NYU TANDON SCHOOL OF ENGINEERING

AI4CE

Robot Perception

Robust Estimation

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ROB-GY 6203, Fall 2024

Overview

+ Feature detection
    - Harris/FAST/DoG
+ Feature description & matching
    - SIFT/SURF
++ Linear & total least square
* RANSAC
    - Intuitions behind RANSAC
    - How RANSAC works
    - Why minimal solution is important
    - More example problems
    - Variations

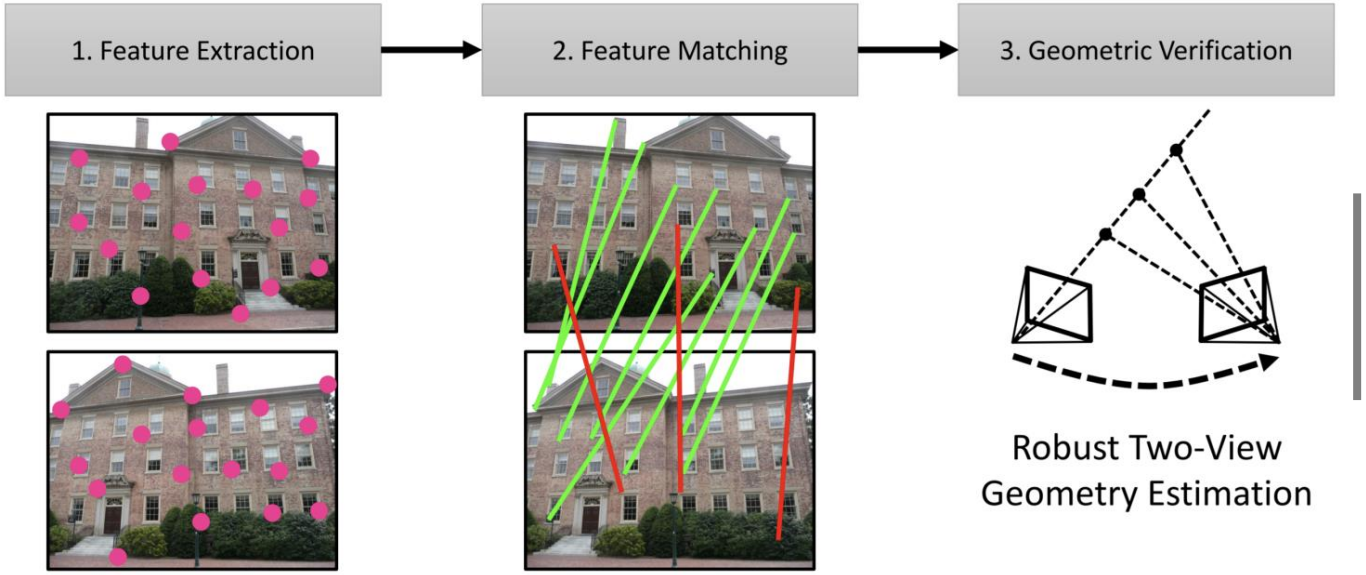
*: know how to code
++: know how to derive
+: know the concept

References

- HZ2003:
  - Section 4.7, 4.8, 11.6
- Corke 2011:
  - Section 14.2.3
- Sz2022:
  - Section 7.1, 7.2, 8.1.4
- DeTone, D., Malisiewicz, T. and Rabinovich, A., 2018. Superpoint: Self-supervised interest point detection and description. In *CVPR workshops* (pp. 224-236).
- Sarlin, P.E., DeTone, D., Malisiewicz, T. and Rabinovich, A., 2020. Superglue: Learning feature matching with graph neural networks. In *CVPR* (pp. 4938-4947).

Data Association: Finding Correspondences Automatically

Pipeline

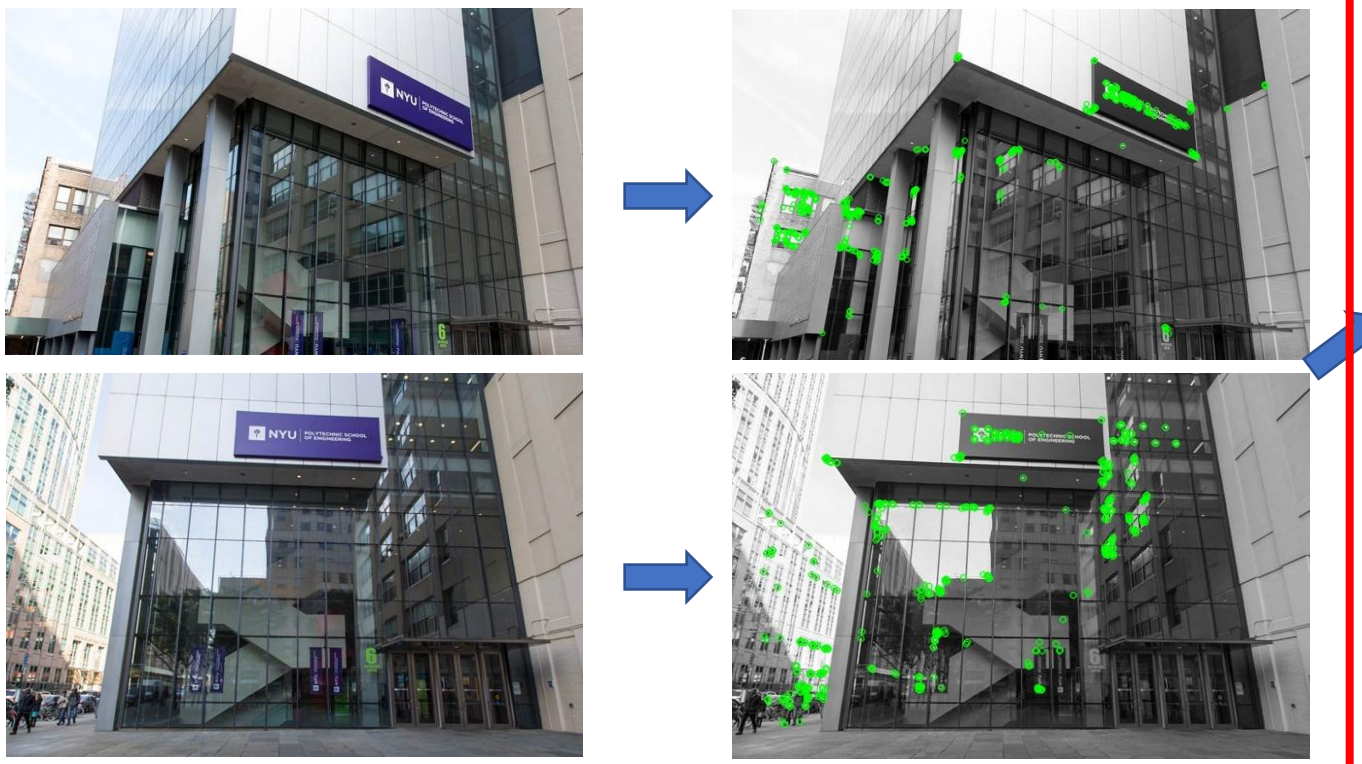


Model Selection

General	Planar	Panoramic
• Fundamental matrix F (*uncalibrated*) • Essential matrix E (*calibrated*)	• Homography H	• Homography H
• 7 correspondences • 5 correspondences	• 4 correspondences	• 4 correspondences

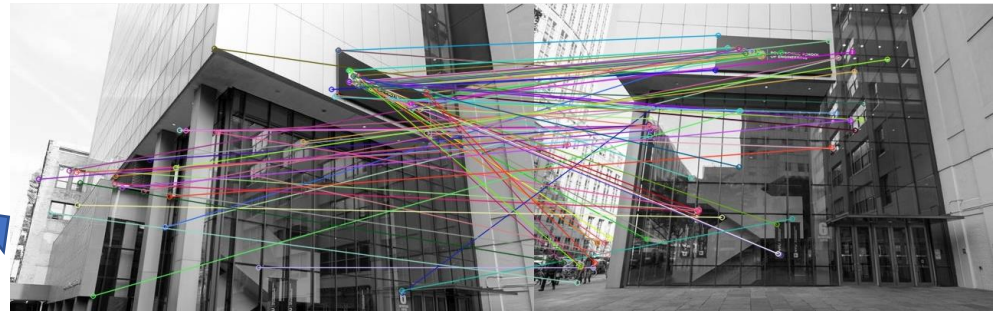
Images from: https://demuc.de/tutorials/cvpr2017/sparse-modeling.pdf

Data Association: Finding Correspondences Automatically



Feature Detection
(Harris/FAST/SIFT/SURF/ORB/LSD)

Feature Description/Matching
(SIFT/SURF/ORB)



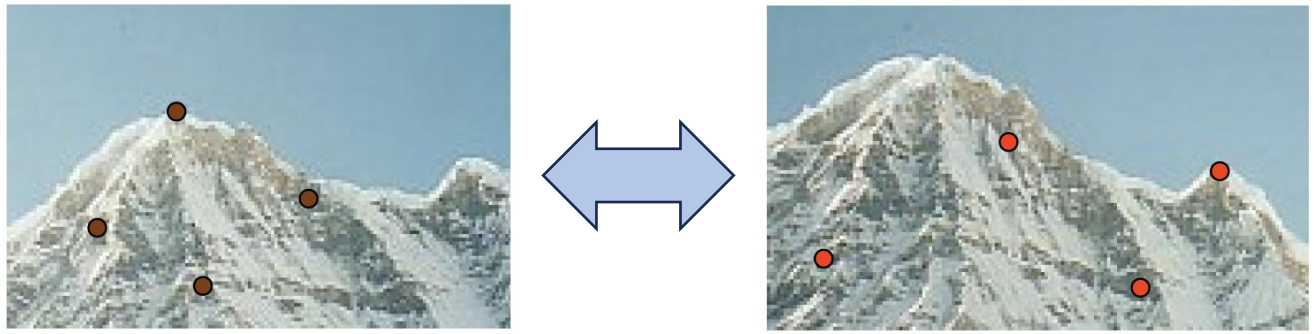
- Homography estimation
- F-matrix estimation
- PnP problem
- …

- RANSAC to reject matching outliers

Image from: https://www.brooklynpaper.com/stories/38/43/dtg-nyu-poly-renaming-2015-10-23-bk.html

Feature Detector Criteria

- **Problem**: How to **detect** the **same** points **independently** in both images?



No chance to match!

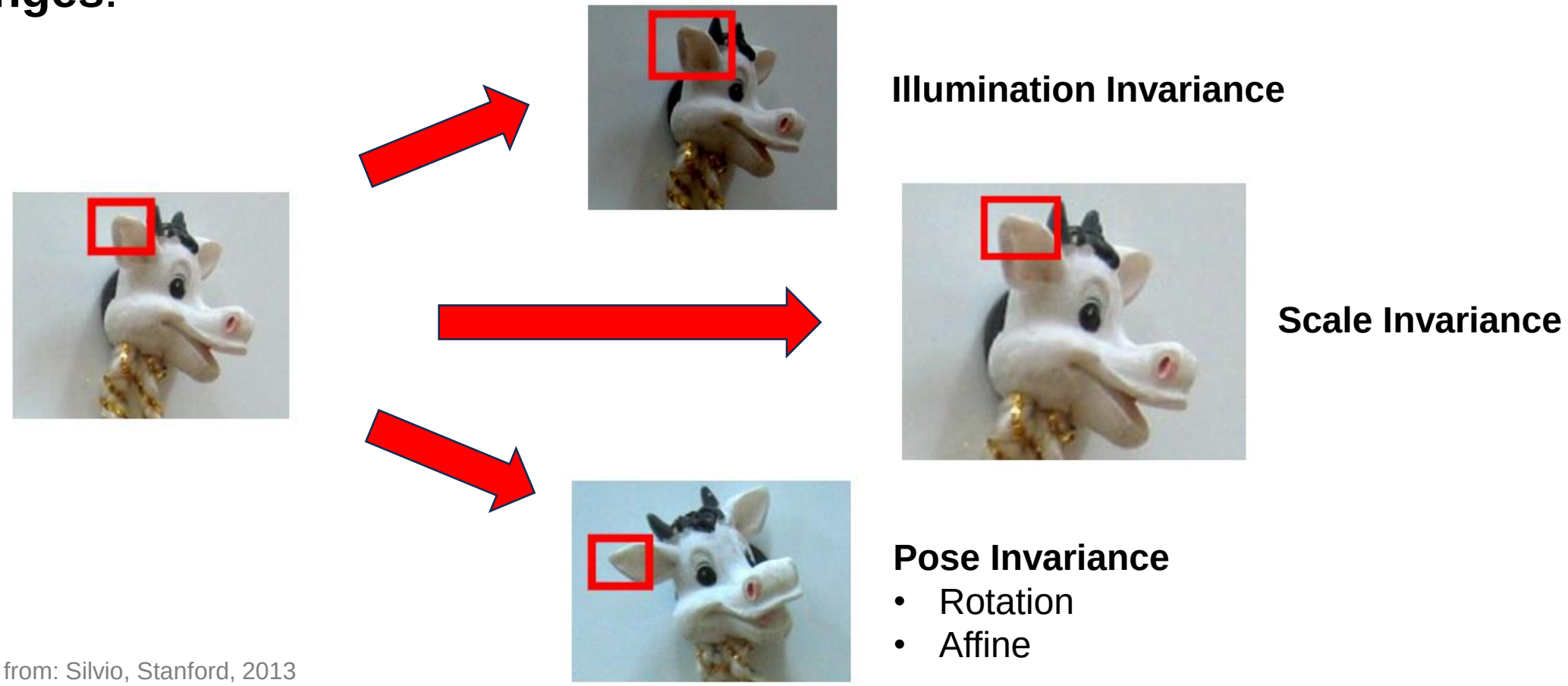
- We need **repeatable feature detector**. Repeatable means that the detector should be able to re-detect the same feature in different images of the same scene.

- This property if called **Repeatability** of a feature detector.

Slide from: Scaeamuzza, UZH, 2019

Feature Detector Criteria Illustrations

- The key to feature detection and matching is to find **repeatable** features and **distinctive** descriptors that are **invariant** to **geometric** and **photometric changes**.



Illumination Invariance

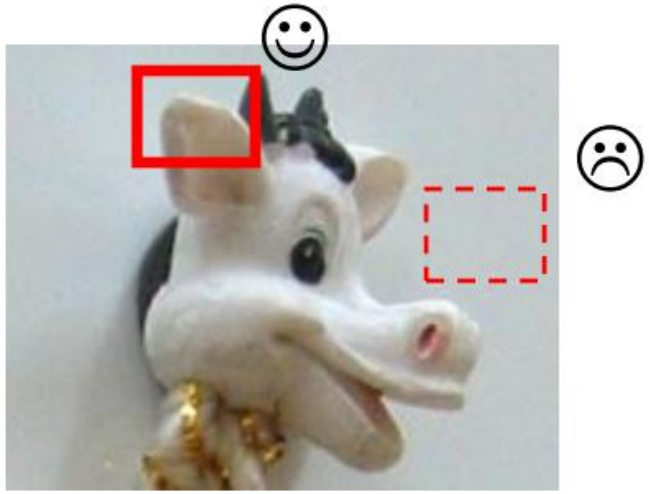
Scale Invariance

Pose Invariance
- Rotation
- Affine

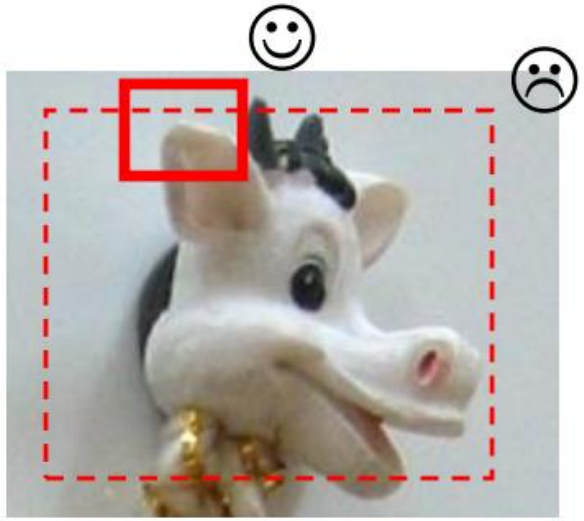
Slide modified from: Silvio, Stanford, 2013

Additional Feature Detector Criteria

- Saliency



- Locality



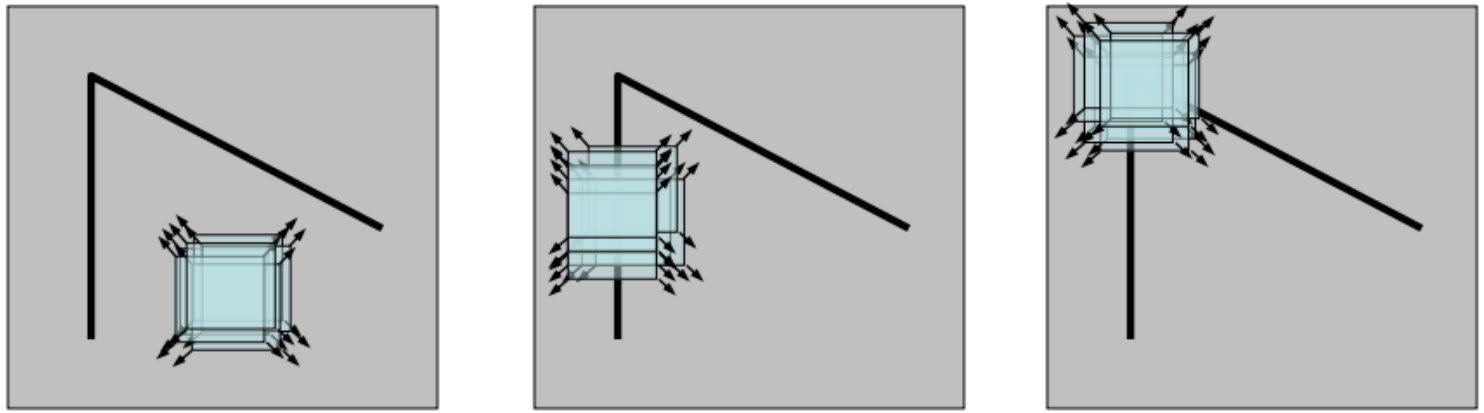
Slide from: Silvio, Stanford, 2013

Harris Corner

- C.Harris and M.Stephens. "A Combined Corner and Edge Detector.“ Proceedings of the 4th Alvey Vision Conference: pages 147--151.

