Session: Linear Regression

One Variable

Gradient Descent

Many Variables

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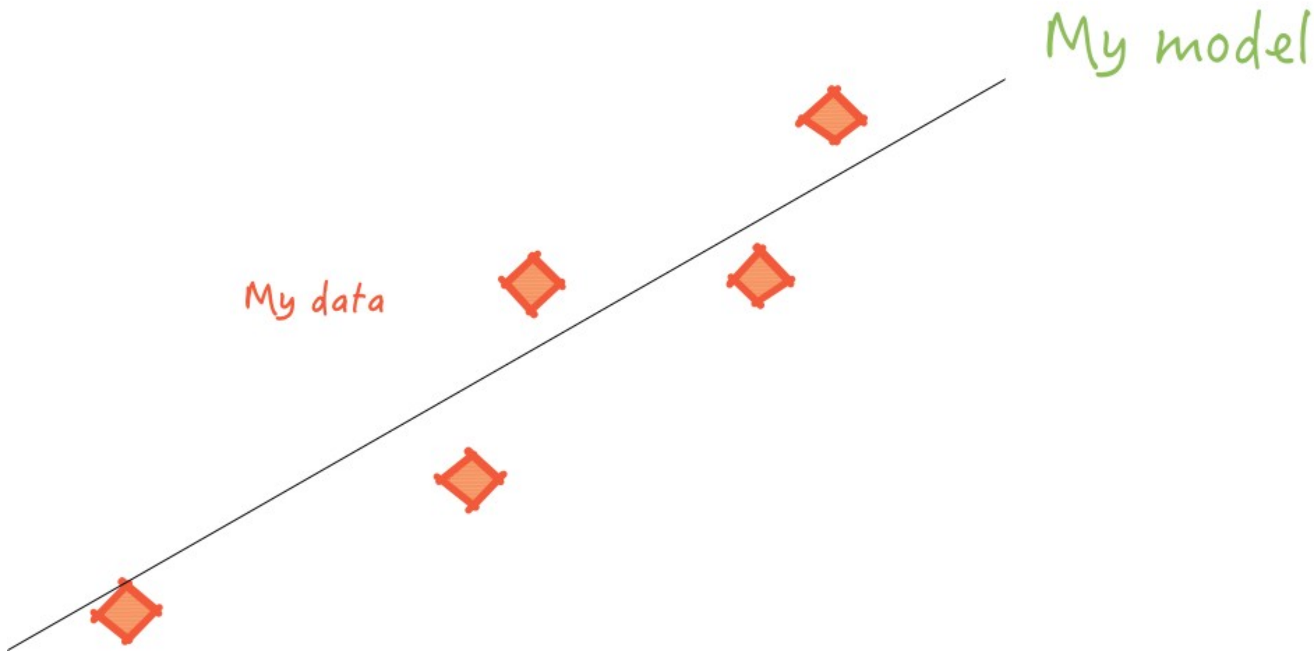
Lesson Objectives

- Understand linear regression with one variable
- Introduce the idea of the model
- Explain the gradient descent solution
- Understand linear regression with many variables

What Is Linear Regression?

