studyguide

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1 Chapter 1

1.1 Propositional Logic

1.1.1 Converse Contrapositive and Inverse (p -> q)

- Coverse
  - $q \rightarrow p$
- Contrapositive
  - $q \rightarrow \neg p$
- Inverse
  - $\neg p \rightarrow \neg q$

1.2 Applications of Propositional Logic

1.2.1 Examples of turning sentences into propositional Logic.

1.3 Propositional Equivalences

1.3.1 Logical Equivalences

p$\wedge T$	p	Identity Laws
p$\vee F$	p	
p$\vee T$	T	Domination Laws
p$\wedge F$	F	
p$\vee p$	p	Idempotent Laws
p$\wedge p$	p	
¬(¬p)	p	Double Negation Laws
p$\vee q$	q$\vee p$	Commutative Laws
p$\wedge q$	q $\wedge p$	
(p$\vee q) \vee r$	p$\vee(q \vee r)$	Associative Laws
(p$\wedge q) \wedge r$	p $\wedge(q \wedge r)$	
p$\vee(q \wedge r)$	(p$\vee q) \wedge (p \vee r)$	Distributive Laws
p $\wedge(q \vee r)$	(p$\wedge q) \vee (p \wedge r)$	
¬(p$\wedge q$)	¬p$\vee q$	De Morgans Laws
¬(p v q)	¬p$\wedge q$	
p$\vee(p \wedge q)$	p	Absorption Laws
p $\wedge(p \vee q)$	p	
p v ¬p	T	Negation Laws
p$\wedge p$	F	

1.3.2 Logical Equivelances Involving Conditional Statements

p $\rightarrow q$	¬p$\vee q$
p $\rightarrow q$	¬q $\rightarrow p$
p$\vee q$	¬p $\rightarrow q$
p$\wedge q$	¬(p $\rightarrow q$)
¬(p $\rightarrow q$)	p$\wedge q$
(p $\rightarrow q) \wedge (p \rightarrow r)$	p $\rightarrow (q \wedge r)$
(p $\rightarrow r) \wedge (q \rightarrow r)$	(p$\vee q) \rightarrow r$
(p $\rightarrow q) \vee (p \rightarrow r)$	p $\rightarrow (q \vee r)$
(p $\rightarrow r) \vee (q \rightarrow r)$	(p$\wedge q) \rightarrow r$

1.3.3 Logical Equivalences Involving Biconditional Statements

p $\leftrightarrow q$	(p $\rightarrow q) \wedge (q \rightarrow p)$
p $\leftrightarrow q$	$\neg$p $\leftrightarrow q$
p $\leftrightarrow q$	(p$\wedge q) \vee (p \wedge q)$
$\neg$(p $\leftrightarrow q$)	p $\leftrightarrow q$

1.4 Predicates and Quantifiers

1.4.1 Quantifiers

- Universal Quantifer
  - $\forall$(x) P(x)
    - Definition: For all x in the universe P(x) is true;
    - Negation
      - $\neg\forall$(x) P(x) can also be written as $\exists\neg$ P(x)
      - There exists an x such that P(x) is false
- Existential Quantifer
  - $\exists$(x) P(x)
    - Definition: For all x in the universe P(x) is true for at least one x;
    - Negation
      - $\neg\exists$(x) P(x) can also be written as $\forall\neg$ P(x)
      - All x in the universe make P(x) false

1.5 Nested Quantifers

1.5.1 Nested Quantifiers can be used when there are multiple variables such as x and y

- $\forall$ (x) $\exists$ (y) (x + y = 0)

1.5.2 Quantification of Two Variables

Statement	When True	When False
$\forall(x)\forall(y)P(x,y)$	P(x,y) is true for every pair	When there is a x,y for which P(x,y) is false
$\forall(x)\exists(y)P(x,y)$	For every x there is a y for which P(x,y) is true	When there is an x such that P(x,y) is false for every y
$\exists(x)\forall(y)P(x,y)$	There is an x for which P(x,y) is true for every y	When for every x there is a y for which P(x,y) is false
$\exists(x)\exists(y)P(x,y)$ $\exists(y)\exists(x)P(x,y)$	There is a pair for x,y for which P(x,y) is true	P(x,y) is false for every pair of x and y

