studyguide

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1 Chapter 1

1.1 Propositional Logic

1.1.1 Converse Contrapositive and Inverse (p -> q)

- Coverse
  - $q \rightarrow p$
- Contrapositive
  - $q \rightarrow \neg p$
- Inverse
  - $\neg p \rightarrow \neg q$

1.2 Applications of Propositional Logic

1.2.1 Examples of turning sentences into propositional Logic.

1.3 Propositional Equivalences

1.3.1 Logical Equivalences

p$\wedge T$	p	Identity Laws
p$\vee F$	p	
p$\vee T$	T	Domination Laws
p$\wedge F$	F	
p$\vee p$	p	Idempotent Laws
p$\wedge p$	p	
$\neg$($\neg$p)	p	Double Negation Laws
p$\vee q$	q$\vee p$	Commutative Laws
p$\wedge q$	q $\wedge p$	
(p$\vee q) \vee r$	p$\vee(q \vee r)$	Associative Laws
(p$\wedge q) \wedge r$	p $\wedge(q \wedge r)$	
p$\vee(q \wedge r)$	(p$\vee q) \wedge (p \vee r)$	Distributive Laws
p $\wedge(q \vee r)$	(p$\wedge q) \vee (p \wedge r)$	
$\neg$(p$\wedge q$)	$\neg$p$\vee q$	De Morgans Laws
$\neg$(p v q)	$\neg$p$\wedge q$	
p$\vee(p \wedge q)$	p	Absorption Laws
p $\wedge(p \vee q)$	p	
p v $\neg$p	T	Negation Laws
p$\wedge p$	F	

1.3.2 Logical Equivelances Involving Conditional Statements

p $\rightarrow q$	$\neg$p$\vee q$
p $\rightarrow q$	$\neg$q $\rightarrow p$
p$\vee q$	$\neg$p $\rightarrow q$
p$\wedge q$	$\neg$(p $\rightarrow q$)
$\neg$(p $\rightarrow q$)	p$\wedge q$
(p $\rightarrow q) \wedge (p \rightarrow r)$	p $\rightarrow (q \wedge r)$
(p $\rightarrow r) \wedge (q \rightarrow r)$	(p$\vee q) \rightarrow r$
(p $\rightarrow q) \vee (p \rightarrow r)$	p $\rightarrow (q \vee r)$
(p $\rightarrow r) \vee (q \rightarrow r)$	(p$\wedge q) \rightarrow r$

1.3.3 Logical Equivalences Involving Biconditional Statements

$\mathrm{p} \leftrightarrow q$	$(\mathrm{p} \rightarrow q) \wedge (q \rightarrow p)$
$\mathrm{p} \leftrightarrow q$	$\neg\mathrm{p} \leftrightarrow q$
$\mathrm{p} \leftrightarrow q$	$(\mathrm{p} \wedge q) \vee (p \wedge q)$
$\neg(\mathrm{p} \leftrightarrow q)$	$\mathrm{p} \leftrightarrow q$

1.4 Predicates and Quantifiers

1.4.1 Quantifiers

- Universal Quantifer
  - ∀(x) P(x)
    - Definition: For all x in the universe P(x) is true;
    - Negation
      - ¬∀(x) P(x) can also be written as ∃¬ P(x)
      - There exists an x such that P(x) is false
- Existential Quantifer
  - ∃(x) P(x)
    - Definition: For all x in the universe P(x) is true for at least one x;
    - Negation
      - ¬∃(x) P(x) can also be written as ∀¬ P(x)
      - All x in the universe make P(x) false

1.5 Nested Quantifers

1.5.1 Nested Quantifiers can be used when there are multiple variables such as x and y

- ∀ (x) ∃ (y) (x + y = 0)