Harris Detector: Basic Idea

Explore intensity changes within a window as the window changes location



"flat" region: no change in all directions

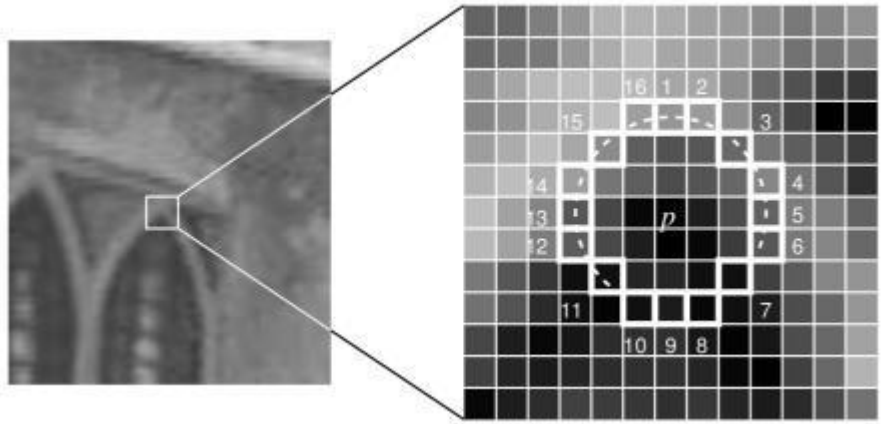
"edge": no change along the edge direction

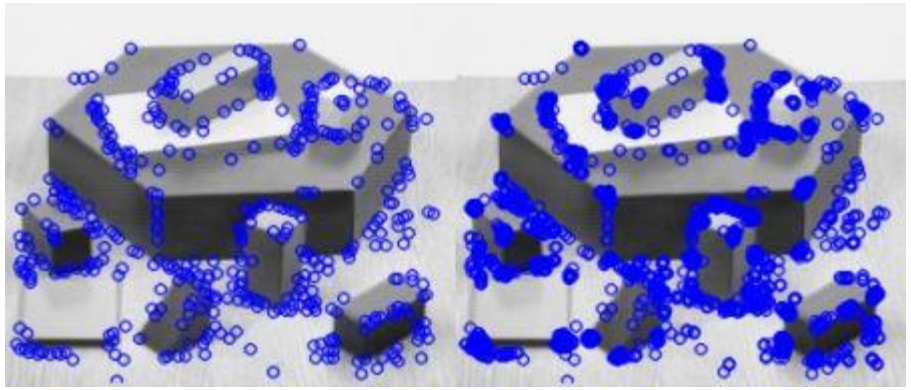
"corner": significant change in all directions

Slide from: Silvio, Stanford, 2013

FAST: Machine Learning for Corner Detection

- FAST (Features from Accelerated Segment Test)
  - Corner: if there exists n=12 contiguous pixels in the circle which are all brighter or all darker than the center for a threshold t
- High-speed test
  - quickly exclude many non-corners
- Decision-tree based improvement
- Non-maximum suppression



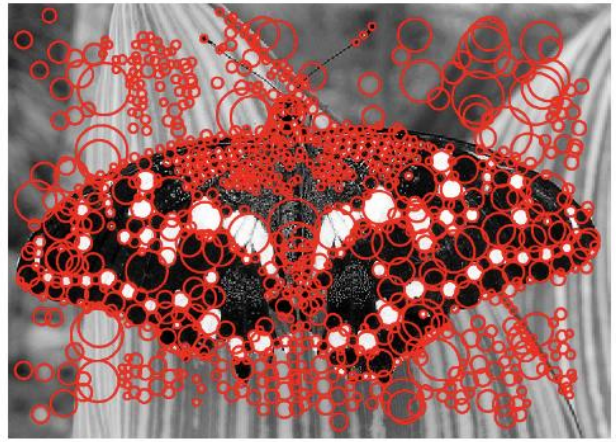


Edward Rosten, Reid Porter, and Tom Drummond, “Faster and better: a machine learning approach to corner detection” in IEEE Trans. Pattern Analysis and Machine Intelligence, 2010, vol 32, pp. 105-119.

Corner vs. Blob Detection

- A corner is defined as the intersection of two or more edges
  - Corners have high localization accuracy – corners are good for VO
  - Corners could be less distinctive than blobs
  - E.g., Harris, Shi-Tomasi, FAST

- A blob is a locally distinctive image pattern that covers a non-negligible area
  - e.g., a connected region of pixels with similar color, a circle, etc.
  - Blobs have less localization accuracy than corners
  - Blobs are more distinctive than corners – blobs are better for place recognition
  - E.g., MSER, LOG, DOG(SIFT), SURF, etc.

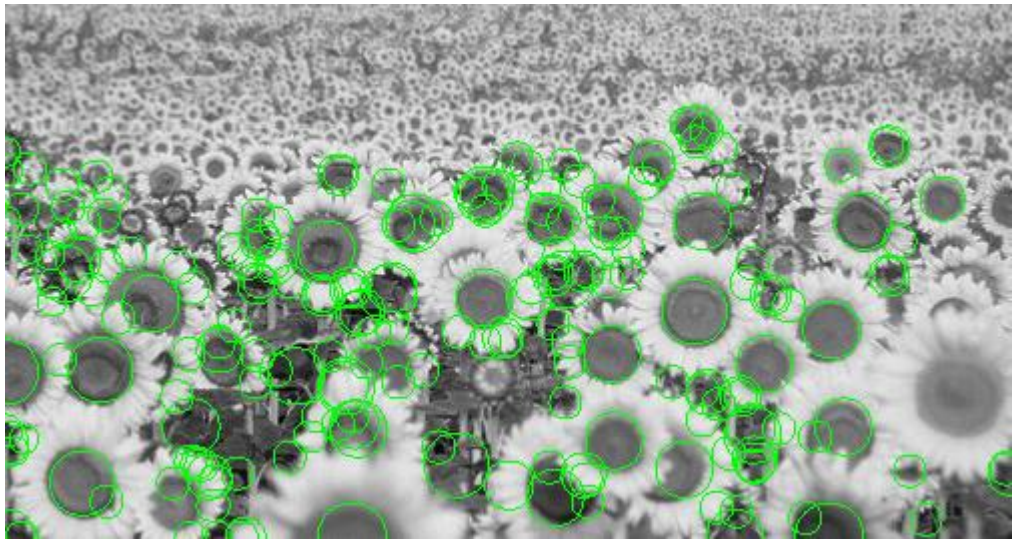


- Blobs become corners when observed from long distances

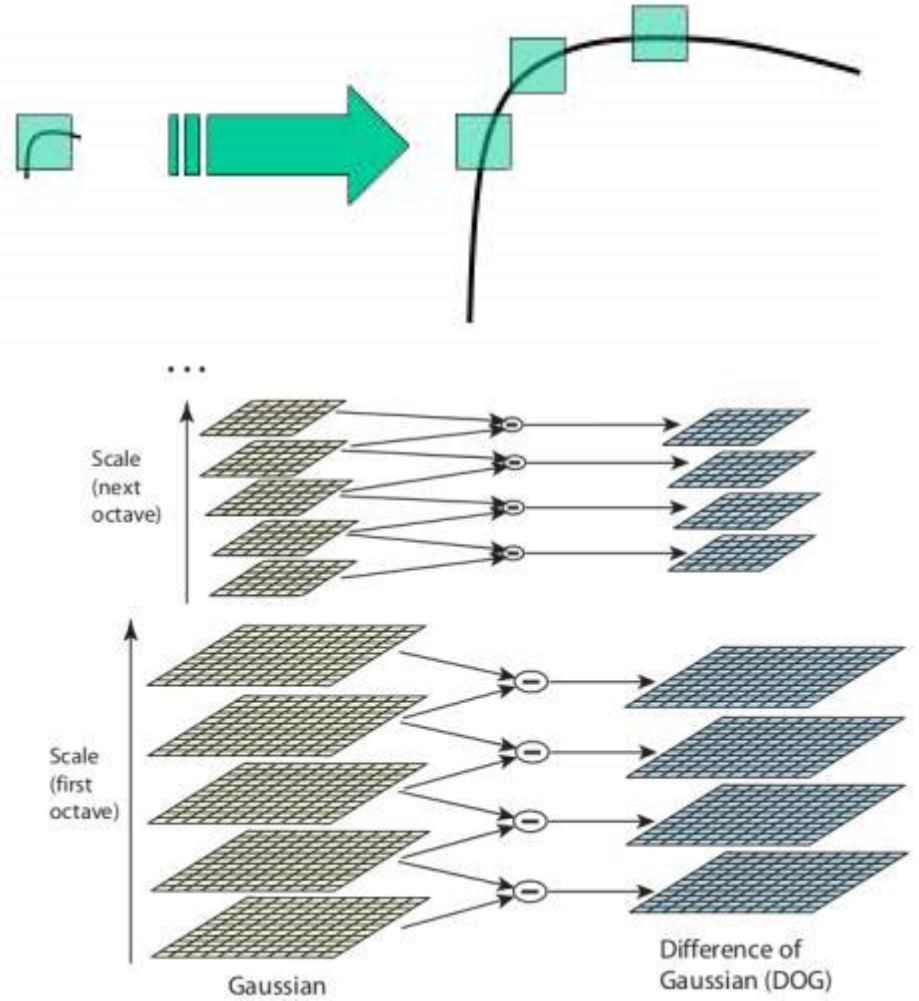
Slide modified from: Scaeamuzza, UZH, 2019

Blob Detector: Difference of Gaussians (DoG)

- Harris corner is rotation invariant
  - But not scale-invariant



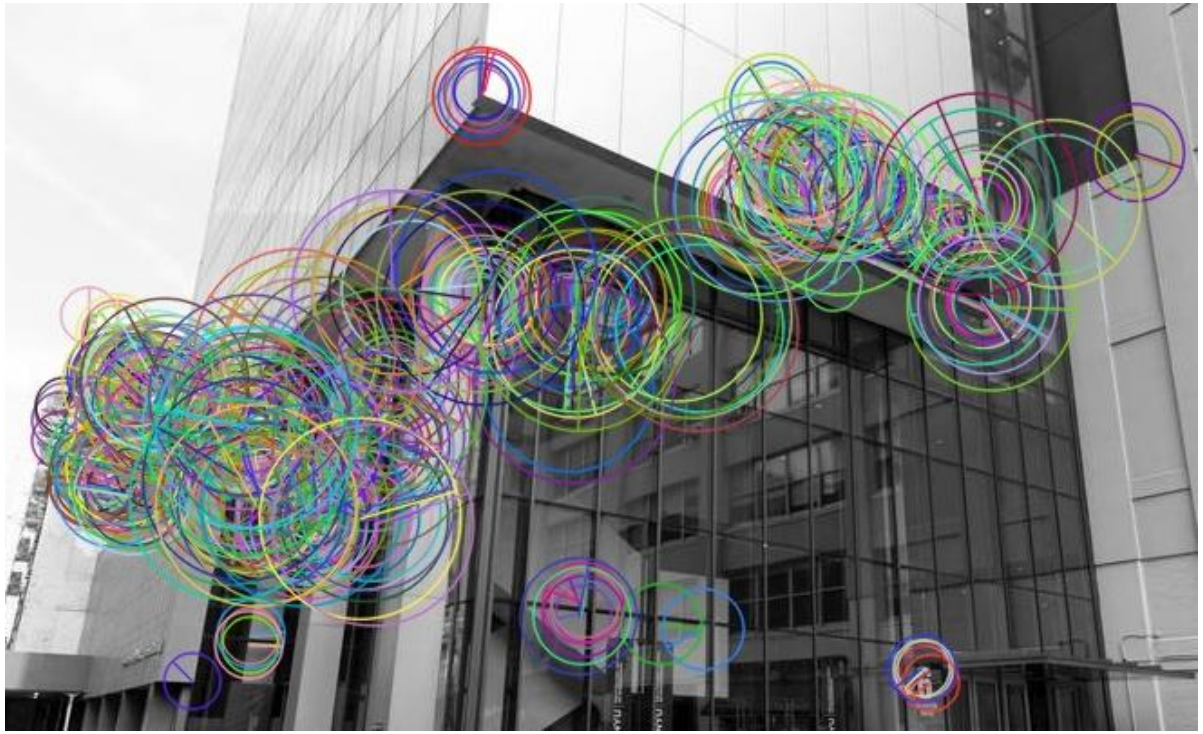
- DoG: find extrema in both 2D-space and scale-space



Lowe, D. G. (2004). Distinctive image features from scale-invariant keypoints. *International journal of computer vision*, *60*(2), 91-110.

DoG Detection Illustration

- Blobs become corners when observed from long distances
- Corners become blobs when observed close enough



A Classic Vision Pipeline: Description & Matching

Feature Detection
(Harris/FAST/SIFT/SURF/ORB
/LSD)

Feature Description/Matching
(SIFT/SURF/ORB)

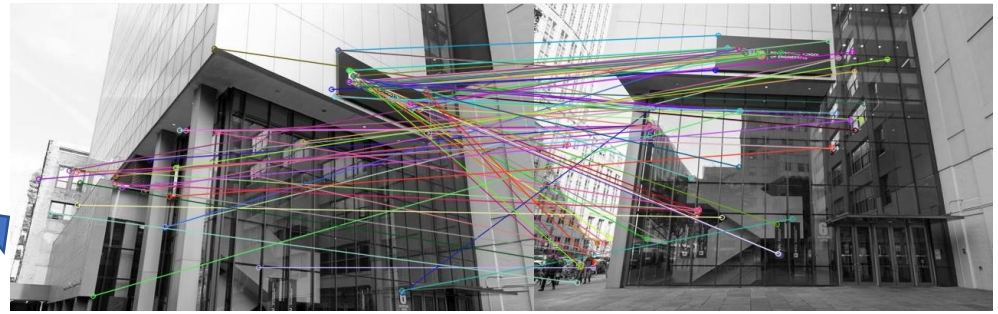


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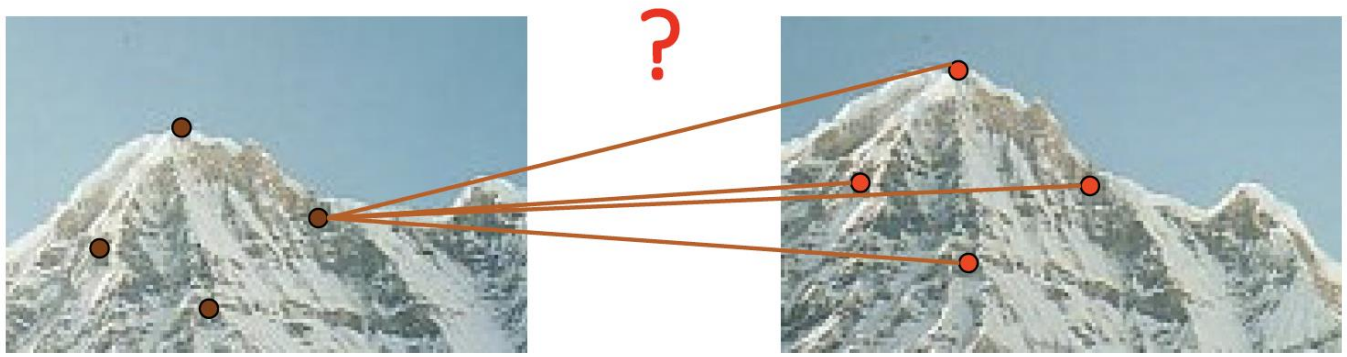
Image from: https://www.brooklynpaper.com/stories/38/43/dtg-nyu-poly-renaming-2015-10-23-bk.html

Feature Descriptor Criteria

- **Problem**: For each point, how to match its corresponding point in the other image

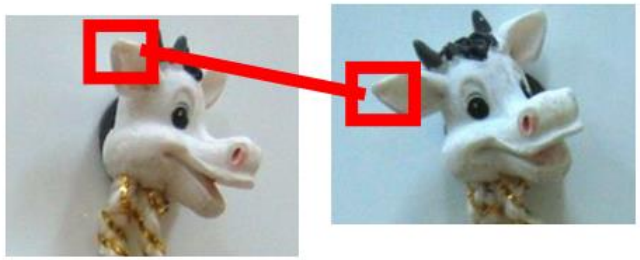


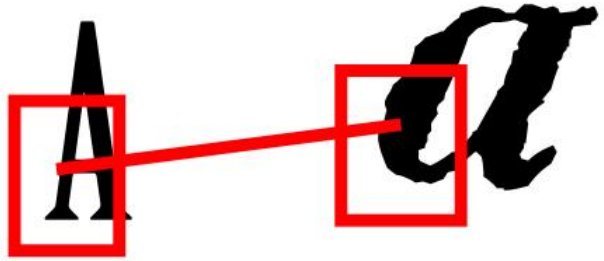
- We need a **distinctive feature descriptor**.
- A descriptor is a “description” of the pixel information around a feature (e.g., patch intensity values, gradient values, etc.).
  - Distinctive means that the descriptor uniquely identifies a feature from other features without ambiguity.
- This property is called **Distinctiveness** of a feature descriptor.
- The descriptor must also be robust to geometric and photometric changes.

