Lab - 6

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CS-302, Modeling and Simulation

In this lab we numerically analyze Malaria spread model with changing parameters and try to modify it, in order to accommodate for effects due to vaccination and fumigation.

I. INTRODUCTION

In a bid to model the spread of vector borne diseases like Malaria or Dengue, we can sub model humans and mosquitoes and define how they interact with each other using differential equations which seek to accurately define the trend based on the parameters. In this report, we will model humans using various compartments and similarly with the mosquitoes. Then we will mathematically formulate a way for the infection to spread to and fro the human - mosquito sub model. In addition to this we shall explore the effect of intervention measures like Vaccinations, fumigation, mosquito repellents and breeding control.

In all of the figures, the population of each compartment has been expressed as fractions of the net population as well as the time on the x-axis has been normalised.

II. MODEL

A. Human

Humans are modelled with three sub-compartments that are given below :

- Uninfected Humans(U_H) - Those who have no immunity from the disease and can become hosts if bitten by a vector
- Host Humans(H_H) - Those who have the disease and can spread it to uninfected mosquitoes that bite them, thus making them vectors. They can either recover from the disease and become uninfected again or can be immunised and never have the disease again or can even die from the disease.
- Immune Humans(I_H) - Those who have recovered and are now immune from it.

We are not counting the deaths due to natural causes and it is to be assumed that the decrease in total human population is only because of the disease because the life expectancy of a human is much greater than that of a mosquito. Moreover we assume that the symptoms appear immediately once a person is bitten thus having no incubation period and we also assume that once a person has recovered from the disease, relapse cannot occur and the only way to fall sick again is when that person is again bitten by a vector.[1]

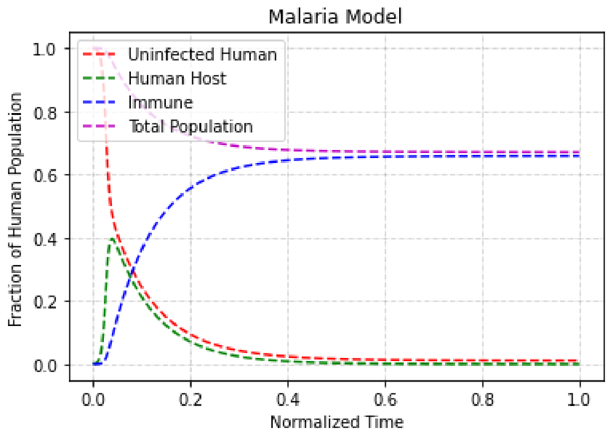

FIG. 1: Human Population with parameters : r = 0.3, p = 0.3, i = 0.01, d = 0.005, mb = 0.01, md = 0.01, N_H = 301, N_M = 300= U_M[0] ,U_H[0]= 300,H_H[0]= 1

B. Mosquito

Mosquitoes are modelled with two sub-compartments that are given below :

- Uninfected Mosquitoes(U_M) - These mosquitoes do not have the disease causing microbes and can either die or become vectors when they bite an infected human host.
- Vectors(V_M) - These are the disease carrying mosquitoes which can either infect humans by biting them or die naturally without spreading the disease.

Because of their relatively short life expectancy, we do consider mosquito births and deaths. We have the assumption that the death rates for infected and uninfected mosquitoes are identical. Similarly, we assume that all mosquitoes reproduce at the same rate. At birth, a mosquito is uninfected. As a simplification for this first version of the model, we suppose that an infected mosquito immediately becomes a host that can infect humans. [1]

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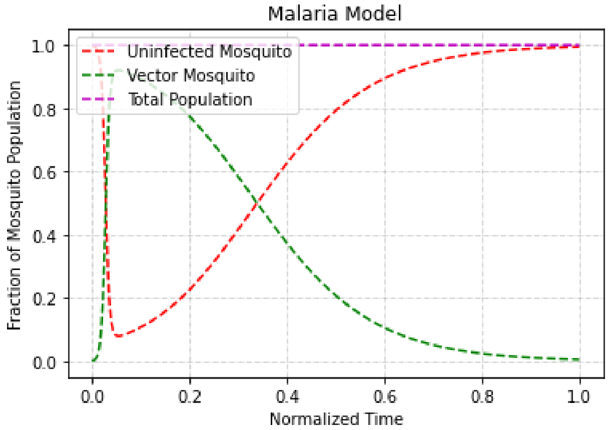

