studyguide

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Contents

1 Chapter 1

1.1 Propositional Logic

1.1.1 Converse Contrapositive and Inverse (p -> q)

- Coverse
  - $q \to p$
- Contrapositive
  - $q \to \neg p$
- Inverse
  - $\neg p \to \neg q$

1.2 Applications of Propositional Logic

1.2.1 Examples of turning sentences into propositional Logic.

1.3 Propositional Equivalences

1.3.1 Logical Equivalences

p$\wedge T$	p	Identity Laws
p$\vee F$	p	
p$\vee T$	T	Domination Laws
p$\wedge F$	F	
p$\vee p$	p	Idempotent Laws
p$\wedge p$	p	
$\neg$($\neg$p)	p	Double Negation Laws
p$\vee q$	q$\vee p$	Commutative Laws
p$\wedge q$	q $\wedge p$	
(p$\vee q) \vee r$	p$\vee(q \vee r)$	Associative Laws
(p$\wedge q) \wedge r$	p $\wedge(q \wedge r)$	
p$\vee(q \wedge r)$	(p$\vee q) \wedge (p \vee r)$	Distributive Laws
p $\wedge(q \vee r)$	(p$\wedge q) \vee (p \wedge r)$	
$\neg$(p$\wedge q$)	$\neg$p$\vee q$	De Morgans Laws
$\neg$(p v q)	$\neg$p$\wedge q$	
p$\vee(p \wedge q)$	p	Absorption Laws
p $\wedge(p \vee q)$	p	
p v $\neg$p	T	Negation Laws
p$\wedge p$	F	

1.3.2 Logical Equivelances Involving Conditional Statements

p $\rightarrow q$	$\neg$p$\vee q$
p $\rightarrow q$	$\neg$q $\rightarrow p$
p$\vee q$	$\neg$p $\rightarrow q$
p$\wedge q$	$\neg$(p $\rightarrow q$)
$\neg$(p $\rightarrow q$)	p$\wedge q$
(p $\rightarrow q) \wedge (p \rightarrow r)$	p $\rightarrow (q \wedge r)$
(p $\rightarrow r) \wedge (q \rightarrow r)$	(p$\vee q) \rightarrow r$
(p $\rightarrow q) \vee (p \rightarrow r)$	p $\rightarrow (q \vee r)$
(p $\rightarrow r) \vee (q \rightarrow r)$	(p$\wedge q) \rightarrow r$

1.3.3 Logical Equivalences Involving Biconditional Statements

p $\leftrightarrow q$	(p $\rightarrow q) \wedge (q \rightarrow p)$
p $\leftrightarrow q$	$\neg$p $\leftrightarrow q$
p $\leftrightarrow q$	(p$\wedge q) \vee (p \wedge q)$
$\neg$(p $\leftrightarrow q$)	p $\leftrightarrow q$

1.4 Predicates and Quantifiers

1.4.1 Quantifiers

- Universal Quantifer
  - $\forall$(x) P(x)
    - Definition: For all x in the universe P(x) is true;
    - Negation
      - $\neg\forall$(x) P(x) can also be written as $\exists\neg$ P(x)
      - There exists an x such that P(x) is false
- Existential Quantifer
  - $\exists$(x) P(x)
    - Definition: For all x in the universe P(x) is true for at least one x;
    - Negation
      - $\neg\exists$(x) P(x) can also be written as $\forall\neg$ P(x)
      - All x in the universe make P(x) false

1.5 Nested Quantifers

1.5.1 Nested Quantifiers can be used when there are multiple variables such as x and y

- $\forall$ (x) $\exists$ (y) (x + y = 0)

1.5.2 Quantification of Two Variables

Statement	When True	When False
$\forall(x)\forall(y)P(x,y)$	P(x,y) is true for every pair	When there is a x,y for which P(x,y) is false
$\forall(x)\exists(y)P(x,y)$	For every x there is a y for which P(x,y) is true	When there is an x such that P(x,y) is false for every y
$\exists(x)\forall(y)P(x,y)$	There is an x for which P(x,y) is true for every y	When for every x there is a y for which P(x,y) is false
$\exists(x)\exists(y)P(x,y)$ $\exists(y)\exists(x)P(x,y)$	There is a pair for x,y for which P(x,y) is true	P(x,y) is false for every pair of x and y

1.6 Rules of Inference

1.6.1 Rules of inference are used to show that an argument is valid.

Rule of Inference	Tautology	Name
p $p \to q$ ———— q	$(p \wedge (p \to q)) \to q$	Modus Ponens
$\neg q$ $p \to q$ ———— $\neg p$	$(\neg q \wedge (p \to q)) \to p$	Modus Tollens
$p \to q$ $q \to r$ ———— $p \to r$	$((p \vee q) \wedge p) \to q$	Hypothetical Syllogism
$p \vee q$ $\neg p$ ———— q	$p\ arrow(p \vee q)$	Disjunctive Syllogism
p ———— $p \vee q$	$p \to (p \vee q)$	Addition
$p \wedge q$ ———— p	$(p \wedge q) \to p$	Simplification
p q ———— $p \wedge q$	$((p) \wedge (q)) \to (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ ———— $q \vee r$	$((p \vee q) \wedge (p \vee r)) \to (q \vee r)$	Conjucntion

1.7 Introduction to Proofs

1.8 Proof methods and Strategy

2 Chapter 2

2.1 Sets

2.2 Set Operations

2.3 Functions

2.4 Sequences and Summations

2.5 Cardinality of Sets

2.6 Matrices

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