_1 - b_1 \iota) + (a_2 + b_2 \iota)(a_2 - b_2 \iota) \tag{40}$$
$$= ({a_1}^2 + {b_1}^2) + ({a_2}^2 + {b_2}^2) \tag{41}$$

By observing above $Eq^n(41)$ we can see that $\langle x, x \rangle$ will always be ≥ 0 and *iff* $x = 0 \Leftrightarrow \langle x, x \rangle = 0$.

Question: 2.

Answer: We are given $\mathbb{P}_3([-1, 1])$ and the inner product $\langle p, q \rangle = \int_{-1}^{1} p(x) q(x)\, dx$. It is also given that,

$$p_0 = 1 \tag{1}$$
$$p_1 = x \tag{2}$$
$$p_2 = \frac{3}{2} x^2 - \frac{1}{2} \tag{3}$$
$$p_3 = \frac{5}{2} x^3 - \frac{3}{2} x \tag{4}$$

We can form the set $p = \{p_0, p_1, p_2, p_3\} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} \\ 0 \\ \frac{3}{2} \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ -\frac{3}{2} \\ 0 \\ \frac{5}{2} \end{bmatrix} \right\}$ First, we check for linear independance $\Rightarrow$

$\begin{bmatrix} 1 & 0 & -\frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{3}{2} \\ 0 & 0 & \frac{3}{2} & 0 \\ 0 & 0 & 0 & \frac{3}{2} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{bmatrix} = 0 \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0 \Rightarrow$ the given set is *Linearly Independant.*

Now we shall check for orthogonality, as per the instruction given in the question $\rightarrow$

$$\langle p_0, p_3 \rangle = \int_{-1}^{1} (1) \cdot \frac{5}{2}x^3 - \frac{3}{2}x \, dx \tag{5}$$
$$= \left[\frac{5}{8}x^4 - \frac{3}{4}x^2 \right]_{-1}^{1} \tag{6}$$
$$= \left(\frac{5}{8} - \frac{3}{4} \right) - \left(\frac{5}{8} - \frac{3}{4} \right) \tag{7}$$
$$= 0 \tag{8}$$

$$\langle p_1, p_2 \rangle = \int_{-1}^{1} x \cdot \left(\frac{3}{2}x^2 - \frac{1}{2} \right) \, dx \tag{9}$$
$$= \int_{-1}^{1} \frac{3}{2}x^3 - \frac{1}{2}x \, dx \tag{10}$$
$$= \left[\frac{3}{8}x^4 - \frac{1}{4}x^2 \right]_{-1}^{1} \tag{11}$$
$$= \left(\frac{3}{8} - \frac{1}{4} \right) - \left(\frac{3}{8} - \frac{1}{4} \right) \tag{12}$$
$$= 0 \tag{13}$$

Hence, we can see that the set p is *Linearly Independant* and that its elements are orthogonal and it spans $\mathbb{P}_3$.

$\therefore$ p forms a orthogonal basis of $\mathbb{P}_3$.

Q.E.D.

Question: 3.

Answer: Given the standard inner product $\langle x, y \rangle = x^T y$ and the vectors,

$$y_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \; y_2 = \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix}, \; y_3 = \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix} \tag{1}$$

We shall apply the Gram Schmidt Procedure to the given set of vectors. We know that,

$$v_k = y_k - \sum_{j=1}^{k-1} \frac{\langle y_k, v_j \rangle}{||v_j||^2} \cdot v_j \tag{2}$$

So,

$$v_1 = y_1 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \tag{3}$$

$$\begin{aligned} v_2 &= y_2 - \frac{\langle y_2, v_1 \rangle}{||v_1||^2} \cdot v_1 \\ &= \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix} - \left(\frac{\begin{bmatrix} 4 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}} \cdot \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right) \\ &= \begin{bmatrix} 4 \\ 0 \\ -1 \end{bmatrix} - 0.5 \cdot \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3.5 \\ 1 \\ -1.5 \end{bmatrix} \end{aligned} \tag{4}$$

$$\begin{aligned} v_3 &= y_3 - \frac{\langle y_3, v_1 \rangle}{||v_1||^2} \cdot v_1 - \frac{\langle y_3, v_2 \rangle}{||v_2||^2} \cdot v_2 \\ &= \begin{bmatrix} -2 \\ 2 \\ 3 \end{bmatrix} - \left(\frac{\begin{bmatrix} -2 & 2 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}}{\begin{bmatrix} 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}} \cdot \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \right) - \left(\frac{\begin{bmatrix} -2 & 2 & 3 \end{bmatrix} \begin{bmatrix} 3.5 \\ 1 \\ -1.5 \end{bmatrix}}{\begin{bmatrix} 3.5 & 1 & -1.5 \end{bmatrix} \begin{bmatrix} 3.5 \\ 1 \\ -1.5 \end{bmatrix}} \cdot \begin{bmatrix} 3.5 \\ 1 \\ -1.5 \end{bmatrix} \right) \\ &= \begin{bmatrix} 0.6452 \\ 1.6129 \\ 2.5806 \end{bmatrix} \end{aligned} \tag{5}$$

$$\therefore \; v = \{v_1, v_2, v_3\} = \left\{ \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3.5 \\ 1 \\ -1.5 \end{bmatrix}, \begin{bmatrix} 0.6452 \\ 1.6129 \\ 2.5806 \end{bmatrix} \right\}$$

Question: 4.(a)

Answer: We are to prove that $(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}$.