FIG. 2: Mosquito Population with parameters : r = 0.3, p = 0.3, i = 0.01, d = 0.005, mb = 0.01, md = 0.01, N_H = 301, N_M = 300= $U_M[0]$,$U_H[0]$= 300,$H_H[0]$= 1

We can model the humans and mosquito together as a system using the following differential equations:

$$\frac{dU_H}{dt} = r \cdot H_H - \frac{p \cdot V_M \cdot U_H}{T_M}$$

$$\frac{dH_H}{dt} = \frac{p \cdot V_M \cdot U_H}{T_M} - r \cdot H_H - i \cdot H_H - d \cdot H_H$$

$$\frac{dI_H}{dt} = i \cdot H_H$$

$$\frac{dU_M}{dt} = mb \cdot T_M - md \cdot U_M - \frac{p \cdot U_M \cdot H_H}{T_H}$$

$$\frac{dV_M}{dt} = \frac{p \cdot U_M \cdot H_H}{T_H} - md \cdot V_M$$

$$T_H = U_H + H_H + I_H$$

$$T_M = U_M + V_M$$

Various constants used in the above equations are defined as,

- r — Recovery rate for host humans becoming uninfected again.
- p — Probability of a mosquito biting a human .
- i — Rate of immunisation for host humans.
- d — Death rate among infected humans as a result of the disease.
- mb — mosquito birth rate.
- md — mosquito death rate.
- N_H - initial human population.
- N_M - initial mosquito population.
- T_H - Current human population.
- T_M - Current mosquito population.

We shall consider the normalized value of the above compartments while visualization for a more holistic representation. Furthermore, time has been normalized so as highlight the events happening in unit cycle of the disease.

We have next generation reproduction number(R) defined as,

$$R = \sqrt{\frac{U_H[0] \cdot U_M[0] \cdot p^2}{(i+d+r) \cdot md \cdot N_H \cdot N_M}}$$

where $U_M[0]$ and $U_H[0]$ are the initial uninfected mosquito and human population respectively.

Putting the values from Fig 1,

$$R \approx 5.226$$

The above expression has been calculated by using the next generation model. The next-general model is based on the number of infected mosquitoes produced by a single human .The equations for infected humans/ human hosts and the vector mosquitoes have been considered in their derivation.

Since R0 > 1, we can consider the model determined by the above parameters to be an epidemic. In a bid to reduce this number we shall discuss 4 control measures :

- Fumigation - increases death rate of mosquitoes (md)
- Vaccination - Reduces Susceptible Population
- Use of Mosquito Repellents by individuals - decrease probability of biting (p_{bit})
- Vector Control - decrease mosquito birth rate so that in the next cycle we have lower mosquito population.

The control measures use the parameter values in Fig 1 and 2, unless mentioned otherwise.

III. RESULT

1. **Total Fumigation**
   Here, we assume the fumigation to take place the entire duration of time. Fumigation results in an increase in death rate of the mosquitoes by the addition of fumigation rate 'f'. This can be seen in the figures below.

