HW CH5 Solution

Craig 4th ed. Prob.: 5.4, 5.8, 5.11, 5.13, 5.16, 5.20

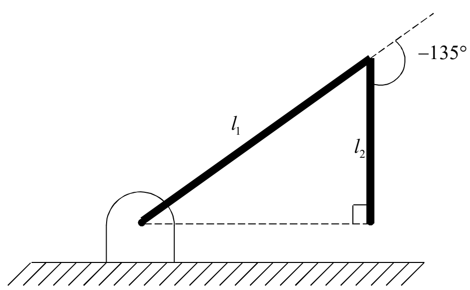
5.4) The mapping which potentially can be singular is: $\mathbf{V} = J(\mathbf{q})\dot{\mathbf{q}}$ for the "position domain," and $\boldsymbol{\tau} = J^T(\mathbf{q})\mathbf{F}$ for the "force domain." Now since transposition has nothing to do with the rank of a (square) matrix, it is clear that the singularities of $J(\mathbf{q})$ are the same as those of $J^T(\mathbf{q})$.

5.8) The Jacobian of this 2-link is: ${}^2J(\mathbf{q}) = \begin{bmatrix} l_1 s_2 & 0 \\ l_1 c_2 + l_2 & l_2 \end{bmatrix}$.

Method 1) An isotropic point exists if ${}^2J(\mathbf{q}) = \begin{bmatrix} l_2 & 0 \\ 0 & l_2 \end{bmatrix}$ => $l_1 s_2 = l_2$ and $l_1 c_2 + l_2 = 0$; or, $s_2 = \frac{l_2}{l_1}$ $c_2 = \frac{-l_2}{l_1}$

Now $s_2^2 + c_2^2 = 1$ → $\left(\frac{l_2}{l_1}\right)^2 + \left(\frac{-l_2}{l_1}\right)^2 = 1$ → $l_1^2 = 2l_2^2$ → $l_1 = \sqrt{2}l_2$ => $s_2 = \frac{1}{\sqrt{2}} = \pm 0.707$ and $c_2 = -0.707$

$\therefore$ An isotropic point exists if $l_1 = \sqrt{2}l_2$, and in that case it exists when $\theta_2 = \pm 135°$. In this configuration, the manipulator looks momentarily like a Cartesian manipulator.



Method 2) $l_1 c_2 + l_2 = 0$ and $l_2^2 = l_1^2 + l_2^2 + 2l_1 l_2 c_2$

$2l_2 c_2 + l_1 = 0$ => $c_2 = \frac{-l_1}{2l_2}$ (where $\left|\frac{l_1}{2l_2}\right| \le 1$) and $l_1 c_2 + l_2 = 0$ => $c_2 = \frac{-l_2}{l_1}$ (where $\left|\frac{l_2}{l_1}\right| \le 1$)

=> $l_1^2 = 2l_2^2$ => $l_1 = \sqrt{2}l_2$ => $c_2 = -\frac{l_2}{\sqrt{2}l_2} = -\frac{1}{\sqrt{2}}$

Method 3) Use the Jacobian ${}^0J(\mathbf{q}) = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} \end{bmatrix}$.

→ $s_{12}(l_1 s_1 + l_2 s_{12}) + c_{12}(l_1 c_1 + l_2 c_{12}) = 0$ → $l_1 c_2 + l_2 = 0$ → $l_2^2 = l_1^2 + l_2^2 + 2l_1 l_2 c_2$

5.11) From Equation (5.100): ${}^B\mathbf{V} = \begin{bmatrix} {}^BR_A & -{}^BR_A\,{}^A\mathbf{P}_{BORG}\times \\ 0 & {}^BR_A \end{bmatrix} {}^A\mathbf{V}$

$$
{}^{B}R_{A}\,{}^{A}\mathbf{P}_{BORG}\times = \begin{bmatrix} 0.866 & 0.5 & 0 \\ -0.5 & 0.866 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -5 & 0 \\ 5 & 0 & -10 \\ 0 & 10 & 0 \end{bmatrix} = \begin{bmatrix} 2.5 & -4.3 & -5.0 \\ 4.3 & 2.5 & -8.6 \\ 0 & 10 & 0 \end{bmatrix}
$$

$$
{}^{B}\mathbf{V} = \begin{bmatrix} 0.866 & 0.5 & 0 & -2.5 & 4.3 & 5.0 \\ -0.5 & 0.866 & 0 & -4.3 & -2.5 & 8.6 \\ 0 & 0 & 1 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0.866 & 0.5 & 0 \\ 0 & 0 & 0 & -0.5 & 0.866 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ -3 \\ 1.41 \\ 1.41 \\ 0 \end{bmatrix} \Rightarrow \therefore\ {}^{B}\mathbf{V} = \begin{bmatrix} 3.52 \\ -7.80 \\ -17.1 \\ 1.91 \\ 0.51 \\ 0 \end{bmatrix}
$$

5.13) $\boldsymbol{\tau} = {}^{0}J^{T}(\mathbf{q})\,{}^{0}\mathbf{F}$ => $\boldsymbol{\tau} = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & l_1 c_1 + l_2 c_{12} \\ -l_2 s_{12} & l_2 c_{12} \end{bmatrix} \begin{bmatrix} 10 \\ 0 \end{bmatrix}$

$\therefore\ \tau_1 = -10 s_1 l_1 - 10 l_2 s_{12}$, $\tau_2 = -10 l_2 s_{12}$

5.16) See the procedure from Craig's Equation (5.38) - (5.42). This problem is identical to Example 5.2 and the answer is given by Equation (5.42):

$$
\boldsymbol{\Omega} = \begin{bmatrix} 0 & -s_1 & c_1 s_2 \\ 0 & c_1 & s_1 s_2 \\ 1 & 0 & c_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} \Rightarrow J(\mathbf{q}) = E_{Z'Y'Z'} = \begin{bmatrix} 0 & -s_1 & c_1 s_2 \\ 0 & c_1 & s_1 s_2 \\ 1 & 0 & c_2 \end{bmatrix}
$$

5.20) At a singularity, an *n*-DOF robot only has *n*-1 DOF remaining. Hence, it can move freely in some *n*-1 dimensional subspace. However, it is still true that it has *n* joints. Therefore, we have a device which has one more joint than the dimensionality of the space it's described in–and that's what a redundant manipulator is.