Slide from: Scaeamuzza, UZH, 2019

Feature Descriptor Criteria

- Similar to feature detection, a feature descriptor algorithm must produce a description of a feature that is:
  - Invariant w.r.t:
    - Illumination
    - Pose
    - Scale
    - Intraclass variability



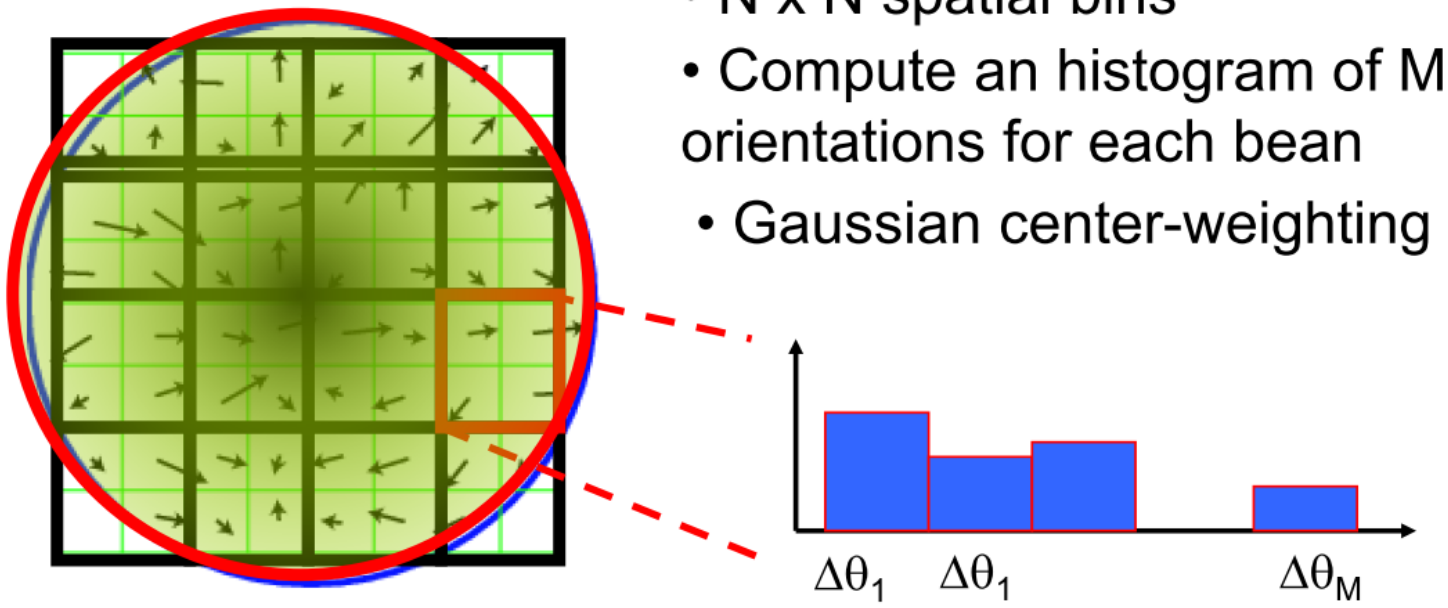


- Highly distinctive (allows a single feature to find its correct match with good probability in a large database of features)

Slide from: Silvio, Stanford, 2013

SIFT Descriptor

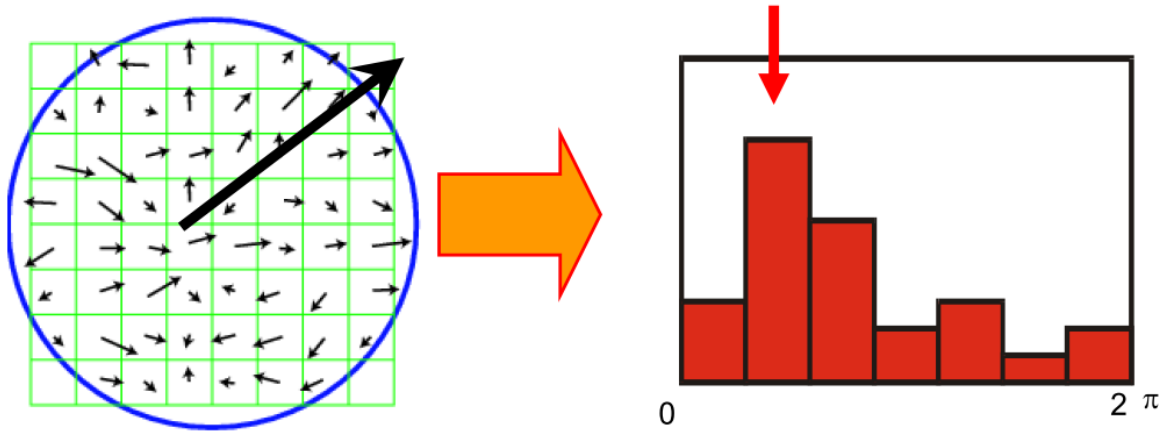
- A standard (but non-free) descriptor
- Location and scale given by DoG detector (SIFT keypoints)



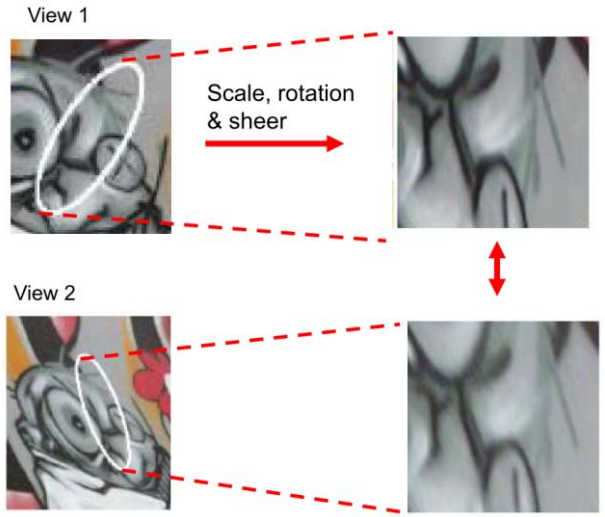
Slide modified from: Silvio, Stanford, 2013

SIFT Descriptor is Robust to Small Variations

- Illumination
  - gradient & normalization
- Pose (small affine variation)
  - orientation histogram
- Scale
  - fixed by DOG
- Intra-class variability
  - histograms (small variations)



This makes the SIFT descriptor rotational invariant



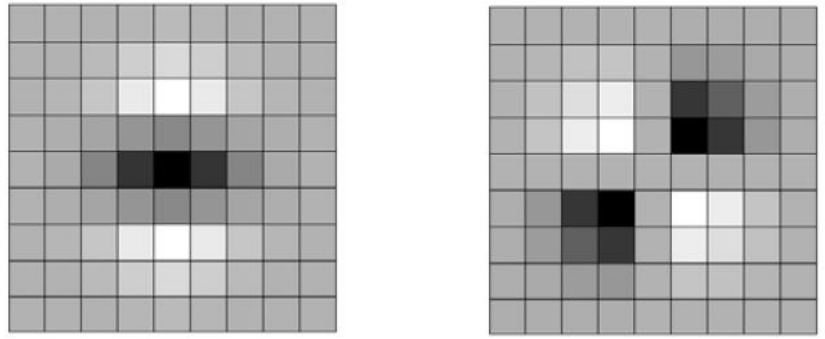
Slide modified from: Silvio, Stanford, 2013

From SIFT to SURF

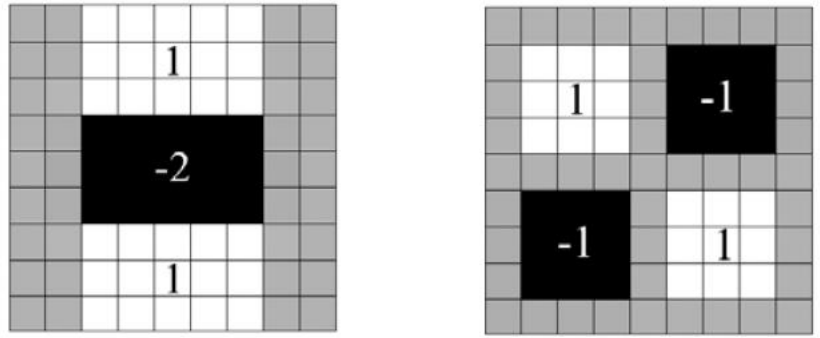
- SIFT is good in terms of matching quality
  - But it is too slow for real-time applications

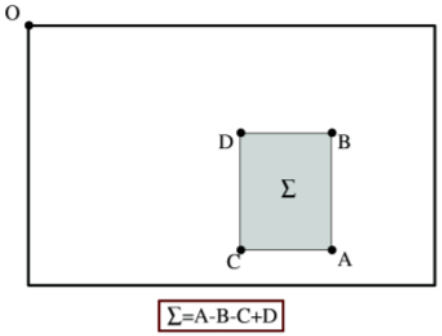
- **S**peeded **U**p **R**obust **F**eatures
  - Based on ideas similar to SIFT
  - Approximated computation for detection and descriptor
  - Results comparable with SIFT, plus:
    - Faster Computation by using **integral image**
    - Generally shorter descriptors

Gaussian second partial derivatives



Approximation using **box filter**

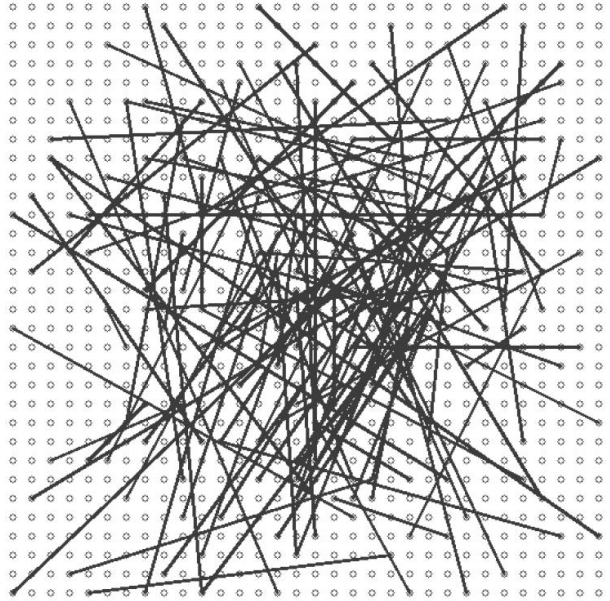


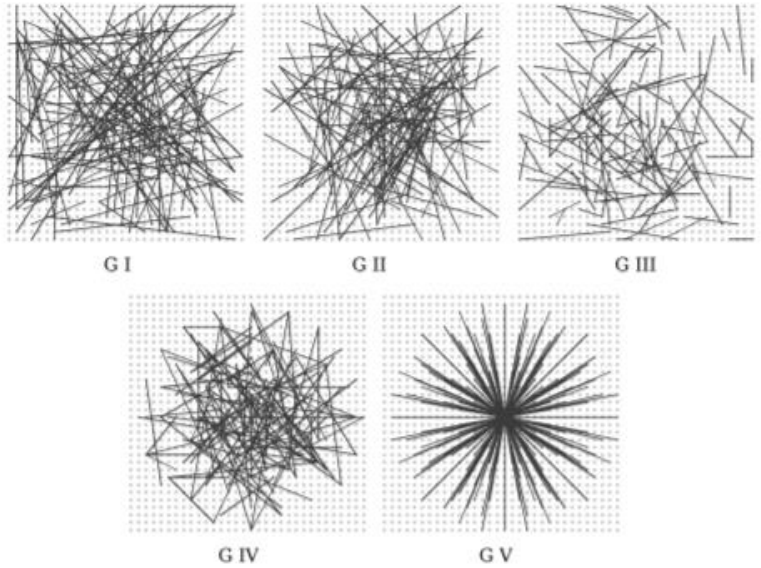


Bay, H., Tuytelaars, T. and Van Gool, L., 2006, May. Surf: Speeded up robust features. In European conference on computer vision (pp. 404-417). Springer, Berlin, Heidelberg.

Even Faster? BRIEF/ORB

- **B**inary **R**obust **I**ndependent **E**lementary **F**eatures
- Goal: high speed (in description and matching)
- **Binary** descriptor formation:
  - smooth image
  - **for each** detected keypoint (e.g. FAST)
  - **sample** 256 intensity pairs (p_1^i, p_2^i) $(i = 1, \dots, 256)$ within a squared patch around the keypoint
  - create an empty 256-element descriptor
  - **for each i^{th} pair**
    - **If $I_{p_1^i} < I_{p_2^i}$ then set i^{th}** bit of descriptor to 1
    - **else to 0**
- The pattern is generated randomly (or by machine learning) only once; then, the same pattern is used for all patches
- Pros: Binary descriptor: allow very fast Hamming distance matching (count the number of bins that are different in the descriptor matched)
- Cons: Not scale/rotation invariant





Calonder, Michael, et al. "Brief: Binary robust independent elementary features." European conference on computer vision. Springer, Berlin, Heidelberg, 2010.

Even Faster? BRIEF**/ORB**

- **O**riented FAST and **R**otated **B**RIEF
- Keypoint detector based on FAST
- **BRIEF** descriptors are rotated according to keypoint orientation (to provide rotation invariance)
- Good binary features are learned by minimize the correlation on a set of training patches.

Rublee,Rabaud Konolige , Bradski , 2011). " ORB: an efficient alternative to SIFT or SURF " (PDF). IEEE International Conference on Computer Vision (ICCV).

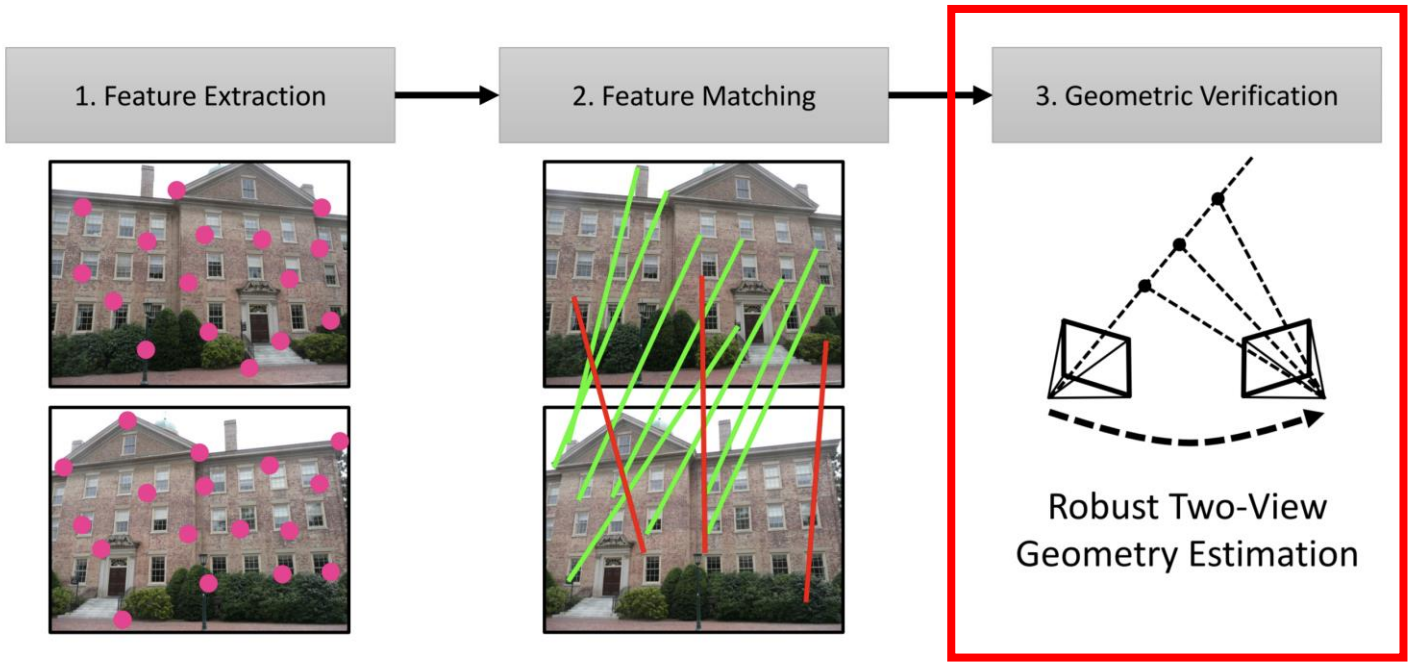
Slide from: Scaeamuzza, UZH, 2019

Recap

Detector	Descriptor that can be used	Localization Accuracy of the detector	Relocalization & Loop closing	Efficiency
Harris	Patch SIFT BRIEF ORB BRISK	++++	+ +++++ +++ ++++ +++	+++ + ++++ ++++ +++
Shi-Tomasi	Patch SIFT BRIEF ORB BRISK	++++	+ +++++ +++ ++++ +++	++ + ++++ ++++ +++
FAST	Patch SIFT BRIEF ORB BRISK	++++	+++ +++++ +++ ++++ +++	++++ + ++++ ++++ +++
SIFT	SIFT	+++	++++	+
SURF	SURF	+++	++++	++

