6.1 X is periodic with period 2π

$$X(e^{j\omega}) = X(e^{j\omega + 2\pi j}) = X(e^{j\omega})$$

6.2 a)

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, e^{j\omega_0 (n-k)}$$

$$= e^{j\omega_0 n} \sum_{k=-\infty}^{\infty} h[k] e^{-j\omega_0 k} = e^{j\omega_0 n} H(e^{j\omega_0})$$

b)

$$y[n] = x[n] * h[n]$$

$$= \sum_k x[k] h[n-k]$$

$$\Rightarrow Y(e^{j\omega}) = \sum_n y[n] e^{-j\omega n}$$

$$= \sum_n \left(\sum_k x[k] h[n-k] \right) e^{-j\omega n}$$

$$= \left(\sum_k x[k] e^{-j\omega k} \right) \left(\sum_n h[n-k] e^{-j\omega (n-k)} \right)$$

$$= X(e^{j\omega}) H(e^{j\omega})$$

Hence proved

6.3 b) $Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left[\sum_{m=0}^{M} b_m\, x[n-m]\right] e^{-j\omega n}$

$- \sum_{n=-\infty}^{\infty} \left(\sum_{L=1}^{L} a_L\, y[n-L]\right) e^{-j\omega n}$

$Y(e^{j\omega}) = \left(\sum_{m=0}^{M} b_m e^{-j\omega m}\right) X(e^{j\omega}) - \left(\sum_{L=1}^{L} a_L e^{-j\omega L}\right) Y(e^{j\omega})$

$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \left(\sum_{m=0}^{M} b_m e^{-j\omega m}\right) \Big/ \left(1 + \sum_{L=1}^{L} a_L e^{-j\omega L}\right)$

c) $y[n] = x[n] + 0.9\,y[n-1]$

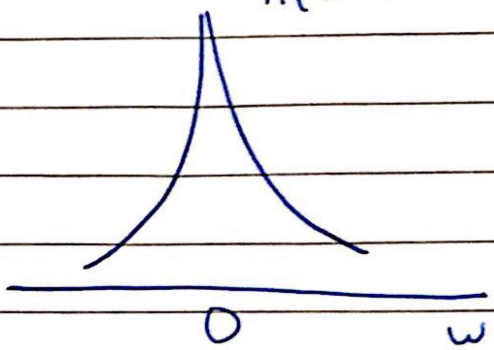
$Y(e^{j\omega}) = X(e^{j\omega}) + 0.9 e^{-j\omega} Y(e^{j\omega})$

$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = 1 \Big/ 1 - 0.9e^{-j\omega}$

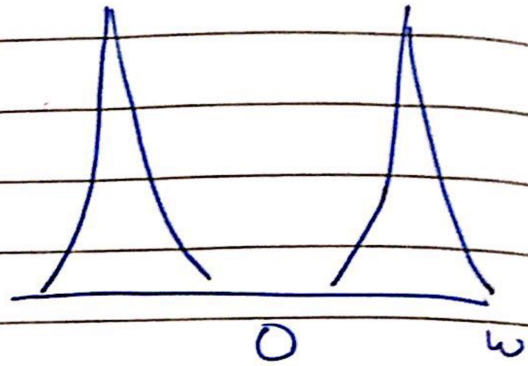
d) $Y(e^{j\omega}) = X(e^{j\omega}) - 0.9 e^{-j\omega}(Y(e^{j\omega}))$

$\Rightarrow \frac{Y(e^{j\omega})}{X(e^{j\omega})} = 1 \Big/ 1 + 0.9e^{-j\omega}$

e) Part (c) $H(e^{j\omega})$



Part (d) $H(e^{j\omega})$



(c) is Low pass filter

(d) is High pass filter

f) Impulse response

$\Rightarrow X(e^{j\omega}) = 1$

$\Rightarrow H(e^{j\omega}) =$ Impulse response

for part (c)

Impulse response $= H(e^{j\omega}) = \dfrac{1}{1-0.9e^{-j\omega}}$

$h[n] = (0.9)^n u[n]$

For part (d)

Impulse response $= H(e^{j\omega}) = \dfrac{1}{1+0.9e^{-j\omega}}$

$h[n] = (-0.9)^n u[n]$