GATE AEROSPACE FORMULA BOOK

Advice Engineer
EDUCATIONAL SERVICES

First Edition

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Preface

This book contains all the formula related to all the five subjects which comes under GATE Aerospace Engineering.
Those Subjects are:

(a) Aerodynamics
(b) Flight Mechanics
(c) Aircraft Structures
(d) Aircraft Propulsion
(e) Space Mechanics

Many illustrations have been given which will help the student to understand clearly the concepts of the subject.

This book is not for a new person who is preparing for GATE Aerospace Engineering. The person who have completed the whole syllabus at least once in their life will get the most out of this book.

Before reading this book, please complete your whole syllabus. This book will be a revision before exam when you will solve questions only.

Best of Luck

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AIRCRAFT PROPULSION

1. Thrust, $F = (\dot{m}_a + \dot{m}_f)U_e - \dot{m}_a U + (P_e - P_i)A_e$

Momentum Thrust $= (\dot{m}_a + \dot{m}_f)U_e - \dot{m}_a U$

Pressure Thrust $= (P_e - P_i)A_e$

In almost every case, $\dot{m}_f < \dot{m}_a$

2. Propulsive Efficiency $\eta_p = \frac{Thrust\ Power}{Exit\ KE - Inlet\ KE} = \frac{2U}{U_e + U}$

3. Thermal Efficiency $\eta_t = \frac{Exit\ KE - Inlet\ KE}{Heat\ given} = \frac{\frac{1}{2}(\dot{m}_a + \dot{m}_f)U_e^2 - \frac{1}{2}\dot{m}_a U^2}{\dot{m}_f Q_R}$

4. Overall Efficiency $\eta_o = \eta_p \times \eta_t$

5. Specific Impulse $I_{sp} = \frac{Thrust}{g \times \dot{m}_f}$

6. Range $= U I_{sp}\left(\frac{L}{D}\right)\ln\left(\frac{m_{ac}}{m_{ac} - m_{fuel}}\right)$

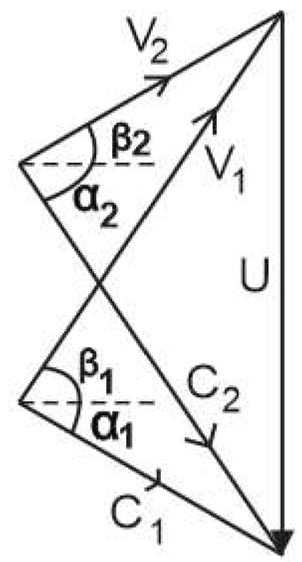
7. Specific fuel Consumption, $sfc = \frac{fuel\ to\ air\ ratio}{Specific\ Thrust}$

8. Bypass Ratio $= \boldsymbol{\alpha} = \frac{\dot{\boldsymbol{m}}_{\boldsymbol{Fan}}}{\dot{\boldsymbol{m}}_{\boldsymbol{Core}}}$

9. Diffuser Efficiency = $\eta_d = \frac{(P_d)^{\frac{\gamma-1}{\gamma}}\left(1+\frac{\gamma-1}{2}M_a^2\right)-1}{\frac{\gamma-1}{2}M_a^2}$ where P_d is pressure ratio across diffuser.

AXIAL COMPRESSOR:

10. Velocity Triangle of Compressor:



11. U and C are blade velocity and axial velocity respectively. V is relative velocity.

12. $\frac{U}{C} = \tan\alpha_1 + \tan\beta_1 = \tan\alpha_2 + \tan\beta_2$

13. Work done per unit mass $= UC(\tan\alpha_2 - \tan\alpha_1) = UC(\tan\beta_1 - \tan\beta_2)$

14. Compressor Efficiency $= \eta_c = \frac{\left((nP_c)^{\frac{\gamma-1}{\gamma}}-1\right)T_{02}}{T_{03}-T_{02}}$ where P_c is compressor pressure ratio and n is number of stage.

15. Degree of Reaction $\lambda = 1 - \frac{C}{2U} \times (tan\,\alpha_2 + tan\,\alpha_1)$

16. Case 1. $\lambda = 0$, Impulse Turbine. No pressure rise in rotor.

17. Case 2. $\lambda = 0.5$, Symmetric velocity Triangles. $\alpha_2 = \beta_1, \alpha_1 = \beta_2$ Equal pressure Rise in Rotor and Stator

18. Case 3. $\lambda = 1$, No Contribution of Stator.

19. Polytropic Efficiency of Compressor $\eta_c = \frac{(P_c)^{\frac{\gamma-1}{\gamma}}-1}{(P_c)^{\frac{\gamma-1}{\gamma\eta_{poly}}}-1}$

TURBINE:

20. Turbine Efficiency $= \frac{T_{01}-T_{03}}{T_{01}-T_{03}'} = \frac{1-\left(\frac{T_{03}}{T_{01}}\right)}{1-(P_t)^{\frac{(\gamma-1)}{\gamma}}}$

21. In turbine, $\frac{U}{C_a} = \tan\alpha_2 - \tan\beta_2 = \tan\beta_3 - \tan\alpha_3$

22. In turbine, $W = UC(\tan\alpha_2 + \tan\alpha_3)$, from above $\tan\alpha_2 + \tan\alpha_3 = \tan\beta_2 + \tan\beta_3$

23. Degree of Reaction, $\lambda = \frac{C_a}{2U}(\tan\beta_3 - \tan\beta_2)$

24. $\eta_t = \frac{1-(P_t)^{\frac{\gamma-1\times\eta_{poly}}{\gamma}}}{1-(P_t)^{\frac{\gamma-1}{\gamma}}}$

NOZZLE:

25. Mass flow rate in nozzle $= \frac{AP_0}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}} \times \frac{M}{\{1+\frac{\gamma-1}{2}M^2\}^{\frac{\gamma+1}{2(\gamma-1)}}}$

26. Throat to Exit Area Ratio $= \frac{A_t}{A_e} = \frac{P_{0e}}{P_{0t}} \times \frac{M_e}{M_t} \times \left[\frac{1+\left(\frac{\gamma-1}{2}\right)M_t^2}{1+\left(\frac{\gamma-1}{2}\right)M_e^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$

ROCKET PROPULSION:

27. Linear Burning Rate $r = k(P_c)^{n_1}$, P_c is combustion chamber pressure

28. Propellant Consumption rate, $\dot{m_c} = \rho r A$

29. Propellant flow rate $=\ \dot{m_p} = \dot{m_c} - \frac{d}{dt}(\rho V)$

30. Specific Impulse $= I_{sp} = \frac{Thrust}{Propellant\ weigh\ \ flow\ rate} = \frac{T}{\dot{m_p} g} = \frac{U_e}{g}$

31. Specific propellant consumption $= \frac{1}{I_{sp}}$

32. Weight flow coefficient $= C_w = \frac{\dot{w_p}}{P_o A^*}$

33. Thrust Coefficient $= C_F = \frac{F}{P_0 A^*}$

34. Characteristic Velocity $V^* = \frac{U_e}{C_F}$

35. Rocket Equation: $\dot{m_p} = -\frac{dm_p}{dt}$

36. At $t = 0, u = 0, m = m_1$ and $t = t_p, u = u_p, m = m_2$

$$u_p + gt_p = gI_{sp} \ln \frac{m_1}{m_2}$$

Since t_p is very less, so we have $u_p = U_e \ln \frac{m_1}{m_2}$

AIRCRAFT STRUCTURES

ELASTICITY:

1. Stress = $\sigma = \frac{Force}{Cross\ Sectional\ Area} = \frac{F}{A}$

2. Strain $= \epsilon = \frac{Chang\quad in\ Leng}{Original\ Len} = \frac{\Delta L}{L}$

3. Modulus of Elasticity $= E = \frac{Stress}{Strain} = \frac{\sigma}{\epsilon}$

4. Stresses in normal directions are $\boldsymbol{\sigma_{xx}}, \boldsymbol{\sigma_{yy}}, \boldsymbol{\sigma_{zz}}$ and are called **Normal Stresses**.