Slide from: Scaeamuzza, UZH, 2019

Data Association: Wrong Matches are Inevitable



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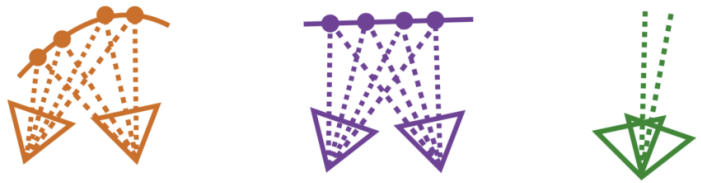
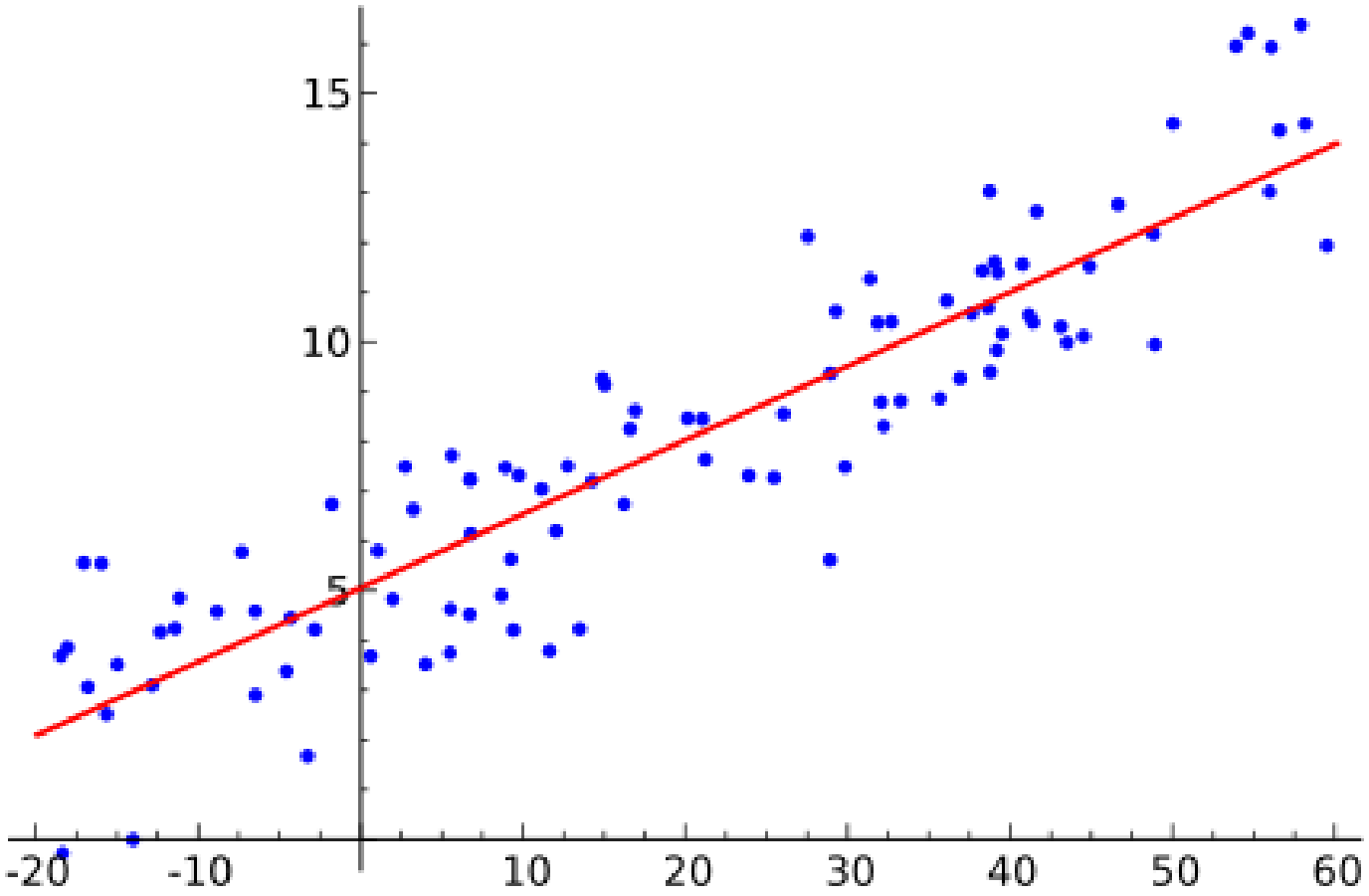
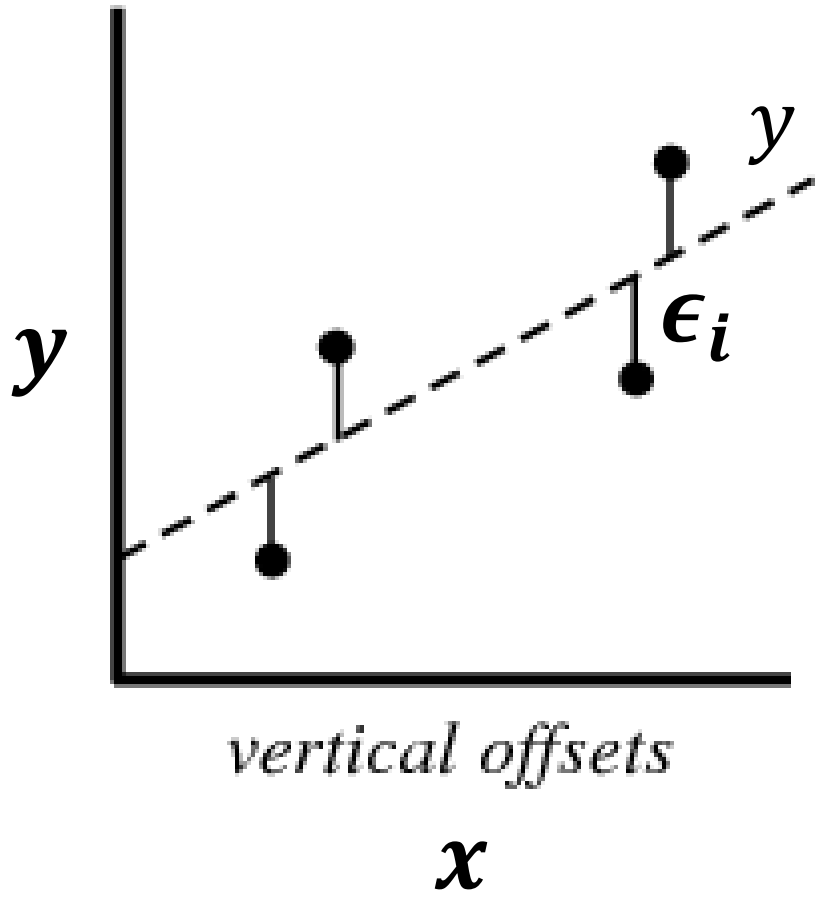


Images from: https://demuc.de/tutorials/cvpr2017/sparse-modeling.pdf

Fitting a 2D Line



Linear Regression (Linear Least Squares)



$$\boldsymbol{\epsilon} = \boldsymbol{y} - a\boldsymbol{x} - b$$

$$[a, b]^* = \underset{a,b}{\operatorname{argmin}} \|\boldsymbol{\epsilon}\|^2$$

Image from: http://mathworld.wolfram.com/LeastSquaresFitting.html

Orthogonal Regression (Total Least Squares)

$$\epsilon_i = \boldsymbol{n}^T(\boldsymbol{p_i} - \boldsymbol{q})$$

$$[\boldsymbol{n}, \boldsymbol{q}]^* = \underset{\boldsymbol{n},\boldsymbol{q}}{\operatorname{argmin}} \sum_{\boldsymbol{i}} \|\epsilon_i\|^2$$
$$s.t. \|\boldsymbol{n}\|^2 = 1$$

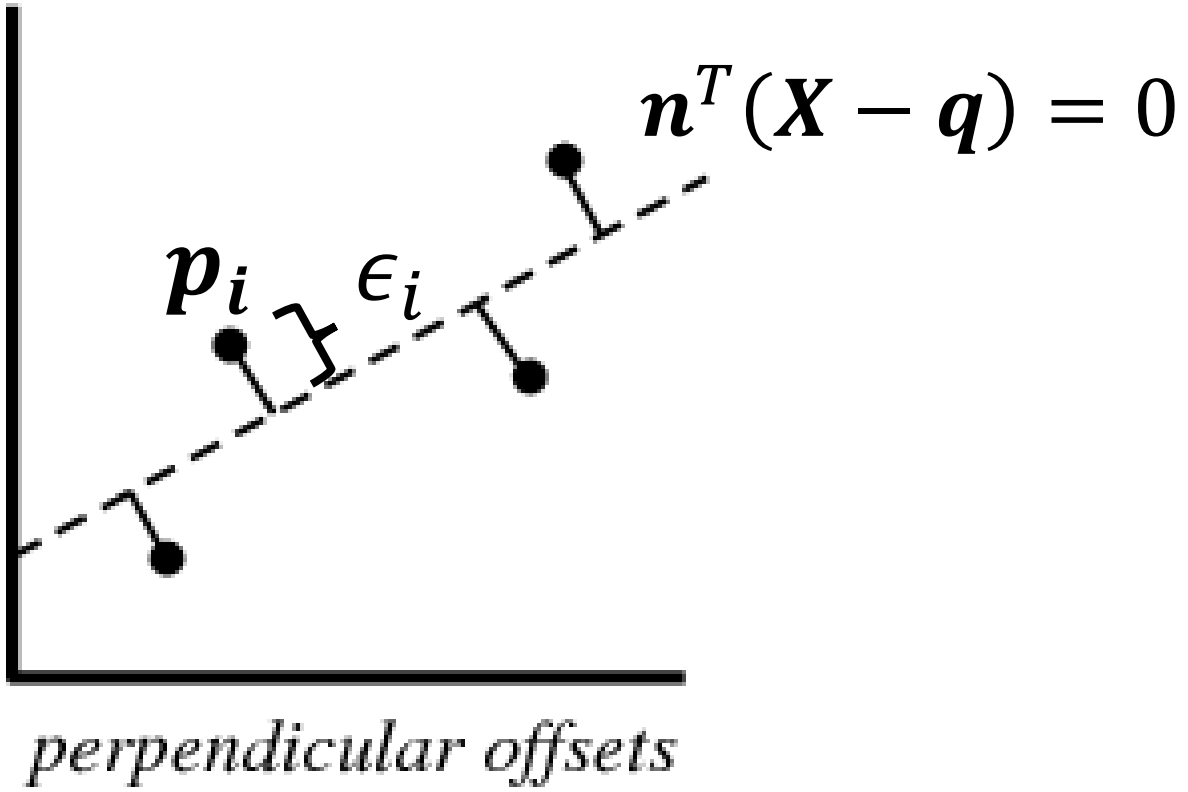


Image from: http://mathworld.wolfram.com/LeastSquaresFitting.html

Orthogonal Regression for Line Estimation

- Given a set of 3D points $\{\boldsymbol{p}_i\}$, we want to find out a line/plane (i.e., unit normal **n** and center **q**) that describes this set of points as

$$n^\mathsf{T}(p_i - q) = 0, \forall i$$

$$\begin{aligned}
\text{cost}(n, q) &\triangleq \sum_i \text{dist}^2(p_i; n, q) \\
&= \sum_i (n^\mathsf{T}(p_i - q))^2 \\
&= n^\mathsf{T}[\cdots, p_i - q, \cdots][\cdots, p_i^\mathsf{T} - q^\mathsf{T}, \cdots]^\mathsf{T} n \\
&= n^\mathsf{T} A(q) A(q)^\mathsf{T} n
\end{aligned}$$

Orthogonal Regression for Line Estimation

- Solving **q**

$$\mathbf{0} = \frac{\partial \text{cost}(n, q)}{\partial q} \equiv \sum_i (2nn^\intercal q - 2nn^\intercal p_i)$$

$$q^* = \frac{1}{|\{p_i\}|} \sum_i p_i$$

Orthogonal Regression for Line Estimation

- Solving **n**

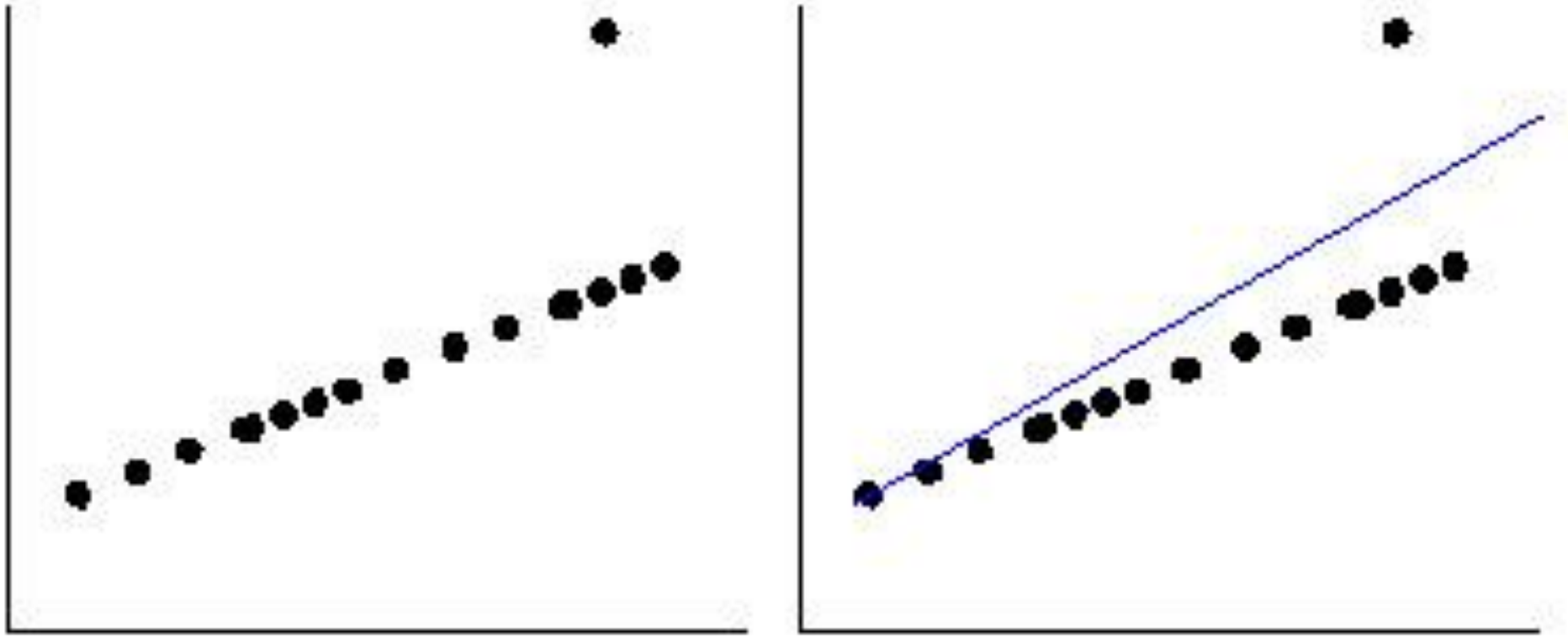
$$\text{cost}(n; q^*) \triangleq n^\intercal A(q^*)A(q^*)^\intercal n = n^\intercal B(q^*)n$$

- Equivalent to solve:

$$n^* = \arg\min_n \quad n^\intercal B(q^*)n$$
$$\text{s.t.} \quad n^\intercal n = 1$$

- Solve by SVD
  - Optimal **n** is **B**'s eigenvector corresponding to the smallest eigenvalue.
  - So, this is also referred to as the PCA-based solution.

