Lab 6 – Discrete-time signals and systems

6.1 Discrete-time Fourier transform (DTFT)

Write a function dtft() that takes as inputs

- x, a discrete-time signal of finite duration (it will be assumed that the signal is zero outside the specified time)
- N_0, location of the sample x[0] in the given input signal x (note that $1 \leq N_0 \leq length(x)$ because matlab indexing starts from 1)
- ω, a vector of frequencies at which to sample and plot the DTFT

The function should return X, a complex vector corresponding to the DTFT sampled at the points in w. Make use of the template function below which also plots the DTFT vs. ω.

```
function X = dtft(x, N0, w)
% Evaluate the DTFT sum directly for each of the w samples
for i=1:length(w)
    X(i) = 0;
    for n=
        X(i) = X(i) + ...
    end
end
% Plot the DTFT magnitude and phase
figure;
subplot(211); plot(w, );
subplot(212); plot(w, );
% Plot the DTFT real and imaginary components
figure;
subplot(211); plot(w, );
subplot(212); plot(w, );
end
```

You can set $\omega = -3\pi: 0.01: 3\pi$ for plotting. Is X periodic? With what period?

Call your function with various discrete-time signals x as input including unit impulse ($\delta[n]$), shifted unit impulse ($\delta[n-k]$), rectangular pulse, sinusoid ($\sin(\omega_0 n)$), exponential ($a^n u[n]$), etc. Note that you should consider sinusoid and exponential signals for only some finite duration.

6.2 Eigenfunctions of LTI systems

a) For a discrete-time LTI system with impulse response $h[n]$, what is the output when the input is $x[n] = e^{j\omega_0 n}$?
b) Prove the convolution property for discrete-time LTI systems i.e. if
$$y[n] = x[n] * h[n], \text{ then}$$
$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega})$$
where $X(e^{j\omega})$ is the discrete-time Fourier transform (DTFT) of $x[n]$, etc.

6.3 Discrete-time LTI systems corresponding to difference equations

Consider a discrete-time system with input x[n] and output y[n] given by the input-output relation

$$y[n] = \sum_{m=0}^{M} b_m x[n-m] - \sum_{l=1}^{L} a_l y[n-l]$$

i.e. these are systems described by constant coefficient difference equations, a_l, b_m are constants.

a) Verify (analytically) that this system is an LTI system.
b) By taking Fourier transform on both sides, find the frequency response of this system.
c) Consider the following difference equation
$$y[n] = x[n] + 0.9\, y[n-1]$$
Find and plot the frequency response (magnitude and phase) of this system for frequency ω in the range $\omega = -3\pi{:}\,0.01{:}\,3\pi$.
d) Repeat (c) for the system corresponding to the difference equation
$$y[n] = x[n] - 0.9\, y[n-1].$$
e) The LTI systems in c) and d) can be interpreted as filters as well. Comment on the nature of the filters obtained in c) and d) i.e. low-pass, high-pass, band-pass, etc.
f) What are the impulse responses corresponding to these filters?