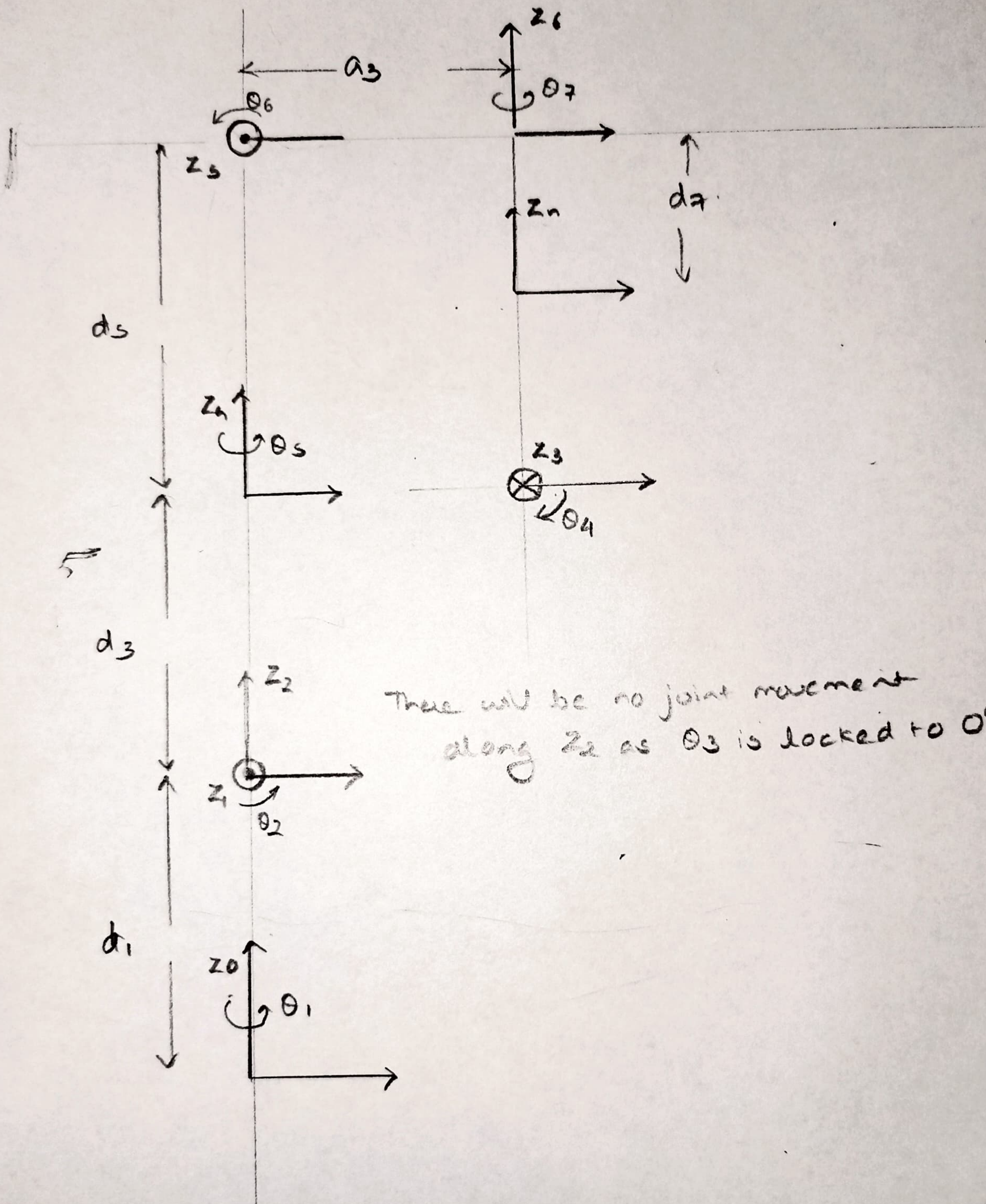


There will be no joint movement along Z_2 as θ_3 is locked to 0°

sparab_hw4_report

November 14, 2022

0.0.1 Importing Important Libraries

[25]:
```
import math
import sympy  as sp
from sympy.matrices import Matrix
from functools import partial
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits import mplot3d
from math import pi
import pprint
pp=pprint.PrettyPrinter(indent=5)
```

Declaring Variables

[26]:
```
theta_i, alpha_i, d_i, a_i, A_i, a_3, d_1, d_3, d_5, d_7 = sp.symbols('theta_i␣
 ↪alpha_i d_i a_i A_i a_3 d_1, d_3, d_5, d_7')
theta_1,theta_2,theta_3,theta_4,theta_5,theta_6,theta_7 = sp.symbols␣
 ↪('theta_1,theta_2, theta_3, theta_4, theta_5, theta_6, theta_7')
```

Rotation and Translation Matrices

[27]:
```
Rot_z = sp.Matrix([ [sp.cos(theta_i), -sp.sin(theta_i),0,0], [sp.
 ↪sin(theta_i),sp.cos(theta_i),0,0], [0,0,1,0], [0,0,0,1] ]);
Rot_x = sp.Matrix([ [1,0,0,0], [0,sp.cos(alpha_i), -sp.sin(alpha_i),0], [0, sp.
 ↪sin(alpha_i), sp.cos(alpha_i), 0], [0,0,0,1] ]);
Tran_z = sp.Matrix([[1,0,0,0], [0,1,0,0], [0,0,1,d_i], [0,0,0,1]]);
Tran_x = sp.Matrix([[1,0,0,a_i], [0,1,0,0], [0,0,1,0], [0,0,0,1]]);
print("\nRotation Matrix for rotation about Z")
pp.pprint(Rot_z)
print("\nRotation Matrix for rotation about X")
pp.pprint(Rot_x)
print("\nTranslation Matrix for translation about Z")
pp.pprint(Tran_z)
print("\nTranslation Matrix for translation about X")
pp.pprint(Tran_x)
```

```
Rotation Matrix for rotation about Z
```

```
Matrix([
[cos(theta_i), -sin(theta_i), 0, 0],
[sin(theta_i),  cos(theta_i), 0, 0],
[           0,             0, 1, 0],
[           0,             0, 0, 1]])

Rotation Matrix for rotation about X
Matrix([
[1,            0,             0, 0],
[0, cos(alpha_i), -sin(alpha_i), 0],
[0, sin(alpha_i),  cos(alpha_i), 0],
[0,            0,             0, 1]])

Translation Matrix for translation about Z
Matrix([
[1, 0, 0,   0],
[0, 1, 0,   0],
[0, 0, 1, d_i],
[0, 0, 0,   1]])

Translation Matrix for translation about X
Matrix([
[1, 0, 0, a_i],
[0, 1, 0,   0],
[0, 0, 1,   0],
[0, 0, 0,   1]])
```

