Markov Chain Model for Market Share Evolution

Problem Setup

We model customer X's preference-switching behavior using a two-state Markov chain:

- **State 1:** First preference (e.g., holiday destination or restaurant type)
- **State 2:** Second preference

Given parameters:

- Initial market shares:
$$p_1(0) = 0.55, p_2(0) = 0.45$$
- Transition probabilities per choice:
$$P(1 \to 1) = 0.75, P(1 \to 2) = 0.25$$
$$P(2 \to 1) = 0.55, P(2 \to 2) = 0.45$$
- Decision frequency:
Customers make **2 choices per year**, so over **2 years** we observe
$$4 \text{ total choices.}$$

1) **Problem parameters (compact)**

Parameter	Value
Preference 1 retention	0.75
Preference 2 retention	0.55
Decision frequency	2 choices / year
Initial share (Pref 1)	0.55
Initial share (Pref 2)	0.45

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2) **Transition Matrix**

$$P = \begin{bmatrix} 0.75 & 0.25 \\ 0.55 & 0.45 \end{bmatrix}$$

(Each row represents the current state; each column represents the next state.)

State Evolution Across 4 Steps
The initial state vector is:

$$s(0) = [0.55 \quad 0.45]$$

We compute:

$$s(4) = s(0)\,P^4.$$

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3) Computation of P^2(showing component sums)

Step 1: Compute P^2

$$P^2 = \begin{bmatrix} 0.75 & 0.25 \\ 0.55 & 0.45 \end{bmatrix} \begin{bmatrix} 0.75 & 0.25 \\ 0.55 & 0.45 \end{bmatrix} = \begin{bmatrix} 0.5625 + 0.1375 & 0.1875 + 0.1125 \\ 0.4125 + 0.2475 & 0.1375 + 0.2025 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0.700 & 0.300 \\ 0.660 & 0.340 \end{bmatrix}$$

Step 2: Compute $P^4 = (P^2)^2$

$$P^4 = \begin{bmatrix} 0.700 & 0.300 \\ 0.660 & 0.340 \end{bmatrix}^2 = \begin{bmatrix} 0.490 + 0.198 & 0.210 + 0.102 \\ 0.462 + 0.224 & 0.198 + 0.116 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 0.688 & 0.312 \\ 0.686 & 0.314 \end{bmatrix}$$

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4) Final Market Share After 4 Choices

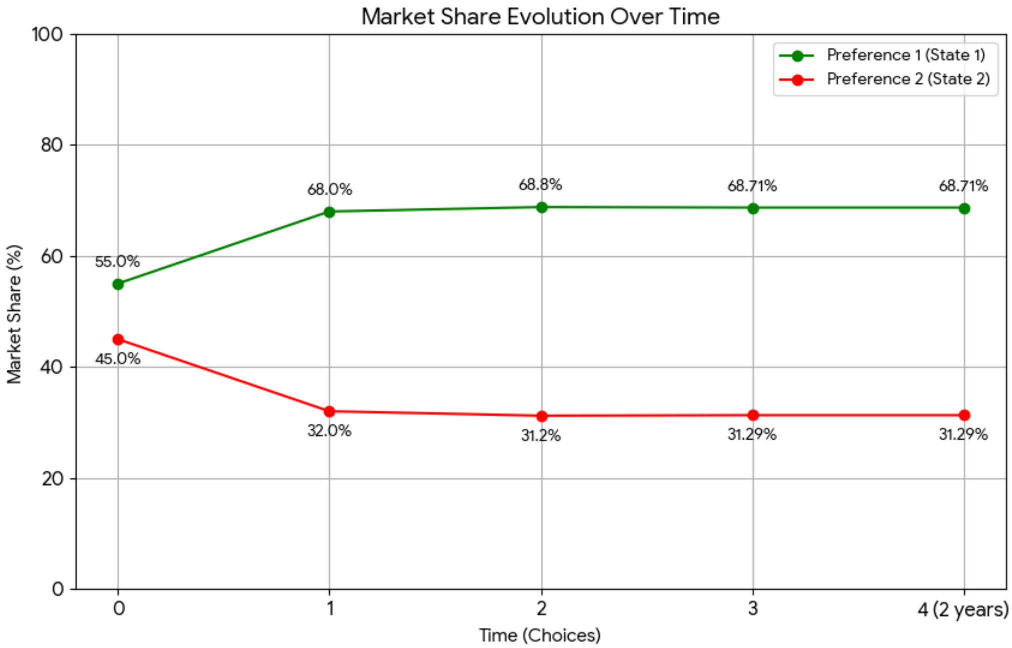
$$s(4) = [0.55 \quad 0.45] \begin{bmatrix} 0.688 & 0.312 \\ 0.686 & 0.314 \end{bmatrix}$$

$$s(4) = [0.3784 + 0.3087 \quad 0.1716 + 0.1413] = [0.6871 \quad 0.3129]$$

Thus, after 2 years:

- **Preference 1 share: 68.71%**
- **Preference 2 share: 31.29%**

Transition Diagram:



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5)Market Share Evolution Over Time

Time (Choices)	Pref 1 Share	Pref 2 Share
0	55.00%	45.00%
1	68.00%	32.00%
2	68.80%	31.20%

Time (Choices)	Pref 1 Share	Pref 2 Share
3	68.71%	31.29%
4 (2 years)	68.71%	31.29%

Long-Term Equilibrium (Stationary Distribution)
Solve for $\pi P = \pi$, with $\pi_1 + \pi_2 = 1$:

$$\pi_1 = 0.75\pi_1 + 0.55\pi_2 \Rightarrow 0.25\pi_1 = 0.55\pi_2 \Rightarrow \pi_1 = 2.2\pi_2$$

Using the normalization condition:

$$\pi_1 + \pi_2 = 1 \Rightarrow 2.2\pi_2 + \pi_2 = 1 \Rightarrow 3.2\pi_2 = 1 \Rightarrow \pi_2 = 0.3125$$
$$\pi_1 = 2.2 \times 0.3125 = 0.6875$$

Thus:

- **Long-run Preference 1: 68.75%**
- **Long-run Preference 2: 31.25%**

6)Sensitivity analysis table (as in your report)

P1 Ret.	P2 Ret.	P1 Steady	P2 Steady
0.75	0.55	64.29%	35.71%
0.8	0.55	69.23%	30.77%
0.75	0.6	60.00%	40.00%
0.7	0.55	60.00%	40.00%

Conclusion
The Markov chain converges quickly to its steady-state distribution. After just 4 choice events (2 years), the market share practically reaches its long-term equilibrium values, with Preference 1 stabilizing around **68.7%** and Preference 2 around **31.3%**.