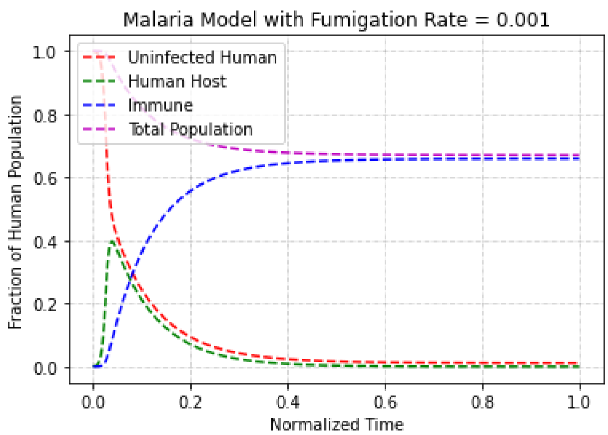

FIG. 3: Human Population with f = 0.001

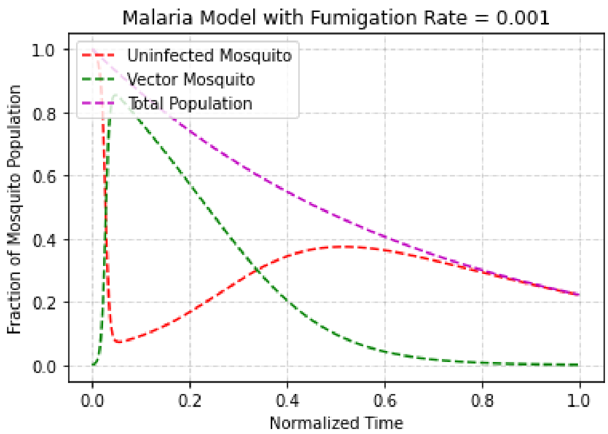

FIG. 4: Mosquito Population with f = 0.001

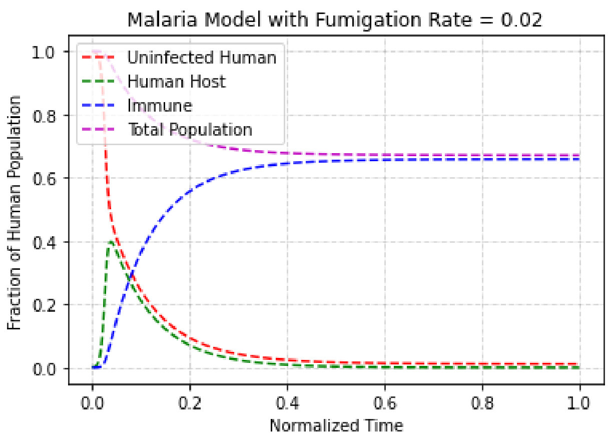

FIG. 5: Human Population with f = 0.02

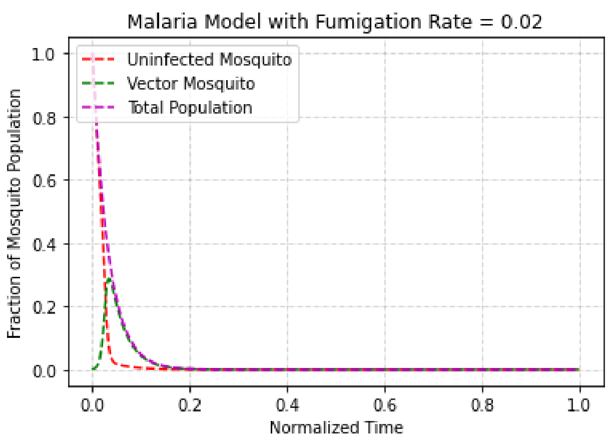

FIG. 6: Mosquito Population with f = 0.02

As long as the fumigation happens we see a significant decrease in the mosquito population. However, this model isn't realistic enough as we can't fumigate for the entire duration.

But in theory, it has the capability to significantly reduce mosquito population.

As the rate of fumigation increases, the mosquito population decreases and it also results in lesser deaths in case of humans. Human host count gradually decreases as fumigation increases. The change in infected human population is slower as compared to the mosquito population on account of the choice of the parameters. However, it is essential that we don't let the vector mosquito count as well as the net mosquito population increase incessantly.

2. **Partial Fumigation**
A more realistic scenario could be the possibility of fumigation happening during a particular interval of time. Here we have taken this interval to be from t = $\frac{1}{4}$ to t =$\frac{3}{4}$.

We observe that in that interval the vector population decreases faster along with the net mosquito population as well. Once the fumigation is over the mosquito population slowly settles down at a lower value. As the fumigation rate increases, the rate of decrease for vector mosquito's increases.