General Homogeneous Matrix

[28]:
```
A_i=Rot_z*Tran_z*Tran_x*Rot_x;
pp.pprint(A_i)
```

```
Matrix([
[cos(theta_i), -sin(theta_i)*cos(alpha_i),  sin(alpha_i)*sin(theta_i),
a_i*cos(theta_i)],
[sin(theta_i),  cos(alpha_i)*cos(theta_i), -sin(alpha_i)*cos(theta_i),
a_i*sin(theta_i)],
[           0,               sin(alpha_i),               cos(alpha_i),
d_i],
[           0,                          0,                          0,
1]])
```

0.0.2 DH Parameter Table for Fixed θ_3

Link	a_i	θ_i	α_i	d_i
1	0	θ_1^*	90	d_1
2	0	θ_2^*	-90	0
3	a_3	0	-90	d_3

Link	a_i	θ_i	α_i	d_i
4	$-a_3$	θ_4^*	90	0
5	0	θ_5^*	90	d_5
6	a_3	θ_6^*	-90	0
7	0	θ_7^*	0	$-d_7$

Homogenous Matrix A1

[202]:
```
A1=A_i.subs([(theta_i,theta_1),(alpha_i,math.radians(90)),(a_i,0),(d_i,d_1)])
A1
```

[202]:

$$\begin{bmatrix} \cos(\theta_1) & -6.12323399573677 \cdot 10^{-17} \sin(\theta_1) & 1.0 \sin(\theta_1) & 0 \\ \sin(\theta_1) & 6.12323399573677 \cdot 10^{-17} \cos(\theta_1) & -1.0 \cos(\theta_1) & 0 \\ 0 & 1.0 & 6.12323399573677 \cdot 10^{-17} & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Homogenous Matrix A2

[ ]:
```
A2=A_i.subs([(theta_i,theta_2),(alpha_i,math.radians(-90)),(a_i,0),(d_i,0)])
A2
```

Homogenous Matrix A3

[ ]:
```
A3=A_i.subs([(theta_i,math.radians(0)),(alpha_i,math.
 ↪radians(-90)),(a_i,a_3),(d_i,d_3)])
A3
```

Homogenous Matrix A4

[ ]:
```
A4=A_i.subs([(theta_i,theta_4),(alpha_i,math.radians(90)),(a_i,-a_3),(d_i,0)])
A4
```

Homogenous Matrix A5

[ ]:
```
A5=A_i.subs([(theta_i,theta_5),(alpha_i,math.radians(90)),(a_i,0),(d_i,d_5)])
A5
```

Homogenous Matrix A6

[ ]:
```
A6=A_i.subs([(theta_i,theta_6),(alpha_i,math.radians(-90)),(a_i,a_3),(d_i,0)])
A6
```

Homogenous Matrix A7

[ ]:
```
A7=A_i.subs([(theta_i,theta_7),(alpha_i,math.radians(0)),(a_i,0),(d_i,-d_7)])
A7
```

0.0.3 Steps to Get the Jacobian Matrix usinh Method 2

Transformation Matrix T_e^0 from O_0 to O_e

[217]:
```
T1=A1
T2=A1*A2
T3=A1*A2*A3
T4=A1*A2*A3*A4
T5=A1*A2*A3*A4*A5
T6=A1*A2*A3*A4*A5*A6
T7=A1*A2*A3*A4*A5*A6*A7
T = T7.subs([(theta_1,0),(theta_2,0),(theta_4,math.radians(90))
             ,(theta_5,0),(theta_6, math.radians(180)),(theta_7,0)])
T=T.subs([(d_1,0.3330),(d_3,0.3160),(d_5,0.3840),(d_7,0.2070),(a_3,0.0880)]);
print("Transformation Matrix with joint angle set to required position")
T
```

```
Transformation Matrix with joint angle set to required position
```

[217]:

$$\begin{bmatrix} 6.12323399573677 \cdot 10^{-17} & -1.83697019872103 \cdot 10^{-16} & -1.0 & 0.679 \\ -6.12323399573676 \cdot 10^{-17} & 1.0 & -1.83697019872103 \cdot 10^{-16} & 5.07616098246578 \cdot 10^{-17} \\ 1.0 & 6.12323399573677 \cdot 10^{-17} & 6.12323399573677 \cdot 10^{-17} & 0.825 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Calculating the Z vector for all links

[218]:
```
print ("The Z0 matrix is given:")
Z0 = T1[:3,2]
pp.pprint(Z0)
```

```
The Z0 matrix is given:
Matrix([
[    1.0*sin(theta_1)],
[   -1.0*cos(theta_1)],
[6.12323399573677e-17]])
```

[219]:
```
print ("The Z1 matrix is given:")
Z1 = T2[:3,2]
pp.pprint(Z1)
```

```
The Z1 matrix is given:
Matrix([
[-6.12323399573677e-17*sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_1) - 1.0*sin(theta_2)*cos(theta_1)],
[-1.0*sin(theta_1)*sin(theta_2) + 6.12323399573677e-17*cos(theta_1)*cos(theta_2)
- 6.12323399573677e-17*cos(theta_1)],
[
1.0*cos(theta_2) + 3.74939945665464e-33]])
```