   Let u, v and w are displacement in x, y and z directions therefore,

   Normal strains are:

$$\boldsymbol{\epsilon_{xx}} = \frac{\partial u}{\partial x}, \qquad \boldsymbol{\epsilon_{yy}} = \frac{\partial v}{\partial y}, \qquad \boldsymbol{\epsilon_{zz}} = \frac{\partial w}{\partial z}$$

5. Stresses in parallel directions are $\boldsymbol{\tau_{xy}}, \boldsymbol{\tau_{yz}}, \boldsymbol{\tau_{zx}}$ and are called **Shear Stresses**.

   Let u, v and w are displacement in x, y and z directions therefore,

   Shear strains are:

$$\boldsymbol{\gamma_{xy}} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \qquad \boldsymbol{\gamma_{yz}} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \qquad \boldsymbol{\gamma_{xz}} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

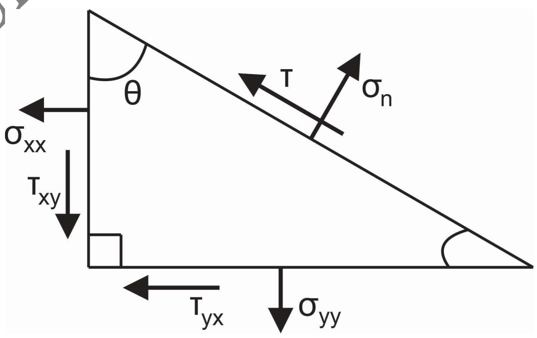
6. Equilibrium Equations:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} = 0$$

$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0$$

7. Stresses in Inclined Plane under Equilibrium:



$$\sigma_n = \sigma_{xx}\cos^2\theta + \sigma_{yy}\sin^2\theta + \tau_{xy}\sin 2\theta$$

$$\tau = \left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)\sin 2\theta - \tau_{xy}\cos 2\theta$$

$$2\theta = \tan^{-1}\left(\frac{2\tau_{xy}}{\sigma_{xx} - \sigma_{yy}}\right)$$

8. Principal Stresses:

$$\sigma_{1,2} = \left(\frac{\sigma_{xx} + \sigma_{yy}}{2}\right) \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right) + \tau_{xy}^2}$$

9. For unidirectional stresses, let the stress is in x direction,

Along with normal strain, there is also existence of shear strain.

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E}, \qquad \epsilon_{yy} = -\nu_{xy}\frac{\sigma_{xx}}{E}, \qquad \epsilon_{zz} = -\nu_{xz}\frac{\sigma_{xx}}{E}$$

where ν_{xy} and ν_{xz} are poisson ratios.

9. Poisson's Ratio = $\nu = \frac{Lateral\ strain}{Normal\ Strain}$. Its range is -1 to 3.

10. Ratio of Shear Stress to Shear Strain is called Modulus of Rigidity and represented by G.

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

11. Till now we have assumed that mechanical properties E, ν and G are independent of direction. But they can also become direction dependent and it varies by material.

12. Stress Strain Relations in 3 Dimensions:

In matrix form it can be written as:

$$[\epsilon] = [a][\sigma]$$

where $[\epsilon]$ is a 6×1 matrix containing six strain components, $[a]$ is a 6×6 matrix containing 36 components called elastic compliances. Similarly, $[\sigma]$ is also a 6×1 matrix containing all six stress components.

13. There are maximum 21 elastic constants in any linear elastic material.

The material which contains all 21 elastic constants are called **anisotropic materials**.

14. **Orthotropic Materials** have 9 elastic constants and **Isotropic materials** have only 2 elastic constants.

15. Elastic Strain Energy:

$$\Delta U = \frac{1}{2}\Delta V\left(\sigma_{xx}\epsilon_{xx} + \sigma_{yy}\epsilon_{yy} + \sigma_{zz}\epsilon_{zz}\right)$$

Elastic Strain Energy per unit volume is called **Strain Energy Density** and represented by W.

16. For any isotropic material,

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{zz}}{E}$$

$$\epsilon_{yy} = \frac{\sigma_{yy}}{E} - \nu\frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{zz}}{E}$$

$$\epsilon_{zz} = \frac{\sigma_{zz}}{E} - \nu\frac{\sigma_{xx}}{E} - \nu\frac{\sigma_{yy}}{E}$$

17. With reference to point 16, the case of **Plain Strain**, is as follows:

If strains in Z direction is zero, then this condition is called **Plain Strain**.

$$\epsilon_{zz} = \gamma_{yz} = \gamma_{xz} = 0$$

$$\therefore \tau_{yz} = \tau_{xz} = 0$$

Use these conditions in point 16 to get respective equations of plain strain.

18. With reference to point 16, the case of **Plain Stress**, is as follows:

If stresses in Z direction is zero, then this condition is called **Plain Stress**.

$$\sigma_{zz} = \tau_{yz} = \tau_{xz} = 0$$

$$\gamma_{yz} = \gamma_{xz} = 0$$

Use these conditions in point 16 to get respective equations of plain stress.

19. **Compatibility Equations**:

$$\frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \frac{\partial^2 \epsilon_{yy}}{\partial x^2} + \frac{\partial^2 \epsilon_{xx}}{\partial y^2}$$

$$\frac{\partial^2 \gamma_{yz}}{\partial y \partial z} = \frac{\partial^2 \epsilon_{zz}}{\partial y^2} + \frac{\partial^2 \epsilon_{yy}}{\partial z^2}$$

$$\frac{\partial^2 \gamma_{xz}}{\partial x \partial y} = \frac{\partial^2 \epsilon_{zz}}{\partial x^2} + \frac{\partial^2 \epsilon_{xx}}{\partial z^2}$$

$$2\frac{\partial^2 \epsilon_{xx}}{\partial y \partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}\right)$$

$$2\frac{\partial^2 \epsilon_{yy}}{\partial x \partial z} = \frac{\partial}{\partial x}\left(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{zx}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}\right)$$

$$2\frac{\partial^2 \epsilon_{zz}}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{zx}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right)$$

20. **Airy Stress Function** $\phi(x, y)$ is defined for plane stress case.

$$\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2}, \sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2}, \tau_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}$$

If, $\nabla^4 \phi = 0$ then it will be valid airy stress function.

21. Relation between E, G and ν:

$$G = \frac{E}{2(1+\nu)}$$

22. Relation between, E, K and ν:

$$E = 3K(1 - 2\nu)$$

Where K is bulk modulus of body.

TORSION:

1. Compatibility equation for torsion:

$$\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y} = 2\theta$$

Where θ is rate of twist.

2. **Prandtl's Stress Function:** Prandtl defined a stress function $\phi(x, y)$ such that

$$\tau_{xz} = \frac{\partial \phi}{\partial y}, \tau_{yz} = \frac{\partial \phi}{\partial x}$$

3. To find **Torque,** T we have:

$$T = 2 \iint_A \phi dxdy$$

Where ϕ is stress function. ϕ is zero at lateral free surface of a bar.

4. To find **Polar Moment of Inertia:**

$$J = -\frac{4}{\nabla^2 \phi} \iint_A \phi dxdy$$

5. For any arbitrary cross section,

$$\frac{T}{J} = \frac{\tau}{r} = G\theta$$

Here θ is rate of twist per unit length.

6. **Shear Flow:** It is product of shear stress and thickness.

$$q = \tau \times t$$

It is a vector quantity.

7. **Breth – Bratho Theorem:** The net torque produced by constant shear flow (q) in a thin walled section of area A is given by

$$T = q(2A)$$

8. From Breth Bratho theorem, product moment of inertia is given by

$$J = \frac{4A^2}{\oint \frac{ds}{t}}$$

Where ds is length of section.

BENDING:

1. **Centroid** is given by,

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + \cdots ..}{A_1 + A_2 + \cdots}, \qquad \bar{y} = \frac{A_1y_1 + A_2y_2 + \cdots ..}{A_1 + A_2 + \cdots}$$

$(\bar{x}, \bar{y})$ is the location of centroid.

