# MATRIX OPERATOR FOR VECTOR PRODUCT

The vector product of two vectors **a** = $[a_x, a_y, a_z]^T$ and **b** = $[b_x, b_y, b_z]^T$ can be expressed also as the product of a matrix operator $S(\mathbf{a})$ by the vector **b**. The matrix operator is a skew-symmetric matrix as follows:

$$S(\mathbf{a}) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

This matrix operator is sometimes called the Dual matrix of vector **a**, and is also sometimes denoted as $\tilde{\mathbf{a}}$.

If $\alpha$ and $\beta$ are scalars, **0** is the 3x1 zero vector, and $I$ is a 3x3 identity matrix, it can be shown that:

- $\mathbf{a}\mathrm{x}\mathbf{b} = S(\mathbf{a})\mathbf{b} = -S(\mathbf{b})\mathbf{a}$
- $S(\mathbf{a})\mathbf{a} = S(\mathbf{b})\mathbf{b} = \mathbf{0}$
- $S(\alpha\mathbf{a} + \beta\mathbf{b}) = \alpha S(\mathbf{a}) + \beta S(\mathbf{b})$
- $S(\mathbf{a})S(\mathbf{b}) = \mathbf{b}\mathbf{a}^T - \mathbf{a}^T\mathbf{b}I$
- $S(S(\mathbf{a})\mathbf{b}) = \mathbf{b}\mathbf{a}^T - \mathbf{a}\mathbf{b}^T$