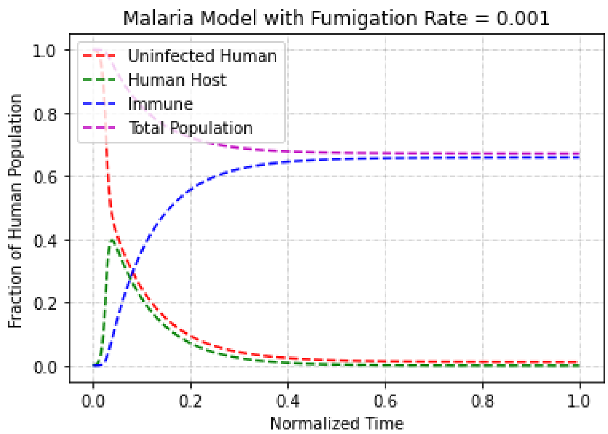

FIG. 7: Human Population with f = 0.001

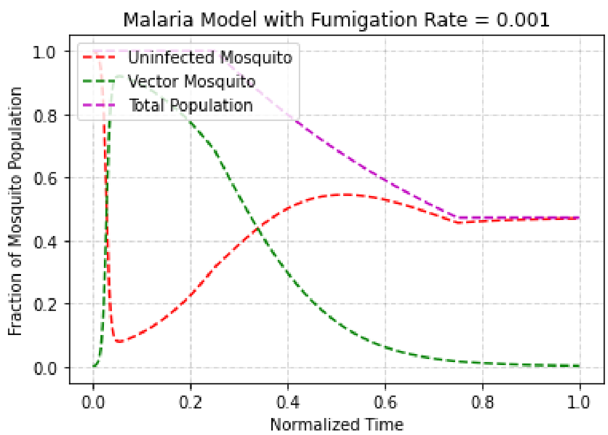

FIG. 8: Mosquito Population with f = 0.001

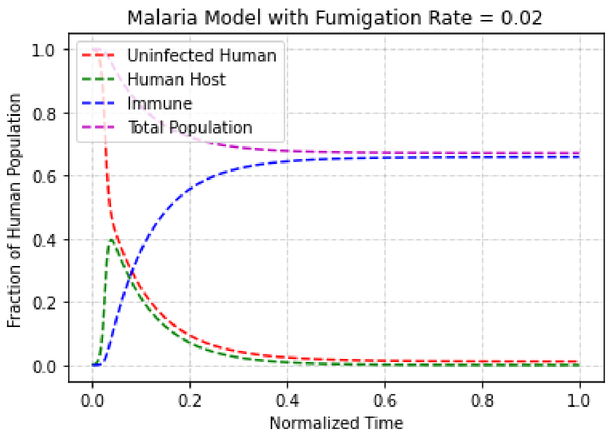

FIG. 9: Human Population with f = 0.02

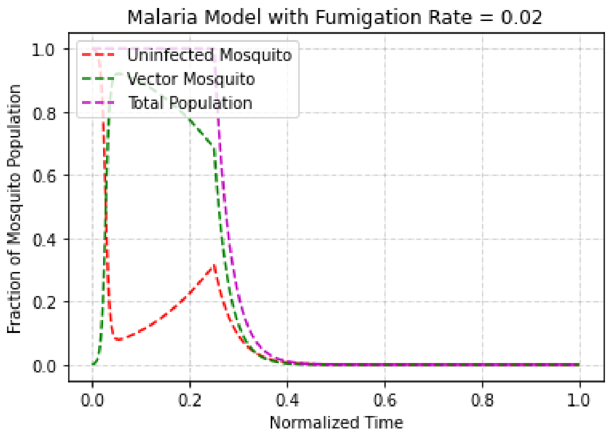

FIG. 10: Mosquito Population with f = 0.02

With a proper strategy, the net effect still remains the same. One efficient way could be to keep this interval in the beginning, when the vector mosquitoes start to breed and increase. If we could decrease the population early, then we can expect it to be a viable control measure.

3. **Vaccination**
It results in better immunity for uninfected humans.