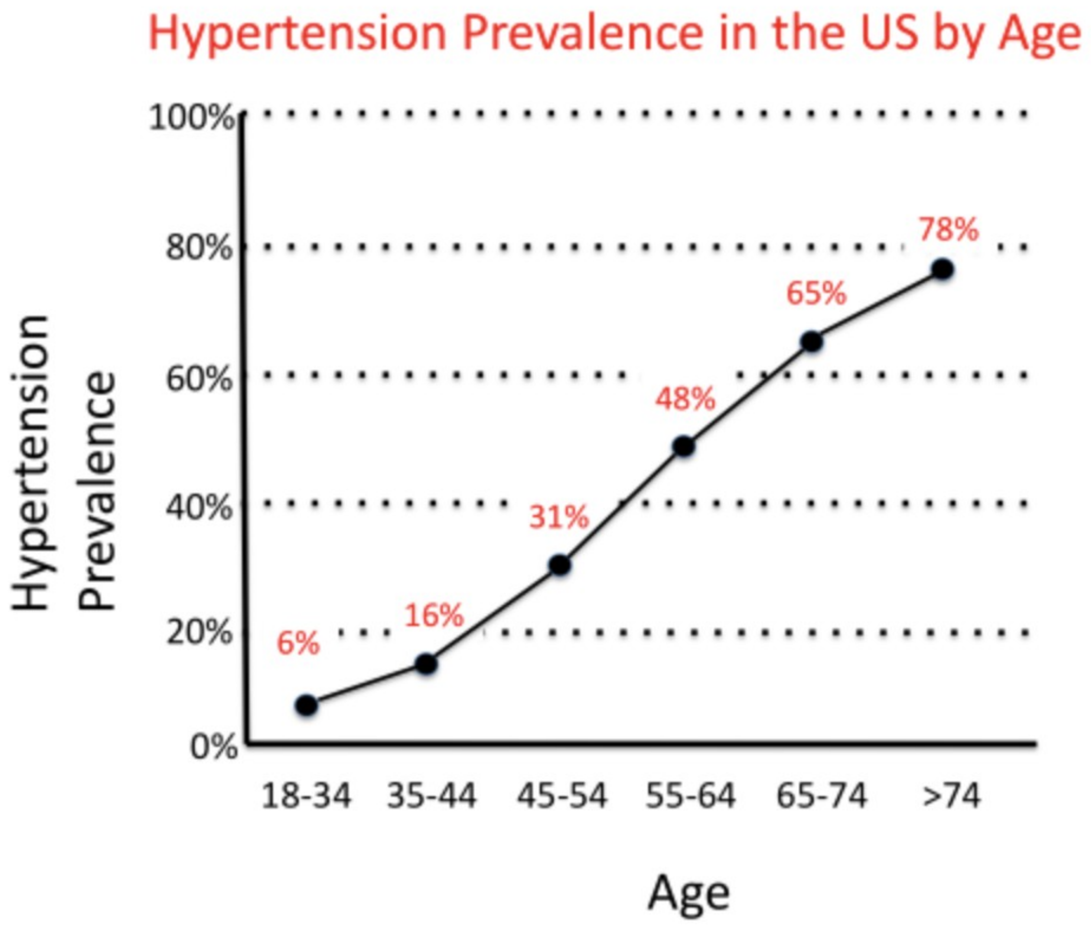
- Data



- Input variables ➔ continuous output

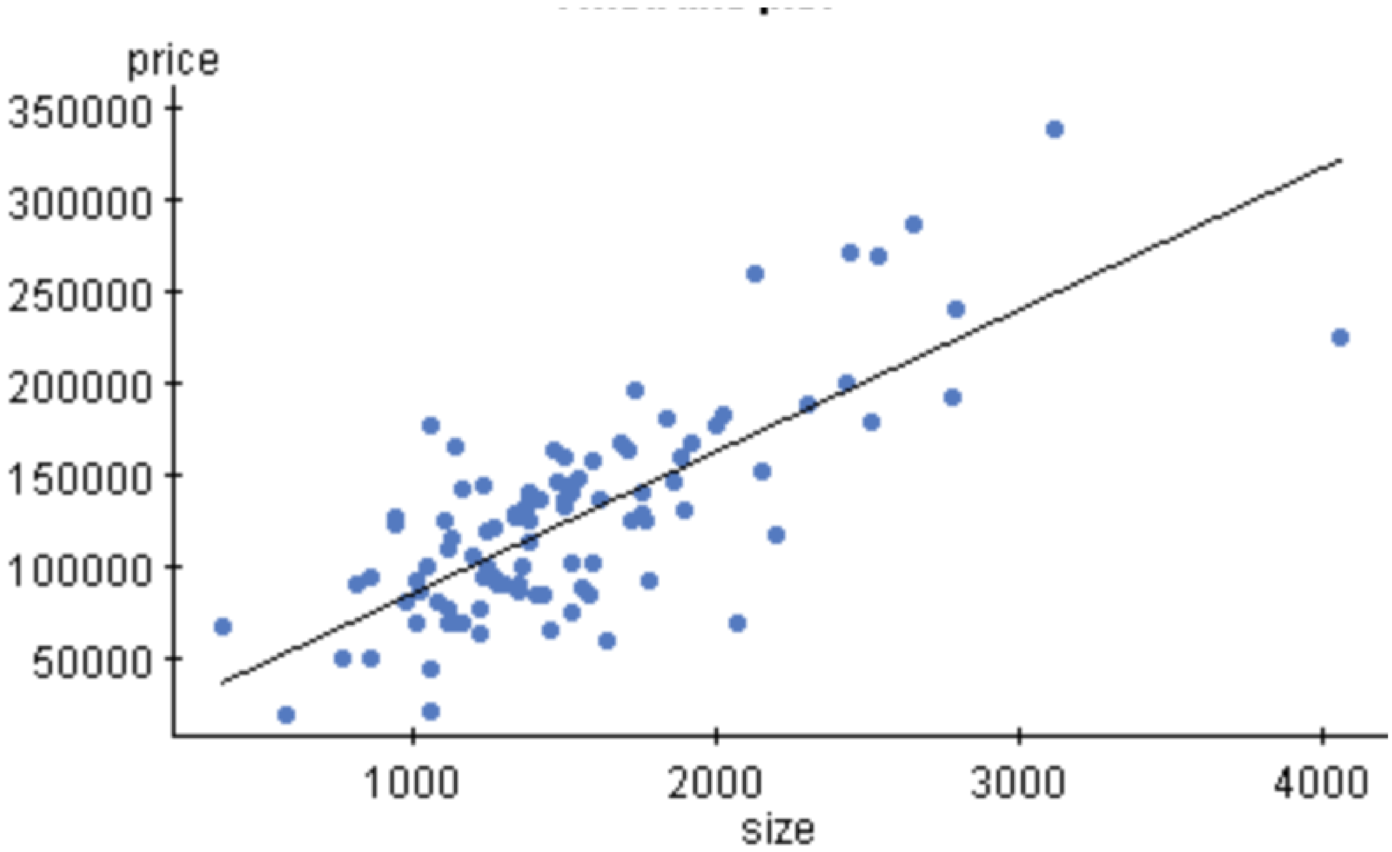
Linear Regression Example

- Blood pressure as function of age



Fields LE et al (2004) Hypertension 44:398-404

House Price vs Size



- Not a perfect fit
- More predictors may be needed

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One Variable

➔ **One Variable**