2. **Moment of Inertia**:

$$I_{xx} = \sum y^2 dA, \qquad I_{yy} = \sum x^2 dA$$

Where I_{xx} and I_{yy} is moment of Inertia in x and y direction.

$I = AK^2$ where K is radius of gyration.

3. **Parallel Axis Theorem:**

Moment of Inertia about any axis in the plane of element is equal to sum of moment of inertia of element about centroidal axis and product of area and square of distance between two axes.

$$I_{xx} = I_{\bar{x}\bar{x}} + A\bar{y}^2$$
$$I_{yy} = I_{\bar{y}\bar{y}} + A\bar{x}^2$$

4. **Perpendicular Axis Theorem:**

$$I_{zz} = I_{xx} + I_{yy}$$

5. **Simple Bending** occurs when bending is not accompanied by Torsion.

6. **Pure Bending** occurs when shear force produced in beam is zero.

7. **Bending Equation** is given by:

$$\frac{M}{I} = \frac{E}{R} = \frac{\sigma}{y}$$

Where M is bending moment, I is moment of Inertia, E is Young's modulus, R is radius of curvature, σ is bending stress and y is distance of element from Neutral Axis.

8. The axis where bending stress is zero in an element is called **Neutral Axis.**

9. To find deflection of a beam we use formula:

$$EI\frac{d^2y}{dx^2} = M$$

Where bending moment according to a condition is written on right hand side and two times integrated to get the value of y (deflection).

Deflection of Beam varies according to the beam used.

10. To find the shear stress in bending of beam, we have

$$\tau = P\frac{\bar{y}A}{Ib}$$

Where P is the load applied.

11. **Product of Inertia:**

$$I_{xy} = \int xy dA$$

12. **Castigliano's Theorem:**

"Rate of change of Strain Energy with respect to statically independent force, gives you the component of deflection of this force in the direction of force."

Applications of Castigliano Theorem:

13. Deflection under axial load:

$$\delta = \int \frac{W dx}{AE}$$

14. Deflection under Torsion:

$$\delta = \int \frac{T}{GJ} \cdot \frac{\partial T}{\partial W} dx$$

16. Deflection under bending:

$$\delta = \int \frac{M}{EI} \cdot \frac{\partial M}{\partial W} dx$$

17. Rotation under Torsion:

$$\theta = \int \frac{T}{GJ} dx$$

18. Deflection under Shear:

$$\delta = \int \frac{F dx}{AG}$$

COLUMNS:

Euler's theory of buckling of columns:

1. **Columns hinged at both Ends:**

$$P_e = \frac{\pi^2 EI}{l_e^2}$$

Where l_e is equivalent length of column. Here $l_e = l$.

2. **Columns hinged at one end and fixed at other end:**

$$P_e = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{\left(\frac{l}{\sqrt{2}}\right)^2}$$

Where l_e is equivalent length of column. Here $l_e = \frac{l}{\sqrt{2}}$.

3. **Columns fixed at both ends:**

$$P_e = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{\left(\frac{l}{2}\right)^2}$$

Where l_e is equivalent length of column. Here $l_e = \frac{l}{2}$

4. **Column Fixed at one end and free at the other:**

$$P_e = \frac{\pi^2 EI}{l_e^2} = \frac{\pi^2 EI}{(2l)^2}$$

Where l_e is equivalent length of column. Here $l_e = 2l$

5. **Rankine Formula**:

$$\frac{1}{P_{R,B}} = \frac{1}{P_C} + \frac{1}{P_{E,B}}$$

$P_{R,B}$ = Rankine Buckling Load

P_C = Compressive or Crippling Load

$P_{E,B} = P_e$ = Euler Load

THEORY OF FAILURES:

1. Total strain energy per unit volume:

$$U = \frac{1}{2E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)\right)$$

Total Strain Energy can be split into two parts:

I. Volumetric Strain Energy
II. Shear Strain Energy

2. Volumetric Strain Energy per unit volume:

$$U_{volumetric} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)^2 . \frac{1 - 2\nu}{2E}$$

3. Shear Strain Energy:

$$U_{shear} = \frac{1}{12G}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

4. **Maximum Principal Stress (Rankine Theory):**

According to this, a material fails when value of maximum principal stress σ_1 reaches a stress value at elastic limit σ as found in simple test.

$$\sigma_1 = \sigma$$

5. **The Maximum Principal Strain Theory (St. Venant's theory):**

According to this, a material fails when value of maximum principal strain reaches a strain value as calculated under some complex stress in a simple tensile test.

$$\epsilon = \frac{1}{\mathrm{E}}\left[\sigma_1 - \nu(\sigma_2 + \sigma_3)\right] \leq \epsilon_{\mathrm{yield}} = \frac{\sigma_{\mathrm{yield}}}{E}$$

6. **Maximum Shear stress theory or Guest – Coulomb's theory or Tresca's theory:**

According to this, a material fails when value of maximum shear stress exceeds a shear stress value as calculated in a simple tensile test.

$$\sigma_1 - \sigma_3 = \sigma$$

7. **Maximum Strain Energy Theory:**

$$\frac{1}{2E}\left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_1\sigma_3)\right) = \frac{\sigma^2}{2E}$$

8. **Distortion Energy Theory or Von Mises Theory:**

$$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \frac{\sigma_0^2}{2}$$

VIBRATIONS:

1. Spring Force $F = kx$ where k is spring constant and x is the elongation of spring.
2. If two springs are in parallel, then $k_{eq} = k_1 + k_2 + \cdots$.
3. If two springs are in series, then

$$k_{eq} = \frac{1}{k_1} + \frac{1}{k_2} + \cdots$$

4. **Harmonic Motion:**

Let $\boldsymbol{X_1} = A_1 \cos \omega t$ and $\boldsymbol{X_2} = A_2 \cos(\omega t + \phi)$

Magnitude of resultant vector $\boldsymbol{X} = \sqrt{(A_1 + A_2 \cos\theta)^2 + (A_2 \sin\theta)^2}$

Angle between $\boldsymbol{X_1}$ **and** $\boldsymbol{X}$ is given by

$$\alpha = \tan^{-1}\frac{(A_2\, sin\,\theta)}{(A_1 + A_2\, cos\,\theta)}$$

5. **Spring Mass System in Single Degree of Freedom:**

For $\boldsymbol{X} = \boldsymbol{A}\cos\omega t, \quad \boldsymbol{X}'' = -\omega^2\boldsymbol{X}$

$$\omega = \sqrt{\frac{k}{m}}$$

6. **Vibration of Undamped System:**

$$m\ddot{x} + kx = 0$$

The solution of above differential equation is:

$$x(t) = Ce^{st}$$
$$ms^2 + k = 0$$

Here $s = \pm i\omega_n$

$$\omega_n = \sqrt{\frac{k}{m}}$$

Where k is spring constant and m is mass of spring.

7. **Vibration with viscous damping:**

$$m\ddot{x} + c\dot{x} + kx = 0$$

The solution of above differential equation is:

$$x(t) = Ce^{st}$$

$$ms^2 + cs + k = 0$$

$$s_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \frac{k}{m}}$$

8. **Critical Damping and Damping Ratio:**

The value of c for which term in square root is equal to zero is called critical damping constant.

$$\left(\frac{c_c}{2m}\right)^2 - \frac{k}{m} = 0$$

$$c_c = 2m\omega_n$$

Damping Ratio is given by,

$$\zeta = \frac{c}{c_c}$$

$$\zeta\omega_n = \frac{c}{2m}$$

$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

If $\zeta < 1$, then it is called underdamped system.
If $\zeta = 1$, then it is called critically damped system.
If $\zeta > 1$, then it is called overdamped system.

