Mathematics for Robotics
ROB-GY 6103
Homework 5 Answers

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Question: 2.

Answer: We are given the matrix,

$$A = \begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 2 & 0 \\ \sqrt{2} & 0 & 0 \end{bmatrix} \tag{1}$$

It gives us the three eigenvalues,

$$\begin{aligned} \lambda_1 &= 2 \\ \lambda_2 &= 2 \\ \lambda_3 &= -1 \end{aligned} \tag{2}$$

$$\Rightarrow \Lambda = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{3}$$

Using MATLAB, we can find the three eigenvectors to be,

$$v^1 = \begin{bmatrix} 0.57 \\ 0 \\ -0.81 \end{bmatrix},\ v^2 = \begin{bmatrix} 0.81 \\ 0 \\ 0.57 \end{bmatrix},\ v^3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{4}$$

$$V = \begin{bmatrix} 0.57 & 0.81 & 0 \\ 0 & 0 & 1 \\ -0.81 & 0.57 & 0 \end{bmatrix} \tag{5}$$

We can see that,

$$V^{-1} = \begin{bmatrix} 0.57 & 0 & -0.81 \\ 0.81 & 0 & 0.57 \\ 0 & 1 & 0 \end{bmatrix} = V^T \Rightarrow V \ \textit{is Orthogonal} \tag{6}$$

By multiplying the matrices $V\Lambda V^T \rightarrow$

$$V\Lambda V^T = \begin{bmatrix} 0.57 & 0.81 & 0 \\ 0 & 0 & 1 \\ -0.81 & 0.57 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0.57 & 0 & -0.81 \\ 0.81 & 0 & 0.57 \\ 0 & 1 & 0 \end{bmatrix} \tag{7}$$

$$= \begin{bmatrix} 1 & 0 & \sqrt{2} \\ 0 & 2 & 0 \\ \sqrt{2} & 0 & 0 \end{bmatrix} \tag{8}$$

$$= A \tag{9}$$

$\therefore$ we can see that the statement *even with repeated e-values, we can still diagonalize a symmetric matrix using orthogonal matrices* is true using above numerical example.

Question: 4.a.

Answer: n = 99

Question: 4.b.

Answer: Given below is the plot of the Norm error in x-hat using Batch Process -

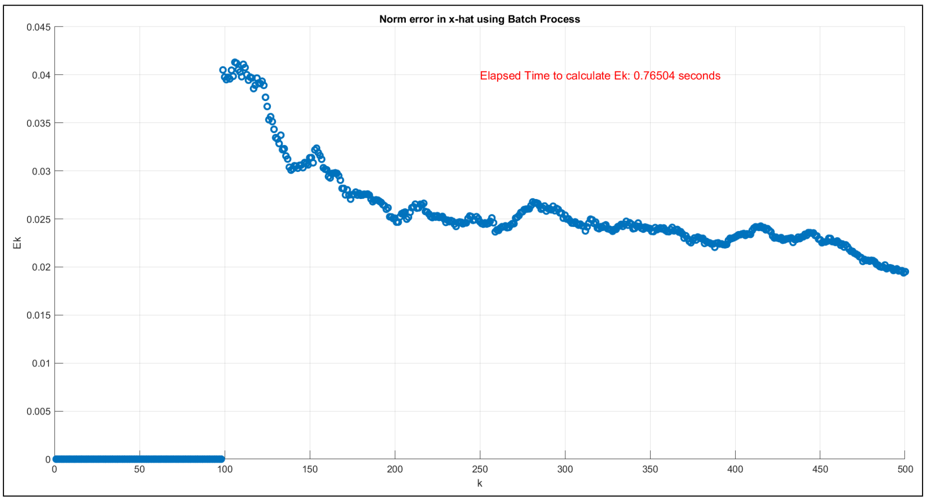

Figure 1: Norm error in x-hat using Batch Process

Question: 4.c.