But Least Squares is NOT Robust to Outliers!



http://www.unige.ch/ses/sococ/cl//////stat/action/nonlin5.jpg

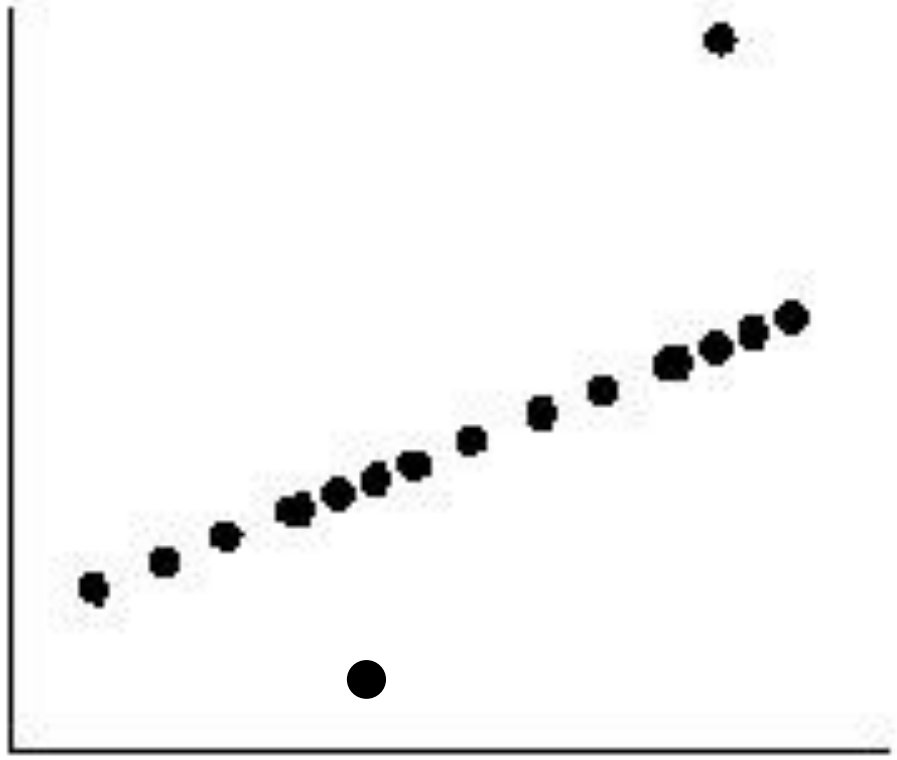
How to Solve This Intuitively?

- Mitigating outlier's influence/weight in the estimation
  - Iteratively Re-weighted Least Squares (IRLS)

- Detect outlier and remove it from estimation
  - RANdom SAmple Consensus (RANSAC)

How to Detect an Outlier?

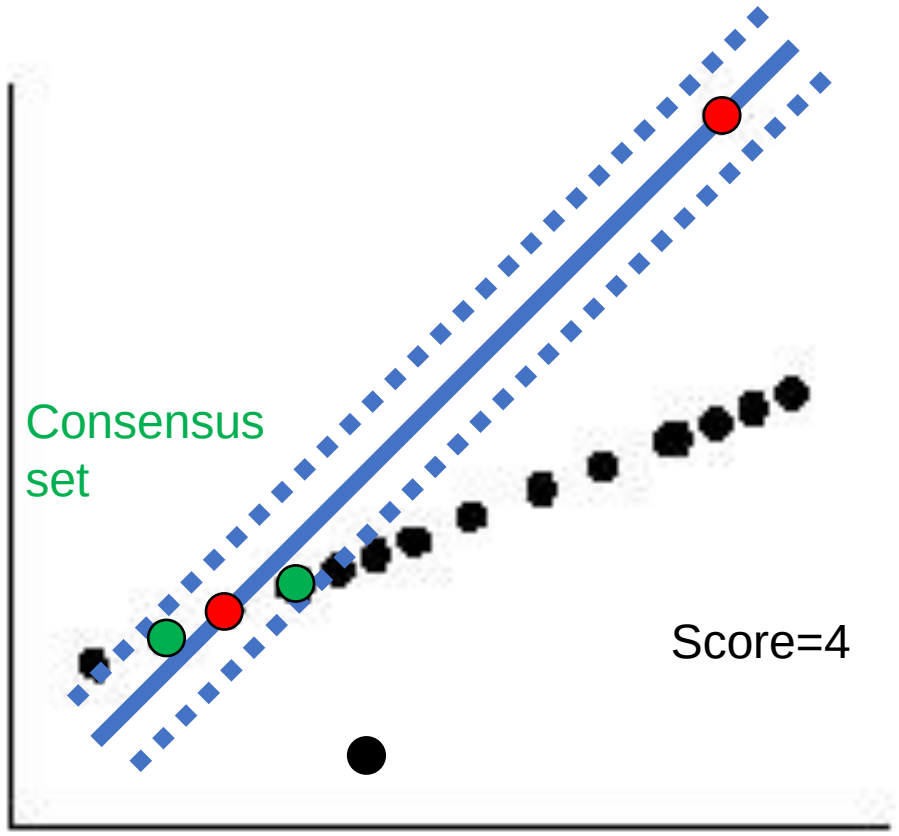
- Enumeration strategy
  - Leave one out
- Voting strategy
  - Hypothesis and test



http://www.unige.ch/ses/sococ/cl//////stat/action/nonlin5.jpg

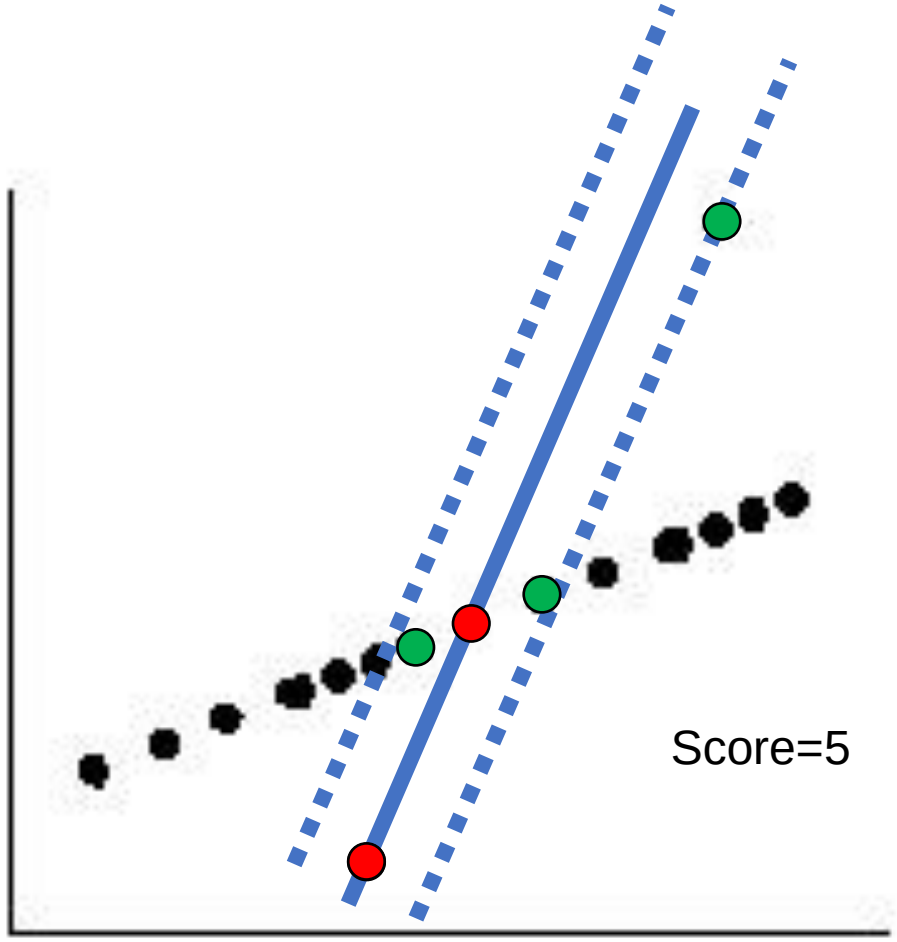
Hypothesis and Test

- Randomly select two points to generate a hypothesis line
- Test how good the current hypothesis is
  - Consensus set = {supporting points}
  - Score by #(supporting points)= |Consensus set|



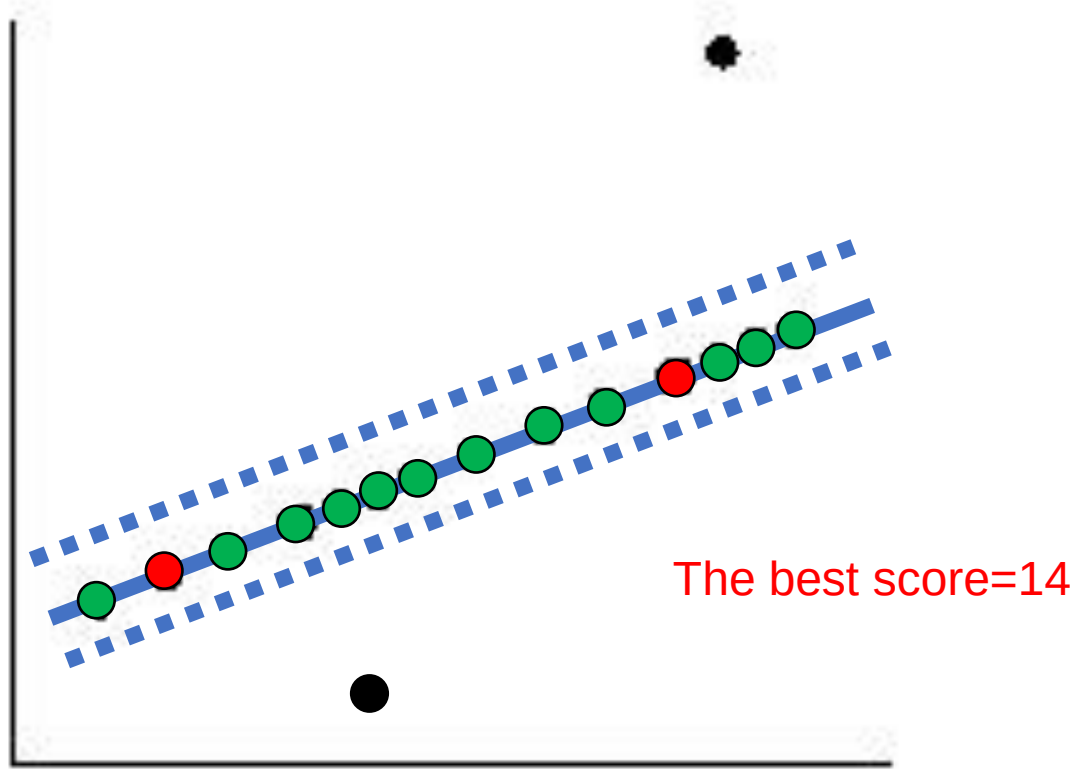
Hypothesis and Test

- Randomly select two points to generate a hypothesis line
- Test how good the current hypothesis is



Hypothesis and Test

- Randomly select two points to generate a hypothesis line
- Test how good the current hypothesis is



RANSAC Framework

Objective

Robust fit of a model to a data set S which contains outliers.

Algorithm

(i) Randomly select a sample of s data points from S and instantiate the model from this subset.

(ii) Determine the set of data points S_i which are within a distance threshold t of the model. The set S_i is the consensus set of the sample and defines the inliers of S.

(iii) If the size of S_i (the number of inliers) is greater than some threshold T, re-estimate the model using all the points in S_i and terminate.

(iv) If the size of S_i is less than T, select a new subset and repeat the above.

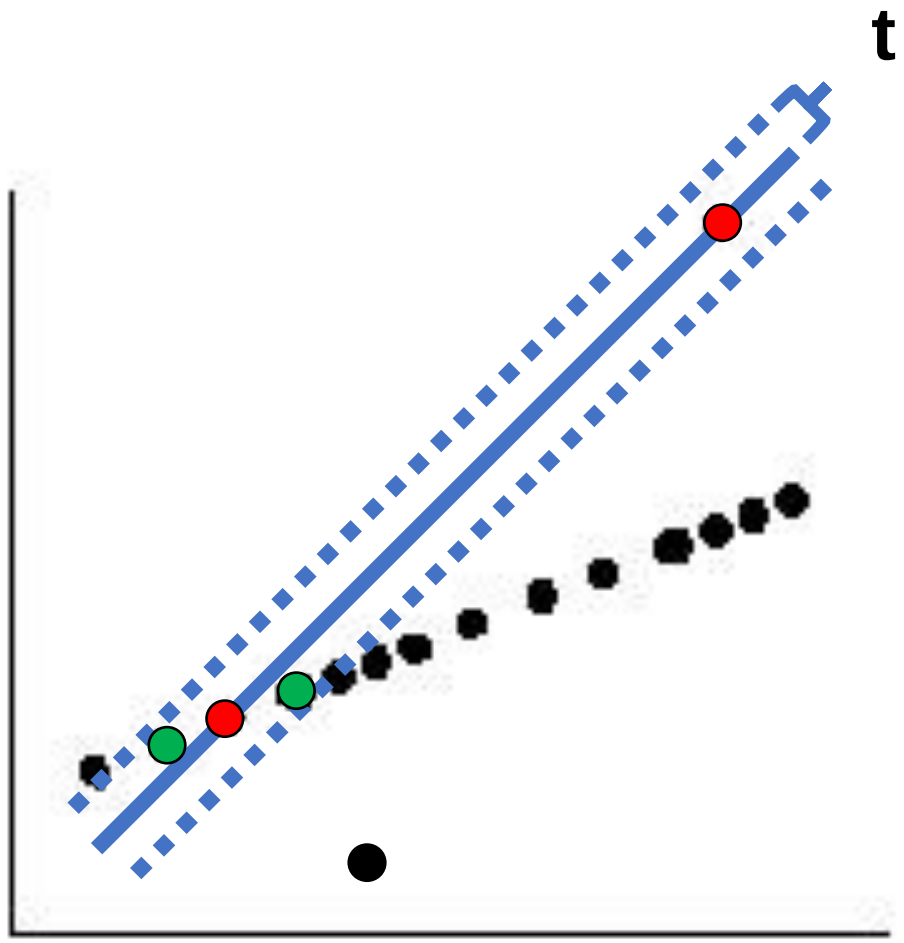
(v) After N trials the largest consensus set S_i is selected, and the model is re-estimated using all the points in the subset S_i.

Often omitted in implementation

Hartley & Zisserman 2003

RANSAC Details – Distant Threshold t

- If data is known to be distributed as a Gaussian of standard deviation σ:
  - Use the 3σ rule
- Otherwise:
  - Determined manually from experience
  - Try-and-error



RANSAC Details – #Samples N

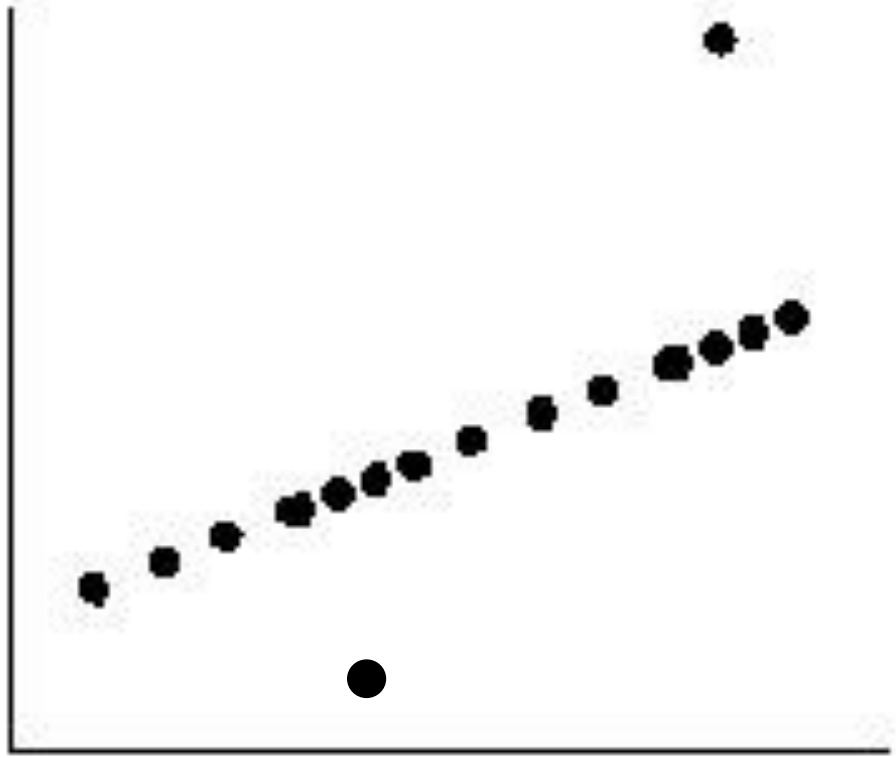
- Basic idea: max the probability p of **at least one** successful sampling
  - Success sample: all the s sampled data points are inliers
- Assume outlier ratio ϵ is known
- Probability of one successful sampling (sampling with replacement): $(1-\epsilon)^s$
- Probability of at least one successful sampling: $1-[1-(1-\epsilon)^s]^N$
- Thus: $\mathrm{N} = \ln(1-p) \,/\, \ln[1-(1-\epsilon)^s]$

Why Minimal Solution is Important

- For p=0.99

Sample size	Proportion of outliers ϵ						
s	5%	10%	20%	25%	30%	40%	50%
2	2	3	5	6	7	11	17
3	3	4	7	9	11	19	35
4	3	5	9	13	17	34	72
5	4	6	12	17	26	57	146
6	4	7	16	24	37	97	293
7	4	8	20	33	54	163	588
8	5	9	26	44	78	272	1177

e



Hartley & Zisserman 2003

RANSAC Details – Adaptive Sampling

- What if outlier ratio ϵ is NOT known?