1.5.2 Quantification of Two Variables

Statement	When True	When False
$\forall(x)\forall(y)P(x,y)$	P(x,y) is true for every pair	When there is a x,y for which P(x,y) is false
$\forall(x)\exists(y)P(x,y)$	For every x there is a y for which P(x,y) is true	When there is an x such that P(x,y) is false for every y
$\exists(x)\forall(y)P(x,y)$	There is an x for which P(x,y) is true for every y	When for every x there is a y for which P(x,y) is false
$\exists(x)\exists(y)P(x,y)$ $\exists(y)\exists(x)P(x,y)$	There is a pair for x,y for which P(x,y) is true	P(x,y) is false for every pair of x and y

1.6 Rules of Inference

1.6.1 Rules of inference

Rule of Inference	Tautology	Name
p p $\rightarrow q$ ———— q	(p$\wedge(p \rightarrow q)) \rightarrow q$	Modus Ponens
¬q p $\rightarrow q$ ———— ¬p	(¬q $\wedge(p \rightarrow q)) \rightarrow p$	Modus Tollens
p $\rightarrow q$ q $\rightarrow r$ ———— p $\rightarrow r$	((p $\vee q) \wedge p) \rightarrow q$	Hypothetical Syllogism
p$\vee q$ ¬p ———— q	p $\rightarrow (p \vee q)$	Disjunctive Syllogism
p ———— p$\vee q$	p $\rightarrow (p \vee q)$	Addition
p$\wedge q$ ———— p	(p$\wedge q) \rightarrow p$	Simplification
p q ———— p$\wedge q$	((p)$\wedge(q)) \rightarrow (p \wedge q)$	Conjunction
p$\vee q$ ¬p$\vee r$ ———— q$\vee r$	((p$\vee q) \wedge (p \vee r)) \rightarrow (q \vee r)$	Conjucntion

1.6.2 Rules of Inference for Quantifies Statements

Rules of Inference	Name
$\forall(x)P(x)$ ————- P(c)	Universal Instantiation
P(x) for an Arbitrary c ————- $\forall(x)P(x)$	Universal Generalization
$\exists(x)P(x)$ ————- P(c) for some element c	Existential Instantiation
P(x) for some element c ————- $\exists P(x)$	Existential Generalization

1.7 Introduction to Proofs

1.7.1 There are 3 main methods for proofs.

- Direct Proof
  - Construct a conditional statement p → q
  - Assume p to be true
  - Use rules of inference to then show that when p is true q must be true (p true and q false can not happen)
- Proof by Contraposition
  - Construct a conditional statement p → q
  - Set up the contrapositive to be ¬q → ¬p
  - Prove that if ¬q is true then ¬p has to be true
- Proof by Contradiction
  - If we want to prove p is true set up the contradiction to be ¬p → q
  - Because q is false but ¬p → q is true we know is false which means p is true
  - You are assuming your premise to be false then attempting to show that the conditional statement is then false

1.8 Proof methods and Strategy

1.8.1 Exhaustive Proof

- Sometimes w cannot prove a theorem using a single argument that holds for all possible cases.
- Proofs by exhaustion use proof by cases for every element and check examples.
- Example
  - Prove that $(n+1)^3 \geq 3^n$ if n is a positive integer ≤ 4
  - To do this test for n = {1,2,3,4}
  - Since for n = {1,2,3,4} 3^n is greater we proved this statement through Proof by Exhaustion
- Proof by Cases
  - A proof by cases must cover all possible cases that arise in a theorem.
  - Example
    - Prove that if n is an integer than $n^2 \geq n$
    - We check 3 cases
    - 1) $n = 0$; $0^2 = 0$ which follows $n^2 \geq n$
    - 2) $n \geq 1$; Multiply both sides of the inequality $n \geq 1$ by the positive integer n. We get $n^2 > n*1$ for $n \geq 1$
    - 3) $n \leq 1$; Since $n^2 \geq 0$ it follows $n^2 \geq n$
- Leveraging Proof by Cases
  -
- Existence Proofs
  -
- Proof Stratagies
  - Forward and Backward Reasoning
  - Adapting Existing Proofs
  - Looking for Counterexamples

