Downlink OFDMA System

Final Project
EEE 552

Shantanu Patne

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1 Description

This project implements a Downlink OFDMA system with one Base Station (BS) and two User Equipment (UEs). The system operates with three consecutive OFDM symbols, each containing 20 subcarriers, transmitted over frequency-selective channels with 5 independent Rayleigh fading taps. Starting with a baseline system, we incrementally add features to enhance the performance. The final system includes Channel Estimation, Diversity (using Repetition Coding), support for two UEs, and the 16QAM modulation scheme.

Baseline

The baseline system consists of a single UE, and the channel is assumed to be known at the receiver, so no channel estimation is needed. All transmissions are uncoded, meaning that no diversity techniques are applied, and the BPSK modulation scheme is used for transmission. The primary goal of this baseline is to create a functional system to which additional features can be added in subsequent stages.

In the baseline system, we begin by generating a random bitstream. Each bit is mapped to its corresponding symbol, which is then transformed into the time domain. A cyclic prefix (CP) is added to each OFDM symbol. Given that the channel has $L = 5$ taps, the CP length is $L-1 = 4$. As a result, the last 4 subcarriers of each OFDM symbol are appended to the beginning of the frame to mitigate inter-symbol interference.

After generating the signal, it is passed through the channel, and White Gaussian Noise (WGN) is added to simulate real-world conditions. The noise variance, N_o, is derived from the expected Signal-to-Noise Ratio (SNR) of the system as follows:

$$N_o = \frac{E_s}{SNR}$$

where E_s is the average symbol energy per subcarrier. For BPSK (and other similar schemes), the symbols are normalized such that the average symbol energy is 1. Therefore, the noise variance simplifies to:

$$N_o = \frac{1}{SNR}$$

At the receiver, the system uses the known channel information to perform equalization, correcting for channel distortions. The received signal is then decoded based on the constellation decoding regions. The reconstructed bitstream, *hatbitstring*, is compared with the transmitted bitstream to calculate the Bit Error Rate (BER). For BPSK modulation, we expect the BER to be approximately 2% at 10 dB SNR and 0.2% at 20 dB SNR. To obtain accurate BER results, the system runs 10^5 iterations for each SNR value, targeting a BER of around 1%. This number of iterations is chosen to capture at least 1000 bit errors. The results, shown in Figure 1, confirm that the baseline system performs as expected.

Additional Features

1. **Advanced Modulation**:

The first enhancement to the baseline system is the introduction of more advanced modulation schemes. Specifically, QPSK and 16QAM modulations have been implemented, with Gray coding used for both. Gray coding is a technique that minimizes bit errors by ensuring that adjacent symbols in the modulation constellation differ by only one bit. This results in fewer bit errors during transmission. Among these, 16QAM is particularly beneficial because it allows for higher data rates by transmitting more bits per symbol compared to simpler schemes like BPSK. For this reason, 16QAM is selected for future system configurations.

2. **Multiple UEs**:
The next feature added to the baseline system is support for multiple UEs. In this case, the BS transmits a single signal that contains information for both UEs, with the data for each UE allocated to different subcarriers. The total of 60 subcarriers across the 3 OFDM symbols are split evenly between the two UEs, with 30 subcarriers assigned to each UE. This ensures that both UEs receive the same bitrate, improving system efficiency and flexibility.

3. **Channel Estimation**:
With multiple UEs, channel estimation becomes essential. Since the channel is unknown at the receiver, pilot symbols must be inserted into the transmission to estimate the channel characteristics. One approach is to add pilots to all 20 subcarriers in each OFDM frame, but this would result in a significant degradation of spectral efficiency. Specifically, if 1 out of the 3 OFDM symbols is used for pilots, the spectral efficiency is reduced by a factor of 2/3.

To address this, we exploit the frequency-selective nature of the channel. By using $L \geq 5$ independent channel taps, we can gather enough information to estimate the channel characteristics accurately with fewer pilots. Max-Likelihood decoding is used to estimate the channels for all 20 subcarriers. By using only 5 pilots over the 60 subcarriers, we achieve a much smaller degradation in spectral efficiency, with a loss of 55/60, which is more acceptable compared to the previous method.