- $N = \infty$, sample_count$= 0$.
- While $N >$ sample_count Repeat
  - Choose a sample and count the number of inliers.
  - Set $\epsilon = 1 - (\text{number of inliers})/(\text{total number of points})$
  - Set N from ϵ and (4.18) with $p = 0.99$.
  - Increment the sample_count by 1.
- Terminate.

Hartley & Zisserman 2003

More Examples – Homography

Objective

Compute the 2D homography between two images.

Algorithm

(i) **Interest points:** Compute interest points in each image.

(ii) **Putative correspondences:** Compute a set of interest point matches based on proximity and similarity of their intensity neighbourhood.

(iii) **RANSAC robust estimation:** Repeat for N samples, where N is determined adaptively as in algorithm 4.5:

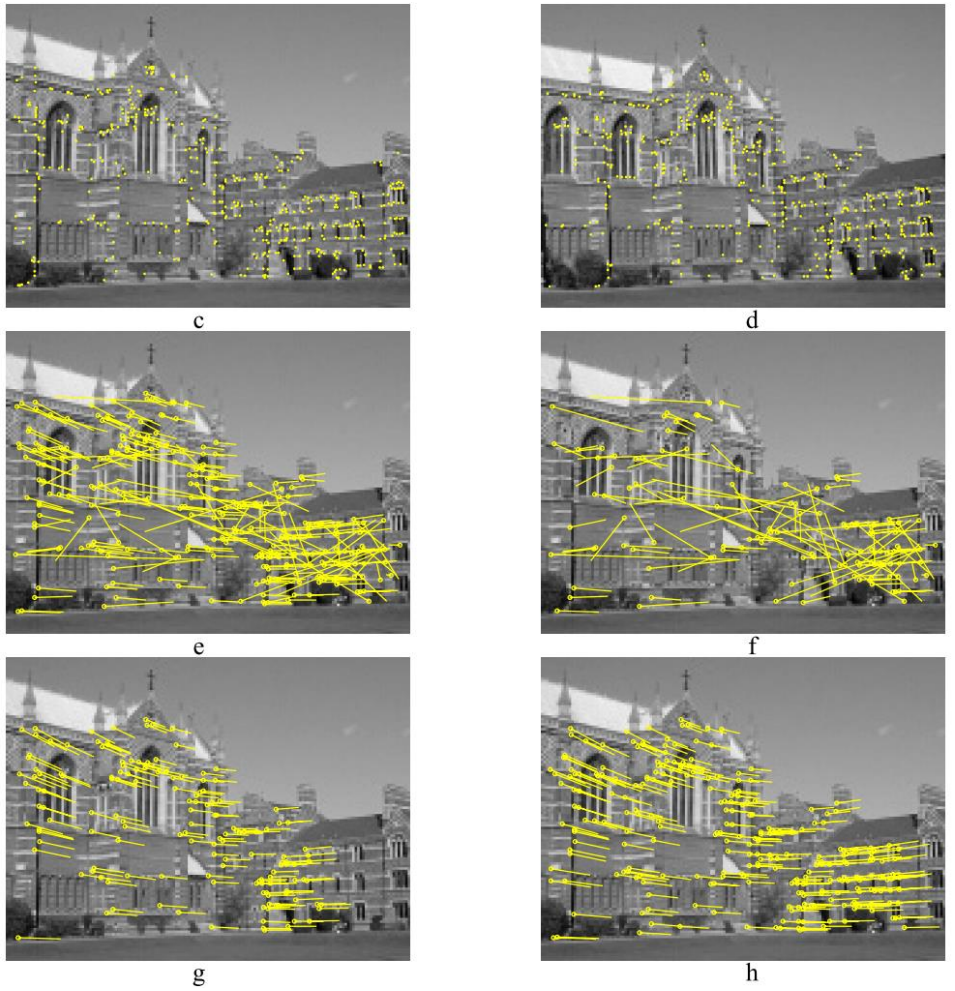
- (a) Select a random sample of 4 correspondences and compute the homography H.
- (b) Calculate the distance $d_\perp$ for each putative correspondence.
- (c) Compute the number of inliers consistent with H by the number of correspondences for which $d_\perp < t = \sqrt{5.99}\,\sigma$ pixels.

Choose the H with the largest number of inliers. In the case of ties choose the solution that has the lowest standard deviation of inliers.

(iv) **Optimal estimation:** re-estimate H from all correspondences classified as inliers, by minimizing the ML cost function (4.8–p95) using the Levenberg–Marquardt algorithm of section A6.2(p600).

(v) **Guided matching:** Further interest point correspondences are now determined using the estimated H to define a search region about the transferred point position.

The last two steps can be iterated until the number of correspondences is stable.



Hartley & Zisserman 2003

More Examples – F-matrix

Objective Compute the fundamental matrix between two images.

Algorithm

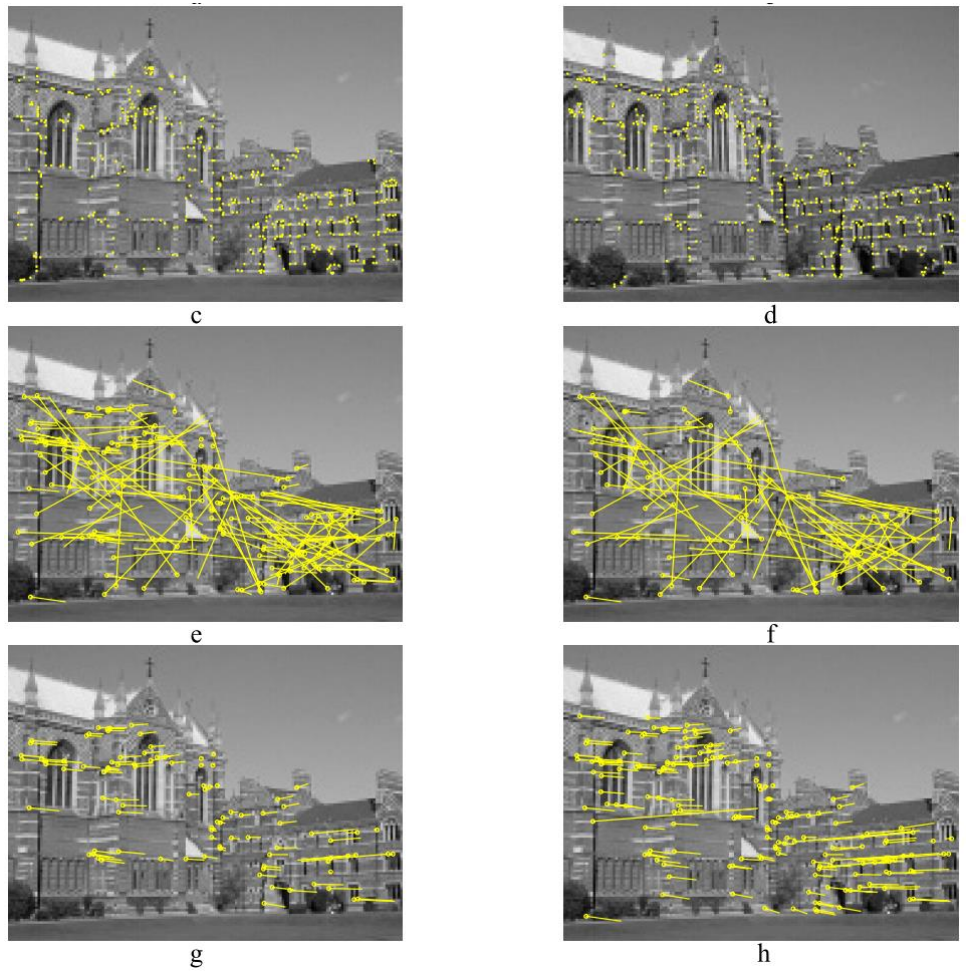
(i) **Interest points:** Compute interest points in each image.
(ii) **Putative correspondences:** Compute a set of interest point matches based on proximity and similarity of their intensity neighbourhood.
(iii) **RANSAC robust estimation:** Repeat for N samples, where N is determined adaptively as in algorithm 4.5(p121):

(a) Select a random sample of 7 correspondences and compute the fundamental matrix F as described in section 11.1.2. There will be one or three real solutions.
(b) Calculate the distance $d_\perp$ for each putative correspondence.
(c) Compute the number of inliers consistent with F by the number of correspondences for which $d_\perp < t$ pixels.
(d) If there are three real solutions for F the number of inliers is computed for each solution, and the solution with most inliers retained.

Choose the F with the largest number of inliers. In the case of ties choose the solution that has the lowest standard deviation of inliers.

(iv) **Non-linear estimation:** re-estimate F from all correspondences classified as inliers by minimizing a cost function, e.g. (11.6), using the Levenberg–Marquardt algorithm of section A6.2(p600).
(v) **Guided matching:** Further interest point correspondences are now determined using the estimated F to define a search strip about the epipolar line.

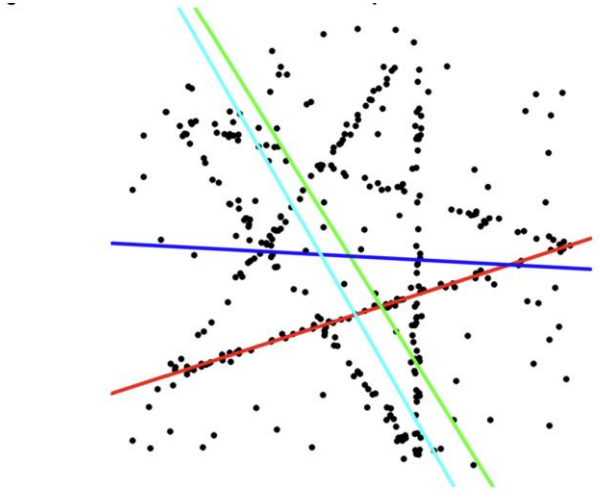
The last two steps can be iterated until the number of correspondences is stable.



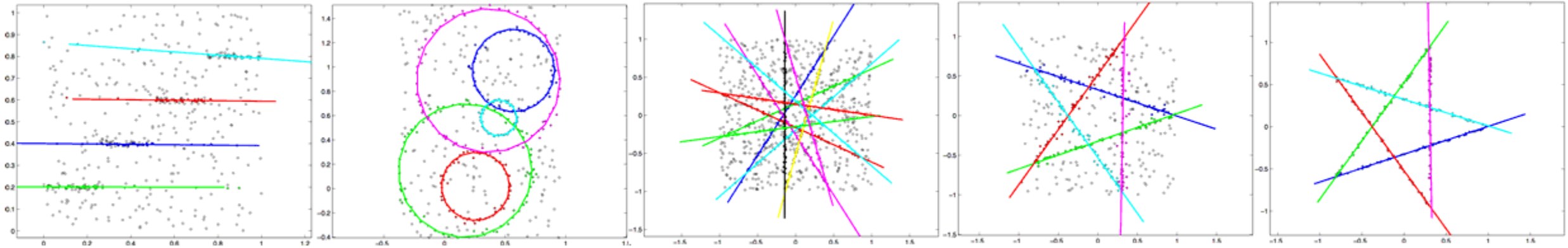
Hartley & Zisserman 2003

Issues in RANSAC

- RANSAC could be **time-consuming**
  - Too many trials
- RANSAC will fail if the problem is **multi-model**
  - i.e., the data is sampled from multiple models (lines/planes/homography/…)

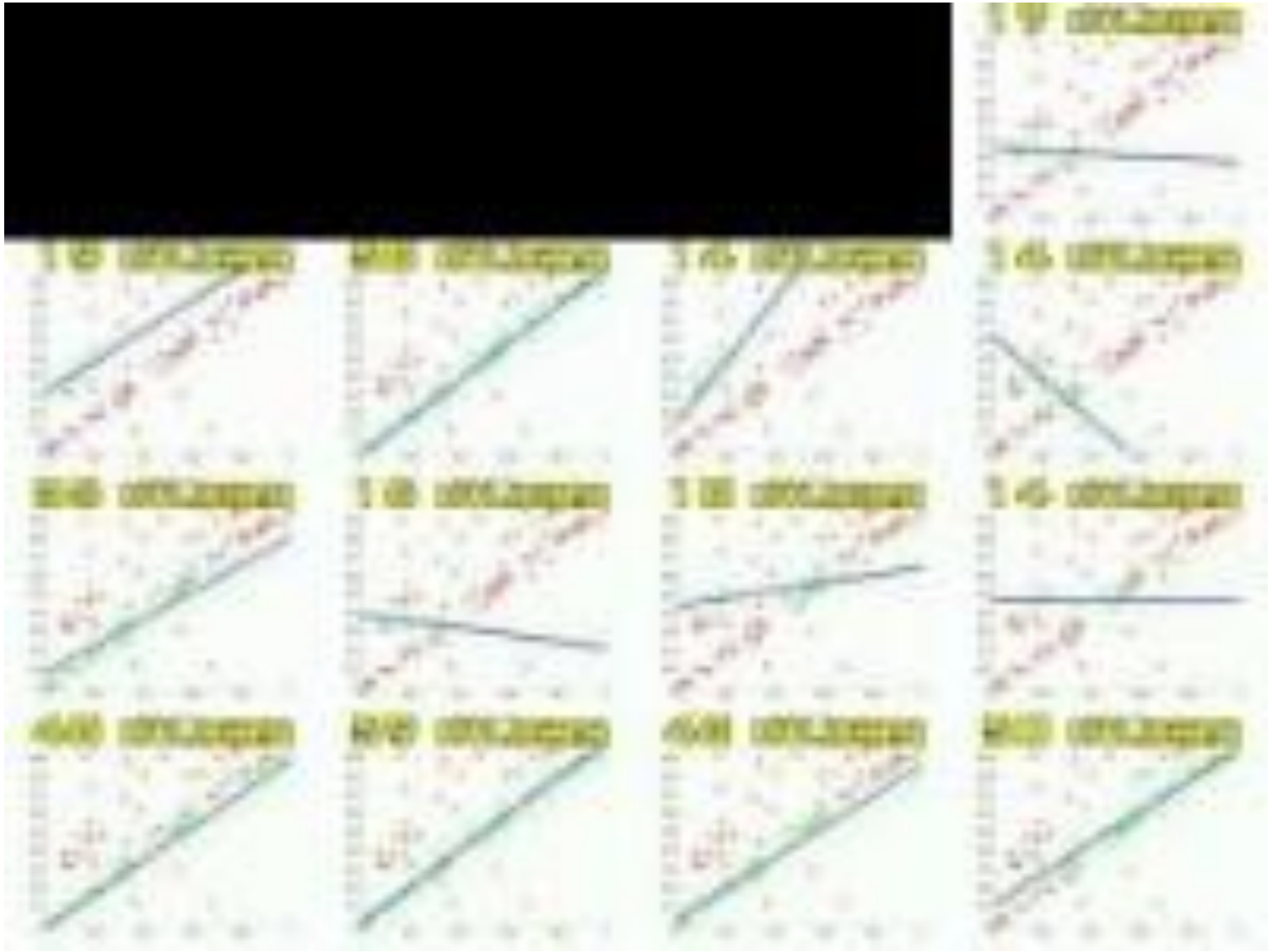


Ordinary Least Squares (O.L.S.), Total Least Squares (T.L.S.) (via PCA), Least Median of Squares (LMedS), Random Sample Consensus (RANSAC)



Images from: R. Toldo and L. Magri (http://www.diegm.uniud.it/fusiello/demo/jlk/), and David F. Fouhey (https://web.eecs.umich.edu/~fouhey/earlier/thesis/dfouhey_thesisPresentation.pdf)

The RANSAC Song



https://youtu.be/1YNjMxxXO-E

Recent Work with Deep Learning

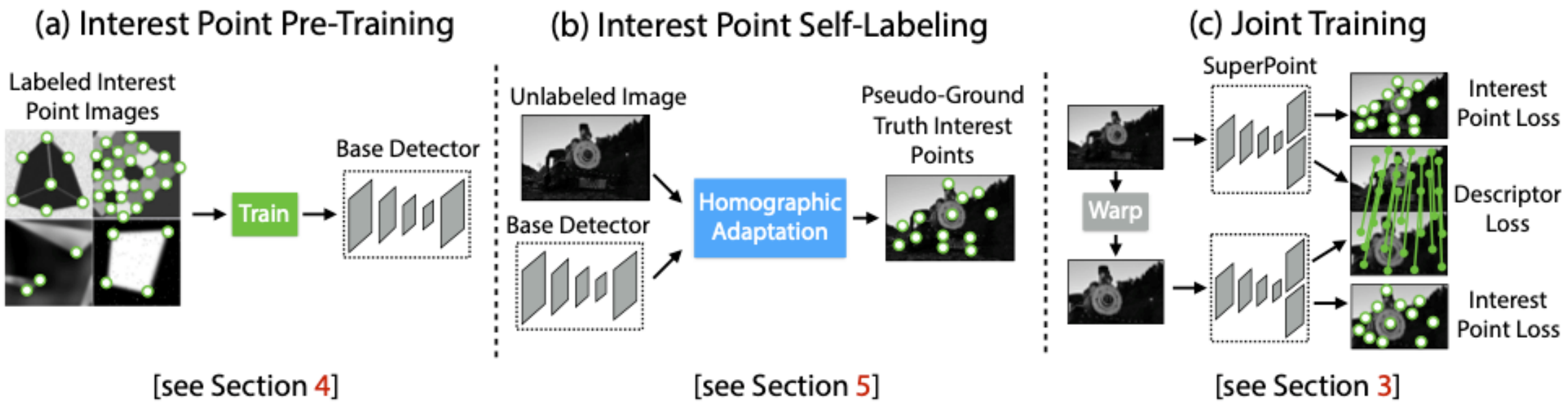


Figure 2. **Self-Supervised Training Overview.** In our self-supervised approach, we (a) pre-train an initial interest point detector on synthetic data and (b) apply a novel Homographic Adaptation procedure to automatically label images from a target, unlabeled domain. The generated labels are used to (c) train a fully-convolutional network that jointly extracts interest points and descriptors from an image.

