Question 16:-

If $((p \Rightarrow q) \wedge (r \Rightarrow q))$ is valid, —— ①

then $(p \vee r) \Rightarrow q)$ is valid. —— ②

Answer:- Proof by contradiction.,

Assume ① is true, but ② is false.

$A'((p \Rightarrow q) \wedge (r \Rightarrow q)) = 1$ —— ③

$A'((p \vee r) \Rightarrow q) = 0$ —— ④

③ $\Rightarrow$ $A'[(\neg p \vee q) \wedge (\neg r \vee q)] = 1$

$A'[\neg p \vee q] = 1$ and $A'(\neg r \vee q) = 1$

$A'(\neg p)$ or $A'(q) = 1$ and $A'(\neg r \vee q) = 1$

$A'(\neg p) = 1$ or $A'(q) = 1$ and $A'(\neg r)$ or $A'(q) = 1$

$A'(p) = 0$ (or) $A'(q) = 1$ and $A'(r) = 0$ or $A'(q) = 1$

From above.,

$A'(p) = 0$

$A'(q) = 1$

$A'(r) = 0$

④ $\Rightarrow$ $A'[\neg(p \vee r) \vee q] = 0$

$A'[(\neg p \wedge \neg r) \vee q] = 0$

$A'(\neg p \wedge \neg r) = 0 \quad \vee \quad A'(q) = 0$

$\neg A'(p \vee r) = 0$

~~$A'(\neg p \wedge \neg r) = 0$~~

$A'(p \vee q) = 1$

~~From the ④ equation.,~~

~~$\therefore A'(p) = 1$~~

~~$\therefore A'(r) = 1$~~

$\therefore A'(q) = 0$

From the equation (2).,
the values are .,

$A'(q) = 0$
$A(p) = 1$
$A'(r) = 0$

$p = 1$ (or) $p = 0$ (or) $p = 1$
$q = 0$ $q = 1$ $q = 1$

Applying the values in equation (1).,

$(1 \text{ or } 0) \wedge (0 \vee 0) \rightarrow$ condition is false

$(0 \text{ or } 0) \wedge (1 \text{ or } 0) \rightarrow$ condition is false

$(1 \text{ or } 0) \wedge (1 \text{ or } 0) \rightarrow$ condition is true

Hence., as per contradiction., the values makes eqn (1) are
$A(q) = 0$, $A(p) = 1$, $A'(r) = 0$.
are valid & hence eqn (2) is false as per contradiction.
Now, both eqn are valid..