9. **Logarithmic Increment:**

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \ln\left(\frac{x_2}{x_3}\right)$$

10. **Energy stored in damper:**

$$\Delta W = \pi c\omega_d \boldsymbol{X^2}$$

AERODYNAMICS

BASIC FLUID MECHANICS:

1. **Continuity Equation:**

$$\frac{\partial \rho}{\partial t} + \nabla.\left(\rho \vec{V}\right) = 0$$

In polar coordinates:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta) + \frac{1}{r}\frac{\partial}{\partial z}(\rho u_z) = 0$$

2. **Streamline Equation:**

$$y = x^t$$

3. **Pathline Equation:**

$$\frac{dx}{dt} = u, \qquad \frac{dy}{dt} = v, \qquad \frac{dz}{dt} = w$$

4. **Newton's Law of Viscosity:**

Shear Stress is proportional to shear strain on every plane.

$$\tau = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

5. **Momentum Equation:**

$$\frac{d\vec{u}}{dt} = g\hat{\jmath} - \frac{\nabla \vec{P}}{\rho} + \frac{1}{\rho V}\overrightarrow{dT} + coriolis\ force$$

6. **Circulation:**

$$\zeta = \oint \vec{u}.\vec{dl}$$

7. **Kelvin's Theorem:**

For some fluid particles, $\zeta = constant$ means,

$$\frac{d\zeta}{dt} = 0$$

Under some assumptions. Those assumptions are:

a. Boundary of fluid particles should not interact with the particles in viscous domain.

b. Flow should be barotropic i.e. pressure should be only function of density.

c. Fluid particles should be in a non-rotating frame of reference.

d. Body force has to be conservative.

8. **Bernoulli's Equation:**

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}.\boldsymbol{\nabla u} = -\frac{1}{\rho}\nabla p$$

$$p + \frac{1}{2}\rho u^2 = constant$$

9. **Vorticity Equation:**

$$\frac{D\boldsymbol{\omega}}{Dt} = \boldsymbol{u}.(\boldsymbol{\omega}.\boldsymbol{\nabla}) + \frac{\mu}{\rho}\boldsymbol{\nabla}^2\boldsymbol{\omega}$$

10. **Biot Savart's Law:**

$$d\boldsymbol{V} = \frac{\zeta}{4\pi}\frac{\boldsymbol{dl} \times \boldsymbol{r}}{r^3}$$

11. **Reynolds number:**

Ratio of Inertia and Viscous Force.

$$R = \frac{\rho V L}{\mu} = \frac{VL}{\nu}$$

12. **Froude Number:**
Ratio of Inertia and Gravity Force.

$$F = \frac{V}{\sqrt{Lg}}$$

13. **Euler Number:**
Ratio of Inertia and Pressure Force.

$$E = \frac{V}{\sqrt{\frac{P}{\rho}}}$$

14. **Mach Number:**
Ratio of Inertia and Elasticity Force.

$$M = \frac{V}{\sqrt{\frac{K}{\rho}}}$$

Where K is bulk modulus of elasticity.

15. **Weber Number:**
Ratio of Inertia Force and Surface Tension.

$$W = \frac{V}{\sqrt{\frac{\sigma}{\rho L}}}$$

POTENTIAL FLOW:

1. **Stream Function** (φ):

$$\rho u = \frac{\partial \varphi}{\partial y}, \qquad \rho v = \frac{\partial \varphi}{\partial x}$$

2. **Potential Function** (ϕ)

$$u = \frac{\partial \phi}{\partial x}, \qquad v = \frac{\partial \phi}{\partial x}$$

3. Stream Function is perpendicular to Potential Function.
4. Complex Potential Function is $\omega = \phi + i\varphi$

$$\frac{d\omega}{dz} = u - iv$$

Where $z = x + iy$
5. Types of Flows:
(a) **Uniform Flow:**

$$\frac{d\omega}{dz} = U$$

$$\phi = U.x, \qquad \varphi = U.y$$

(b) **Uniform Flow inclined at an angle $\boldsymbol{\beta}$:**

$$\phi = Ux\cos\beta + Uy\sin\beta$$
$$\varphi = Uy\cos\beta - Ux\sin\beta$$

(c) **Point Source or Sink:**

$$\phi = \frac{p}{2\pi}\ln r, \qquad \varphi = \frac{p}{2\pi}\theta$$

Here p is strength of source/sink. Positive Sign for Source and Negative Sign for Sink.

(d) **Vortex Flow:**

$$\phi = -\frac{\varphi}{2\pi}\theta, \qquad \varphi = \frac{\varphi}{2\pi}\ln r$$

(e) **Half Rankine Oval:**

Here Uniform flow is overlapped on Source flow which creates Half Rankine Oval.
We can superimpose every parameter except pressure.

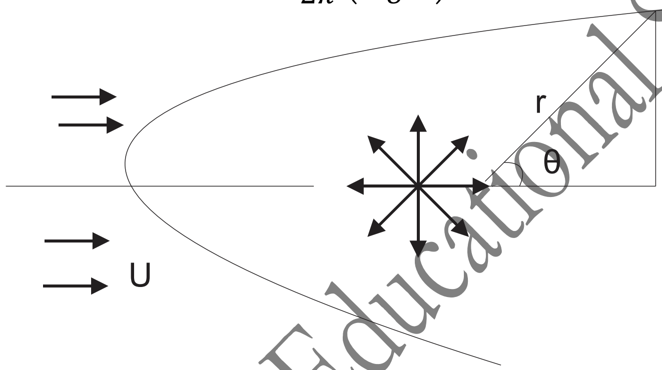
$$\phi = Ur\cos\theta + \frac{p}{2\pi}ln\,r$$
$$\varphi = Ur\sin\theta + \frac{p}{2\pi}\theta$$

Distance between Stagnation point and Source is given by

$$\lambda = \frac{p}{2\pi U}$$

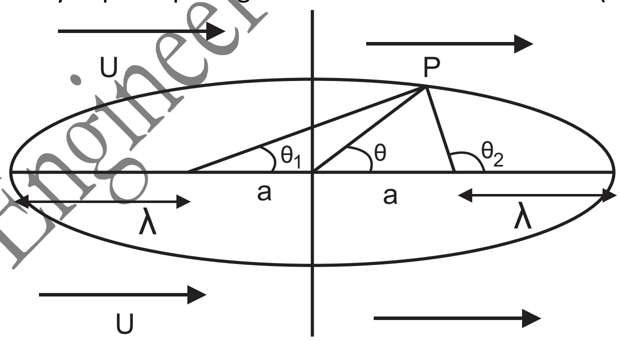
Height of Half Rankine Oval is given by:

$$h = \frac{p}{2\pi}\left(\frac{\pi - \theta}{U}\right)$$



(f) **Full Rankine Oval:**

Full Rankine Oval is created by superimposing Uniform flow with source at (-a, 0) and sink at (a, 0).



$$\omega = Uz + \frac{p}{2\pi}\ln(z+a) - \frac{p}{2\pi}\ln(z-a)$$

(g) **Source – Sink Doublet:**

If source and sink is placed very near to each other, they form a doublet. Let they are separated by a distance 2ϵ when they are at $(-\epsilon, 0)$ and $(\epsilon, 0)$.

$$\omega = \frac{p}{2\pi}\ln(z+\epsilon) - \frac{p}{2\pi}\ln(z-\epsilon)$$
$$\omega = \frac{p\epsilon}{\pi z} = \frac{\mu}{z}$$

Where μ is strength of source/sink doublet.

(h) **Vortex Doublet:**

$$\omega = \frac{\zeta\epsilon}{\pi z} = \frac{\propto}{z}$$

Where α is the strength of vortex doublet.

(i) **Stationary or Non Rotating Cylinder:**

Here uniform flow is superimposed on source sink doublet.

$$\omega = Uz + \frac{\mu}{z}$$

$$u_r = U\left[1 - \frac{\mu}{r^2 U}\right]\cos\theta = U\left[1 - \frac{a^2}{r^2}\right]cos\,\theta$$

$$u_\theta = -U\left[1 + \frac{\mu}{r^2 U}\right]\sin\theta = -U\left[1 + \frac{a^2}{r^2}\right]sin\,\theta$$

Here $a^2 = \frac{\mu}{U}$

At the surface of cylinder, $u_r = 0$ and $u_\theta = -2U\sin\theta$

$$C_p = \frac{P - P_\infty}{\frac{1}{2}\rho U_\infty^2}$$

Where C_p is coefficient of pressure.