[215]:
```python
print ("The Z2 matrix is given:")
Z2 = T4[:3,2]
pp.pprint(Z2)
```

```
The Z2 matrix is given:
Matrix([
[1.0*(-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*sin(theta_4) -
1.0*(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) + 1.0*sin(theta_2)*cos(theta_1))*cos(theta_4)
- 4.59169004331693e-49*sin(theta_1)*cos(theta_2) -
6.12323399573677e-17*sin(theta_1) -
7.49879891330929e-33*sin(theta_2)*cos(theta_1)],
[ 1.0*(sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*sin(theta_4) -
1.0*(1.0*sin(theta_1)*sin(theta_2) -
6.12323399573677e-17*cos(theta_1)*cos(theta_2) +
1.22464679914735e-16*cos(theta_1))*cos(theta_4) -
7.49879891330929e-33*sin(theta_1)*sin(theta_2) +
4.59169004331693e-49*cos(theta_1)*cos(theta_2) +
6.12323399573677e-17*cos(theta_1)],
[
-1.0*(-1.0*cos(theta_2) - 7.49879891330929e-33)*cos(theta_4) +
1.0*sin(theta_2)*sin(theta_4) + 7.49879891330929e-33*cos(theta_2) -
3.74939945665464e-33]])
```

[143]:
```python
print ("The Z4 matrix is given:")
Z4 = T5[:3,2]
pp.pprint(Z4)
```

```
The Z3 matrix is given:
```

[143]:

$$\begin{bmatrix} 1.0\left(\left(-6.12323399573677\cdot 10^{-17}\sin(\theta_1)\sin(\theta_2)+\cos(\theta_1)\cos(\theta_2)\right)\cos(\theta_4)+\left(6.12323399573677\cdot 10^{-17}\sin(\theta_1)\,\mathrm{co}\ldots\right.\right. \\ 1.0\left(\left(\sin(\theta_1)\cos(\theta_2)+6.12323399573677\cdot 10^{-17}\sin(\theta_2)\cos(\theta_1)\right)\cos(\theta_4)+\left(1.0\sin(\theta_1)\sin(\theta_2)-6.1232339957\ldots\right.\right. \\ \end{bmatrix}$$

