studyguide

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October 17, 2016

Contents

1 Chapter 1

1.1 Propositional Logic

1.1.1 Converse Contrapositive and Inverse (p -> q)

- Coverse
  - q → p
- Contrapositive
  - q → ¬p
- Inverse
  - ¬p → ¬q

1.2 Applications of Propositional Logic

1.2.1 Examples of turning sentences into propositional Logic.

1.3 Propositional Equivalences

1.3.1 Logical Equivalences

p$\wedge T$	p	Identity Laws
p$\vee F$	p	
p$\vee T$	T	Domination Laws
p$\wedge F$	F	
p$\vee p$	p	Idempotent Laws
p$\wedge p$	p	
$\neg$($\neg$p)	p	Double Negation Laws
p$\vee q$	q$\vee p$	Commutative Laws
p$\wedge q$	q $\wedge p$	
(p$\vee q) \vee r$	p$\vee(q \vee r)$	Associative Laws
(p$\wedge q) \wedge r$	p $\wedge(q \wedge r)$	
p$\vee(q \wedge r)$	(p$\vee q) \wedge (p \vee r)$	Distributive Laws
p $\wedge(q \vee r)$	(p$\wedge q) \vee (p \wedge r)$	
$\neg$(p$\wedge q$)	$\neg$p$\vee q$	De Morgans Laws
$\neg$(p v q)	$\neg$p$\wedge q$	
p$\vee(p \wedge q)$	p	Absorption Laws
p $\wedge(p \vee q)$	p	
p v $\neg$p	T	Negation Laws
p$\wedge p$	F	

1.3.2 Logical Equivelances Involving Conditional Statements

p $\rightarrow q$	$\neg$p$\vee q$
p $\rightarrow q$	$\neg$q $\rightarrow p$
p$\vee q$	$\neg$p $\rightarrow q$
p$\wedge q$	$\neg$(p $\rightarrow q$)
$\neg$(p $\rightarrow q$)	p$\wedge q$
(p $\rightarrow q) \wedge (p \rightarrow r)$	p $\rightarrow (q \wedge r)$
(p $\rightarrow r) \wedge (q \rightarrow r)$	(p$\vee q) \rightarrow r$
(p $\rightarrow q) \vee (p \rightarrow r)$	p $\rightarrow (q \vee r)$
(p $\rightarrow r) \vee (q \rightarrow r)$	(p$\wedge q) \rightarrow r$

1.3.3 Logical Equivalences Involving Biconditional Statements

p $\leftrightarrow q$	(p $\rightarrow q) \wedge (q \rightarrow p)$
p $\leftrightarrow q$	$\neg$p $\leftrightarrow q$
p $\leftrightarrow q$	(p$\wedge q) \vee (p \wedge q)$
$\neg$(p $\leftrightarrow q$)	p $\leftrightarrow q$

1.4 Predicates and Quantifiers

1.4.1 Quantifiers

- Universal Quantifer
  - $\forall$(x) P(x)
    - Definition: For all x in the universe P(x) is true;
    - Negation
      - $\neg\forall$(x) P(x) can also be written as $\exists\neg$ P(x)
      - There exists an x such that P(x) is false
- Existential Quantifer
  - $\exists$(x) P(x)
    - Definition: For all x in the universe P(x) is true for at least one x;
    - Negation
      - $\neg\exists$(x) P(x) can also be written as $\forall\neg$ P(x)
      - All x in the universe make P(x) false

1.5 Nested Quantifers

1.5.1 Nested Quantifiers can be used when there are multiple variables such as x and y

- $\forall$ (x) $\exists$ (y) (x + y = 0)

1.5.2 Quantification of Two Variables

Statement	When True	When False
$\forall(x)\forall(y)P(x,y)$	P(x,y) is true for every pair	When there is a x,y for which P(x,y) is false
$\forall(x)\exists(y)P(x,y)$	For every x there is a y for which P(x,y) is true	When there is an x such that P(x,y) is false for every y
$\exists(x)\forall(y)P(x,y)$	There is an x for which P(x,y) is true for every y	When for every x there is a y for which P(x,y) is false
$\exists(x)\exists(y)P(x,y)$ $\exists(y)\exists(x)P(x,y)$	There is a pair for x,y for which P(x,y) is true	P(x,y) is false for every pair of x and y