Gradient Descent

Many Variables

SageMaker Linear Learner

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Linear Regression With One Variable

- **Univariate (simple) linear regression**

- One input ➔ One output
- x ➔ y
- Our hypothesis (or model)

$$y = T_0 + T_1 x$$

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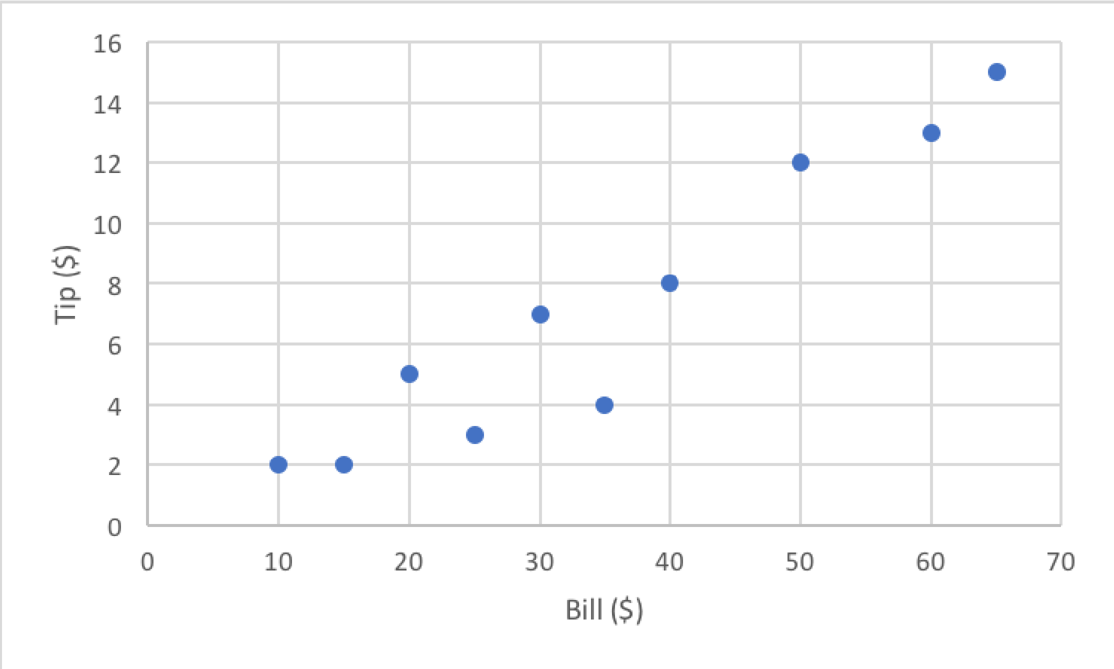
Problem: Tip Calculation