- **SuperPoint** - fairly "old" work by modern ML standard, but very relevant and is widely used in work on Feature Extraction.
- Built upon an earlier work by the same authors (MagicPoint) – extended with self-supervised learning + test-time augmentation (they called it Homographic Adaptation).
- Supersedes handcrafted feature extractor like SIFT / SURF / ORB / etc.

SuperPoint: Self-Supervised Interest Point Detection and Description, Daneil DeTone et al., 2018

Recent Work with Deep Learning

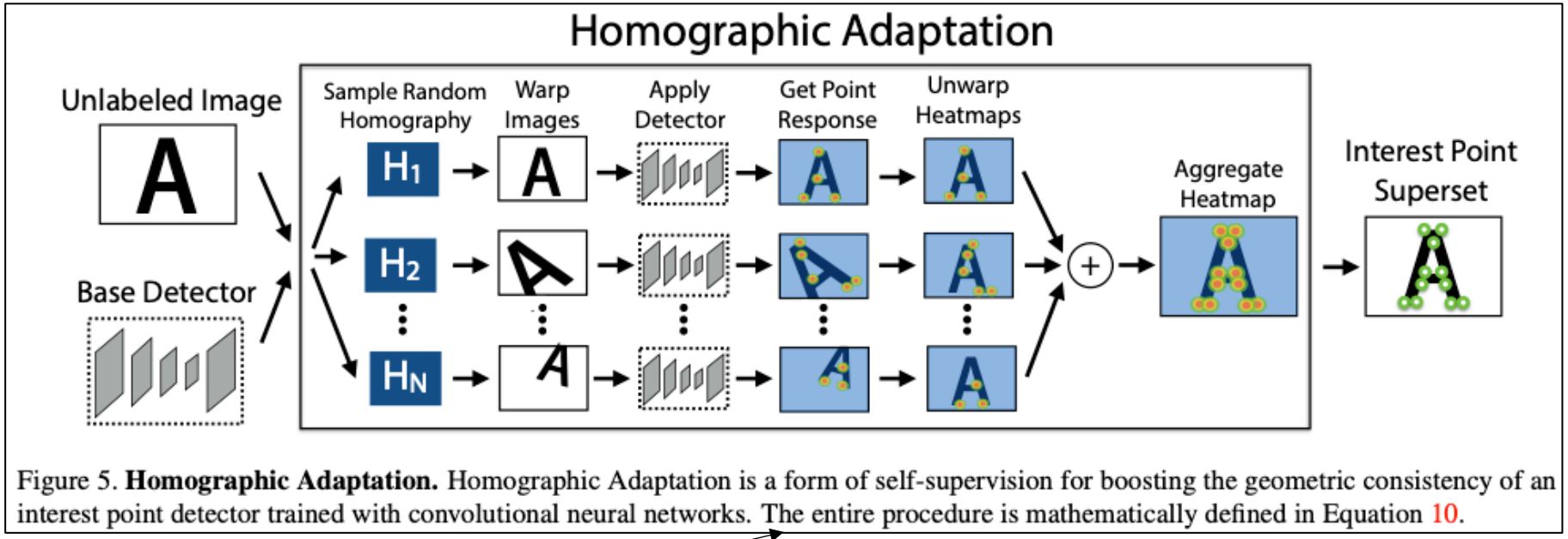


Figure 5. **Homographic Adaptation.** Homographic Adaptation is a form of self-supervision for boosting the geometric consistency of an interest point detector trained with convolutional neural networks. The entire procedure is mathematically defined in Equation 10.

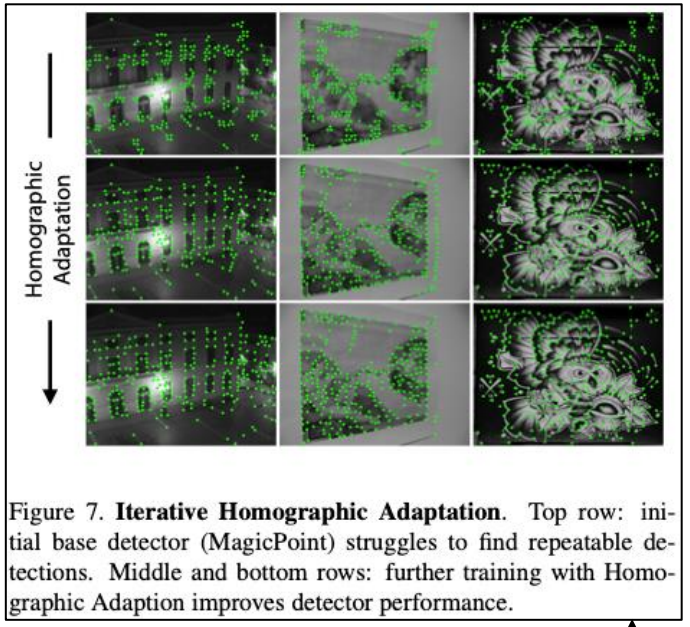
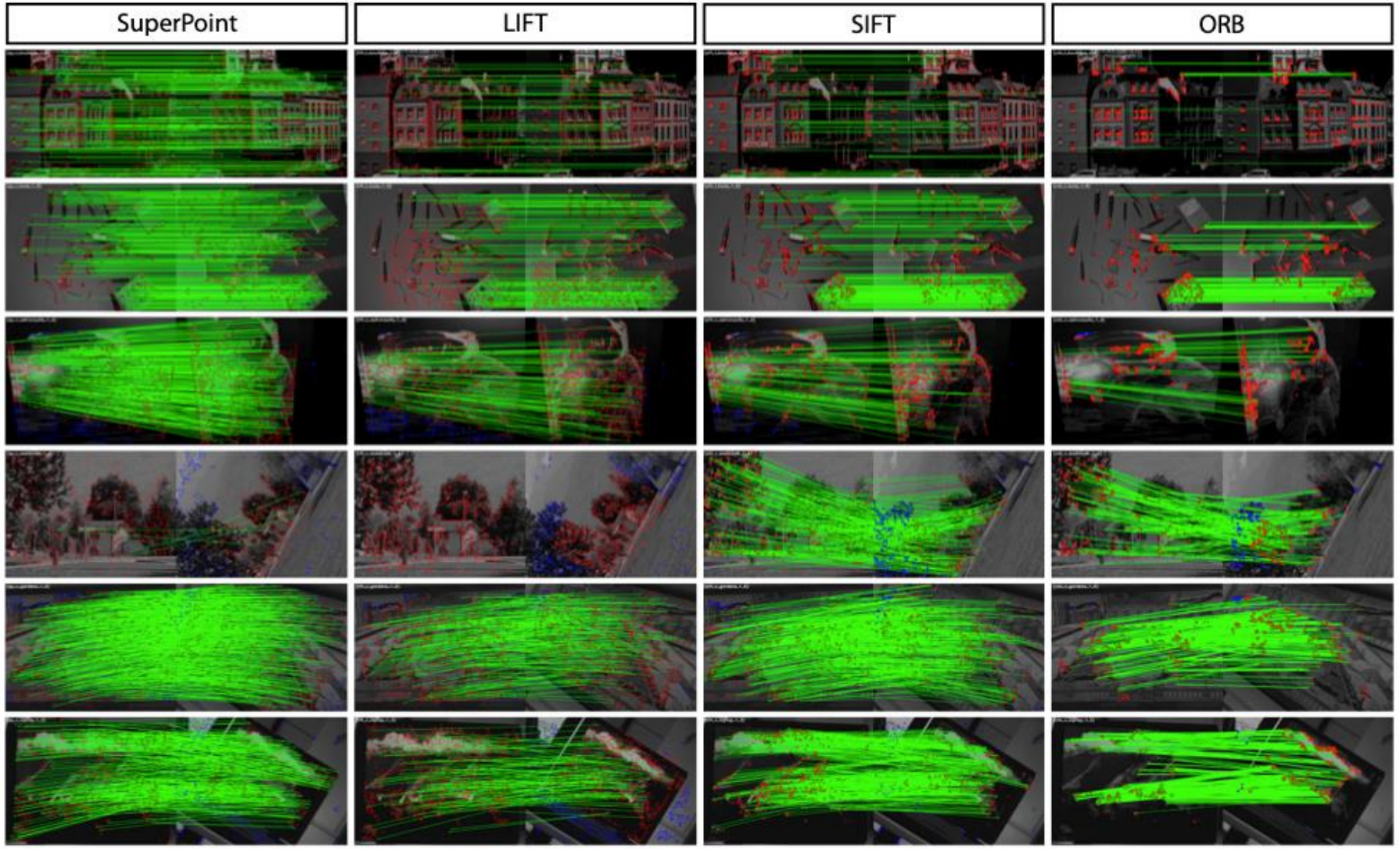


Figure 7. **Iterative Homographic Adaptation.** Top row: initial base detector (MagicPoint) struggles to find repeatable detections. Middle and bottom rows: further training with Homographic Adaption improves detector performance.

- Homograhic Adaptation (test-time augmentation) is used to generate a super-set of features via random homography transformation on a source image.
- The super-set of features are then used to train a final classifier (SuperPoint) in a self-supervised manner.
- This self-supervised approach + augmentation enables more robust and consistent detection compared to MagicPoint alone.

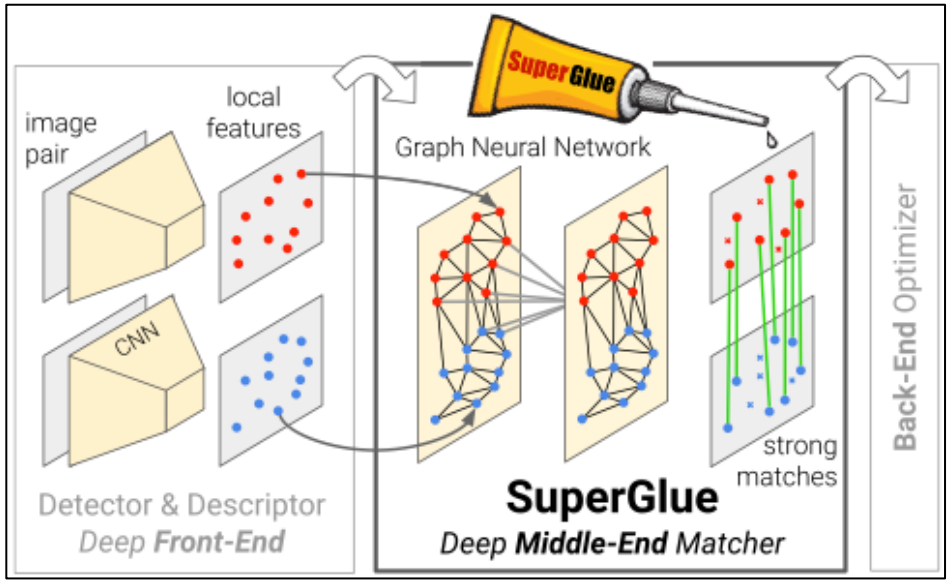
SuperPoint: Self-Supervised Interest Point Detection and Description, Daneil DeTone et al., 2018

Recent Work with Deep Learning



SuperPoint: Self-Supervised Interest Point Detection and Description, Daneil DeTone et al., 2018

Recent Work with Deep Learning



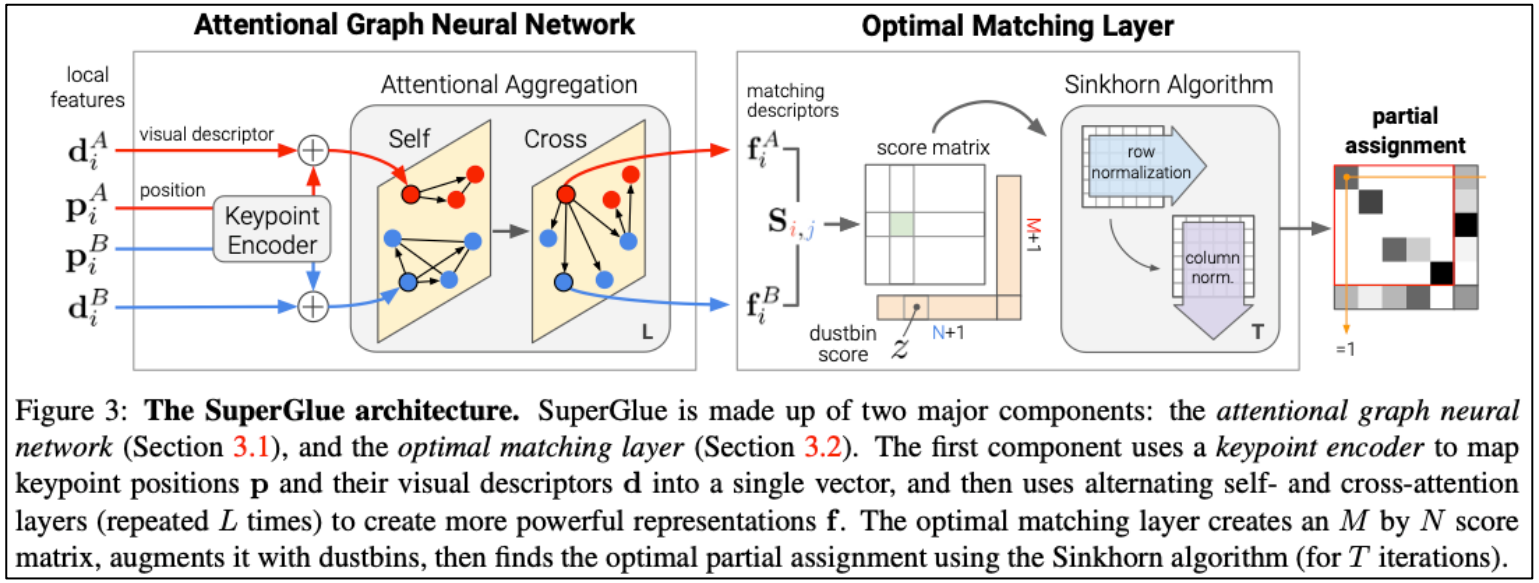
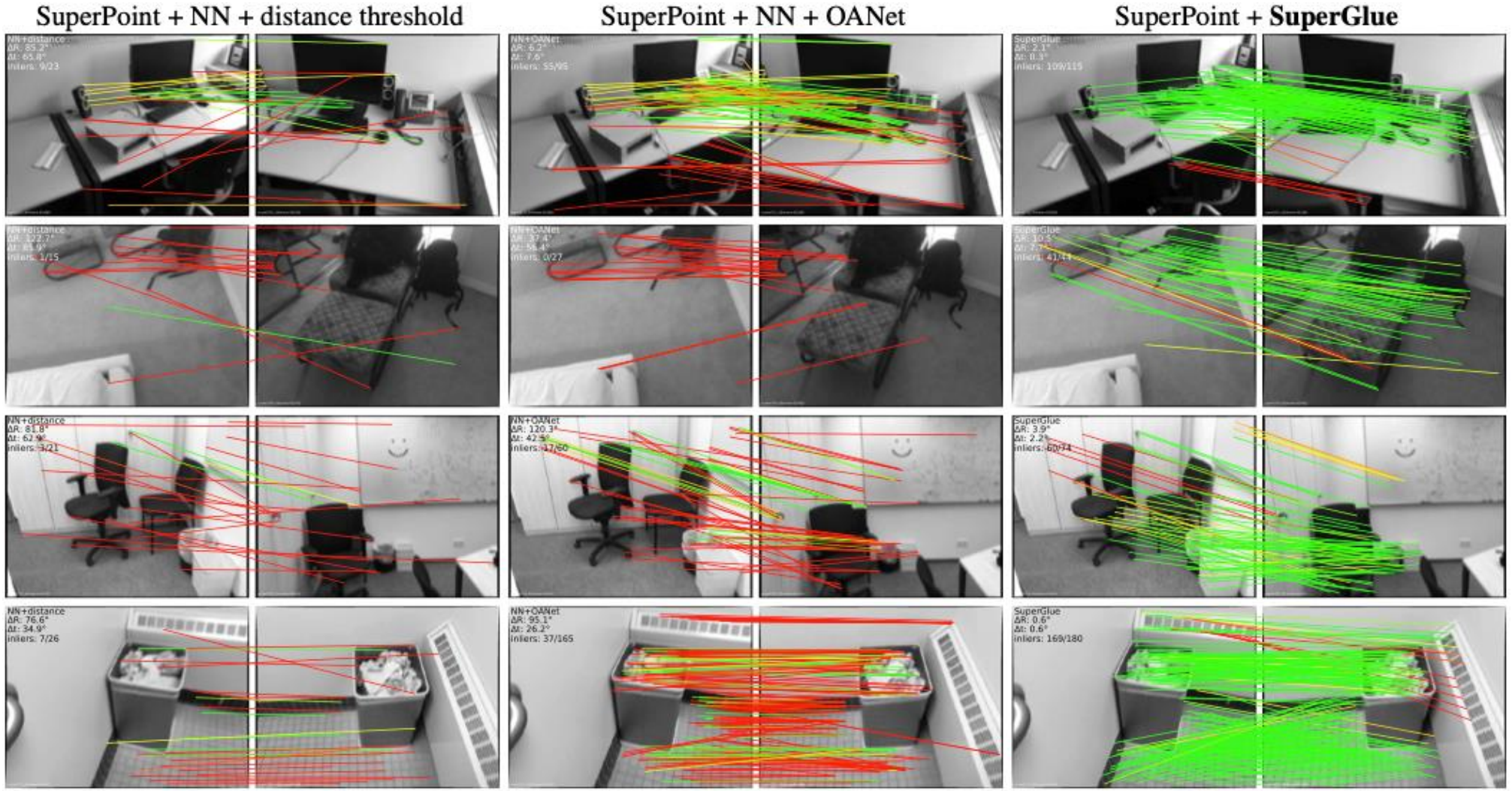


Figure 3: **The SuperGlue architecture.** SuperGlue is made up of two major components: the *attentional graph neural network* (Section 3.1), and the *optimal matching layer* (Section 3.2). The first component uses a *keypoint encoder* to map keypoint positions $\mathbf{p}$ and their visual descriptors $\mathbf{d}$ into a single vector, and then uses alternating self- and cross-attention layers (repeated L times) to create more powerful representations $\mathbf{f}$. The optimal matching layer creates an M by N score matrix, augments it with dustbins, then finds the optimal partial assignment using the Sinkhorn algorithm (for T iterations).