(j) **Rotating Cylinder:**

Here uniform flow is superimposed on source sink doublet and vortex flow.

$$\omega = Uz + \frac{\mu}{z} + \frac{i\zeta}{2\pi}\ln z$$

Put $z = re^{i\theta}$ in the above equation to find all the equations u_r and u_θ.

(k) **Corner Flows:**

$$\omega = Az^n$$

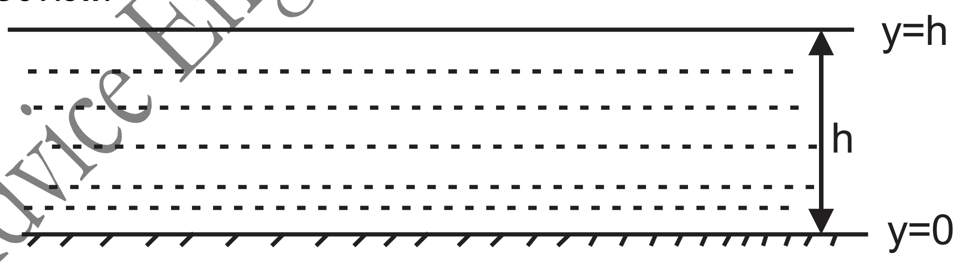
Put $z = re^{i\theta}$ in the above equation to find all the equations u_r and u_θ.

(l) There is no drag or lift on non-rotating cylinder.

(m) On a rotating cylinder, there is no drag and lift is equal to $L = \rho U\zeta$

LAMINAR VISCOUS FLOWS

1. **Couette's Flow:**



Lower plate is stationary here. Upper plate is infinitely long. No pressure gradient.

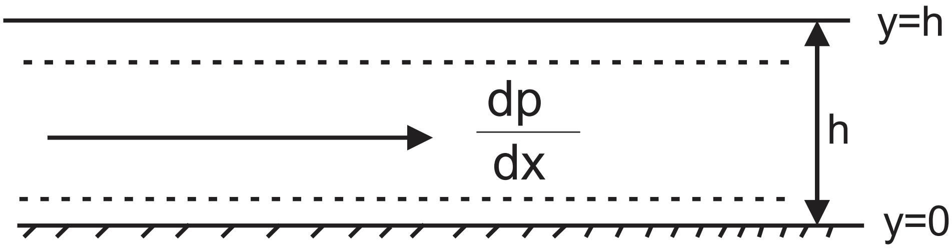
$$u = \frac{V}{h}y$$

Volume flow rate, $Q = \frac{Vh}{2}$

Mass flow rate, $\dot{m} = \rho Q$

Shear Stress, $\tau = \frac{\mu V}{h}$

2. **Plane – Poiseuille Flow:**



$$u = \frac{1}{\mu}\frac{\partial P}{\partial x}\frac{y}{2}\,(y-h)$$

$$Q = -\frac{h^3}{12\mu}\frac{\partial P}{\partial x}$$

$$\tau = \frac{\partial P}{\partial x}\left(y - \frac{h}{2}\right)$$

BOUNDARY LAYER:

1. Boundary layer thickness is given by

$$\delta_x = \frac{0.5x}{\sqrt{R_e}}$$

Where R_e is Reynolds number.

2. Displacement Thickness:

$$\delta^* = \int_0^h \left(1 - \frac{u}{U}\right) dy$$

3. Momentum Thickness:

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

4. Energy Thickness:

$$\delta^{**} = \int_0^\delta \frac{u}{U}\left(1 - \frac{u^2}{U^2}\right) dy$$

5. Von Karman Integral Equation:

$$\frac{UdU}{dx}\delta^* + \frac{\partial}{\partial x}\,[U^2\theta] = \frac{\tau_{wall}}{\rho}$$

6. **Boundary Layer over a flat plate:**

$$\frac{U^2\partial\theta}{\partial x} = \frac{\tau_{wall}}{\rho}$$

Boundary Layer Profile is given by:

$$\boldsymbol{\frac{u}{U} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\frac{y^3}{\delta^3}}$$

$$\theta = \frac{39}{280}\delta$$

$$\delta = \frac{4.62x}{\sqrt{Re_x}}$$

Skin friction Coefficient is given by,

$$C_f = \frac{\mu}{\rho}.\frac{3}{U}.\frac{1}{\delta}$$

$$\delta^* = \frac{1.7325x}{\sqrt{Re_x}}$$

$$\theta = \frac{0.644x}{\sqrt{Re_x}}$$

7. **Prandtl Number:**

$$(\mathrm{Pr})^{\frac{1}{3}} = \frac{\delta_{hydrodynamic}}{\delta_{thermal}}$$

8. **Turbulent Boundary Layer:**

$$\delta_T = \frac{0.37x}{(Re)^{\frac{1}{5}}}$$

$$C_f = \frac{0.074}{(Re)^{\frac{1}{5}}}$$

GAS DYNAMICS

1. Temperature and Pressure Relationship is given by:

$$\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \frac{P_2}{P_1}$$

2. Speed of Sound in an Isothermal Flow:

$$a = \sqrt{RT}$$

3. Speed of Sound in isentropic flow:

$$a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma RT}$$

4. Ratio of Total Temperature to ambient temperature:

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

5. Characteristic Mach Number versus Mach Number

$$M_* = \sqrt{\frac{\frac{\gamma + 1}{2} M^2}{1 + \frac{(\gamma - 1)M^2}{2}}}$$

6. **Normal Shock Wave:**

Here upstream mach number is more than 1 which led to creation of shock wave. Normal shock wave means flow is perpendicular to shock wave. Across normal shock wave, total energy remains conserved but there is an increase in entropy.

The subscript 1 refers to situations before shock wave and subscript 2 refers to situation after shock wave.

Mass Conservation: $\rho_1 V_1 = \rho_2 V_2$

Momentum Equation: $p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2$

Energy Conservation:

$$c_p T_1 + \frac{V_1^2}{2} = c_p T_2 + \frac{v_2^2}{2}$$

Ideal Gas Equation: $P_1 = \rho_1 R T_1$ and $P_2 = \rho_2 R T_2$

Internal Energy: $e_1 = C_v T_1$ and $e_2 = C_v T_2$

Relation between Characteristic Mach Numbers:

$$M_1^*.M_2^* = 1$$

$$M_1^{*2} = \frac{\left(\frac{\gamma+1}{2}\right)M_1^2}{1+\left(\frac{\gamma-1}{2}\right)M_1^2}$$

Relation between Mach Numbers across the shock:

$$M_2^2 = \frac{1+\left(\frac{\gamma-1}{2}\right)M_1^2}{\gamma M_1^2-\left(\frac{\gamma-1}{2}\right)}$$

7. **Oblique Shock Wave:**

For a weak shock wave, it is called a mach wave. It forms a cone which has an angle μ, called mach angle.

$$\sin\mu = \frac{1}{M}$$

Oblique shock wave is inclined at an angle β with the angle of disturbance θ.
If $\beta = 90^0$, it will become the case of normal shock wave.

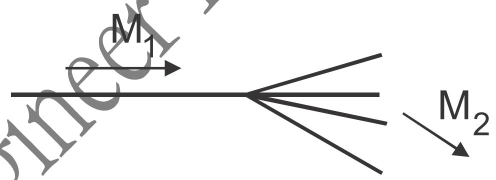
Here $M_{1,n} = M_1 \sin\beta$ and $M_{2,n} = M_2 \sin(\beta-\theta)$

For rest formulaes, just take formula of normal shock wave and replace M_1 by $M_{1,n}$ and M_2 by $M_{2,n}$.