1.6 Rules of Inference

1.6.1 Rules of inference

Rule of Inference	Tautology	Name
p $p \rightarrow q$ ———————— q	$(p \wedge (p \rightarrow q)) \rightarrow q$	Modus Ponens
$\neg$q $p \rightarrow q$ ———————— $\neg$p	$(\neg q \wedge (p \rightarrow q)) \rightarrow p$	Modus Tollens
$p \rightarrow q$ $q \rightarrow r$ ———————— $p \rightarrow r$	$((p \vee q) \wedge p) \rightarrow q$	Hypothetical Syllogism
$p \vee q$ $\neg$p ———————— q	$p \rightarrow (p \vee q)$	Disjunctive Syllogism
p ———————— $p \vee q$	$p \rightarrow (p \vee q)$	Addition
$p \wedge q$ ———————— p	$(p \wedge q) \rightarrow p$	Simplification
p q ———————— $p \wedge q$	$((p) \wedge (q)) \rightarrow (p \wedge q)$	Conjunction
$p \vee q$ $\neg p \vee r$ ———————— $q \vee r$	$((p \vee q) \wedge (p \vee r)) \rightarrow (q \vee r)$	Conjucntion

1.6.2 Rules of Inference for Quantifies Statements

Rules of Inference	Name
$\forall(x)P(x)$ ————- P(c)	Universal Instantiation
P(x) for an Arbitrary c ————- $\forall(x)P(x)$	Universal Generalization
$\exists(x)P(x)$ ————- P(c) for some element c	Existential Instantiation
P(x) for some element c ————- $\exists P(x)$	Existential Generalization

1.7 Introduction to Proofs

1.7.1 There are 3 main methods for proofs.

- Direct Proof
  - Construct a conditional statement p → q
  - Assume p to be true
  - Use rules of inference to then show that when p is true q must be true (p true and q false can not happen)
- Proof by Contraposition
  - Construct a conditional statement p → q
  - Set up the contrapositive to be ¬q → ¬p
  - Prove that if ¬q is true then ¬p has to be true
- Proof by Contradiction
  - If we want to prove p is true set up the contradiction to be ¬p → q
  - Because q is false but ¬p → q is true we know is false which means p is true
  - You are assuming your premise to be false then attempting to show that the conditional statement is then false

1.8 Proof methods and Strategy

1.8.1 Exhaustive Proof

- Sometimes w cannot prove a theorem using a single argument that holds for all possible cases.
- Proofs by exhaustion use proof by cases for every element and check examples.
- Example
  - Prove that $(n+1)^3 \geq 3^n$ if n is a positive integer ≤ 4
  - To do this test for n = {1,2,3,4}
  - Since for n = {1,2,3,4} 3^n is greater we proved this statement through Proof by Exhaustion
- Proof by Cases
  - A proof by cases must cover all possible cases that arise in a theorem.
  - Example
    - Prove that if n is an integer than $n^2 \geq n$
    - We check 3 cases
    - 1) $n = 0$; $0^2 = 0$ which follows $n^2 \geq n$
    - 2) $n \geq 1$; Multiply both sides of the inequality $n \geq 1$ by the positive integer n. We get $n^2 > n*1$ for $n \geq 1$
    - 3) $n \leq 1$; Since $n^2 \geq 0$ it follows $n^2 \geq n$
- Leveraging Proof by Cases
  -
- Existence Proofs
  -
- Proof Stratagies
  - Forward and Backward Reasoning
  - Adapting Existing Proofs
  - Looking for Counterexamples

2 Chapter 2

2.1 Sets

2.1.1 Definition

- A set is an unordered collection of objects. $a \in A$
- Set builder notation
  - Set builder notation is used to describe a set.
  - $O = \{x \mid x$ is a positive integer less than $10\}$
- $O = \{x \mid x < 10 \wedge 2x+1 \in ZZ^(+)\}$

2.1.2 Number sets

- $N = \{0,1,2,..,\}$ - Set of Natural Numbers
- $Z = \{..., -1,0,1,\ldots,\}$ - Set of Intgers
- $Z+ = \{1,2,3,\ldots,\}$ - Set of Positive Integers
- $Q = \{p/q \mid p \in Z$, q /in Z, and $q \neq 0\}$ - Set of Rational Numbers
- $R =$ Set of Real Numbers
- $R+ =$ Set of positive Real Numbers
- $C =$ Set of complex numbers

2.2 Set Operations

2.3 Functions

2.4 Sequences and Summations

2.5 Cardinality of Sets

2.6 Matrices

$\emptyset$