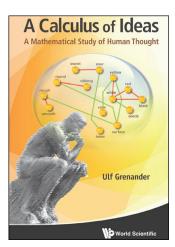
## **Book Review**



## A Calculus of Ideas: A Mathematical Study of Human Thought

Reviewed by Andrew I. Dale

A Calculus of Ideas: A Mathematical Study of Human Thought

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In 1963 Ulf Grenander published his *Probabilities on Algebraic Structures*. That book's introduction describes it as an attempt "to present a unified and coherent theory of the calculus of probability" on things like topological semigroups, topological vector spaces, and algebras. Continuing with some of the ideas emerging from this work (see [6, p. xix]), he presented "a mathematical model intended for the description and analysis of patterns" [4, p. 79] at a conference in Loutraki, Greece, in 1966. This signalled the early days of a study in pattern theory that has lasted for almost fifty years, the latest contribution (in book form) being *A Calculus of Ideas: A Mathematical Study of Human Thought*.

In the preface to this work Grenander states, "This monograph reports a thought experiment with a mathematical structure intended to illustrate the workings of a mind" [p. ix].¹ But his intent is not to present a *general* pattern of human thought. Rather, "We will try to present only a shell, a scheme only, of human thought that will have to be filled with content different for each individual, setting different values to the (many) mind parameters" [p. 4]. The book is thus written

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from a very personal point of view—even the dog Rufsan, whose name appears frequently, was Grenander's own dog.

Whether the human mind can be understood, and if so in what sense of "understood", is a difficult matter, and Grenander's aim here is to explore one facet of this problem. That aim is summarized in the abstract as follows:

We shall reduce mind activities to elementary ideas and mental operations on them. Connecting the ideas according to certain rules we will be led to an algebraic structure, one for each cultural sphere. Within such a sphere individuals are characterized by the usage they make of the mental concepts as expressed by a probability measure resulting in a calculus of ideas. [p. xi]

This is a bold and ambitious goal, but, unperturbed, Grenander pursues and reaches it.

By the way, a similar connection between algebra and mental activity was made in the late nineteenth century by Thomas Huxley, who, in his study of David Hume, wrote, "The nervous system is an apparatus for supplying us with a sort of algebra of fact, based on those symbols" [8, p. 96] (by "symbols" here, Huxley meant sensations).

In his Loutraki conference paper Grenander considered the basic "building blocks": signs, configurations and images, as related by similarity, synonymity, and deformation transformations. Using these blocks he discussed the breaking down of patterns into elementary parts, thus obtaining "a grammatical description of the image presented to the observer" [4, p. 79]. This enabled him "to express many notions we need in terms of simple and well-known concepts in elementary algebra" [4, p. 81]. In *A Calculus of Ideas* this earlier analysis is replaced by a synthesis: "Our goal is to build a model of the mind in pattern theoretic terms"

<sup>&</sup>lt;sup>1</sup> References to page numbers without a citation are to the book being reviewed. All words in italics in quotations from this book are given that way in the original.

[p. 15]. However, the constructions of Patterns of Thought (PoT) in the present work are based on the principles of General Pattern Theory (GPT—Grenander regarded his 1993 book of this name as his chef d'œuvre [7, p. 416]), and Grenander presents a useful summary of the needful.

Grenander's original terms have been changed slightly over the years; thus in the book under review the objects studied are graphs whose vertices are called *generators* and with the vertices joined by *bonds*. The generators, the building blocks whose combination is to be effected, constitute the substance of GPT, the possibility of change in these generators being governed by appropriate operations (e.g., elements of a similarity group).

Of course one also thinks in pictures. Francis Galton [2], for instance, described most carefully the ways (colors, position on a printed page, different fonts, etc.) in which some people "see" numbers in their minds. Plainly, then, ideas need not be presented in words. It is, however, important to note that one should try not to think of the generators as words or the graphs as sentences: "We shall deal with concepts—not words" [p. 21]. Furthermore, "Thinking comes before language, it is the primary mental activity" [p. 22]. Or as Huxley expressed it more than a century ago, "The elements of consciousness and the operations of the mental faculties, under discussion, exist independently of and antecedent to, the existence of language" [8, p. 121].

Grenander's idea that *ideas* and *words* are distinct is, however, somewhat at variance with the conclusions of other philosophers. For example, Bertrand Russell wrote:

Words and ideas are, in fact, interchangeable; both have meaning, and both have the same kind of causal relations to what they mean. The difference is that, in the case of words, the relation to what is meant is in the nature of a social convention, and is learnt by hearing speech, whereas in the case of ideas the relation is "natural", i.e., it does not depend upon the behaviour of other people, but upon intrinsic similarity and (one must suppose) upon physiological processes existing in all human beings, and to a lesser extent in the higher animals. [10, p. 111]

Finding a complete theory of the brain to be absent, Grenander bases his work here "on introspection and on what has been learned over the centuries in a less formal setting about the working of the mind by clinicians and what can be found in novels, poetry and plays...Expressed differently, our approach could perhaps be stated as studying the software of the mind rather than the hardware" [pp. 4–5].