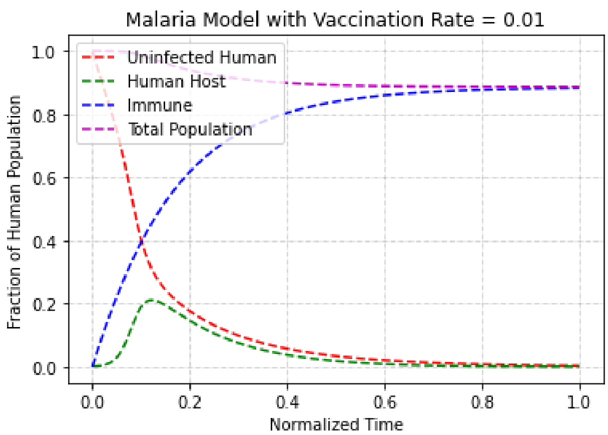

FIG. 11: Human Population with v = 0.01

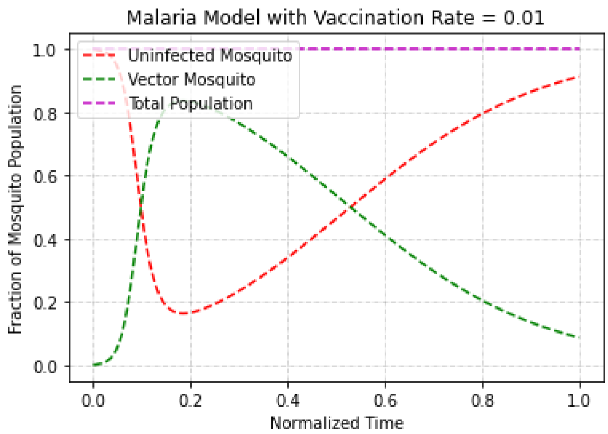

FIG. 12: Mosquito Population with v = 0.01

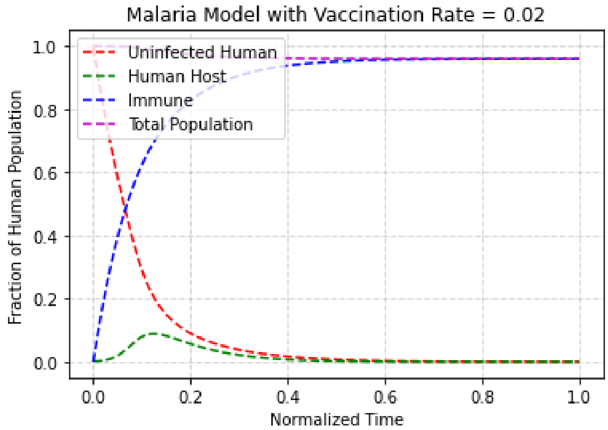

FIG. 13: Human Population with v = 0.02

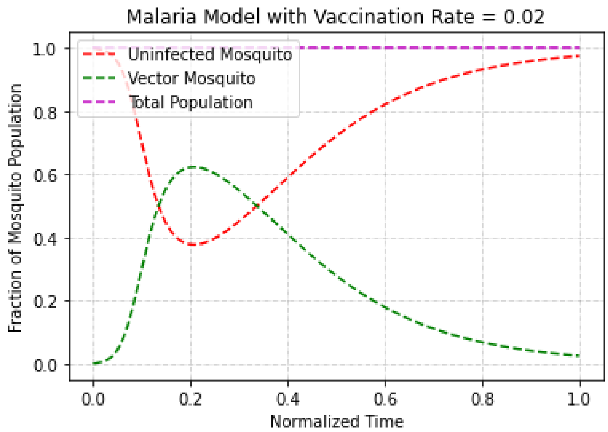

FIG. 14: Mosquito Population with v = 0.02

We see that as the rate of vaccination increases the rate at which the immune population is increasing, increases as more and more humans become immune. At every time step, a certain fraction depicted by v (Vaccination Rate), is moved from the Susceptible Human Compartment to the Immune Compartment.

The deaths and the peak host count reduce as seen in the figures. One can observe that the effect of vaccination in reducing the infected human population is far more effective than most of the other measures.

4. **Mosquito Repellent**
On using mosquito repellent, the probability of an infected mosquito biting a human reduces.

