Exercises 1

- sigmoid neurons simulating perceptrons, part I

let w_i be the weights of all i perceptrons and
x_i be the all the i inputs.
b be the bias

WKT

$$\text{output} = \begin{cases} 0 & \text{if } w.x + b \leq 0 \quad -①\\ 1 & \text{if } w.x + b > 0 \quad -② \end{cases}$$

Now let c be the constant multiplied by weights and the bais

multiplying c on both sides of eqⁿ ① & ②

$$c(w.x) + c(b) \leq c(0) \quad -①$$
$$c(w.x) + c(b) > c(0) \quad -③$$

upon solving we see that the inequality remains the same

∴ the behaviour of neural network does not change.

- part II

Given is a network of perceptrons for which $wx + b \neq 0$ holds true $\forall x$

Replacing perceptrons with sigmoid neurons

$$\frac{1}{1+e^{-z}} \quad ; \quad z = (\Sigma w_i x_i + b)$$

multiplying weights and bais with c

new weights $= c \cdot w_i$

new bias $= c \cdot b$

upon substitution we get

$$\frac{1}{1+e^{-(\Sigma c w_i x_i + cb)}}$$

now

$$\lim_{c\to\infty} = \frac{1}{1+e^{-(\Sigma c w_i x_i + cb)}}$$

$$= \frac{1}{1+e^{-c[\Sigma w_i x_i + b]}}$$

let $\Sigma w_i x_i + b = x$

$$\lim_{c\to\infty} = \frac{1}{1+e^{-cx}}$$

now $e^{-\infty}$ is a very small number

$\therefore$

$$\frac{1}{1+e^{-\infty}} = 1$$

$\therefore$ The behaviour does not change

This will fail for $wx+b=0$ because when e is raised to high negative number the value tends to a very small number. Here it tends to $-\infty$ $\therefore$ it decreases at a very high rate and is a very small number and $\therefore$ upon dividing with 1, it gives out 1 only.

This case would be successful if and if the constant value c was -ve