[207]:
```python
print ("The Z5 matrix is given:")
Z5 = T6[:3,2]
pp.pprint(Z5)
```

```
The Z5 matrix is given:
Matrix([
[-1.0*(((-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*cos(theta_4) +
(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) +
1.0*sin(theta_2)*cos(theta_1))*sin(theta_4))*cos(theta_5) +
```

```
(-6.12323399573677e-17*(-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*sin(theta_4) +
6.12323399573677e-17*(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) + 1.0*sin(theta_2)*cos(theta_1))*cos(theta_4)
- 7.49879891330929e-33*sin(theta_1)*cos(theta_2) - 1.0*sin(theta_1) -
1.22464679914735e-16*sin(theta_2)*cos(theta_1))*sin(theta_5))*sin(theta_6) +
6.12323399573677e-17*((-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*cos(theta_4) +
(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) +
1.0*sin(theta_2)*cos(theta_1))*sin(theta_4))*sin(theta_5) +
3.74939945665464e-33*(-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*sin(theta_4) -
3.74939945665464e-33*(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) + 1.0*sin(theta_2)*cos(theta_1))*cos(theta_4)
- 6.12323399573677e-17*(-6.12323399573677e-17*(-6.12323399573677e-17*sin(theta_1
)*sin(theta_2) + cos(theta_1)*cos(theta_2))*sin(theta_4) +
6.12323399573677e-17*(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) + 1.0*sin(theta_2)*cos(theta_1))*cos(theta_4)
- 7.49879891330929e-33*sin(theta_1)*cos(theta_2) - 1.0*sin(theta_1) -
1.22464679914735e-16*sin(theta_2)*cos(theta_1))*cos(theta_5) +
1.0*(-6.12323399573677e-17*((-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*cos(theta_4) +
(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) +
1.0*sin(theta_2)*cos(theta_1))*sin(theta_4))*sin(theta_5) +
1.0*(-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*sin(theta_4) -
1.0*(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) + 1.0*sin(theta_2)*cos(theta_1))*cos(theta_4)
+ 6.12323399573677e-17*(-6.12323399573677e-17*(-6.12323399573677e-17*sin(theta_1
)*sin(theta_2) + cos(theta_1)*cos(theta_2))*sin(theta_4) +
6.12323399573677e-17*(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) + 1.0*sin(theta_2)*cos(theta_1))*cos(theta_4)
- 7.49879891330929e-33*sin(theta_1)*cos(theta_2) - 1.0*sin(theta_1) -
1.22464679914735e-16*sin(theta_2)*cos(theta_1))*cos(theta_5) -
4.59169004331693e-49*sin(theta_1)*cos(theta_2) -
6.12323399573677e-17*sin(theta_1) -
7.49879891330929e-33*sin(theta_2)*cos(theta_1))*cos(theta_6) -
1.72160801535391e-81*sin(theta_1)*cos(theta_2) -
2.29584502165847e-49*sin(theta_1) -
2.81159925711243e-65*sin(theta_2)*cos(theta_1)],
[        -1.0*(((sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*cos(theta_4) +
(1.0*sin(theta_1)*sin(theta_2) - 6.12323399573677e-17*cos(theta_1)*cos(theta_2)
+ 1.22464679914735e-16*cos(theta_1))*sin(theta_4))*cos(theta_5) +
(-6.12323399573677e-17*(sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*sin(theta_4) +
```

```
6.12323399573677e-17*(1.0*sin(theta_1)*sin(theta_2) -
6.12323399573677e-17*cos(theta_1)*cos(theta_2) +
1.22464679914735e-16*cos(theta_1))*cos(theta_4) -
1.22464679914735e-16*sin(theta_1)*sin(theta_2) +
7.49879891330929e-33*cos(theta_1)*cos(theta_2) +
1.0*cos(theta_1))*sin(theta_5))*sin(theta_6) +
6.12323399573677e-17*((sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*cos(theta_4) +
(1.0*sin(theta_1)*sin(theta_2) - 6.12323399573677e-17*cos(theta_1)*cos(theta_2)
+ 1.22464679914735e-16*cos(theta_1))*sin(theta_4))*sin(theta_5) +
3.74939945665464e-33*(sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*sin(theta_4) -
3.74939945665464e-33*(1.0*sin(theta_1)*sin(theta_2) -
6.12323399573677e-17*cos(theta_1)*cos(theta_2) +
1.22464679914735e-16*cos(theta_1))*cos(theta_4) -
6.12323399573677e-17*(-6.12323399573677e-17*(sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*sin(theta_4) +
6.12323399573677e-17*(1.0*sin(theta_1)*sin(theta_2) -
6.12323399573677e-17*cos(theta_1)*cos(theta_2) +
1.22464679914735e-16*cos(theta_1))*cos(theta_4) -
1.22464679914735e-16*sin(theta_1)*sin(theta_2) +
7.49879891330929e-33*cos(theta_1)*cos(theta_2) + 1.0*cos(theta_1))*cos(theta_5)
+ 1.0*(-6.12323399573677e-17*((sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*cos(theta_4) +
(1.0*sin(theta_1)*sin(theta_2) - 6.12323399573677e-17*cos(theta_1)*cos(theta_2)
+ 1.22464679914735e-16*cos(theta_1))*sin(theta_4))*sin(theta_5) +
1.0*(sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*sin(theta_4) -
1.0*(1.0*sin(theta_1)*sin(theta_2) -
6.12323399573677e-17*cos(theta_1)*cos(theta_2) +
1.22464679914735e-16*cos(theta_1))*cos(theta_4) +
6.12323399573677e-17*(-6.12323399573677e-17*(sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*sin(theta_4) +
6.12323399573677e-17*(1.0*sin(theta_1)*sin(theta_2) -
6.12323399573677e-17*cos(theta_1)*cos(theta_2) +
1.22464679914735e-16*cos(theta_1))*cos(theta_4) -
1.22464679914735e-16*sin(theta_1)*sin(theta_2) +
7.49879891330929e-33*cos(theta_1)*cos(theta_2) + 1.0*cos(theta_1))*cos(theta_5)
- 7.49879891330929e-33*sin(theta_1)*sin(theta_2) +
4.59169004331693e-49*cos(theta_1)*cos(theta_2) +
6.12323399573677e-17*cos(theta_1))*cos(theta_6) -
2.81159925711243e-65*sin(theta_1)*sin(theta_2) +
1.72160801535391e-81*cos(theta_1)*cos(theta_2) +
2.29584502165847e-49*cos(theta_1)],
[
-1.0*(((-1.0*cos(theta_2) - 7.49879891330929e-33)*sin(theta_4) +
1.0*sin(theta_2)*cos(theta_4))*cos(theta_5) +
(6.12323399573677e-17*(-1.0*cos(theta_2) - 7.49879891330929e-33)*cos(theta_4) -
```

```
6.12323399573677e-17*sin(theta_2)*sin(theta_4) +
1.22464679914735e-16*cos(theta_2) -
6.12323399573677e-17)*sin(theta_5))*sin(theta_6) +
6.12323399573677e-17*((-1.0*cos(theta_2) - 7.49879891330929e-33)*sin(theta_4) +
1.0*sin(theta_2)*cos(theta_4))*sin(theta_5) -
3.74939945665464e-33*(-1.0*cos(theta_2) - 7.49879891330929e-33)*cos(theta_4) -
6.12323399573677e-17*(6.12323399573677e-17*(-1.0*cos(theta_2) -
7.49879891330929e-33)*cos(theta_4) -
6.12323399573677e-17*sin(theta_2)*sin(theta_4) +
1.22464679914735e-16*cos(theta_2) - 6.12323399573677e-17)*cos(theta_5) +
1.0*(-6.12323399573677e-17*((-1.0*cos(theta_2) -
7.49879891330929e-33)*sin(theta_4) + 1.0*sin(theta_2)*cos(theta_4))*sin(theta_5)
- 1.0*(-1.0*cos(theta_2) - 7.49879891330929e-33)*cos(theta_4) +
6.12323399573677e-17*(6.12323399573677e-17*(-1.0*cos(theta_2) -
7.49879891330929e-33)*cos(theta_4) -
6.12323399573677e-17*sin(theta_2)*sin(theta_4) +
1.22464679914735e-16*cos(theta_2) - 6.12323399573677e-17)*cos(theta_5) +
1.0*sin(theta_2)*sin(theta_4) + 7.49879891330929e-33*cos(theta_2) -
3.74939945665464e-33)*cos(theta_6) +
3.74939945665464e-33*sin(theta_2)*sin(theta_4) +
2.81159925711243e-65*cos(theta_2) - 1.40579962855621e-65]])
```