2 Chapter 2

2.1 Sets

2.1.1 Definition

- A set is an unordered collection of objects. a $\in$ A
- Set builder notation
  - Set builder notation is used to describe a set.
  - O = {x | x is a positive integer less than 10}
  - O = {x | x < 10 $\wedge$ 2x+1 $\in$ Z+ }

2.1.2 Number sets

- N = {0,1,2,..,} - Set of Natural Numbers
- Z = {..., -1,0,1,. . . ,} - Set of Intgers
- Z+ = {1,2,3,. . . ,} - Set of Positive Integers
- Q = {p/q | p $\in$ Z, q /in Z, and q $\neq$ 0} - Set of Rational Numbers
- R = Set of Real Numbers
- R+ = Set of positive Real Numbers
- C = Set of complex numbers

2.1.3 Interval Notation

[a,b]	{a $\leq x \leq b$}
[a,b)	{a $\leq x$}b}
(a,b]	{a x $\leq b$}
(a,b)	{a x b}

2.1.4 Empty Set

- Denoted by $\emptyset$
- All sets contain the empty set although it does not count as a element when measuring cardinality

2.1.5 Subsets

- The set A is a subset of B iff every element of A is also in B. Denoted by A $\subseteq B$

2.1.6 Proper Subset

- The set A is a subset of B iff every element of A is also in B but A $\neq$ B. Denoted by A $\subset$ B

2.1.7 Set Cardinality

- If S is a set the cardinality of the set denoted by |S| is the number of UNIQUE elements.
  - Example S = {1,2,3,3,4,4,5} |S| = 5

2.1.8 Power Sets

- Definition: Given the set S the power set of S i the set of all subsets of the set S. The powerset of S is denoted by P(S)
  - Example: What is the powerset of {0,1,2}
  - P(S) = {$\emptyset$, {0}, {1}, {2}, {0,1}, {0,2}, {1,2},{0,1,2},}

2.1.9 Cartesian Product

- Definition: The ordered n-tuple (a1,a2..an) is the ordered collection that has a1 as its first element, a2 as its second..and an as its nth element.
- A x B = {(a,b) | a $\in$ A $\wedge$ b $\in$ B}
- Example: What is the cartesian Product of A = {1,2} B = {a,b,c}
  - A x B = {(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)}
  - B x A = {(a, 1), (a, 2), (b, 1), (b, 2), (c, 1), (c, 2)}
- Example: What is the cartesian Product of A = {0,1} B = {1,2} C = {0,1,2}
  - A $\times$ B $\times$ C = {(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 0), (0, 2, 1), (0, 2, 2),(1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2)}.

2.1.10 Truth Set

- The truth set for a predicate P, and domain D the truth set of P is $\{x \in D \mid P(x)\}$
- Another way to phrase this is the Truth set is the set that makes a predicate true in the domain.

2.2 Set Operations

2.2.1 Union

- If A and B are sets the union of the sets A and B, denoted by $A \cup B$ is the set that contains elements in either or both A and B
- $A \cup B = \{x \mid x \in A \lor x \in B\}$

2.2.2 Intersection

- If A and B are sets, the intersection of the sets A and B, denoted by $A \cap B$, is the set containing elements in both A and B
- $A \cap B = \{x \mid x \in A \land x \in B\}$.

2.2.3 Disjoint

- Two sets are considered disjoint if they have no intersection or their intersection set is empty

2.2.4 Difference

- If A and B are sets, the difference of A and B denoted by A - B is the set elements in A that are not in B
- $A - B = \{x \mid x \in A \land x \notin B \}$

2.2.5 Complement

- If U is the universal set, the complement of the set A denoted by A(bar) is the set of U - A or all the elements in the universe not in A
- $A(bar) = \{x \in U \mid x \notin A\}$

