Preliminaries:
Review logic symbols here: `http://en.wikipedia.org/wiki/List_of_logic_symbols`
Read about truth tables here:
`http://sites.millersville.edu/bikenaga/math-proof/truth-tables/truth-tables.html`

1. Negate the following statements. For each one, construct a truth table to prove that your negation is correct. Note that "or" in logic is always non-exclusive!
   (a) (P $\wedge$ Q)
   (b) (P $\vee$ Q)
2. Negate the following statements. No truth table is required. You do not have to provide your steps. It is enough to just give an answer.
   (a) For every integer n, $2n + 1$ is odd.
   (b) For some integer n, $2^n + 1$ is prime.
   (c) Let A be an $n \times n$ real matrix and $\lambda \in \mathbb{R}$. Statement: $\exists\ v \in \mathbb{R}^n,\ v \neq 0$, such that $Av = \lambda v$.
   (d) Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Statement: $\forall\ \eta > 0,\ \exists\ \delta > 0$ such that $|x| \leq \delta \implies |f(x)| \leq \eta|x|$
3. Prove that $\sqrt{7}$ is irrational. In your proof, you may use as **true** the following statement: "Let m be an integer. If 7 divides m^2, then 7 also divides m."
4. Let A be a square matrix. Prove: If $\det(A) = 0$, then A is not invertible.

   Remark:
   (a) As part of your solution, look up the definition of an inverse of a matrix and write it down carefully.
   (b) You may use as known: $\det(AB) = \det(A)\det(B)$ for square matrices of the same size.
5. Prove that, for all integers $n \geq 1$, $\sum\limits_{k=1}^{n} \frac{1}{k(k+1)} = \frac{n}{n+1}$.
6. These use strong induction:
   (a) Prove that, for all integers $n \geq 12$, there exist non-negative integers k_1 and k_2 such that $n = k_1 4 + k_2 5$. Is the same statement true for $n \geq 8$?
   (b) Prove that, for all <u>even</u> integers $n \geq 6$, there exist non-negative integers k_1 and k_2 such that $n = k_1 3 + k_2 5$.