[208]:
```python
print ("The Z6 matrix is given:")
Z6 = T7[:3,2]
pp.pprint(Z6)
```

```
The Z6 matrix is given:
Matrix([
[-1.0*(((-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*cos(theta_4) +
(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) +
1.0*sin(theta_2)*cos(theta_1))*sin(theta_4))*cos(theta_5) +
(-6.12323399573677e-17*(-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*sin(theta_4) +
6.12323399573677e-17*(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) + 1.0*sin(theta_2)*cos(theta_1))*cos(theta_4)
- 7.49879891330929e-33*sin(theta_1)*cos(theta_2) - 1.0*sin(theta_1) -
1.22464679914735e-16*sin(theta_2)*cos(theta_1))*sin(theta_5))*sin(theta_6) +
6.12323399573677e-17*((-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*cos(theta_4) +
(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) +
1.0*sin(theta_2)*cos(theta_1))*sin(theta_4))*sin(theta_5) +
3.74939945665464e-33*(-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*sin(theta_4) -
3.74939945665464e-33*(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) + 1.0*sin(theta_2)*cos(theta_1))*cos(theta_4)
```

```
- 6.12323399573677e-17*(-6.12323399573677e-17*(-6.12323399573677e-17*sin(theta_1
)*sin(theta_2) + cos(theta_1)*cos(theta_2))*sin(theta_4) +
6.12323399573677e-17*(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) + 1.0*sin(theta_2)*cos(theta_1))*cos(theta_4)
- 7.49879891330929e-33*sin(theta_1)*cos(theta_2) - 1.0*sin(theta_1) -
1.22464679914735e-16*sin(theta_2)*cos(theta_1))*cos(theta_5) +
1.0*(-6.12323399573677e-17*((-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*cos(theta_4) +
(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) +
1.0*sin(theta_2)*cos(theta_1))*sin(theta_4))*sin(theta_5) +
1.0*(-6.12323399573677e-17*sin(theta_1)*sin(theta_2) +
cos(theta_1)*cos(theta_2))*sin(theta_4) -
1.0*(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) + 1.0*sin(theta_2)*cos(theta_1))*cos(theta_4)
+ 6.12323399573677e-17*(-6.12323399573677e-17*(-6.12323399573677e-17*sin(theta_1
)*sin(theta_2) + cos(theta_1)*cos(theta_2))*sin(theta_4) +
6.12323399573677e-17*(6.12323399573677e-17*sin(theta_1)*cos(theta_2) -
1.22464679914735e-16*sin(theta_1) + 1.0*sin(theta_2)*cos(theta_1))*cos(theta_4)
- 7.49879891330929e-33*sin(theta_1)*cos(theta_2) - 1.0*sin(theta_1) -
1.22464679914735e-16*sin(theta_2)*cos(theta_1))*cos(theta_5) -
4.59169004331693e-49*sin(theta_1)*cos(theta_2) -
6.12323399573677e-17*sin(theta_1) -
7.49879891330929e-33*sin(theta_2)*cos(theta_1))*cos(theta_6) -
1.72160801535391e-81*sin(theta_1)*cos(theta_2) -
2.29584502165847e-49*sin(theta_1) -
2.81159925711243e-65*sin(theta_2)*cos(theta_1)],
[        -1.0*(((sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*cos(theta_4) +
(1.0*sin(theta_1)*sin(theta_2) - 6.12323399573677e-17*cos(theta_1)*cos(theta_2)
+ 1.22464679914735e-16*cos(theta_1))*sin(theta_4))*cos(theta_5) +
(-6.12323399573677e-17*(sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*sin(theta_4) +
6.12323399573677e-17*(1.0*sin(theta_1)*sin(theta_2) -
6.12323399573677e-17*cos(theta_1)*cos(theta_2) +
1.22464679914735e-16*cos(theta_1))*cos(theta_4) -
1.22464679914735e-16*sin(theta_1)*sin(theta_2) +
7.49879891330929e-33*cos(theta_1)*cos(theta_2) +
1.0*cos(theta_1))*sin(theta_5))*sin(theta_6) +
6.12323399573677e-17*((sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*cos(theta_4) +
(1.0*sin(theta_1)*sin(theta_2) - 6.12323399573677e-17*cos(theta_1)*cos(theta_2)
+ 1.22464679914735e-16*cos(theta_1))*sin(theta_4))*sin(theta_5) +
3.74939945665464e-33*(sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*sin(theta_4) -
3.74939945665464e-33*(1.0*sin(theta_1)*sin(theta_2) -
6.12323399573677e-17*cos(theta_1)*cos(theta_2) +
1.22464679914735e-16*cos(theta_1))*cos(theta_4) -
```

```
6.12323399573677e-17*(-6.12323399573677e-17*(sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*sin(theta_4) +
6.12323399573677e-17*(1.0*sin(theta_1)*sin(theta_2) -
6.12323399573677e-17*cos(theta_1)*cos(theta_2) +
1.22464679914735e-16*cos(theta_1))*cos(theta_4) -
1.22464679914735e-16*sin(theta_1)*sin(theta_2) +
7.49879891330929e-33*cos(theta_1)*cos(theta_2) + 1.0*cos(theta_1))*cos(theta_5)
+ 1.0*(-6.12323399573677e-17*((sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*cos(theta_4) +
(1.0*sin(theta_1)*sin(theta_2) - 6.12323399573677e-17*cos(theta_1)*cos(theta_2)
+ 1.22464679914735e-16*cos(theta_1))*sin(theta_4))*sin(theta_5) +
1.0*(sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*sin(theta_4) -
1.0*(1.0*sin(theta_1)*sin(theta_2) -
6.12323399573677e-17*cos(theta_1)*cos(theta_2) +
1.22464679914735e-16*cos(theta_1))*cos(theta_4) +
6.12323399573677e-17*(-6.12323399573677e-17*(sin(theta_1)*cos(theta_2) +
6.12323399573677e-17*sin(theta_2)*cos(theta_1))*sin(theta_4) +
6.12323399573677e-17*(1.0*sin(theta_1)*sin(theta_2) -
6.12323399573677e-17*cos(theta_1)*cos(theta_2) +
1.22464679914735e-16*cos(theta_1))*cos(theta_4) -
1.22464679914735e-16*sin(theta_1)*sin(theta_2) +
7.49879891330929e-33*cos(theta_1)*cos(theta_2) + 1.0*cos(theta_1))*cos(theta_5)
- 7.49879891330929e-33*sin(theta_1)*sin(theta_2) +
4.59169004331693e-49*cos(theta_1)*cos(theta_2) +
6.12323399573677e-17*cos(theta_1))*cos(theta_6) -
2.81159925711243e-65*sin(theta_1)*sin(theta_2) +
1.72160801535391e-81*cos(theta_1)*cos(theta_2) +
2.29584502165847e-49*cos(theta_1)],
[
-1.0*(((-1.0*cos(theta_2) - 7.49879891330929e-33)*sin(theta_4) +
1.0*sin(theta_2)*cos(theta_4))*cos(theta_5) +
(6.12323399573677e-17*(-1.0*cos(theta_2) - 7.49879891330929e-33)*cos(theta_4) -
6.12323399573677e-17*sin(theta_2)*sin(theta_4) +
1.22464679914735e-16*cos(theta_2) -
6.12323399573677e-17)*sin(theta_5))*sin(theta_6) +
6.12323399573677e-17*((-1.0*cos(theta_2) - 7.49879891330929e-33)*sin(theta_4) +
1.0*sin(theta_2)*cos(theta_4))*sin(theta_5) -
3.74939945665464e-33*(-1.0*cos(theta_2) - 7.49879891330929e-33)*cos(theta_4) -
6.12323399573677e-17*(6.12323399573677e-17*(-1.0*cos(theta_2) -
7.49879891330929e-33)*cos(theta_4) -
6.12323399573677e-17*sin(theta_2)*sin(theta_4) +
1.22464679914735e-16*cos(theta_2) - 6.12323399573677e-17)*cos(theta_5) +
1.0*(-6.12323399573677e-17*((-1.0*cos(theta_2) -
7.49879891330929e-33)*sin(theta_4) + 1.0*sin(theta_2)*cos(theta_4))*sin(theta_5)
- 1.0*(-1.0*cos(theta_2) - 7.49879891330929e-33)*cos(theta_4) +
6.12323399573677e-17*(6.12323399573677e-17*(-1.0*cos(theta_2) -
7.49879891330929e-33)*cos(theta_4) -
```

```
6.12323399573677e-17*sin(theta_2)*sin(theta_4) +
1.22464679914735e-16*cos(theta_2) - 6.12323399573677e-17)*cos(theta_5) +
1.0*sin(theta_2)*sin(theta_4) + 7.49879891330929e-33*cos(theta_2) -
3.74939945665464e-33)*cos(theta_6) +
3.74939945665464e-33*sin(theta_2)*sin(theta_4) +
2.81159925711243e-65*cos(theta_2) - 1.40579962855621e-65]])
```

