1. Tomando $b=\bar{y}-a\bar{x}$, basta que $a=\frac{cov(x,y)}{var(x)}$

$$cov(x,y)=\frac{1}{n}\sum_{i=1}^{n}(y_i-\bar{y})(x_i-\bar{x})$$

$$=\frac{1}{n}\sum_{i=1}^{n}(ax_i+b-(a\bar{x}+b))(x_i-\bar{x})$$

$$=\frac{1}{n}\sum_{i=1}^{n}(ax_i-a\bar{x})(x_i-\bar{x})$$

$$=\frac{a}{n}\sum_{i=1}^{n}(x_i-\bar{x})(x_i-\bar{x})=a\,Var(x)$$

$$\Rightarrow a=\frac{cov(x,y)}{var(x)}.$$

Como $\bar{y}=\sum_{i=1}^{n}\frac{y_i}{n}=\sum_{i=1}^{n}\frac{ax_i+b}{n}=a\sum_{i=1}^{n}\frac{x_i}{n}+\frac{nb}{n}=a\bar{x}+b.$

$\Rightarrow b=\bar{y}-a\bar{x}$ lo anterior es suficiente..

4.2).a) $x'=0$ y $y'=0$.

$$\bullet\ x'=\frac{1}{n}\sum_{i=1}^{n}x_i'=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})=\frac{1}{n}\sum_{i=1}^{n}x_i-\frac{n\bar{x}}{n}=\bar{x}-\bar{x}=0.$$

y' es análogo a lo anterior.

4.2)b). $Var(x)=Var(x')$ y $Var(y)=Var(y')$

$$Var(x)=\frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2=\frac{1}{n}\sum_{i=1}^{n}(x_i'-0)^2=Var(x')$$

y' es análogo.

4.2) c). $\operatorname{cov}(x', y') = \operatorname{cov}(x', y) = \operatorname{cov}(x, y') = \operatorname{Cov}(x, y)$.

$$\operatorname{Cov}(x', y') = \frac{1}{n}\sum^{n}(x_i')^2 (y_i')^2.$$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 (y_i')^2 = \operatorname{Cov}(x, y')$$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 (y_i - \bar{y})^2 = \operatorname{Cov}(x, y).$$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i')^2 (y_i - \bar{y})^2 = \operatorname{Cov}(x', y).$$

4.2.d)

$$R(x, y) = R(x', y')$$

$$R(x, y) = \frac{\operatorname{Cov}(x, y)}{\sqrt{\operatorname{var} X}\sqrt{\operatorname{var} Y}} = \frac{\operatorname{cov}(x', y')}{\sqrt{\operatorname{Var} X}\sqrt{\operatorname{Var} Y}} = R(x', y').$$

4.3). a) $\bar{Z}_x = \bar{Z}_y = 0$.

$$\bar{Z}_x = \frac{1}{n}\sum_{i=1}^{n}\frac{x_i - \bar{x}}{\sqrt{\operatorname{var} x}} = \frac{1}{\sqrt{\operatorname{var} x}}\left(\sum_{i=1}^{n}\frac{x_i}{n} - \sum_{i=1}^{n}\frac{\bar{x}}{n}\right) = \frac{1}{\sqrt{\operatorname{var} X}}(\bar{x} - \bar{x}) = 0$$

y es análogo.

b) $\operatorname{Var}(Z_x) = \operatorname{Var}(Z_y) = 1$.

$$\operatorname{Var}(Z_x) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{(x_i - \bar{x} - 0)}{\sqrt{\operatorname{var} X}}\right)^2 = \frac{1}{\operatorname{Var}(x)}\sum_{i=1}^{n}\frac{(x_i - \bar{x})^2}{n}$$

$$= \frac{\operatorname{Var}(X)}{\operatorname{var}(X)} = 1$$

y es análogo

c)

$$R(x,y) = Cor(Z_x, Z_y)$$

$$R(x,y) = \frac{1}{n}\sum_{i=1}^{n} \frac{(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{Var X}\sqrt{Var Y}} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i-\bar{x}}{\sqrt{Var X}}-0\right)\left(\frac{y_i-\bar{y}}{\sqrt{Var Y}}\right)$$

$$= \frac{1}{n}\sum_{i=1}^{n}(Z_{x_i}-0)(Z_{y_i}-0) = Cov(Z_x, Z_y).$$

4)

1) $\langle x,y\rangle = \langle x'+\bar{x}, y'+\bar{y}\rangle = \langle x',y'\rangle + \langle x',\bar{y}\rangle + \langle \bar{x},y'\rangle + \langle \bar{x},\bar{y}\rangle$

$= \langle x',y'\rangle + \bar{y}\langle x',1\rangle + \bar{x}\langle 1,y'\rangle + \bar{x}\bar{y}\langle 1,1\rangle.$

2) $\langle z,1\rangle = \langle 1,z\rangle = \frac{1}{n}\sum_{i=1}^{n} z_i \cdot 1 = \bar{z}$

$\Rightarrow \langle x,y\rangle = cov(x',y') + \bar{y}\bar{x'} + \bar{x}\bar{y'} + \bar{x}\bar{y} = cov(x,y) + \bar{x}\bar{y}.$

pues $\bar{x'} = \bar{y'} = 0$ por d).

También tenemos que.

$\|x\|^2 = \langle x,x\rangle = \langle x'+\bar{x}, x'+\bar{x}\rangle = \langle x',x'\rangle + \langle x',\bar{x}\rangle + \langle\bar{x},\bar{x}\rangle + \langle\bar{x},x'\rangle.$

$= Var(x') + 2\bar{x}\bar{x'} + \bar{x}^2 = Var(x) + \bar{x}^2$; por a) y b).

Cálculo es análogo para y.

$$\Rightarrow Cor(x,y) = \frac{cov(x,y) + \bar{x}\bar{y}}{\|x\|\|y\|} = \frac{cov(x,y)+\bar{x}\bar{y}}{\sqrt{\|x\|^2}\sqrt{\|y\|^2}}$$

$$= \frac{cov(x,y)+\bar{x}\bar{y}}{\sqrt{var(x)+\bar{x}^2}\sqrt{var(y)+\bar{y}^2}}$$