Relation between θ and β is given by:

$$\tan\theta = \frac{2\cot\beta\,[M^2\sin^2\beta-1]}{M_1^2(\gamma+\cos 2\beta)+2}$$

8. **Expansion Waves:**



Here $T_{01} = T_{02}$, $P_{01} = P_{02}$, $S_{02} = S_{01}$ because there are no shocks present here.
Mach number increases after expansion fan.

$$\nu(M) = \sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\left(\sqrt{\frac{\gamma+1}{\gamma-1}(M^2-1)}\right) - \tan^{-1}(\sqrt{(M^2-1)}$$

9. **Pressure distribution along a Converging Diverging Nozzle**

(a) Area Ratio in the C-D Nozzle:

$$\frac{A}{A^*} = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\left(\frac{\gamma-1}{2}\right)M_1^2\right)\right]^{\frac{\gamma+1}{\gamma-1}}$$

(b) **Mass Flow Rate inside nozzle:**

$$\dot{m} = \frac{P_0 A^*}{\sqrt{T_0}}\sqrt{\frac{\gamma}{R}\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

SPACE MECHANICS

1. Acceleration due to gravity, $g_e = \sqrt{\frac{GM_e}{Re}}$

2. Gravitational Potential Energy, $U = -\frac{GMm}{r}$

3. In a circular orbit, Velocity $V_o = \sqrt{\frac{GM}{R}}$. Its direction is along the tangent to the path.

4. Work done by satellite in a full orbit is zero.

5. Time period for circular orbit is calculated as $T = \frac{2\pi R}{V_o}$

6. Total energy of the satellite in circular orbit $TE = KE + PE = \frac{1}{2}mV_O^2 + \left(-\frac{GMm}{r}\right)$

7. Binding energy for satellite in circular orbits $= -(Total\ Energy)$

8. Escape Velocity $= \sqrt{\frac{2GM}{R}}$

9. In an elliptical orbit, perigee is the nearest point to the earth and apogee is the distant point from the earth.

Perigee distance $= a(1-e)$ and Apogee Distance $= a(1+e)$ where e is eccentricity of orbit.

10. From Kepler's second law, $\frac{dA}{dt} = constant$

11. Time period of circular orbit $T^2 \propto r^3$

13. Time period of elliptical orbit $T^2 \propto a^3$ or $T^2 \propto b^6$

14. Eccentricity of elliptical orbit $e = \frac{R_A - R_P}{R_A + R_P}$

15. Velocity at perigee $V_P = \sqrt{\frac{2GM}{R_A + R_P} \times \frac{R_A}{R_P}}$

16. Velocity at perigee $V_A = \sqrt{\frac{2GM}{R_A + R_P} \times \frac{R_P}{R_A}}$

17. $V_A R_A = V_P R_P$

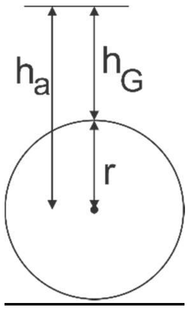
18. When plane is changed in Hoffman transfer at an angle θ, the change in velocity

$$\Delta v = 2v_i \sin\frac{\theta}{2}$$

v_i is the velocity of satellite.

19. Time period of geostationary satellite is 24 hours.

FLIGHT MECHANICS

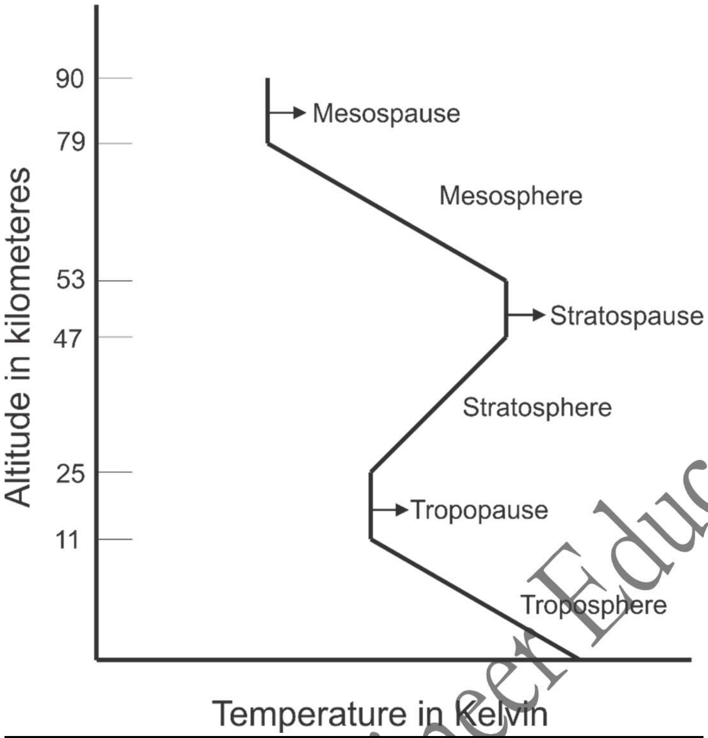


$$g = g_0\left(\frac{r}{h_G}\right)^2$$

$$h_a = r + h_G$$

$$h = \left(\frac{r}{r+h_G}\right) \times h_G$$

h_G is geometric altitude, h_a is absolute altitude, h is geopotential altitude.



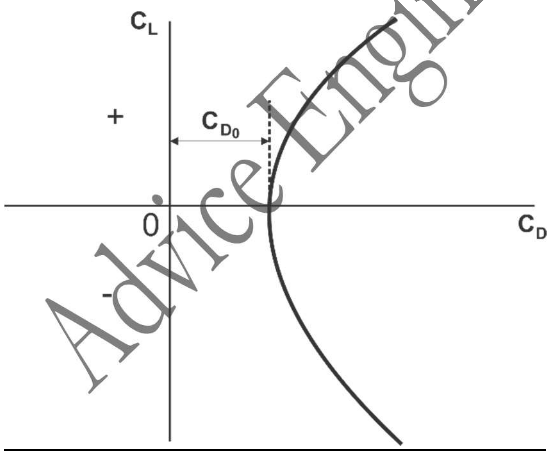
For gradient layers:

$$\frac{P}{P_1} = \left(\frac{T}{T_1}\right)^{(-\frac{g_0}{aR})}$$

$$\frac{\rho}{\rho_1} = \left(\frac{T}{T_1}\right)^{-[\left(\frac{g_0}{aR}\right)+1]}$$

For isothermal layers:

$$\frac{P}{P_1} = \frac{\rho}{\rho_1} = e^{-\left[\frac{g_0}{RT}\right](h-h_1)}$$

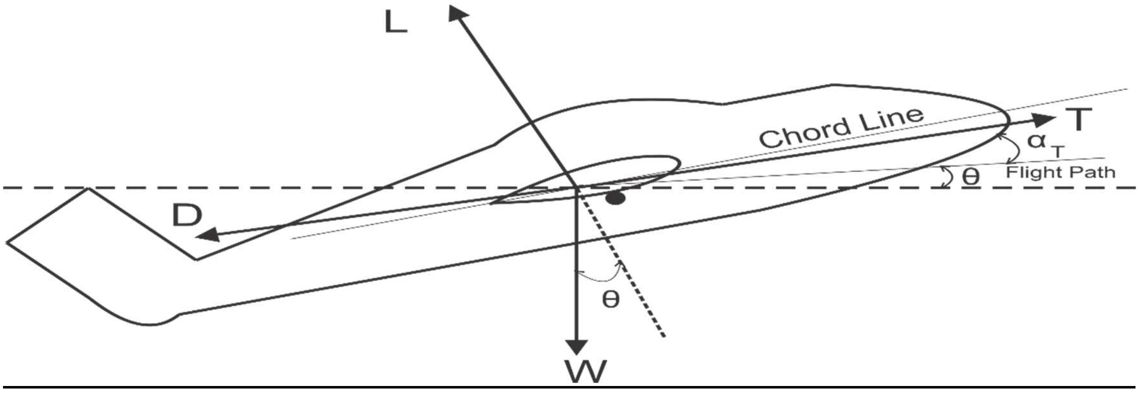


$$C_D = C_{D0} + \frac{C_L^2}{\pi e AR}$$

Where e is Oswald efficiency and AR is aspect ratio of the aircraft.

