Calculus A Complete Course NINTH EDITION ADAMS ESSEX

$\log(a) + \log(b) = \log(ab)$

$\log(a) - \log(b) = \log(a/b)$

Practice Exercise #2.

2)

$T(1) = a$; $T(n) = 7T(\frac{n}{2}) + bn^2$

$T(\frac{n}{2}) = 7T(\frac{n}{2^2}) + b(\frac{n}{2})^2$

$T(\frac{n}{4}) = 7T(\frac{n}{8}) + b(\frac{n}{4})^2$

$2^{2 \cdot 2}$

① Recurrence

$$T(n) = 7T(\tfrac{n}{2}) + bn^2$$

$$= 7(7T(\tfrac{n}{4}) + b(\tfrac{n}{2})^2) + bn^2$$

$$= 7^2 T(\tfrac{n}{2^2}) + 7b\frac{n^2}{2^2} + bn^2$$

$$= 7^2(7T(\tfrac{n}{2^3}) + b(\tfrac{n}{4})^2) + \frac{7bn^2}{2^2} + bn^2$$

$$= 7^3 T(\tfrac{n}{2^3}) + 7^2 b\frac{(n^2)}{(2^2)^2} + \frac{7^1 bn^2}{(2^2)^1} + bn^2$$

$$= 7^i T(\tfrac{n}{2^i}) + b(n^2)\sum_{j=0}^{i-1}\left(\frac{7}{2^2}\right)^j$$

$$= 7^i T(\tfrac{n}{2^i}) + bn^2\left(\frac{(\frac{7}{4})^i - 1}{\frac{7}{4} - 1}\right) = 7^i T(\tfrac{n}{2^i}) + \frac{4}{3}bn^2((\tfrac{7}{4})^i - 1)$$

② $i = c$

$\frac{n}{2^c} = 1$

$n = 2^c$

$c = \log_2 n$

$$T(n) = 7^{\log_2 n} a + \frac{4}{3}bn^2(\tfrac{7}{4}^{\log_2 n} - 1)$$

$$T(n) = n^{\log_2 7} a + \frac{4}{3}bn^2(n^{\log_2 \frac{7}{4}} - 1)$$

$$T(n) = n^{\log_2 7} a + \frac{4}{3}bn^2(n^{\log_2 \frac{7}{4}}) - \frac{4}{3}bn^2$$

$O(n^2)$

$$T(n) = n^{\log_2 7} a + \frac{4}{3}b\, n^{2 + \log_2 \frac{7}{4}} - O(n^2)$$

$$\boxed{T(n) = O(n^{\log_2 7})}$$

$n^{\log_2 7}$