Answer: Given below is the plot of the Norm error in x-hat using Reecursive Least Squares without Matrix Inversion Lemma

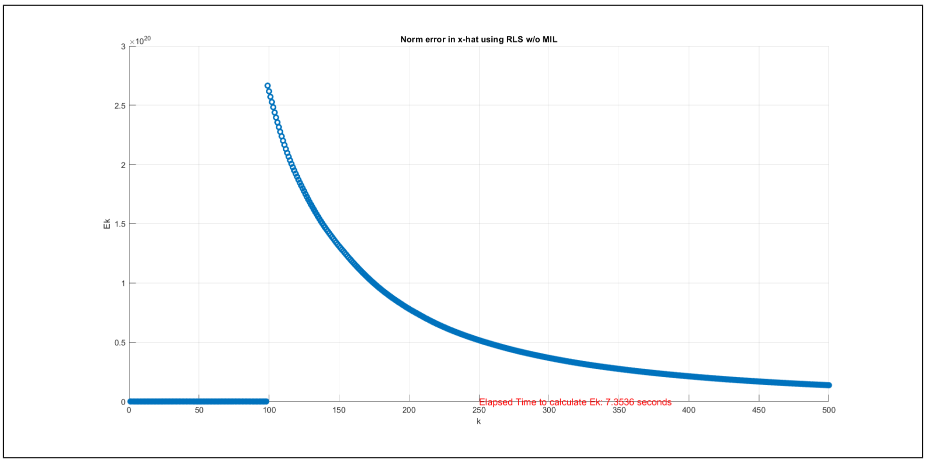

Figure 2: Norm error in x-hat using RLS w/o MIL

Question: 4.d.

Answer: Given below is the plot of the Norm error in x-hat using Recursive Least Squares with using Matrix Inversion Lemma

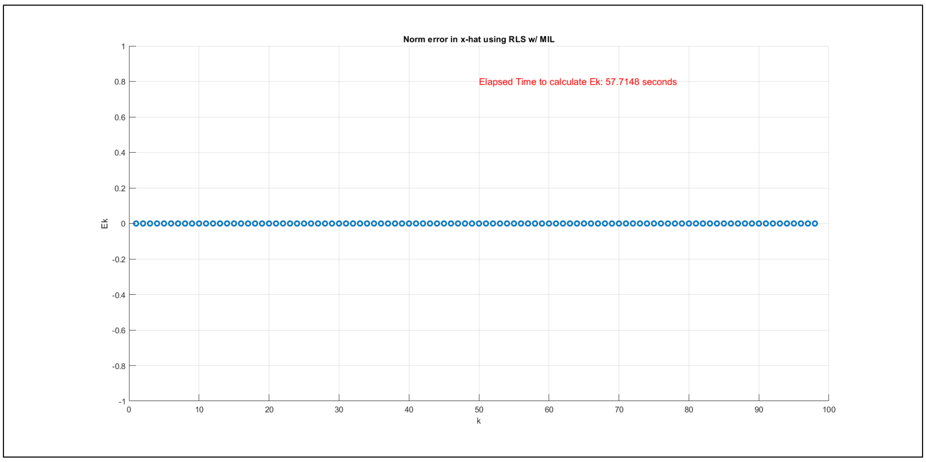

Figure 3: Norm error in x-hat using Recursive Least Squares with using Matrix Inversion Lemma

Question: 5.a.

Answer: n = 18

Question: 5.b.