4. **Repetition Coding**:
Repetition coding is a diversity technique that enhances the robustness of the system by repeating symbols over multiple coherence periods (time or frequency) to better capture the channel's behavior. This increases the reliability of the transmission by mitigating the effects of fading and other impairments, as weak signal paths can be compensated for by stronger paths.

In our system, repetition coding is implemented by repeating each symbol at intervals of N/L subcarriers, ensuring that adjacent symbols are not correlated due to the channel. The repeated symbols are then decoded using Max-Ratio Combining (MRC), which is an optimal technique for combining signals from different paths to maximize the received signal power. This technique provides a maximum diversity gain of L, improving the system's performance significantly by increasing its resistance to channel impairments and noise.

2 Numerical Results

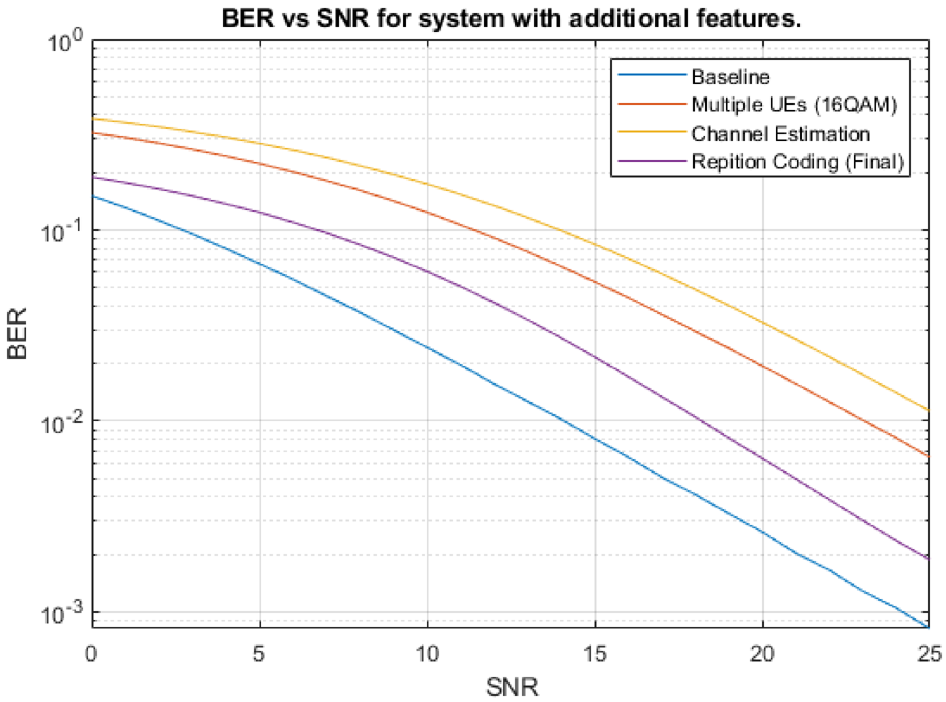

Figure 1: BER vs SNR curves for incremental changes

In this section, we present the BER curves for the baseline system and its improvements, and discuss the results. Figure 1 shows the BER curves for all configurations. Starting from the baseline, each additional feature is added step-by-step, with its own corresponding BER curve.

1. **Baseline System (BPSK Modulation)**:
The baseline system, which uses BPSK modulation, achieves the best performance overall. BPSK is a robust modulation scheme because it has a relatively simple signal structure, which reduces the likelihood of errors in decoding. In this configuration, the system benefits from larger decoding regions and fewer transmitted bits per symbol, which contributes to a lower bit error rate. The performance closely matches the theoretical expectations, with a BER of 2.4% at 10 dB, which is very close to the expected 2%. At 20 dB, the observed BER is 0.23%, near the anticipated 0.2%. This strong performance serves as an effective baseline to compare the next system configurations and improvements.

2. **16QAM Modulation (Multiple UEs)**:
The 16QAM modulation scheme performs worse than the BPSK baseline. It offers higher data rates by encoding more bits per symbol (4 bits per symbol), but this increases the sensitivity to noise and interference, leading to a higher likelihood of errors. In this case, the BER for both single UE and multiple UE scenarios with 16QAM are nearly identical and difficult to distinguish, so they are plotted as a single curve. This indicates that the number of UEs does not significantly affect the performance in this configuration. The increased BER compared to the BPSK baseline can be attributed to

the smaller decoding regions in 16QAM, which means that the system is more susceptible to errors as the SNR decreases.