AR = b.c = wing span × chord length

$C_{D,0}$ is zero lift drag coefficient



Equations of Motion:

$$T\cos\alpha_T - D - W\sin\theta = m\frac{dV}{dt}$$

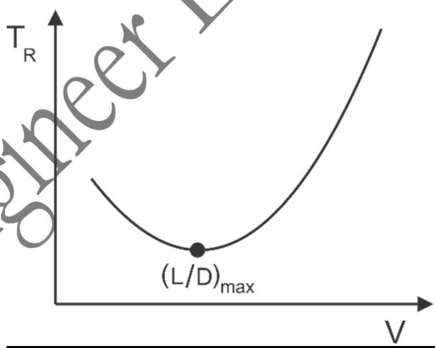
$$L + T\sin\alpha_T - W\ \cos\theta = \frac{mV^2}{R_c}$$

In a steady straight and level flight $L = W$ and $T = D$

Thrust Required for Level Unacclerated Flight:

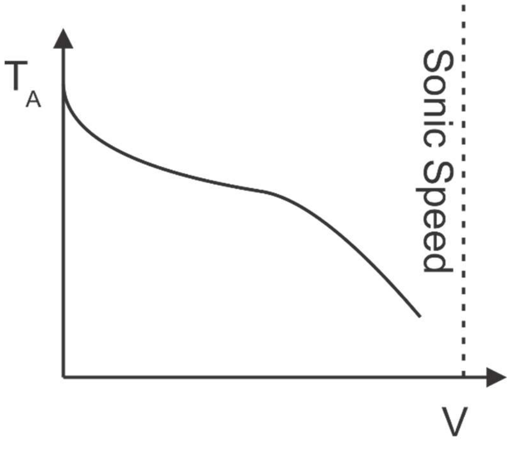
$$T_R = \frac{W}{\left(\frac{C_L}{C_D}\right)}$$

For minimum thrust required, $C_{D0} = C_{Di}$

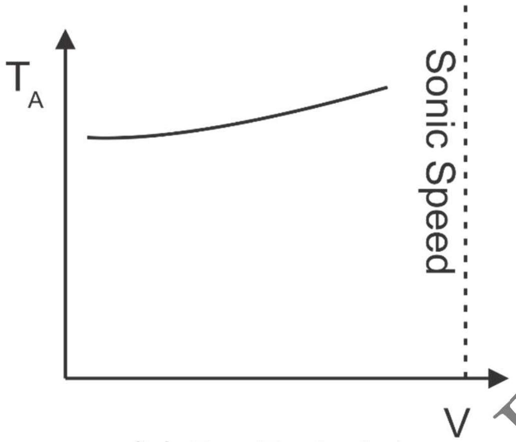


THRUST AVAILABLE

Thrust available is the property of engine used. It varies from propeller engine to Turbojet Engine.



(a) For Propeller Engine



(b) For Turbojet

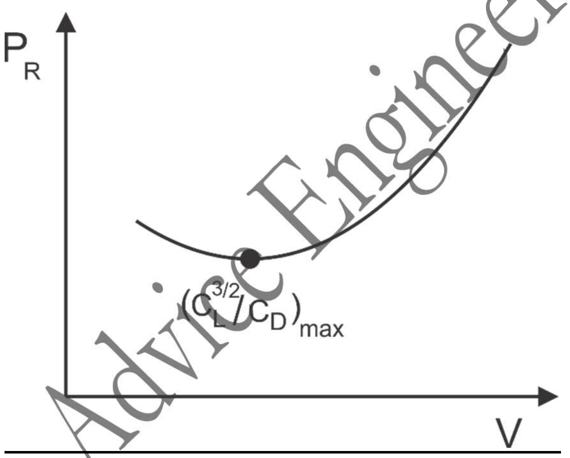
Power Required for Level Unacclerated Flight:

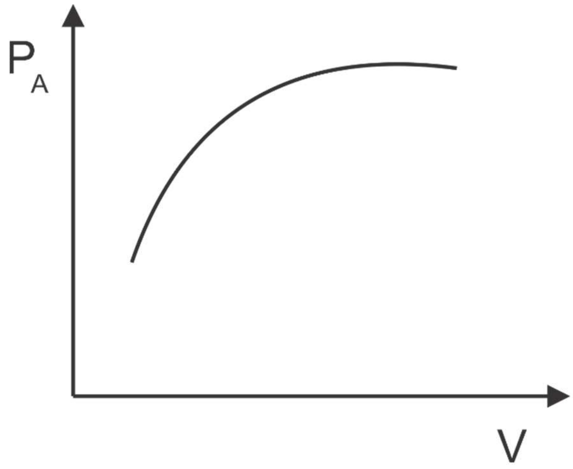
$$P_R = T_R \times V_\infty$$

$$P_R \propto \frac{1}{\left(\frac{C_L^{\frac{3}{2}}}{C_D}\right)}$$

For minimum power required,

$$C_{D,0} = \frac{1}{3} C_{Di}$$





(a) For Propeller Engine

POWER AVAILABLE

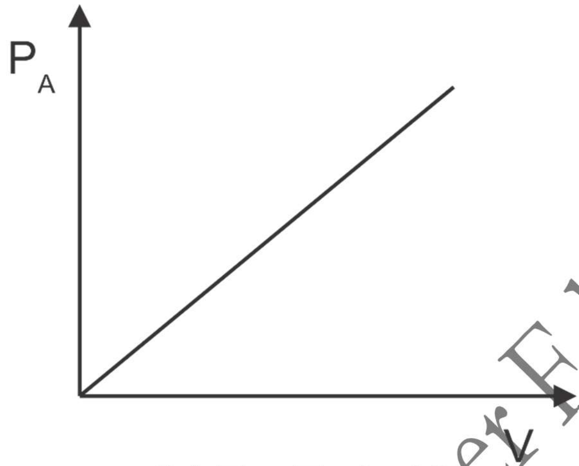
For Propeller Engine,

$$P_A = \eta P$$

Where P is the Brake power of shaft.

For Jet Engine,

$$P_A = T_A \times V_\infty$$



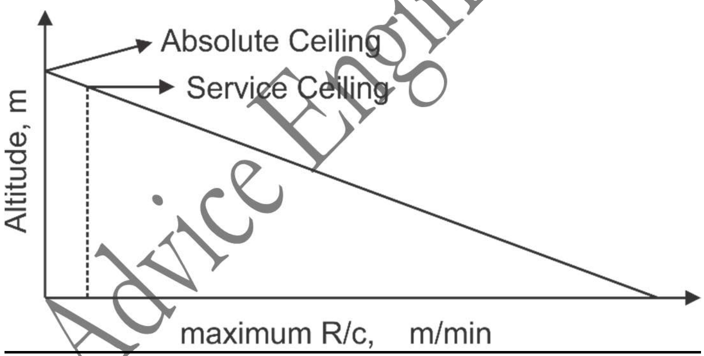
(b) For Turbojet

RATE OF CLIMB:

$$Rate\ of\ Climb = \frac{Power\ Avaailable - Power\ Required}{Weight}$$

$$Rate\ of\ Climb = V \sin\theta$$

Where θ is angle of climb.



Absolute Ceiling is the height at which rate of climb is zero.

Service Ceiling is where most flights take place.

For maneuvering, aircraft fly between Absolute Ceiling and Service Ceiling

GLIDING FLIGHT:
Equilibrium Glide Angle:

$$\tan\theta = \frac{1}{\left(\frac{L}{D}\right)}$$

For minimum equilibrium glide angle, glide ratio should be maximum.
Time of Flight:

$$\int dt = \int \frac{dh}{Rate\ of\ Climb}$$

Range and Endurance of Propeller Aircraft:

Range of Aircraft:

$$R = \frac{\eta}{c}\frac{C_L}{C_D}\ln\frac{W_0}{W_1}$$

Where c is specific fuel consumption.
For maximum range of propeller aircraft, C_L/C_D should be maximum.