In "An Architecture for the Mind", the first of the four parts into which the main text is divided, Grenander considers an Algebra of Thinking, pointing out that his intention is to search for answers to the following questions: "What are the mental objects that make up the mind? What are the mental operations that act upon these objects? How do these objects combine to form thoughts?" [p. 16]. Subsequently, he outlines his Algebra of Human Thought with an axiomatic description of the algebra, with thoughts being framed as compositions of generators (primitive ideas). These generators are perhaps best represented by an organizational tree. Thus, for instance, the sentence "John asks what is the cat's name" is represented by a tree with "question3" at the top (the "3" indicating that three arrows emerge from this generator), with arrows leading to "John", "cat", and "name". As further illustration, Grenander presents some trees drawn from passages in Goethe's Die Wahlverwandtschaften (Elective affinities).

More specifically, we have a group G of generators, a collection M of modalities (bond values), and a set of regular thoughts (definition omitted in this review) of regularity R (and how many of us, given the generators "agent" and "James", wouldn't feel an almost irresistible urge to choose a bond value of 007). In this setting the set of all such regular thoughts is called  $\{MIND(R), P\}$ , where P is a probability on MIND(R). MIND then "represents all the thoughts that are possible currently, whether likely to occur or not" [p. 41].

In part two, "Personality of a Mind", Grenander introduces an idea function Q on G taking on positive values. A large value of Q(q) indicates that the primitive idea g is likely. A positive acceptor or association function  $A(q_1, q_2)$  is defined which measures the likeliness of direct association between ideas  $g_1$  and  $g_2$  in the thinking of MIND. In terms of these two functions and a normalizing function Z (the partition function of statistical mechanics), the *mind equation* is set up for the prior probability p(thought). Note that, in [5, p. 2] Grenander wrote, in connection with a very similar formula, "Z is notoriously difficult to calculate except for the simplest cases." Note too that "the Q's and A's determine the character of an individual mind" [p. 45].

"Two Personalities" is the title of the third part. Here Grenander explores the building of a thinking machine called a GOLEM. (In Jewish folklore the Golem is an animated being created from inanimate matter and having human form. Many stories about such creatures exist, but in general Golems are seen as rather unintelligent "beings" which, when given instructions, carry them out literally.) The GOLEM code (using MATLAB as the programming language) is given in an appendix. While the best way to

understand GOLEM is to work through the program, Grenander describes in the main text some features of the functioning of MIND in this case: free associations, inferential thinking, associations driven by themes and continuous thought are all investigated. Grenander is refreshingly honest when, in considering the behavior of GOLEM, he writes, "The code is working but it does not work well" [p. 99], but he nevertheless concludes that "The human mind can be understood" [p. 100].

Some consideration is given to the selection of the mind parameters G, Q, A, ... used in the programming, and Grenander points out that the representation he uses is that of his own mind. Here he introduces more software, called LEGACY.

In his fourth and final part Grenander considers the relation of GOLEM and LEGACY to actual human thinking (or, if you prefer, between MIND and brain). We build our way from simple ideas and their connections through thoughts, weight functions Q, and acceptor functions A to joint probability measures for thoughts. On the way, the conclusion is reached that "MIND achieves conceptual inference via conditional probabilities" [p. 128].

In Chapter 7 Grenander indulges himself, becoming "less systematic, more free-wheeling" [p. 153] than before. He considers matters such as introspection, as well as the thinking (better: logical) machine developed in the thirteenth century by Raymond Lully (Raymundus Lullus, Ramon Llull). This device comprised a number of concentric circular discs that, on rotation, yielded theological statements.

The final chapter contains some doubts and certainties and also lists "14 basic rules for thinking about thinking" [p. 165], of which we mention the following: "[1.] Thoughts are made up of discrete entities: ideas. [10.] Thoughts are created probabilistically by the mind equation. [14.] The high level study of thinking should take place in mind space, not physical space" [pp. 165–6].

Grenander suggests that "we feel that we have proposed a cohesive theory with some credibility of how the human mind works" [p. 165]. He might take some consolation from the words of Alan Turing: "We can only see a short distance ahead, but we can see plenty there that needs to be done" [11, p. 460].

A number of papers on matters of conjecture on physical and mathematical topics were published in European journals in the seventeenth and eighteenth centuries in sections entitled *Classe de Philosophie spéculative*. In some respects *A Calculus of Ideas* can also be seen as "speculative philosophy". Indeed, speaking in general of his work in pattern theory, Grenander once said, "My

present approach is completely speculative, based on introspection rather than hard data" [7, p. 417].

In several instances Grenander illustrates his investigations here with literary references, and if we go away after having read this book with nothing more than an awakened interest in ourselves, let me suggest one further such reference, this time from the conclusion of J. B. Priestley's essay "All the News", as given in [9, p. 231]: "I am also wondering what, after all, *is* the pattern of my mind."

Readers of the book will encounter ideas from a wide range of areas, including mathematics, artificial intelligence, psychology, neural networks, computer science, and philosophy. But the book is not really a text in any of these disciplines. In his interview with Nitis Mukhopadhyay, Grenander said, "I refer to pattern theory as the intellectual adventure of my life" [7, p. 416]. The adventure is clearly by no means over, and the reader of *A Calculus of Ideas* will fully enjoy the excitement of joining Grenander in his enthralling enterprise.

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