Forming the columns J_1 to J_6 of the Jacobian Matrix

[220]:
```
Xp=T7[:3,3]
diff_thet_1 = Xp.diff(theta_1) #Partially differentiating Xp wrt  1

diff_thet_2 = Xp.diff(theta_2) #Partially differentiating Xp wrt  2

diff_thet_4 = Xp.diff(theta_4) #Partially differentiating Xp wrt  4

diff_thet_5 = Xp.diff(theta_5) #Partially differentiating Xp wrt  5

diff_thet_6 = Xp.diff(theta_6) #Partially differentiating Xp wrt  6

diff_thet_7 = Xp.diff(theta_7) #Partially differentiating Xp wrt  7

print("The initial jacobian matrix for home position is given by:")
J =␣
↪Matrix([[diff_thet_1[0],diff_thet_2[0],diff_thet_4[0],diff_thet_5[0],diff_thet_6[0],diff_th
        ␣
↪[diff_thet_1[1],diff_thet_2[1],diff_thet_4[1],diff_thet_5[1],diff_thet_6[1],diff_thet_7[1]]
        ␣
↪[diff_thet_1[2],diff_thet_2[2],diff_thet_4[2],diff_thet_5[2],diff_thet_6[2],diff_thet_7[2]]
        ␣
↪[Z0[0],Z1[0],Z2[0],Z4[0],Z5[0],Z6[1]],[Z0[1],Z1[1],Z2[1],Z4[1],Z5[1],Z6[1]],[Z0[2],Z1[2],Z2

J1=J.subs([(theta_1,0),(theta_2,0),(theta_4,math.radians(90))
             ,(theta_5,0),(theta_6, math.radians(180)),(theta_7,0)])
J1=J1.subs([(d_1,0.3330),(d_3,0.3160),(d_5,0.3840),(d_7,0.2070),(a_3,0.0880)])
J1
```

```
The initial jacobian matrix for home position is given by:
```

[220]:

$$\begin{bmatrix} -5.07616098246578 \cdot 10^{-17} & -0.492 & 0.176 & 5.38844591624835 \cdot 10^{-18} \\ 0.679 & 4.15767588310526 \cdot 10^{-17} & 3.61883129148043 \cdot 10^{-17} & -0.088 \\ 0 & 0.679 & -0.591 & 1.99617428261019 \cdot 10^{-17} \\ 0 & 0 & 1.0 & 1.22464679914735 \cdot 10^{-16} \\ -1.0 & 0 & 6.12323399573677 \cdot 10^{-17} & -1.0 \\ 6.12323399573677 \cdot 10^{-17} & 1.0 & 6.12323399573677 \cdot 10^{-17} & -6.12323399573677 \cdot 10^{-17} \end{bmatrix}$$

0.0.4 Substitutng Link Lengths

[188]:
```
J=J.subs([(d_1,0.3330),(d_3,0.3160),(d_5,0.3840),(d_7,0.2070),(a_3,0.0880)]);J
```