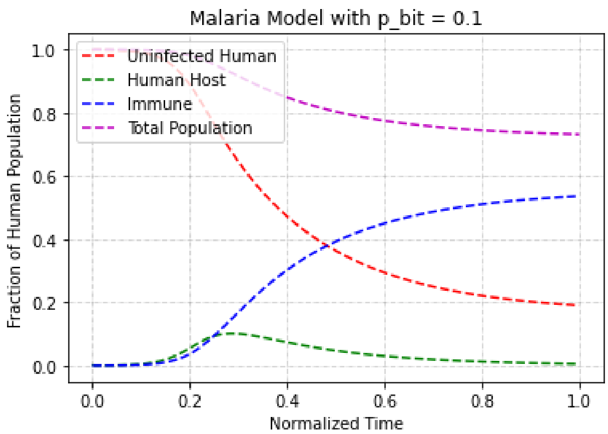

FIG. 15: Human Population

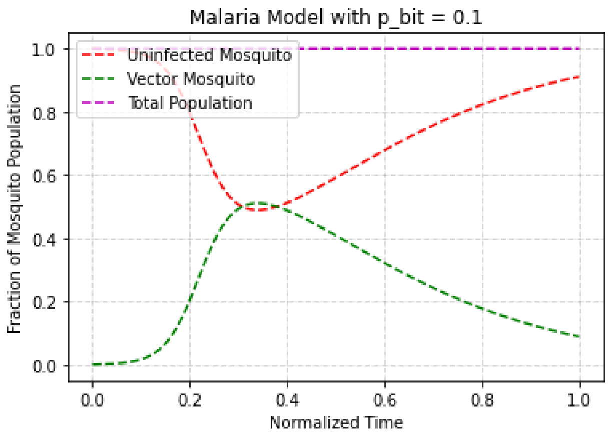

FIG. 16: Mosquito Population

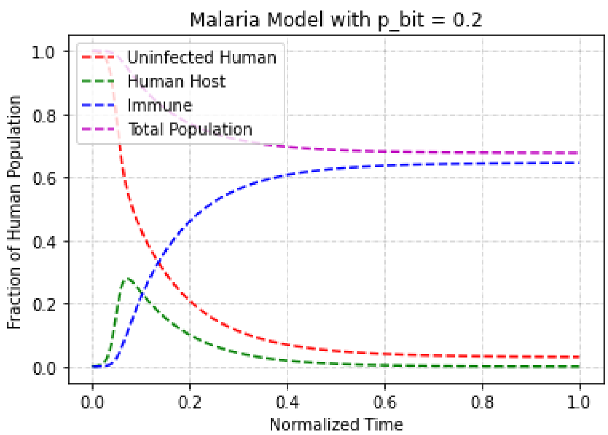

FIG. 17: Human Population

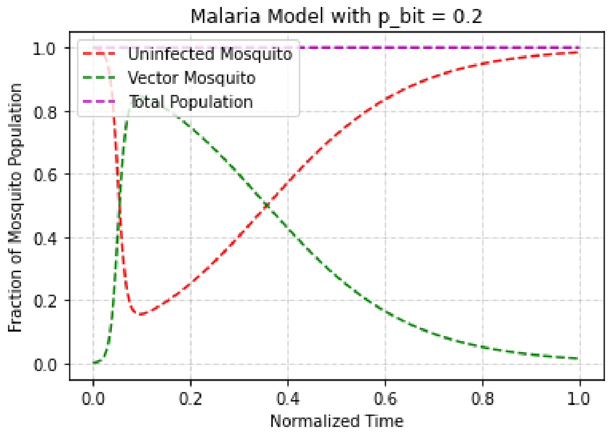

FIG. 18: Mosquito Population

As we can see from the graph, the peak vector count reduces as the biting probability reduces. Thus, the human host count also flattens out towards the right resulting in lesser deaths among humans as they become immune.

5. **Vector Control**
Until now, we had assumed the birth rate and death rate of the mosquitoes to remain same thereby keeping the mosquito population constant. Here however we seek to limit the birth rate by finding the breeding sites for mosquitoes.

As the birth rate is less than the death rate, the mosquito population gradually starts decreasing with time. This significantly reduces the vector population as seen in the figures given below. Furthermore when the death rate is more than the birth rate, the total mosquito population decreases slowly as compared to the uninfected mosquitoes. Thus in the next cycle the ratio $U_M[0]/N_M$ lowers, thus decreasing the reproduction number.