1.6 Rules of Inference

1.6.1 Rules of inference

Rule of Inference	Tautology	Name
p p $\to q$ ———— q	(p$\wedge(p \to q)) \to q$	Modus Ponens
¬q p $\to q$ ———— ¬p	(¬q $\wedge(p \to q)) \to p$	Modus Tollens
p $\to q$ q $\to r$ ———— p $\to r$	((p $\vee q) \wedge p) \to q$	Hypothetical Syllogism
p$\vee q$ ¬p ———— q	p $\to (p \vee q)$	Disjunctive Syllogism
p ———— p$\vee q$	p $\to (p \vee q)$	Addition
p$\wedge q$ ———— p	(p$\wedge q) \to p$	Simplification
p q ———— p$\wedge q$	((p)$\wedge(q)) \to (p \wedge q)$	Conjunction
p$\vee q$ ¬p$\vee r$ ———— q$\vee r$	((p$\vee q) \wedge (p \vee r)) \to (q \vee r)$	Conjucntion

1.6.2 Rules of Inference for Quantifies Statements

Rules of Inference	Name
$\forall(x)P(x)$ ————- P(c)	Universal Instantiation
P(x) for an Arbitrary c ————- $\forall(x)P(x)$	Universal Generalization
$\exists(x)P(x)$ ————- P(c) for some element c	Existential Instantiation
P(x) for some element c ————- $\exists P(x)$	Existential Generalization

1.7 Introduction to Proofs

1.7.1 There are 3 main methods for proofs.

- Direct Proof
  - Construct a conditional statement p $\rightarrow$ q
  - Assume p to be true
  - Use rules of inference to then show that when p is true q must be true (p true and q false can not happen)
- Proof by Contraposition
  - Construct a conditional statement p $\rightarrow$ q
  - Set up the contrapositive to be $\neg$q $\rightarrow$ $\neg$p
  - Prove that if $\neg$q is true then $\neg$p has to be true
- Proof by Contradiction
  - If we want to prove p is true set up the contradiction to be $\neg$p $\rightarrow$ q
  - Because q is false but $\neg$p $\rightarrow$ q is true we know is false which means p is true
  - You are assuming your premise to be false then attempting to show that the conditional statement is then false

1.8 Proof methods and Strategy

1.8.1 Exhaustive Proof

- Sometimes w cannot prove a theorem using a single argument that holds for all possible cases.
- Proofs by exhaustion use proof by cases for every element and check examples.
- Example
  - Prove that $(n+1)^3 \geq 3^n$ if n is a positive integer ≤ 4
  - To do this test for n = {1,2,3,4}
  - Since for n = {1,2,3,4} 3^n is greater we proved this statement through Proof by Exhaustion
- Proof by Cases
  - A proof by cases must cover all possible cases that arise in a theorem.
  - Example
    * Prove that if n is an integer than $n^2 \geq n$
    * We check 3 cases
    * 1) $n = 0$; $0^2 = 0$ which follows $n^2 \geq n$
    * 2) $n \geq 1$; Multiply both sides of the inequality $n \geq 1$ by the positive integer n. We get $n^2 > n*1$ for $n \geq 1$
    * 3) $n \leq 1$; Since $n^2 \geq 0$ it follows $n^2 \geq n$
- Leveraging Proof by Cases
  -
- Existence Proofs
  -
- Proof Stratagies
  - Forward and Backward Reasoning
  - Adapting Existing Proofs
  - Looking for Counterexamples

2 Chapter 2

2.1 Sets

2.1.1 Definition

- A set is an unordered collection of objects. a $\in$ A
- Set builder notation
  - Set builder notation is used to describe a set.
  - O = {x | x is a positive integer less than 10}
  - O = {x | x $<$ 10 $\wedge$ 2x+1 $\in$ Z+ }

2.1.2 Number sets

- N = {0,1,2,..,} - Set of Natural Numbers
- Z = {..., -1,0,1,. . . ,} - Set of Intgers
- Z+ = {1,2,3,. . . ,} - Set of Positive Integers
- Q = {p/q | p $\in$ Z, q /in Z, and q $\neq$ 0} - Set of Rational Numbers
- R = Set of Real Numbers
- R+ = Set of positive Real Numbers
- C = Set of complex numbers

2.1.3 Interval Notation

[a,b]	{a $\leq x \leq b$}
[a,b)	{a $\leq x\}b\}$
(a,b]	{a x $\leq b$}
(a,b)	{a x b}

2.1.4 Empty Set

- Denoted by $\emptyset$
- All sets contain the empty set although it does not count as a element when measuring cardinality

2.1.5 Subsets

- The set A is a subset of B iff every element of A is also in B. Denoted by A $\subseteq B$

2.1.6 Proper Subset

- The set A is a subset of B iff every element of A is also in B but A $\neq$ B. Denoted by A $\subset$ B

2.1.7 Set Cardinality

- If S is a set the cardinality of the set denoted by |S| is the number of UNIQUE elements.
  - Example S = {1,2,3,3,4,4,5} |S| = 5

2.2 Set Operations

2.3 Functions

2.4 Sequences and Summations

2.5 Cardinality of Sets

2.6 Matrices

$\emptyset$