0.0.5 Circle equations

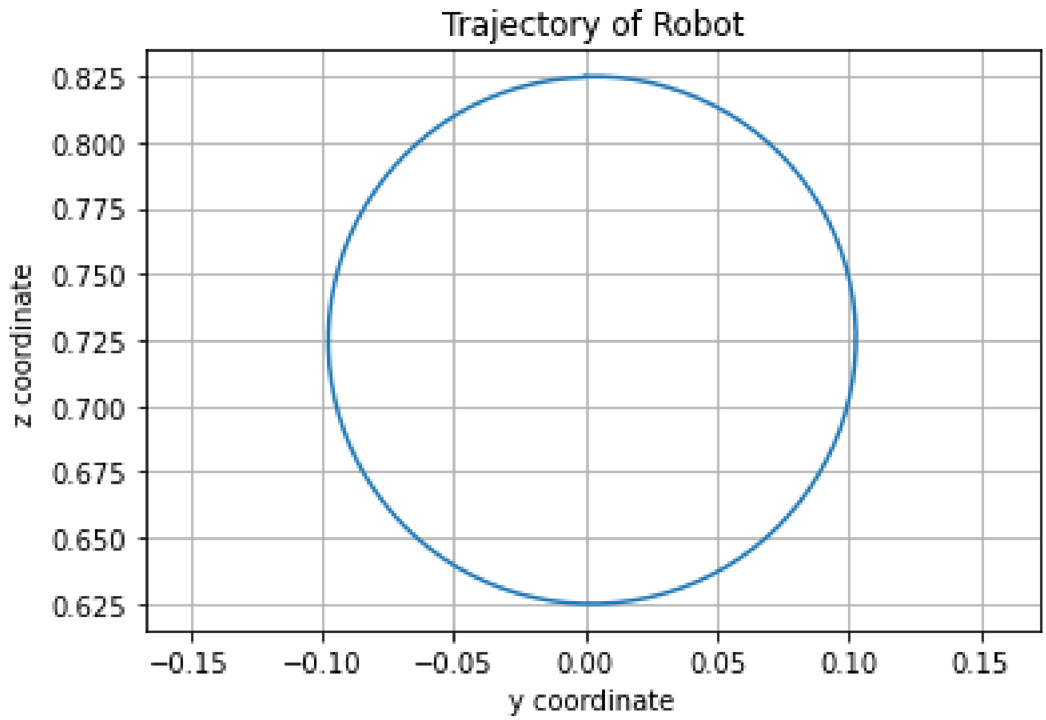
The equation of ciecle is given by $y^2 + (z - 72.5)^2 = 100$
Using the polar form
$y = r\cos(\theta), z = r\sin(\theta)$
Therefore,
$\dot{y} = r\cos(\theta)\dot{\theta}$
$\dot{z} = r\sin(\theta)\dot{\theta}$
Also, $\dot{\theta} = \frac{2\pi}{5}$

Using the above equations and the Jacobian matrix we compute the tool positions over a time period using Numerical Integration

[196]:
```
import numpy as np
import sympy as sp
x,y,z,r,o=sp.symbols("x y z r theta")
dt=0.05  #Time difference
t_1=[math.radians(0)]  #Theta 1
t_2=[math.radians(0)]  #Theta 2
t_4=[math.radians(90)]  #Theta 4
t_5=[math.radians(0)]  #Theta 5
t_6=[math.radians(180)]  #Theta 6
t_7=[math.radians(0)]  #Theta 7

T=T7.subs([(d_1,0.3330),(d_3,0.3160),(d_5,0.3840),(d_7,0.2070),(a_3,0.0880)]);T
x_tool=[]
y_tool=[]
z_tool=[]
#Tool Velocity Matrix
X=sp.Matrix([[0],[0.1*sp.cos((2*np.pi*o)/100)*(2*np.pi)/5],[-0.1*sp.sin((2*np.
 ↪pi*o)/100)*(2*np.pi)/5],[0],[0],[0]])
i=0
print("Computing Trajectory")
while i<=100:
    X_eval=X.subs(o,i)
    T_eval=T.subs([(theta_1,t_1[i]),(theta_2,t_2[i]),(theta_3,math.
 ↪radians(0)),(theta_4,t_4[i]),
              (theta_5,t_5[i]),(theta_6,t_6[i]),(theta_7,t_7[i])])
    x_tool.append(T_eval[3])
    y_tool.append(T_eval[7])
    z_tool.append(T_eval[11])
    J_eval=J.subs([(theta_1,t_1[i]),(theta_2,t_2[i]),(theta_3,math.
 ↪radians(0)),(theta_4,t_4[i]),
```

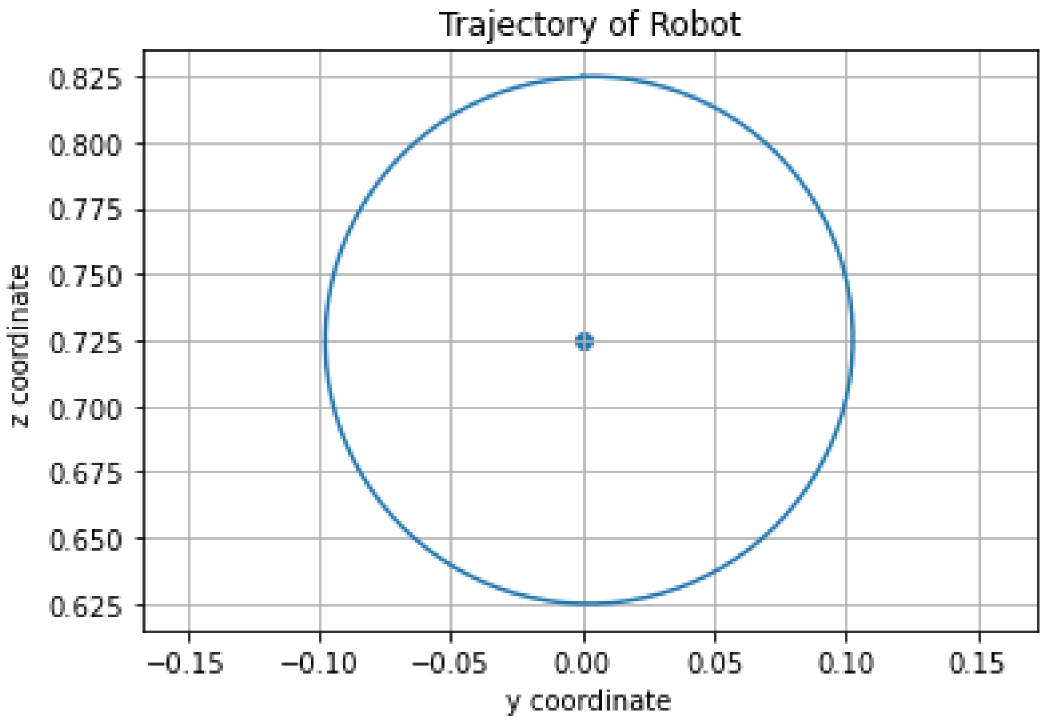
```
                (theta_5,t_5[i]),(theta_6,t_6[i]),(theta_7,t_7[i])])
    q=J_eval.inv('LU')*X_eval
    q=q*dt

    t_1.append(q[0]+t_1[i])
    t_2.append(q[1]+t_2[i])
    t_4.append(q[2]+t_4[i])
    t_5.append(q[3]+t_5[i])
    t_6.append(q[4]+t_6[i])
    t_7.append(q[5]+t_7[i])
    print(".",end="")
    i=i+1


import matplotlib.pyplot as plt
plt.plot(y_tool,z_tool)
plt.plot(0,0.725)
plt.xlabel("y coordinate")
plt.ylabel("z coordinate")
plt.axis("equal")
plt.title("Trajectory of Robot")
plt.grid(True)
plt.show()
```

```
Computing Inverse Kinematics
…
…
```



