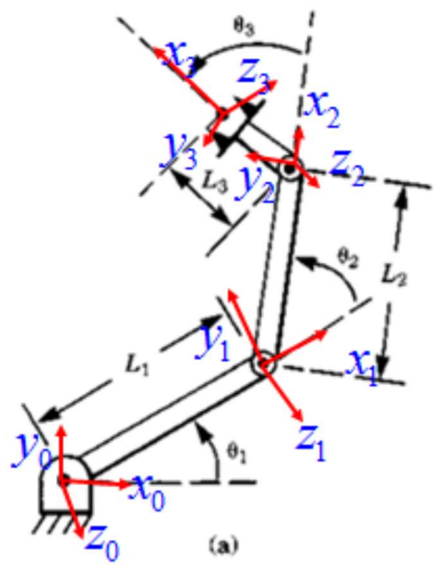
HW CH3 Solution

Craig 4th ed. Prob.: 3.1, 3.4 (regard $\{S\}$ as $\{0\}$, and $\{T\}$ as $\{3\}$), 3.8, 3.12, 3.16, 3.17

- Note: If a reference configuration is not given in the problem, then it can be assigned arbitrarily.

3.1) DH table (according to standard DH convention)

	θ_i	d_i	a_i	α_i	Variable
1	$\theta_1 = 0 + q_1$	0	L_1	0	q_1
2	$\theta_2 = 0 + q_2$	0	L_2	0	q_2
3	$\theta_3 = 0 + q_3$	0	L_3	0	q_3

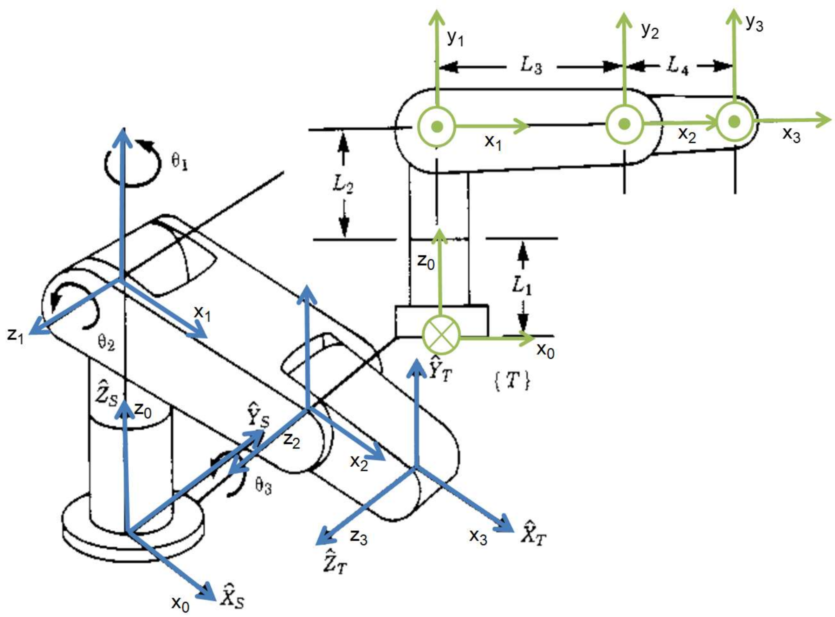


$$
{}^{0}T_1 = \begin{bmatrix} C_1 & -S_1 & 0 & L_1C_1 \\ S_1 & C_1 & 0 & L_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ {}^{1}T_2 = \begin{bmatrix} C_2 & -S_2 & 0 & L_2C_2 \\ S_2 & C_2 & 0 & L_2S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ {}^{2}T_3 = \begin{bmatrix} C_3 & -S_3 & 0 & L_3C_3 \\ S_3 & C_3 & 0 & L_3S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$${}^{0}T_3 = {}^{0}T_1 {}^{1}T_2 {}^{2}T_3$$

3.4) DH table (standard DH convention)

	θ_i	d_i	a_i	α_i	Variable
1	$\theta_1 = 0 + q_1$	L_1+L_2	0	90	q_1
2	$\theta_2 = 0 + q_2$	0	L_3	0	q_2
3	$\theta_3 = 0 + q_3$	0	L_4	0	q_3



$$
{}^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1\cos(90^\circ) & \sin\theta_1\sin(90^\circ) & 0\cdot\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1\cos(90^\circ) & -\cos\theta_1\sin(90^\circ) & 0\cdot\sin\theta_1 \\ 0 & \sin(90^\circ) & \cos(90^\circ) & L_1+L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & L_1+L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2\cos 0 & \sin\theta_2\sin 0 & L_3\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2\cos 0 & -\cos\theta_2\sin 0 & L_3\sin\theta_2 \\ 0 & \sin 0 & \cos 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_3\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_3\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

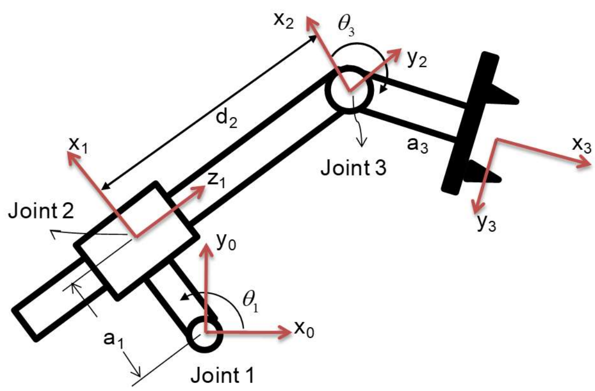
$$
{}^{2}T_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3\cos 0 & \sin\theta_3\sin 0 & L_4\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3\cos 0 & -\cos\theta_3\sin 0 & L_4\sin\theta_3 \\ 0 & \sin 0 & \cos 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & L_4\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & L_4\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

3.8) When $\{G\} = \{T\}$ → ${}^{B}T_W {}^{W}T_T = {}^{B}T_S {}^{S}T_G \;\therefore\; {}^{W}T_T = {}^{B}T_W^{-1}\, {}^{B}T_S\, {}^{S}T_G$

3.12) No. An arbitrary transformation of a rigid-body in a 3-dimensional space requires six parameters (i.e., degrees of freedom).

3.16) DH table (standard DH convention)

Joint	θ_i	d_i	a_i	α_i	Variable
1	$\theta_1 = 0 + q_1$	0	a_1	90	q_1
2	0	$d_2 = 0 + q_2$	0	90	q_2
3	$\theta_3 = 90 + q_3$	0	a_3	0	q_3



*In this problem, the reference configuration was not given. Therefore, the reference configuration can be assigned arbitrarily, including other than those in the above DH table.

3.17)