DISCLAIMER: The provision of this formula sheet is solely to ensure that you do not spend unnecessary effort to memorize certain theorems or formulas. The inclusion (or exclusion) of theorems or formulas on this sheet DOES NOT imply the inclusion (or exclusion) of these theorems or formulas on the actual midterm exam problems.

Given the conditional statement $p \to q$ **Converse:** $q \to p$ **Contrapositive:** $\sim q \to \sim p$

Definition 2.1 *(Chen, 2nd edition, page 8) : A* ***field*** *consists of a set, denoted by $\mathcal{F}$, of elements called* ***scalars*** *and two operations called addition "+" and multiplication "·"; the two operations are defined over $\mathcal{F}$ such that they satisfy the following conditions:*

1. *To every pair of elements α and β in $\mathcal{F}$, there correspond an element $\alpha + \beta$ in $\mathcal{F}$ called the* sum *of α and β, and an element $\alpha \cdot \beta$ (or simply $\alpha\beta$) in $\mathcal{F}$ called the* product *of α and β.*
2. *Addition and multiplication are respectively commutative: For any α and β in $\mathcal{F}$,*
$$\alpha + \beta = \beta + \alpha \qquad \alpha \cdot \beta = \beta \cdot \alpha$$
3. *Addition and multiplication are respectively associative: For any α, β, γ in $\mathcal{F}$,*
$$(\alpha + \beta) + \gamma = \alpha + (\beta + \gamma) \qquad (\alpha \cdot \beta) \cdot \gamma = \alpha \cdot (\beta \cdot \gamma)$$
4. *Multiplication is distributive with respect to addition: For any α, β, γ in $\mathcal{F}$,*
$$\alpha \cdot (\beta + \gamma) = (\alpha \cdot \beta) + (\alpha \cdot \gamma)$$
5. *$\mathcal{F}$ contains an element, denoted by 0, and an element, denoted by 1, such that $\alpha + 0 = \alpha$ and $1 \cdot \alpha = \alpha$ for every α in $\mathcal{F}$.*
6. *To every α in $\mathcal{F}$, there is an element β in $\mathcal{F}$ such that $\alpha + \beta = 0$. The element β is called the* additive inverse.
7. *To every α in $\mathcal{F}$ which is not the element 0, there is an element γ in $\mathcal{F}$ such that $\alpha \cdot \gamma = 1$. The element γ is called the* multiplicative inverse.

Definition 2.2 *(Chen 2nd Edition, page 9) A* ***vector space (or, linear space)*** *over a field $\mathcal{F}$, denoted by $(\mathcal{X}, \mathcal{F})$, consists of a set, denoted by $\mathcal{X}$, of elements called* ***vectors****, a field $\mathcal{F}$, and two operations called* ***vector addition*** *and* ***scalar multiplication****. The two operations are defined over $\mathcal{X}$ and $\mathcal{F}$ such that they satisfy all the following conditions:*

1. *To every pair of vectors v^1 and v^2 in $\mathcal{X}$, there corresponds a vector $v^1 + v^2$ in $\mathcal{X}$, called the sum of v^1 and v^2* [1].
2. *Addition is commutative: For any v^1, v^2 in $\mathcal{X}$, $v^1 + v^2 = v^2 + v^1$.*
3. *Addition is associative: For any v^1, v^2, and v^3 in $\mathcal{X}$, $\left(v^1 + v^2\right) + v^3 = v^1 + \left(v^2 + v^3\right)$.*
4. *$\mathcal{X}$ contains a vector, denoted by $\mathbf{0}$, such that $\mathbf{0} + v = v$ for every v in $\mathcal{X}$. The vector $\mathbf{0}$ is called the zero vector or the origin.*
5. *To every v in $\mathcal{X}$, there is a vector $\bar{v}$ in $\mathcal{X}$, such that $v + \bar{v} = 0$.*
6. *To every α in $\mathcal{F}$, and every v in $\mathcal{X}$, there corresponds a vector $\alpha \cdot v$ in $\mathcal{X}$ called the* scalar product *of α and v.*
7. *Scalar multiplication is associative: For any α, β in $\mathcal{F}$ and any v in $\mathcal{X}$, $\alpha \cdot (\beta \cdot x) = (\alpha \cdot \beta) \cdot x$*
8. *Scalar multiplication is distributive with respect to vector addition: For any α in $\mathcal{F}$ and any v^1, v^2 in $\mathcal{X}$, $\alpha \cdot \left(v^1 + v^2\right) = \alpha \cdot v^1 + \alpha \cdot v^2$.*
9. *Scalar multiplication is distributive with respect to scalar addition: For any α, β in $\mathcal{F}$ and any v in $\mathcal{X}$, $(\alpha + \beta) \cdot v = \alpha \cdot v + \beta \cdot v$.*
10. *For any v in $\mathcal{X}$, $1 \cdot v = v$, where 1 is the element 1 in $\mathcal{F}$.*

[1] We use superscripts v^1, v^2, v^3 to denote different *vectors*. The superscripts *do not* denote powers.

Let $(\mathcal{X},\mathcal{F})$ be a vector space, and let $\mathcal{Y}$ be a subset of $\mathcal{X}$. Then $\mathcal{Y}$ is a **subspace** if using the rules of vector addition and scalar multiplication defined in $(\mathcal{X},\mathcal{F})$, we have that $(\mathcal{Y},\mathcal{F})$ is a vector space.

The following are equivalent:

(a) $(\mathcal{Y},\mathcal{F})$ *is a subspace of* $(\mathcal{X},\mathcal{F})$.

(b) $\forall v^1, v^2 \in \mathcal{Y}, v^1+v^2 \in \mathcal{Y}$ *(closed under vector addition),*

and $\forall y \in \mathcal{Y}$ *and* $\alpha \in \mathcal{F}, \alpha y \in \mathcal{Y}$ *(closed under scalar multiplication).*

(c) $\forall v^1, v^2 \in \mathcal{Y}, \forall \alpha \in \mathcal{F}, \alpha \cdot v^1 + v^2 \in \mathcal{Y}$.

(d) $\forall v^1, v^2 \in \mathcal{Y}, \forall \alpha_1, \alpha_2 \in \mathcal{F}, \alpha_1 \cdot v^1 + \alpha_2 \cdot v^2 \in \mathcal{Y}$.

A **linear combination** is a finite sum of the form: $\alpha_1 v^1 + \alpha_2 v^2 + \ldots + \alpha_n v^n$ where $n \geq 1,\ \alpha_1 \in \mathcal{F},\ v^i \in \mathcal{X},\ v^i \in \mathcal{X},\ 1 \leq i \leq n$

$$[x]_v := \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} \in \mathcal{F}^n$$

$$\begin{tikzcd} \mathcal{X} \arrow[r, "\mathcal{L}"] \arrow[d, "{[x]_u}"'] & \mathcal{Y} \arrow[d] \\ \mathcal{F}^m \arrow[r, "A"] & \mathcal{F}^n \end{tikzcd}$$

$$[x]_{\overline{u}} = P[x]_u$$

$$\forall x \in \mathcal{X}, \left[\mathcal{L}(x)\right]_v = A\left[x\right]_u$$

$$\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc$$

$$\det\left(\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\right) = a\det\left(\begin{bmatrix} e & f \\ h & i \end{bmatrix}\right) - b\det\left(\begin{bmatrix} d & f \\ g & i \end{bmatrix}\right) + c\det\left(\begin{bmatrix} d & e \\ g & h \end{bmatrix}\right)$$

$$Av = \lambda v$$

$$A = M \Lambda M^{-1}$$