Endurance of Aircraft:

$$E = \frac{\eta}{c}\frac{C_L^{\frac{3}{2}}}{C_D}(2\rho S)^{\frac{1}{2}}\left(W_1^{-\frac{1}{2}} - W_0^{-\frac{1}{2}}\right)$$

For maximum endurance of propeller aircraft, $\frac{C_L^{\frac{3}{2}}}{C_D}$ should be maximum.

Range and Endurance of Jet Aircraft:
Endurance of Aircraft:

$$E = \frac{1}{c_t}\frac{C_L}{C_D}\ln\frac{W_0}{W_1}$$

Where c is specific fuel consumption.
For maximum endurance of jet aircraft, C_L/C_D should be maximum.

Range of Aircraft:

$$E = 2\sqrt{\frac{2}{\rho S}}\frac{1}{c_t}\frac{C_L^{\frac{1}{2}}}{C_D}\left(W_0^{\frac{1}{2}} - W_1^{\frac{1}{2}}\right)$$

For maximum range of jet aircraft, $\frac{C_L^{\frac{1}{2}}}{C_D}$ should be maximum.

Takeoff Performance:
Takeoff Distance is given by

$$s_{LO} = \frac{1.44W^2}{g\rho S C_{L,max} T}$$

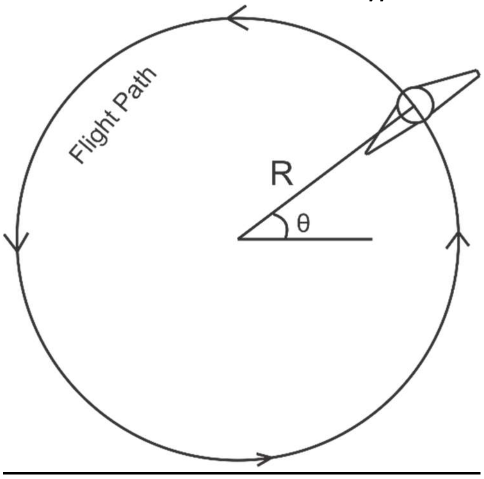
Landing Performance:

$$s_{Landing} = \frac{1.69W^2}{g\rho S C_{L,max} D}$$

TURNING FLIGHT:

$$L\cos\phi = W$$

$$Load\ Factor = n = \frac{L}{W}$$



Here,

$$F_r = \sqrt{L^2 - W^2} = W\sqrt{n^2 - 1} = \frac{mV_\infty^2}{R}$$

Radius of Curvature is given by:

$$R = \frac{V^2}{g\sqrt{n^2 - 1}}$$

Turn Rate is given by:

$$\omega = \frac{g\sqrt{n^2 - 1}}{V_\infty}$$

Case 1: Pull up maneuver:

$$F_r = L - W$$

Convert in the form of n and then find R and ω.

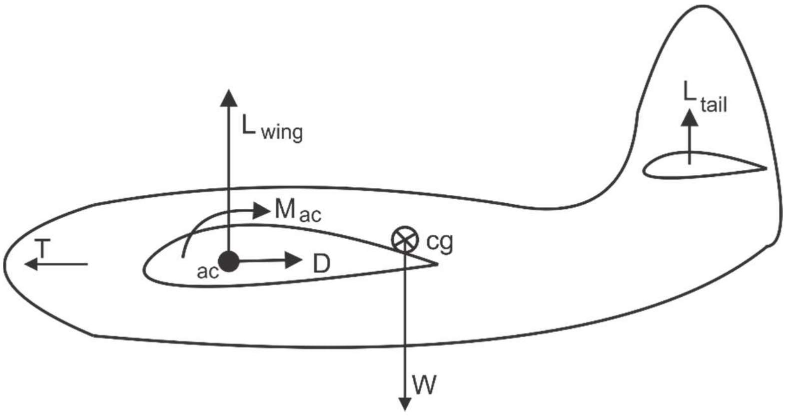
Case 2: Pull Down Maneuver:

$$F_r = L + W$$

Convert in the form of n and then find R and ω.

STABILITY AND CONTROL:

1. Turning Motion about x axis is called **Roll** (Lateral Motion) and **ailerons** control roll.
2. Turning Motion about y axis is called **Pitch** (Longitudinal Motion) and **elevators** control pitch.
3. Turning Motion about z axis is called **Yaw** (Directional Motion) and **rudders** control yaw.



Here, ac is aerodynamic center, a point on the aircraft moment about which is independent of angle of attack.

cg is centre of gravity and weight of aircraft act on this point. There are two airfoils here, one is wing airfoil and other is tail airfoil.

Coefficient of moment $C_m = \frac{M}{qSc}$

When moment abut centre of gravity is zero, then aircraft is said to be **trimmed.**

Necessary Criteria for Longitudinal Stability is:

(a) $C_{M,0}$ must be positive.

(b) $\dfrac{\partial C_{m,cg}}{\partial \alpha}$ must be negative.

Wing Stability:

$$C_{m,cg,w} = C_{m,ac} + C_{L,w}(h - h_{ac})$$

$$C_{m,cg,w} = C_{m,ac} + a_w \alpha_w (h - h_{ac})$$

h is distance from leading edge to centre of gravity and h_{ac} is distance from aerodynamic centre to centre of gravity.

a_w is lift curve slope which is $\frac{dC_L}{d\alpha}$ and α_w is angle of attack on wing only.

Tail Stability:

$$C_{m,cg,t} = -V_H C_{L,t} = -V_H a_t \alpha_t$$

Here V_H is tail volume ratio and is defined as

$$V_H = \frac{l_t S_t}{cS}$$

Where l_t is tail length, S_t is tail area.
a_t is tail curve slope and α_t is angle of attack on tail only.

$$\alpha_t = \alpha_w - i_t - \epsilon$$

$$\epsilon = \epsilon_0 + \left(\frac{\partial \epsilon}{\partial \alpha}\right)\alpha_w$$

$$C_{m,cg,t} = -a_t V_H \alpha_w \left(1 - \frac{\partial \epsilon}{\partial \alpha}\right) + a_t V_H (\epsilon_0 + i_t)$$

Total Longitudinal Stability:

$$C_{m,cg} = C_{m,cg,w} + C_{m,cg,t}$$
$$C_{m,0} = \left(C_{m,cg}\right)_{L=0}$$

Neutral Point:

When $\frac{\partial C_{m,cg}}{\partial \alpha} = 0,$ the value of h is changed to h_n and calculated using above formulae.

For longitudinal stability, the position of centre of gravity must always be forward of neutral point.

Static Margin:

$$\frac{\partial C_{m,cg}}{\partial \alpha} = -a(h_n - h) = -a \times static\ margin$$

Elevator angle to trim:

$$\delta_{trim} = \frac{C_{M,0} + \frac{\partial C_{m,cg}}{\partial \alpha} . \alpha_n}{V_H \frac{\partial C_{L,t}}{\partial \delta_e}}$$

Modes in Longitudinal Motion:

(a) Short Period Mode:
Eigen value having largest imaginary part will correspond to short period mode.

(b) Long Period Mode:
Eigen value having shortest imaginary part will correspond to long period mode.

(c) Phugoid Mode:
This mode has low frequency and a low damping factor.

$$T = \frac{\sqrt{2}\pi V}{g}$$

Modes in Lateral Motion:

(a) Roll Mode and Spiral Mode:
Purely real roots determine these two modes. Larger Eigen value means Roll mode and short eigen value means spiral mode.

(b) Dutch Roll Mode:
The complex pair describe this mode.

Feedback

Write to us at **contact@adviceengineer.in** if you have any suggestions. We welcome all such suggestions to improve this book furthermore.

Please do write a feedback.