2.2.6 Set Identities

p$\wedge U$	A	Identity Laws
p$\vee \emptyset$	A	
A$\vee T$	U	Domination Laws
A$\wedge F$	$\emptyset$	
A$\vee A$	A	Idempotent Laws
A$\wedge A$	A	
(A(bar)bar)	A	Double Negation Laws
A$\vee B$	B$\vee A$	Commutative Laws
A$\wedge B$	B $\wedge A$	
(A$\vee B) \vee C$	p$\vee (B \vee C)$	Associative Laws
(A$\wedge B) \wedge C$	A$\wedge (B \wedge C)$	
p$\vee (B \wedge C)$	(p$\vee B) \wedge (p \vee C)$	Distributive Laws
A$\wedge (B \vee C)$	(p$\wedge B) \vee (A \wedge C)$	
$\neg$(p$\wedge B$)	$\neg$p$\vee B$	De Morgans Laws
$\neg$(Av B)	$\neg$p$\wedge B$	
A$\vee (p \wedge B)$	A	Absorption Laws
A$\wedge (p \vee B)$	A	
A V A(bar)	U	Complement Laws
A & A(bar)	Empty Set	

2.3 Functions

2.3.1 Functions

- Let A and B be nonempty sets. A function f from A to B is an assignment of exactly one element of B to each element of A
- This also means that every element of A gets mapped to at exactly one set of B
- Everything in set A gets mapped to an element of set B
- (f1+f2)(x) = f1(x) + f2(x)

2.3.2 Image

- If f is a function that maps A to B, elements of B are called the image and the elements of A are called the preimage.

2.3.3 One to One (Injective)

- If f is a function that maps A to B, each a $\in$ A maps to a unique b $\in$ B

2.3.4 Onto

- If f is a function that maps A to B, the function is onto if all elements of the codomain are mapped to from values of A

2.3.5 One to one correspondence

- If f is a function that maps A to b, it has one to one correspondance if it is both one to one and onto

2.3.6 Inverse Functions

- If a function f has a one to one correspondance it has an inverse function f-1 that maps the codomain to the domain.b

2.3.7 Composition

- (f o f-1)(x) = x
- (f o g)(x) = f(g(x))
- (f + g)(x) = f(x) + g(x)
- (f * g)(x) = f(x) * g(x)

2.4 Sequences and Summations

2.4.1 Definition: Sequence is a function from a subset of the set of integers to a set S. They're also an ordered set.

2.4.2 Example:

- {1,2,3,4,5,6,. . . ,}

2.4.3 Geometric Sequence

- a, ar, ar^2,. . . ,ar^n
- Where a is the inital term and r is a ration

2.4.4 Arithmetic Progression

- a, a+d, a+2d,...,a+nd
- Where a is the initial term and d is the common difference

2.4.5 Recurrence Relations

- A reccurence relation for the sequence {an} is an equation that expresses an in terms of one or more of the previous terms.
- Example:
  - a0 = 0;
  - a1 = a(n-1)+1

2.4.6 We say that we have solved a recurrence relation together with the initial condition when we find a formula. We call this the closed formula.

- Example:
  - DO THIS

2.4.7 Summations

$\sum(k = 0 \to n)ar^k$	ar$(n+1) - a/(r-1)r \neq 1$
$\sum(k = 1 \to n)k$	n(n+1)/2
$\sum(k = 1 \to n)k^2$	n(n+1)(2n+1)/6
$\sum(k = 1 \to n)k^3$	n$^2(n+1)^2/4$
$\sum(k = 0 \to \text{inf})x^k abs(x) \leq 1$	1/(1-x)
$\sum(k = 0 \to \text{inf})kx(k-1)abs(x) \leq 1$	1/(1-x)^2

2.5 Cardinality of Sets

2.5.1 Definition: The sets A and B have the same cardinality iff there is a one to one correspondance from A to B, we write |A| = |B|

2.5.2 Countability

- A set that is either finite or has the same cardinality as the set of positive integers is called countable..

- If A and B are both countable sets then $A \cup B$ is also countable.