3. **Channel Estimation (Maximum Likelihood Estimation with 6 Pilots)**:
   The Channel Estimation curve shows the poorest performance of all the plotted curves. This is because the channel estimation, which is based on a Maximum Likelihood approach, is impacted by inaccuracies due to noise interference. Channel estimation is a critical step for ensuring that the receiver can correctly decode the transmitted signal, but in this case, the accuracy of the estimation is compromised by the limited number of pilots (6 in this case). These pilots are used to infer the characteristics of the channel, but with fewer pilots, the estimation is less accurate, leading to an increase in BER. By using more pilots, ideally distributed across the entire OFDM frame, the accuracy of the channel estimation could be improved, reducing the overall BER. This is discussed in detail in Section 3.

4. **Diversity Gain (Repetition of Symbols)**:
   The final curve shows the effect of diversity on the system. In this case, the diversity gain is achieved by repeating the symbols, which increases the robustness of the system to channel impairments. Diversity techniques help mitigate the effects of fading, interference, and noise by using multiple signal copies transmitted through different channels or at different times. The repetition of symbols significantly reduces the BER compared to the channel estimation step. This can be seen by the BER being more than halved, showing a clear improvement in performance. At 10 dB SNR, we achieve a diversity gain of 3, meaning that the system's performance is much better than in the channel estimation step, even though the only change is the introduction of symbol repetition. This highlights the effectiveness of diversity in improving the system's reliability in the presence of noise.

3 Trade-Off Study

The provided plot highlights the trade-off between the number of pilots, Bit Error Rate (BER), and Spectral Efficiency (SE) in a 16QAM modulation system operating at a constant SNR of 10 dB. The system uses varying numbers of pilots, ranging from 1 to 20, out of the total subcarriers to perform channel estimation.

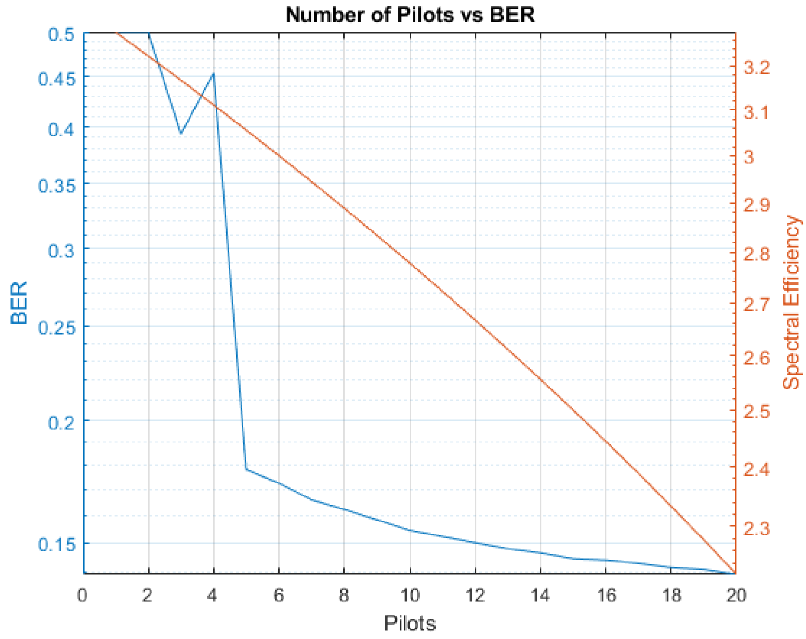

Figure 2: Trade-Off between number of Pilots and BER

Initially, when the number of pilots is very low (fewer than 5), the BER is extremely high. This indicates that the system cannot accurately estimate the channel state information, leading to unreliable communication. As the number of pilots increases, the BER drops significantly, reaching a stable and low value when approximately 10 pilots are used. However, the spectral efficiency steadily decreases as more pilots are added, as these pilots occupy subcarriers that would otherwise be used for data symbols. Herein lies the trade-off: **What is the optimal number of pilots that minimize BER while also maximizing SE?**

The optimal number of pilots lies in a range where BER is sufficiently low, and SE is not compromised. For the given system, this range is between 5 and 10 pilots. Selecting the exact number within this range depends heavily on the channel environment. In noisy or fast-varying channels, more pilots are needed to ensure accurate channel estimation, even if it reduces SE. Conversely, in cleaner or slowly varying channels, fewer pilots can suffice, preserving SE while maintaining acceptable BER. The number of pilots must also meet a minimum criteria determined by the number of channel taps in the frequency-selective channel. For instance, if a channel has five taps, at least five pilots are required to ensure the channel is properly estimated.

