studyguide

Shantanu Vyas

October 17, 2016

Contents

Cha	apter 1	2
1.1	Propositional Logic	2
	1.1.1 Converse Contrapositive and Inverse $(p \rightarrow q) \dots$	2
1.2	Applications of Propositional Logic	3
	1.2.1 Examples of turning sentences into propositional Logic.	3
1.3	Propositional Equivalences	3
	1.3.1 Logical Equivalences	3
	1.3.2 Logical Equivelences Involving Conditional Statements	3
	1.3.3 Logical Equivalences Involving Biconditional Statements	4
1.4	Predicates and Quantifiers	4
	1.4.1 Quantifiers	4
1.5	Nested Quantifers	4
	1.5.1 Nested Quantifiers can be used when there are multiple	
	variables such as x and y	4
	1.5.2 Quantification of Two Variables	4
1.6	Rules of Inference	6
	1.6.1 Rules of inference are used to show that an argument	
	is valid.	6
1.7	Introduction to Proofs	7
1.8	Proof methods and Strategy	7
Cha	apter 2	7
2.1	Sets	7
2.2	Set Operations	7
2.3	Functions	7
2.4	Sequences and Summations	7
2.5	Cardinality of Sets	7
	1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 Cha 2.1 2.2 2.3 2.4	1.1.1 Converse Contrapositive and Inverse (p -> q) 1.2 Applications of Propositional Logic 1.2.1 Examples of turning sentences into propositional Logic. 1.3.1 Propositional Equivalences 1.3.1 Logical Equivalences 1.3.2 Logical Equivelances Involving Conditional Statements 1.3.3 Logical Equivalences Involving Biconditional Statements 1.4 Predicates and Quantifiers 1.4.1 Quantifiers 1.5.1 Nested Quantifiers can be used when there are multiple variables such as x and y 1.5.2 Quantification of Two Variables 1.6 Rules of Inference 1.6.1 Rules of inference are used to show that an argument is valid. 1.7 Introduction to Proofs 1.8 Proof methods and Strategy Chapter 2 2.1 Sets 2.2 Set Operations 2.3 Functions 2.4 Sequences and Summations

1 Chapter 1

1.1 Propositional Logic

- 1.1.1 Converse Contrapositive and Inverse (p -> q)
 - Coverse

$$- q \rightarrow p$$

• Contrapositive

$$- \ q \to \neg p$$

 \bullet Inverse

$$- \neg p \rightarrow \neg q$$

1.2 Applications of Propositional Logic

1.2.1 Examples of turning sentences into propositional Logic.

1.3 Propositional Equivalences

1.3.1 Logical Equivalences

$p \wedge T$	p	Identity Laws
$p \lor F$	p	
$p \lor T$	Τ	Domination Laws
$p \wedge F$	F	
$p \lor p$	р	Idempotent Laws
$p \land p$	p	
$\neg(\neg p)$	p	Double Negation Laws
$p \lor q$	$q \lor p$	Commutative Laws
$p \land q$	$q \wedge p$	
$(p \lor q) \lor r$	$p \lor (q \lor r)$	Associative Laws
$(p \land q) \land r$	$p \land (q \land r)$	
$p \lor (q \land r)$	$(p \lor q) \land (p \lor r)$	Distributive Laws
$p \land (q \lor r)$	$(p \land q) \lor (p \land r)$	
$\neg(p \land q)$	$\neg p \lor q$	De Morgans Laws
$\neg(p \ v \ q)$	$\neg p \land q$	
$p \lor (p \land q)$	p	Absorption Laws
$p \land (p \lor q)$	p	
р v ¬р	Τ	Negation Laws
$p \land p$	F	