- Bill ➔ Tip
- Imagine we do not know 15% rule
- Looks like a linear dependency



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Solution Attempt

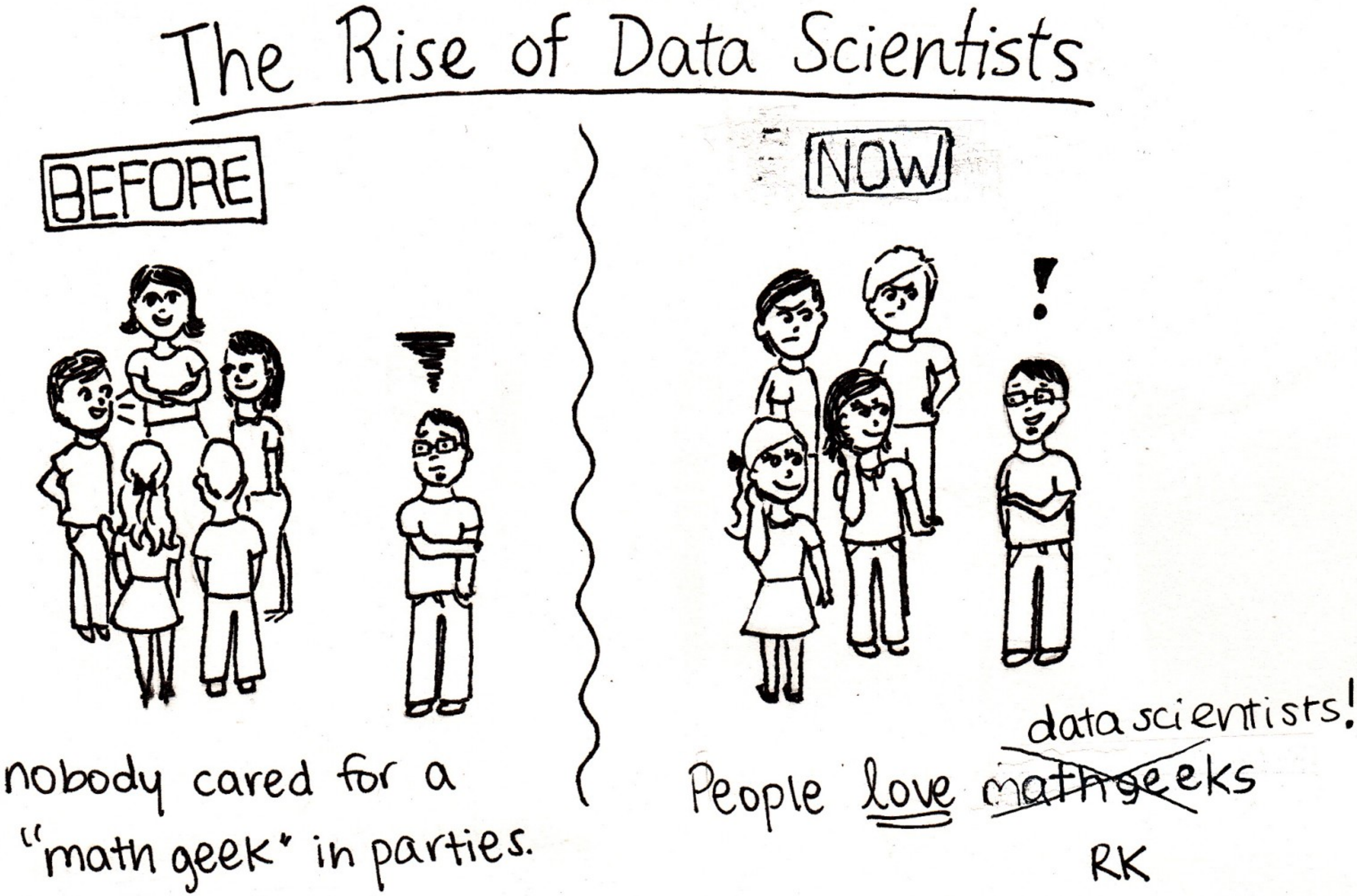
- Hypothesis

  tip = T_0 + T_1 bill

- But what are T_0 and T_1?
- Say T_0=0 and T_1=0.15
- Not a fantastic success
- ☹

Prediction
7.5
4.5
9
6
9.75
3
1.5
2.25
3.75
5.25

We Need a Data Scientist!