Proper pilot allocation ensures reliable communication while efficiently utilizing the available bandwidth. This trade-off between BER and SE highlights the need to carefully balance pilot allocation based on the communication system's specific requirements.

4 Supplemental Information

SE Calculation

For the final scheme, we have 2 UEs with 15 symbols each, 5 Pilots, 20 subcarriers, 3 frames and 16QAM scheme. Therefore we can calculate the total number of bits as follows:

$$\text{Total bits} = 15 \times 2 \times 4 = 120$$

Similarly, the total number of samples transmitted can be found as:

$$\text{Total Samples} = (20 + 4) \times 3 = 72$$

Thus, we get the Spectral Efficiency of the overall system as:

$$SE = \frac{\text{Total Bits}}{\text{Total Samples}} = \frac{120}{72} = 1.6667$$

Constellation

The constellation implemented in the final system is 16QAM using Gray coding. To visualize this, we can use the MATLAB `scatter` function. With some additional modifications, we can output the bits allocated to every constellation point.

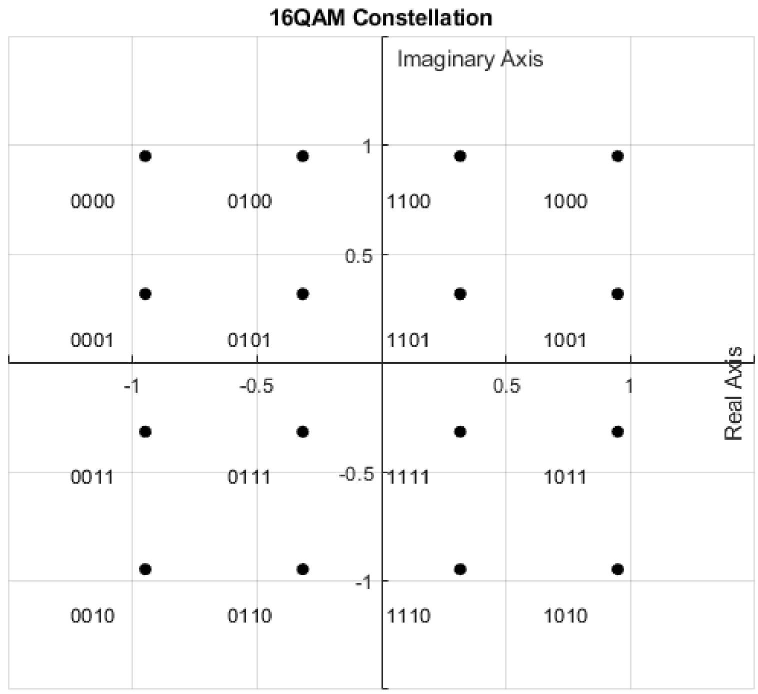

Figure 3: Constellation Diagram with Bit Allocation

OFDM Symbol Structure

The final system implements the following symbols structure. The term 'UxSy' denotes the y^{th} symbol of x UE.

Subcarriers	OFDM 1	OFDM 2	OFDM 3
0	P	U1S4	U2S8
1	U1S1	U1S5	U2S9
2	U1S2	U1S6	U2S10
3	U1S3	U1S7	U2S11
4	P	U2S4	U1S8
5	U2S1	U2S5	U1S9
6	U2S2	U2S6	U1S10
7	U2S3	U2S7	U1S11
8	P	U1S4	U2S8
9	U1S1	U1S5	U2S9
10	U1S2	U1S6	U2S10
11	U1S3	U1S7	U2S11
12	P	U2S4	U1S12
13	U2S1	U2S5	U1S13
14	U2S2	U2S6	U1S14
15	U2S3	U2S7	U1S15
16	P	U1S8	U2S12
17	U1S1	U1S9	U2S13
18	U1S2	U1S10	U2S14
19	U1S3	U1S11	U2S15

Table 1: OFDM Symbol Structure