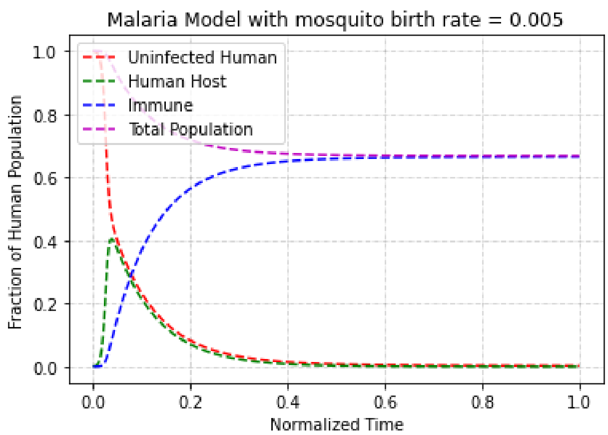

FIG. 19: Human Population

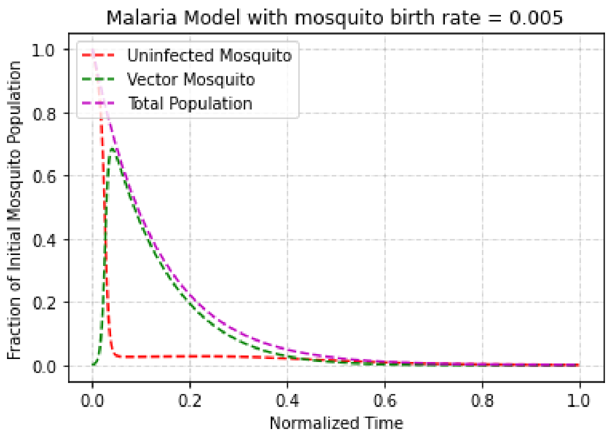

FIG. 20: Mosquito Population

In the event that the birth rate is more than the death rate on account of human negligence, we observe that the net mosquito population increases exponentially as compared to the initial population. This would prove to be a bigger problem in the next disease cycle.

6. **Dengue Outbreak**

DA-IICT witnessed a Dengue outbreak in the after the monsoon of 2019. Despite regular fumigation by the college authorities and preventive measures like mosquito repellents, there were relapses and Dengue persisted. The main reason was that the primary focus was not on identifying the mosquito breeding sites that came into being because of water logging post construction work. This resulted in an increase in birth rate of mosquitoes despite killing the mosquitoes by fumigation or using repellents to lower the probability of getting bitten. The high birth rate kept resulting in relapses of Dengue.

Thus it is very important to control the population of mosquitoes. Only by controlling the population of mosquitoes (not letting it increase), we can incorporate the other aforementioned measures in successfully controlling the spread of this persistent plague. Also, in the event of partial fumigation, it should be done as soon as possible, because our primary goal is to reduce the number of vector mosquitoes which rapidly increases in the first segment itself. This is particularly important because these vectors are itself responsible for the spread of the disease.

Our primary goal is to reduce the reproduction number.The above measures manipulated the parameter values of the mosquito death rate, the susceptible population as well as the mosquito population .By ensuring proper medical facilities we can further increase the recovery rate ('r') for the infected. The reduce in number of mosquitoes is especially important to control the disease in the succeeding cycles.

IV. CONCLUSIONS

We analyzed vector based disease spread model to numerically see how the disease spreads among humans and mosquitoes and we also saw how the two sub models interact with each other. We also analyzed how various interventions done in one of the sub models effects the other sub model and the system as a whole. We observed that the using mosquito repellents to reduce the probability of being bit or getting vaccinated are the two most effective measure to control such diseases in the current cycle itself. We furthermore, saw how changing the birth and death rate affects the mosquito population as well as the infected human population. One could also analyze the grave danger that arises if we take this issue lightly, i.e. it leads to an increase in the net mosquito population,thus making the next cycle for this disease even more deadly.

[1] Module 4.4, A. Shiflet and G. Shiflet, *Introduction to Computational Science: Modeling an Simulation for the Sciences*, Princeton University Press.3, 276 (2006).