Lab 2 – Fourier Series

2.1 Fourier series reconstruction

For a signal x(t) with Fourier series coefficients {a_k}, a partial Fourier sum (of order N) is given as

$$\hat{x}(t) = \sum_{k=-N}^{N} a_k \, \mathrm{e}^{jk\omega_0 t}$$

As the order N is increased the partial sum approaches the original signal x(t).

Write a matlab function, “partialfouriersum”, to compute a partial Fourier sum from a given vector of Fourier series coefficients { a_k }. Your function should take as input

- A, a 2*N+1 vector of Fourier Series coefficients { a(-N), ..., a(-1), a(0), a(1), ... a(N) }
- T, the period of the signal
- t, a time vector

and should return y, which corresponds to the Fourier series reconstruction y(t) obtained from computing the partial Fourier sum from -N to N. You can use the following template:

```
function y = partialfouriersum(A,T,t)
% Compute N based on the length of a
y = zeros(size(t));
for k=-N:N
    y = y + ...
end
end
```

While calling the function you can set T = 1 and t = -2:0.01:2. Call your function with various inputs and plot y(t) and show the output to the TAs. Here y(t) will be real or complex in general?

2.2 Square Wave

What are the Fourier Series coefficients { a_k } for a real, periodic square wave that has amplitude 1 in [-T1, T1] and period T? Note that we require T1 < T/2.

Write a matlab function “squareFS” that takes the following inputs:

- the signal period, T
- the value, T1
- a time vector, t
- number of coefficients, N

and returns the following outputs:

- A, a 2*N+1 vector which contains the Fourier Series coefficients { a(-N), ..., a(-1), a(0), a(1), ... a(N) } for the square wave
- y, where y(t) corresponds to the Fourier series reconstruction obtained by computing the partial sum from -N to N (you can use the function you wrote in Problem 2.1)
- sq, which corresponds to the ideal square wave sq(t) at the samples in t

You can use the following template:

```
function [A,y,sq] = squareFS(T,T1,t,N)
% First, compute A vector (via formula)
A = ...

% Compute Fourier partial sum (see Problem 1.1)
y = zeros(size(t));
for k=-N:N
    y = y + ...
end

% In this problem, we know the signal is real
y = real(y);

% Determine the ideal periodic square pulse at the given
time samples
sq = rectangularPulse...
end
```

(a) Plot and compare Fourier Series coefficients for fixed T1 and various values of T.
(b) For T1 = 0.1, T = 1, t = -0.5:0.01:0.5, plot y(t) and sq(t) in the same plot. Repeat this as N is changed (N = 10,25,100) and note your observations.

2.3 Fourier series approximation error

For a fixed set of input parameters, from the outputs of the above function compute the following two types of error:

- The maximum absolute error between y and sq
- The mean squared error between y and sq

Write a matlab script which computes and plots these two errors as N is increased from 1 to 100. What are your observations? Optional reading: Gibbs Phenomenon.