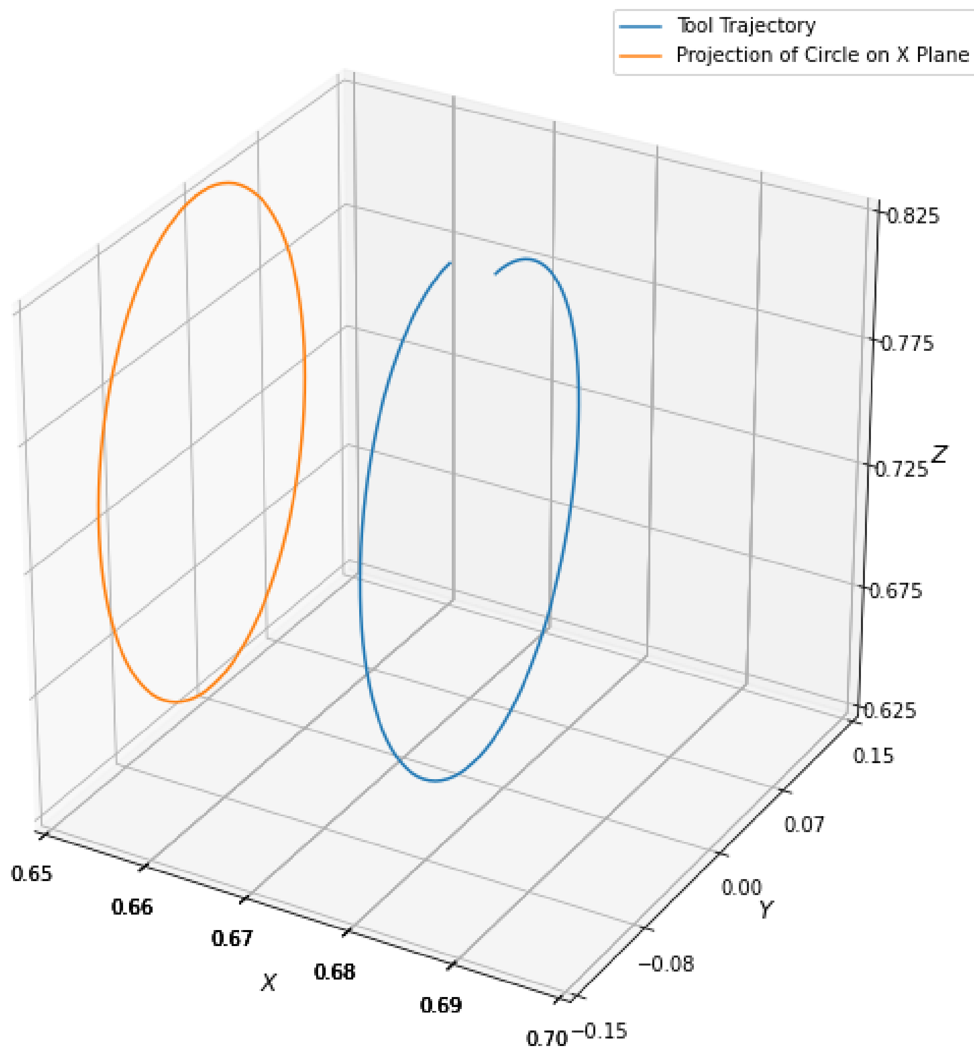
0.0.6 Plotting trajectory in 2D Space

[197]:
```
import matplotlib.pyplot as plt
plt.plot(y_tool,z_tool)
plt.scatter(0,0.725)
plt.xlabel("y coordinate")
plt.ylabel("z coordinate")
plt.axis("equal")
plt.title("Trajectory of Robot")
plt.grid(True)
plt.show()
```



0.0.7 Plotting Trajectory in 3D Space

[201]:
```
ax = plt.figure(figsize=(10,10)).add_subplot(projection='3d')
ax.axes.set_xlim3d(left=0.65, right=0.7)
ax.axes.set_ylim3d(bottom=-0.15, top=0.15)
ax.axes.set_zlim3d(bottom=0.625, top=0.825)
ax.set_xlabel('$X$', fontsize=12)
ax.set_ylabel('$Y$', fontsize=12)
ax.set_zlabel('$Z$', fontsize=12)
ax.set_zticks(np.linspace(0.625,0.825,5).round(3))
ax.set_xticks(np.linspace(0.65,0.7,20).round(2))
ax.set_yticks(np.linspace(-0.15,0.15,5).round(2))
ax.plot(x_tool, y_tool, z_tool, label='Tool Trajectory')
ax.plot(np.full(len(y_tool),0.65), y_tool, z_tool, label='Projection of Circle␣
↪on X Plane')
ax.legend()

plt.show()
```


Tool Trajectory
Projection of Circle on X Plane
0.825
0.775
0.725
0.675
0.625
Z
0.15
0.07
0.00
−0.08
−0.15
Y
0.65
0.66
0.67
0.68
0.69
0.70
X

```
[ ]:
```