Lab 4 – Sampling and interpolation

4.1 Interpolation methods

In this lab we will use Matlab to sample and interpolate continuous-time signals. Since general continuous-time signals cannot be completely represented in Matlab, we will use a very fine time grid to approximate a continuous-time signal. Let `t_fine = 0:0.0001:2` be the fine grid for representing continuous time-signals in Matlab.

We will make use of the `interp1()` matlab function for interpolation. Read up the documentation and sample examples for this function.

Consider the continuous-time signal $x(t) = 1 + \sin(3\pi t) + \cos(5\pi t)$ for analysis. Let this signal be sampled with sampling interval Ts = 0.1s and be denoted by $x[n] = x(nT_s)$. Plot the given signal $x(t)$ (using the fine grid `t_fine`) and its samples $x[n]$ in the same plot. In this lab we will restrict to the time interval [0 2] as indicated by the end points of `t_fine`. In each of the following, plot the interpolated signals on top of this plot.

a) Given the sampled signal $x[n]$, plot the *sample and hold* approximation to $x(t)$. Use `interp1()` function to get this signal with appropriately selected 'method'. Note that the approximation signal should be computed over the fine grid `t_fine`.
b) From the given the sampled signal $x[n]$, plot the *linear interpolated* signal approximation to $x(t)$. Use `interp1()` function to get this signal.
c) From the given samples $x[n]$, find and plot the approximate *sinc interpolated* signal. Recall from the discussion in class, the ideal reconstruction using sinc function is given by the formula

$$x_r(t) = \sum_{n=-\infty}^{\infty} T_s\, x(nT_s) \frac{\sin\,(\omega_c(t - nT_s))}{\pi(t - nT_s)}$$

   For the low-pass filter corresponding to the sinc interpolation use a cut-off frequency of $\omega_c = \frac{\omega_s}{2}$ where $\omega_s = \frac{2\pi}{T_s}$. You might have to write a for-loop to do this interpolation. You can use the Matlab's inbuilt `sinc()` command [with appropriate time scaling to get required cut-off frequency]. Read up the documentation for this function before using it. Since the sinc function is of infinite duration, for computation purpose restrict each of the sinc in the summation to the interval [0 2].
   Is the quality of reconstruction uniform throughout the interval [0 2]? Justify.
d) For each of the above interpolation methods, compute the mean square error between the given signal and the interpolated signal.

4.2 Aliasing

Aliasing occurs when the sampling frequency is less than the Nyquist rate required by the sampling theorem. In this part we will look at the effect of aliasing for the signal $x(t) = \cos(5\pi t)$.

What is the Nyquist rate for this $x(t)$?

Consider samples of $x(t)$ for sampling intervals i) Ts = 0.1s, ii) Ts = 0.2s, and iii) Ts = 0.3s. For each of these cases perform sinc interpolation. What are your observations as Ts is changed?