The simplest way is by multplying $(A + BCD)^{-1}$ with $(A + BCD)$ and it should equal to $I \Rightarrow$

$$(A + BCD)(A + BCD)^{-1} = (A + BCD)\left(A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}\right) \tag{1}$$

$$= \left(I - B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}\right) + \left(BCDA^{-1} - BCDA^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}\right) \tag{2}$$

$$= \left(I + BCDA^{-1}\right) - \left(B(C^{-1} + DA^{-1}B)^{-1}DA^{-1} + BCDA^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}\right) \tag{3}$$

$$= I + BCDA^{-1} - (B + BCDA^{-1}B)(C^{-1} + DA^{-1}B)^{-1}DA^{-1} \tag{4}$$

$$= I + BCDA^{-1} - BC(C^{-1} + DA^{-1}B)(C^{-1} + DA^{-1}B)^{-1}DA^{-1} \tag{5}$$

$$= I + BCDA^{-1} - BCDA^{-1} \tag{6}$$

$$= I \tag{7}$$

Q.E.D.

Question: 4.(b)

Answer: We are given the following $\rightarrow$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix}, \ B = \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \\ 3 \end{bmatrix}, \ C = 0.2, \ D = B^T = \begin{bmatrix} 1 & 0 & 2 & 0 & 3 \end{bmatrix} \tag{1}$$

Now,

$$BC = 0.2 \begin{bmatrix} 1 \\ 0 \\ 2 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0 \\ 0.4 \\ 0 \\ 0.6 \end{bmatrix} \tag{2}$$

$$BCD = \begin{bmatrix} 0.2 \\ 0 \\ 0.4 \\ 0 \\ 0.6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 0.2 & 0 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0.8 & 0 & 1.2 \\ 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0 & 1.2 & 0 & 1.8 \end{bmatrix} \tag{3}$$

$$A + BCD = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} 0.2 & 0 & 0.4 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0 & 0.8 & 0 & 1.2 \\ 0 & 0 & 0 & 0 & 0 \\ 0.6 & 0 & 1.2 & 0 & 1.8 \end{bmatrix} = \begin{bmatrix} 1.2 & 0 & 0.4 & 0 & 0.6000 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0.4 & 0 & 1.3 & 0 & 1.2 \\ 0 & 0 & 0 & 1 & 0 \\ 0.6 & 0 & 1.2 & 0 & 2.3 \end{bmatrix} \tag{4}$$

$$(A + BCD)^{-1} = \begin{bmatrix} 1.2 & 0 & 0.4 & 0 & 0.6000 \\ 0 & 0.5 & 0 & 0 & 0 \\ 0.4 & 0 & 1.3 & 0 & 1.2 \\ 0 & 0 & 0 & 1 & 0 \\ 0.6 & 0 & 1.2 & 0 & 2.3 \end{bmatrix}^{-1} = \begin{bmatrix} 0.9688 & 0 & -0.1250 & 0 & -0.1875 \\ 0 & 2 & 0 & 0 & 0 \\ -0.1250 & 0 & 1.5 & 0 & -0.75 \\ 0 & 0 & 0 & 1 & 0 \\ -0.1875 & 0 & -0.75 & 0 & 0.8750 \end{bmatrix} \tag{5}$$

Question: 5.(a)

Answer:

Question: 5.(b)

Answer:

Question: 6.(a)

Answer:

Question: 6.(b)

Answer:

Question: 8. A norm $||\cdot||$ on a vector space $(\mathcal{X}, \mathbb{R})$ is said to be strict when $||x+y|| = ||x|| + ||y||$ holds if and only if there exists a non-negative constant α such that either $y = \alpha x$ or $x = \alpha y$. One then says that $(\mathcal{X}, \mathbb{R}, ||\cdot||)$ is strictly normed. Suppose that $(\mathcal{X}, \mathbb{R}, ||\cdot||)$ is strictly normed. Let M be a subspace of $\mathcal{X}$ and suppose that $x \in \mathcal{X}$ is such that $d(x, M) > 0$. Show that there exists $m^* \in M$ such that

$$||x - m^*|| = d(x, M) := \inf_{y \in M} ||x - y||$$

then m^* is unique.

Answer:

Question: 9.(a)

Answer: Given $||x||_1 = |x_1| + |x_2|$. Let $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $y = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. First let us find $||x+y||_1 \rightarrow$

$$x + y = \begin{bmatrix} 2+4 \\ 3+5 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix} \tag{1}$$
$$||x+y||_1 = |6| + |8| = 14 \tag{2}$$

Now let us find $||x||_1 + ||y||_1 \rightarrow$

$$||x||_1 = |2| + |3| = 5 \tag{3}$$
$$||y||_1 = |4| + |5| = 9 \tag{4}$$
$$||x||_1 + ||y||_1 = 5 + 9 = 14 \tag{5}$$

From $Eq^n(2) = Eq^n(5)$ and the non-existance of an α s.t. $y = \alpha x$ or $x = \alpha y$, we can say that $||x||_1$ is *not* strictly normed.

Question: 9.(c)

Answer: Given $||x||_\infty = max\{x_1, x_2\}$. Let $x = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ and $y = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$. First let us find $||x+y||_\infty \rightarrow$

$$x + y = \begin{bmatrix} 2+4 \\ 3+5 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix} \tag{1}$$
$$||x+y||_\infty = max\{|6|, |8|\} = 8 \tag{2}$$

Now let us find $||x||_\infty + ||y||_\infty \rightarrow$

$$||x||_\infty = max\{|2|, |3|\} = 3 \tag{3}$$
$$||y||_\infty = max\{|4|, |5|\} = 5 \tag{4}$$
$$||x||_\infty + ||y||_\infty = 3 + 5 = 8 \tag{5}$$

From $Eq^n(2) = Eq^n(5)$ and the non-existance of an α s.t. $y = \alpha x$ or $x = \alpha y$, we can say that $||x||_\infty$ is *not* strictly normed.