1.3.2 Logical Equivelences Involving Conditional Statements

$p \rightarrow q$	$\neg p \lor q$
$p \rightarrow q$	$\neg \mathbf{q} \to p$
$p \lor q$	$\neg \mathbf{p} \to q$
$p \land q$	$\neg(p \to q)$
$\neg(p \to q)$	$p \land q$
$\overline{(p \to q) \land (p \to r)}$	$p \to (q \land r)$
$\overline{(\mathbf{p} \to r) \land (q \to r)}$	$(p \lor q) \to r$
$\overline{(p \to q) \lor (p \to r)}$	$p \to (q \lor r)$
$(p \to r) \lor (q \to r)$	$(p \land q) \to r$

1.3.3 Logical Equivalences Involving Biconditional Statements

$p \leftrightarrow q$	$(p \to q) \land (q \to p)$
$p \leftrightarrow q$	$\neg p \leftrightarrow q$
$p \leftrightarrow q$	$(p \land q) \lor (p \land q)$
$\neg(\mathbf{p}\leftrightarrow q)$	$p \leftrightarrow q$

1.4 Predicates and Quantifiers

1.4.1 Quantifiers

- Universal Quantifer
 - $\forall (x) P(x)$
 - * Definition: For all x in the universe P(x) is true;
 - * Negation
 - $\neg \forall (x) \ P(x) \ can also be written as \exists \neg P(x)$
 - There exists an x such that P(x) is false
- Existential Quantifer
 - $\ \exists (x) \ P(x)$
 - * Definition: For all x in the universe P(x) is true for at least one x;
 - * Negation
 - $\neg \exists (x) P(x) \text{ can also be written as } \forall \neg P(x)$
 - · All x in the universe make P(x) false

1.5 Nested Quantifers

- 1.5.1 Nested Quantifiers can be used when there are multiple variables such as x and y
 - \forall (x) \exists (y) (x + y = 0)
- 1.5.2 Quantification of Two Variables

Statement	When True	When False
$\forall (x) \forall (y) P(x,y)$	P(x,y) is true for every pair	When there is a x,y
		for which $P(x,y)$ is false
$\forall (x) \exists (y) P(x,y)$	For every x there is a y	When there is an x such
	for which $P(x,y)$ is true	that $P(x,y)$ is false for every y
$\exists (x) \forall (y) P(x,y)$	There is an x for which $P(x,y)$	When for every x there is a y
	is true for every y	for which $P(x,y)$ is false
$\exists (x)\exists (y)P(x,y)$	There is a pair for x,y	P(x,y) is false for
$\exists (y) \exists (x) P(x,y)$	for which $P(x,y)$ is true	every pair of x and y

1.6 Rules of Inference

1.6.1 Rules of inference are used to show that an argument is valid.

Rule of Inference	Tautology	Name
$\begin{array}{c} \mathbf{p} \\ \mathbf{p} \to q \end{array}$	$(p \land (p \to q)) \to q$	Modus Ponens
q ¬q	$(\neg q \land (p \to q)) \to p$	Modus Tollens
$\begin{array}{c} q \\ p \to q \\ \hline \end{array}$	$(\neg q \land (p \to q)) \to p$	Modus Tollens
$\frac{\neg p}{p \to q}$		
$p \to q$ $q \to r$	$((p \lor q) \land p) \to q$	Hypothetical Syllogism
$p \to r$		
$\frac{\mathbf{p} \to r}{\mathbf{p} \vee q}$	p $arrow(p \lor q)$	Disjunctive
¬p ————		Syllogism
q		
p 	$\mathbf{p} \to (p \vee q)$	Addition
$ \begin{array}{c} p \lor q \\ p \land q \end{array} $	$(p \land q) \to p$	Simplification
p		
p	$((p) \land (q)) \to (p \land q)$	Conjunction
q 		
$p \land q$		
$ \begin{array}{c} p \lor q \\ \neg p \lor r \end{array} $	$((p \lor q) \land (p \lor r)) \to (q \lor r)$	Conjuention
$q \lor r$		

- 1.7 Introduction to Proofs
- 1.8 Proof methods and Strategy
- 2 Chapter 2
- 2.1 Sets
- 2.2 Set Operations
- 2.3 Functions
- 2.4 Sequences and Summations
- 2.5 Cardinality of Sets
- 2.6 Matrices

 \emptyset