7.1 $H(z) = \frac{z}{z-p}$

c) For $p \in (0,1)$ we get Low pass Filter

As $p \to 1$, slope becomes steep

For $p \in (-1,0)$ we get High pass filter

As $p \to -1$, slope becomes steep.

d) There are 2 types of impulse responses possible because 2 different ROCs are possible.

For $\frac{z}{z-p}$, $p^n u[n]$ for $|z| > |p|$

$-p^n u[-n-1]$ for $|z| < |p|$

Matlab gives impulse response $p^n u[n]$ for $p = 0.9$

We get Low pass filter

Scanned with CamScanner

For $H(z) = \frac{z - p^{-1}}{z - p}$

c) For $p \in (0,1)$,

We get All pass filter

As $p \to 1$, we get some disturbance

Otherwise a straight line.

e) All pass filter

7.2 a) $\frac{z}{z^2 - (2r\cos\theta)z + r^2}$

Zeros $z = 0$

Poles $z^2 - (2r\cos\theta)z + r^2 = 0$

$$z = \frac{2r\cos\theta \pm \sqrt{4r^2\cos^2\theta - 4r^2}}{2}$$

$$z = r\cos\theta \pm r\sqrt{\cos^2\theta - 1}$$

$$z = r\left(e^{\pm i\theta}\right) = re^{i\theta}, re^{-i\theta}$$

Scanned with CamScanner

7.3

$$y[n] = 2.5\,y[n-1] - y[n-2] + x[n] - 5x[n-1] + 6x[n-2]$$

$$Y(z)\left(1 - 2.5z^{-1} + z^{-2}\right) = X(z)\left(1 - 5z^{-1} + 6z^{-2}\right)$$

$$H(z) = \frac{6z^{-2} - 5z^{-1} + 1}{z^{-2} - 2.5z^{-1} + 1}$$

a) Zeros $\quad 6 - 5z + z^2 = 0$

$$z = 2, 3$$

Poles $\quad 1 - 2.5z + z^2 = 0$

$$z = 2, \tfrac{1}{2}$$

b) $$H(z) = \frac{(z-2)(z-3)}{(z-2)(z-\frac{1}{2})} = \frac{z-3}{z-\frac{1}{2}}$$

Reduced-order difference eq^n

$$Y(z)\left(z - \tfrac{1}{2}\right) = X(z)(z-3)$$

$$Y(z)\left(1 - \tfrac{1}{2}z^{-1}\right) = X(z)\left(1 - 3z^{-1}\right)$$

$$y[n] - \tfrac{1}{2}y[n-1] = x[n] - 3x[n-1]$$

Scanned with CamScanner

c) $H(z) = \frac{z-3}{z-1/2} = \frac{z}{z-1/2} - 3\left(\frac{1}{z-1/2}\right)$

$h[n] = \left(\frac{1}{2}\right)^n u[n] - 3\left(\frac{1}{2}\right)^{n-1} u[n-1]$

ROC: $\boxed{|z| > 1/2}$

This ROC is required for causal system.

Scanned with CamScanner