- **SuperGlue** – robust feature matching via attentional Graph Neural Networks.
- SuperPoint introduces a method to find salient features, SuperGlue introduces a method to find robust correspondences with said features.
- Traditionally, this is usually done by some sort of Nearest Neighbor algorithm, which is prone to outliers, thereby necessitating additional post-processing – e.g. mutual check, Lowe's Ratio Test.
- How does SuperGlue tackle this?
  - By representing it via a Graph Neural Network which renders it end-to-end differentiable, and
  - By leveraging self and cross attentions to better find correspondences in a learnable way.
- Predictably, SuperPoint is used here for Feature Extraction.

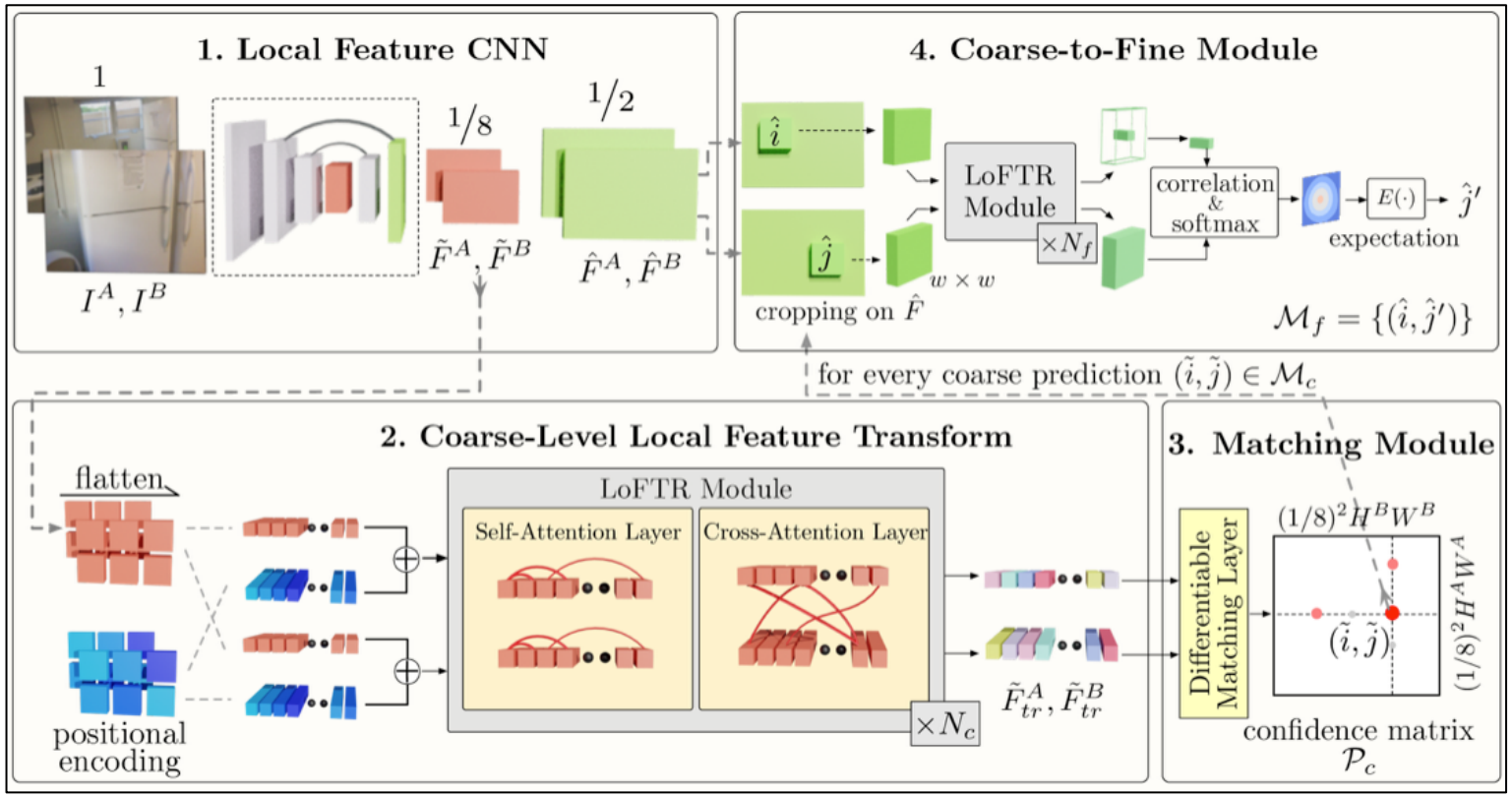
SuperGlue: Learning Feature Matching with Graph Neural Networks, Daneil DeTone et al., 2020

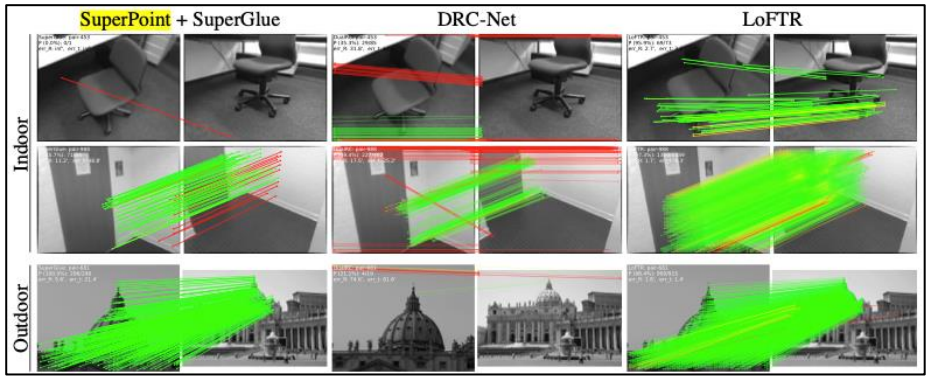
Recent Work with Deep Learning



SuperGlue: Learning Feature Matching with Graph Neural Networks, Daneil DeTone et al., 2020

Recent Work with Deep Learning





Category	Method	Homography est. AUC @3px	@5px	@10px	#matches
Detector-based	D2Net [11]+NN	23.2	35.9	53.6	0.2K
	R2D2 [32]+NN	50.6	63.9	76.8	0.5K
	DISK [47]+NN	52.3	64.9	78.9	1.1K
	SP [9]+SuperGlue [37]	53.9	68.3	81.7	0.6K
Detector-free	Sparse-NCNet [33]	48.9	54.2	67.1	1.0K
	DRC-Net [19]	50.6	56.2	68.3	1.0K
	LoFTR-DS	**65.9**	**75.6**	**84.6**	1.0K

- **LoFTR** – An ”detector-free” Multi-View matching algorithm; think of it as SuperPoint + SuperGlue in one package.
- Utilizes attention as well via Transformers. It’s “detector-free” since there are no explicit feature extraction like SuperPoint.
- Since feature extraction and matching is done end-to-end in a single learnable pipeline, it seems to be able to find better features and correspondences compared to piecewise methods. Also utilizes a course-to-fine approach with 2 stages of attentional modules to help achieve this.

LoFTR: Detector-Free Local Feature Matching with Transformers, Jiaming Sun et al., 2021

Recent Work with Deep Learning

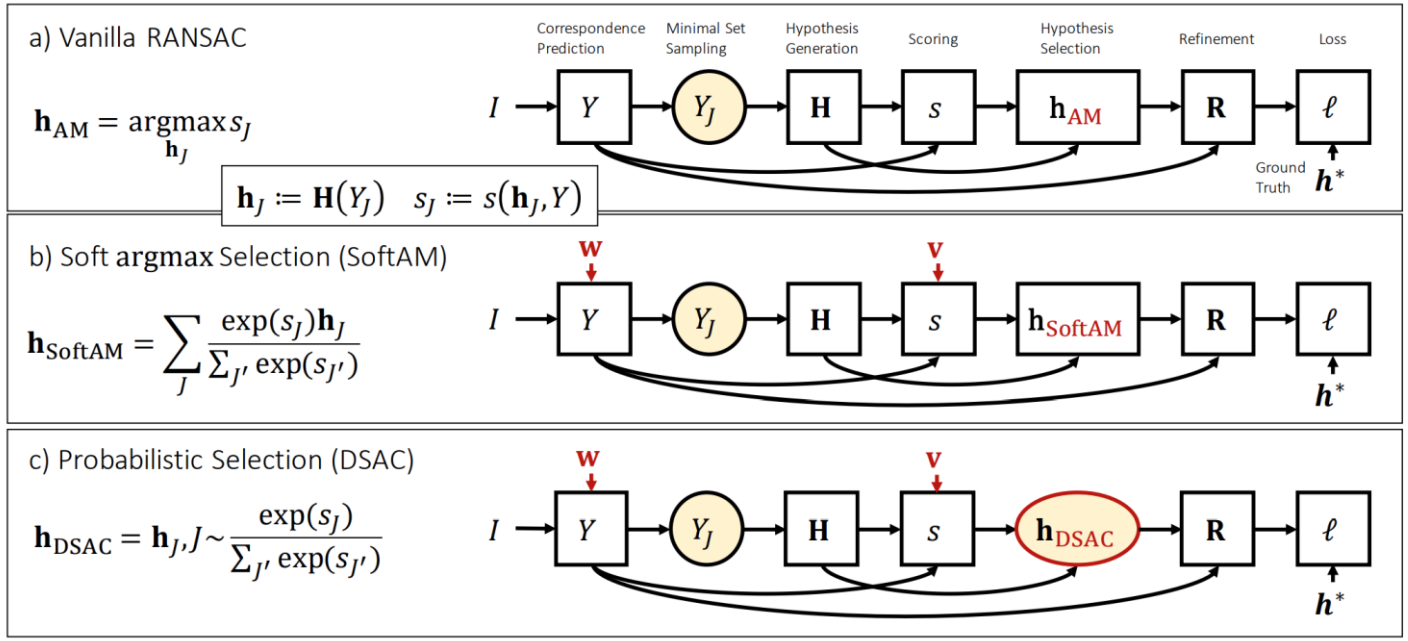
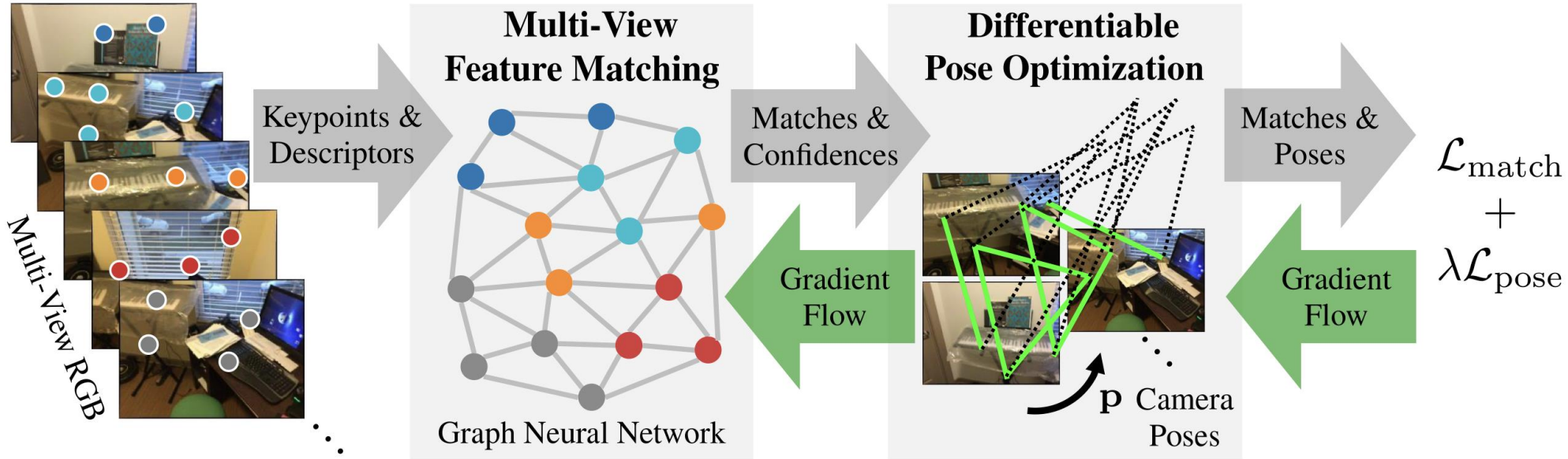


Figure 1. **Stochastic Computation Graphs [34].** A graphical representation of three RANSAC variants investigated in this work. The variants differ in the way they select the final model hypothesis: **a)** non-differentiable, vanilla RANSAC with hard, deterministic argmax selection; **b)** differentiable RANSAC with deterministic, soft argmax selection; **c)** differentiable RANSAC with hard, probabilistic selection (named DSAC). Nodes shown as boxes represent deterministic functions, while circular nodes with yellow background represent probabilistic functions. Arrows indicate dependency in computation. All differences between a), b) and c) are marked in red.

- RANSAC can be viewed as a stochastic computation graph
- RANSAC itself is not directly differentiable, i.e., the partial derivative of the final loss w.r.t. any intermediate variables before hypothesis selection cannot be computed from backpropagation due to the use of argmax
- The DSAC paper proposed two possible solutions, Soft argmax, or even better, a probabilistic selection inspired by policy gradient approaches in reinforcement learning

DSAC - Differentiable RANSAC for Camera Localization, Eric Brachmann, Alexander Krull, Sebastian Nowozin, 2017

Recent Work with Deep Learning



- An end-to-end framework that combines matching + outlier filtering. No RANSAC required. Uses SuperPoint for extraction. (Wait for next team to combine extraction as well for all-in-one!).
- Does so via joint-learning of multi-view matching + pose estimation. The task of pose estimation essentially constraints and direct the network to find proper correspondences across views.
- Uses a Graph Neural Network just like SuperGlue, and introduces a learnable pose-estimation schema for robust matching.

End2End Multi-View Feature Matching with Differentiable Pose Optimization, Barbara Roessle and Matthias Nießner, 2023

Next Week

+ Depth from Stereo (Stereo Calibration and Stereo Matching)

+ Monocular Depth Estimation by CNN

* ICP

++ Procrustes Analysis

+ TSDF & Kinect Fusion

+ Fast Plane Extraction (PEAC)

*: know how to code
++: know how to derive
+: know the concept

References for next week

- Co2011:
  - Section 14.3
- Sz2022:
  - Chapter 12
- FP2011
  - Chapter 7, Section 12.1, 14.3
- HZ2003
  - Section 11.12
- Umeyama, S. (1991). Least-squares estimation of transformation parameters between two point patterns. IEEE Transactions on Pattern Analysis & Machine Intelligence, (4), 376-380.
- Low, K.L., 2004. Linear least-squares optimization for point-to-plane icp surface registration. *University of North Carolina Chapel Hill*, *4*(10).
- Garg, Ravi, Vijay Kumar Bg, Gustavo Carneiro, and Ian Reid. "Unsupervised cnn for single view depth estimation: Geometry to the rescue." In European conference on computer vision, pp. 740-756. Springer, Cham, 2016.
- Feng, C., Taguchi, Y. and Kamat, V.R., (2014). Fast plane extraction in organized point clouds using agglomerative hierarchical clustering. In *IEEE International Conference on Robotics and Automation (ICRA)* (pp. 6218-6225).
- Park, J., Zhou, Q. Y., & Koltun, V., (2017). Colored point cloud registration revisited. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition* (pp. 143-152).