Answer: Given below is the plot of the Norm error in x-hat using Batch Process -

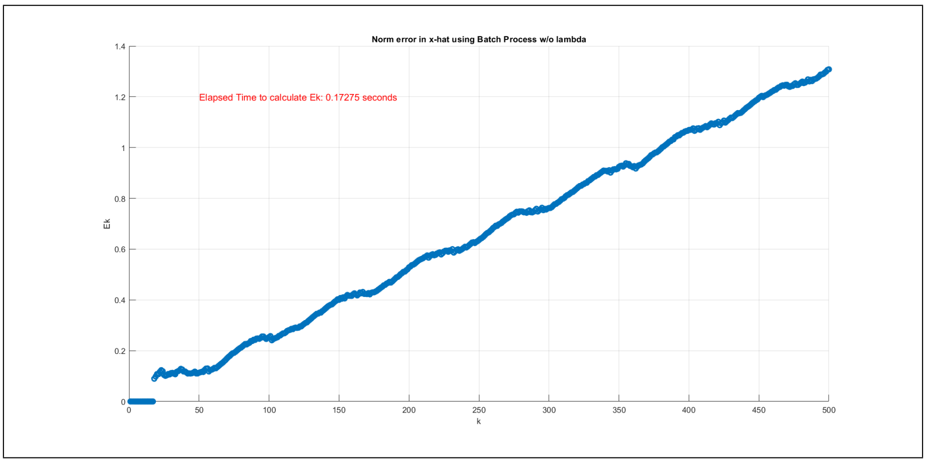

Figure 4: Norm error in x-hat using Batch Process

Question: 5.c.

Answer: Given below is the plot of the Norm error in x-hat using Batch Process but using Forgetting Factor

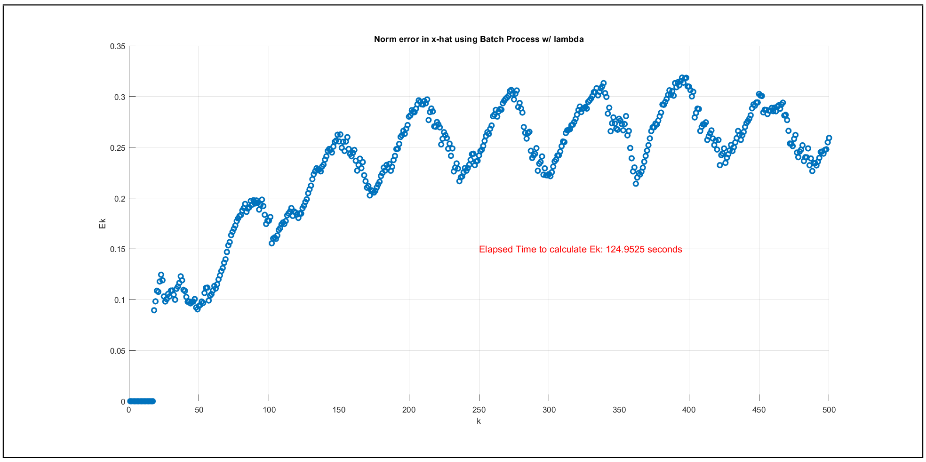

Figure 5: Norm error in x-hat using Batch Process and Forgetting Factor

Question: 5.d.

Answer: Given below is the plot of the Norm error in x-hat using Reecursive Least Squares with using Matrix Inversion Lemma but using Forgetting Factor

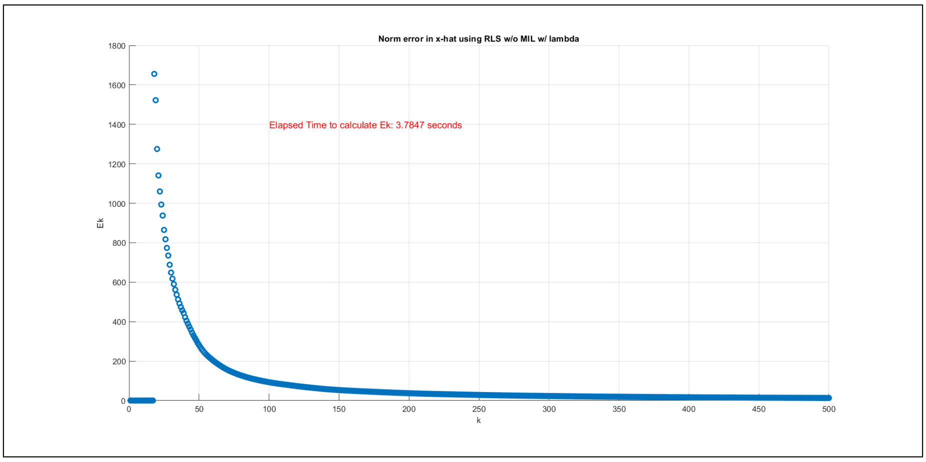

Figure 6: Norm error in x-hat using Reecursive Least Squares with using Matrix Inversion Lemma but using Forgetting Factor