Cost Function

- Cost of replacing data with our model
- Measures the accuracy of our hypothesis (model)

$$C(T_0, T_1) = \frac{1}{2m} \sum_{1}^{m} (y'_i - y_i)^2$$

- Where
  - y' is estimated
  - y is real value

- Where
  - y' is estimated
  - y is real value

Cost Function Explained

- Error

- Error

$$h_T(x_i) - y_i$$

- Square error

- Square error

$$\sum_{1}^{m} (h_T(x_i) - y_i)^2$$

- Special multiplier

- Special multiplier

$$\frac{1}{2m}$$

- To do **mean** and to cancel out in subsequent gradient descent

- To do **mean** and to cancel out in subsequent gradient descent

Cost Function Breakdown

-

- Better, use function h_T instead of

$$C(T_0, T_1) = \frac{1}{2m}\sum_{1}^{m}(y'_i - y_i)^2$$

- Better, use function h_T instead of y'_i

$$C(T_0, T_1) = \frac{1}{2m}\sum_{1}^{m}(h_T(xi) - y_i)^2$$

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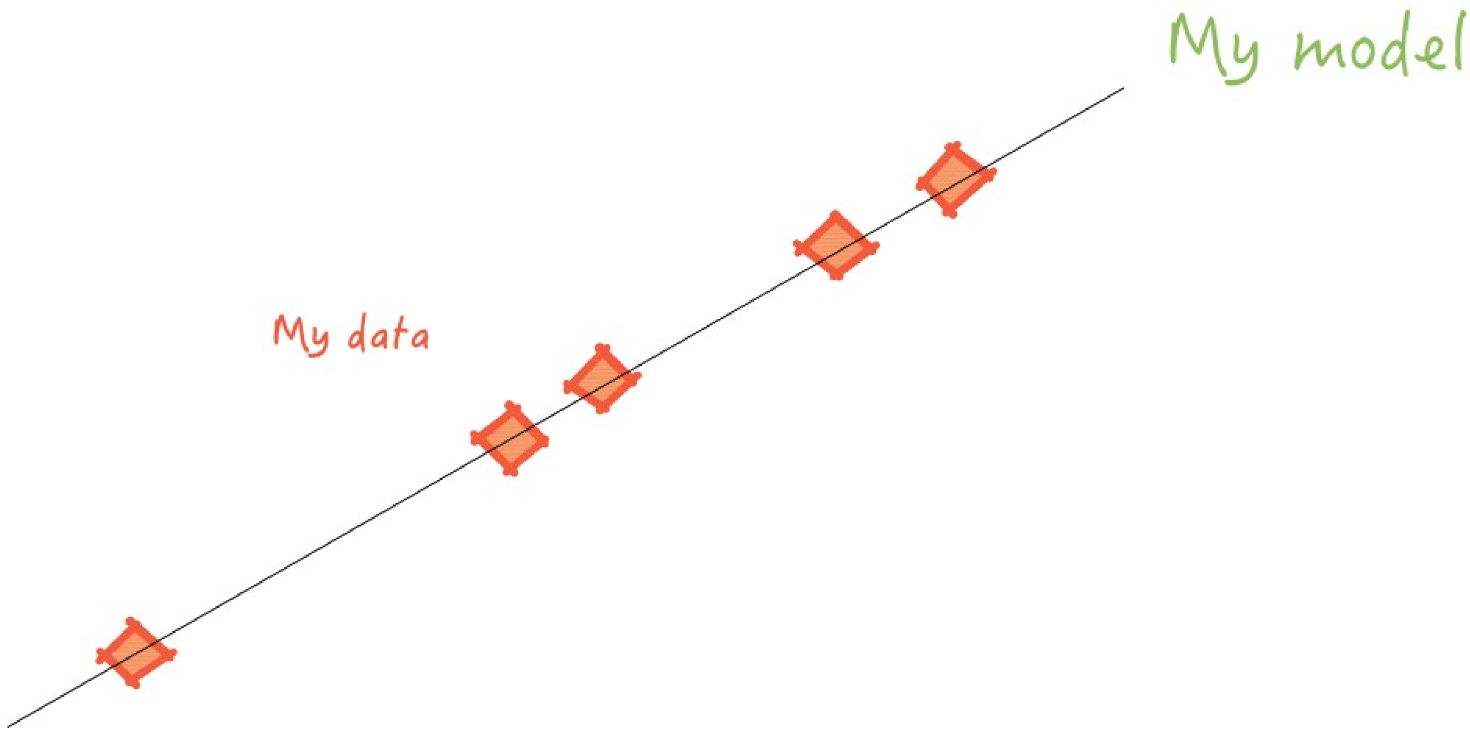
Here the Cost Would Be Zero

- If the line passes through all points
- The the cost would be zero

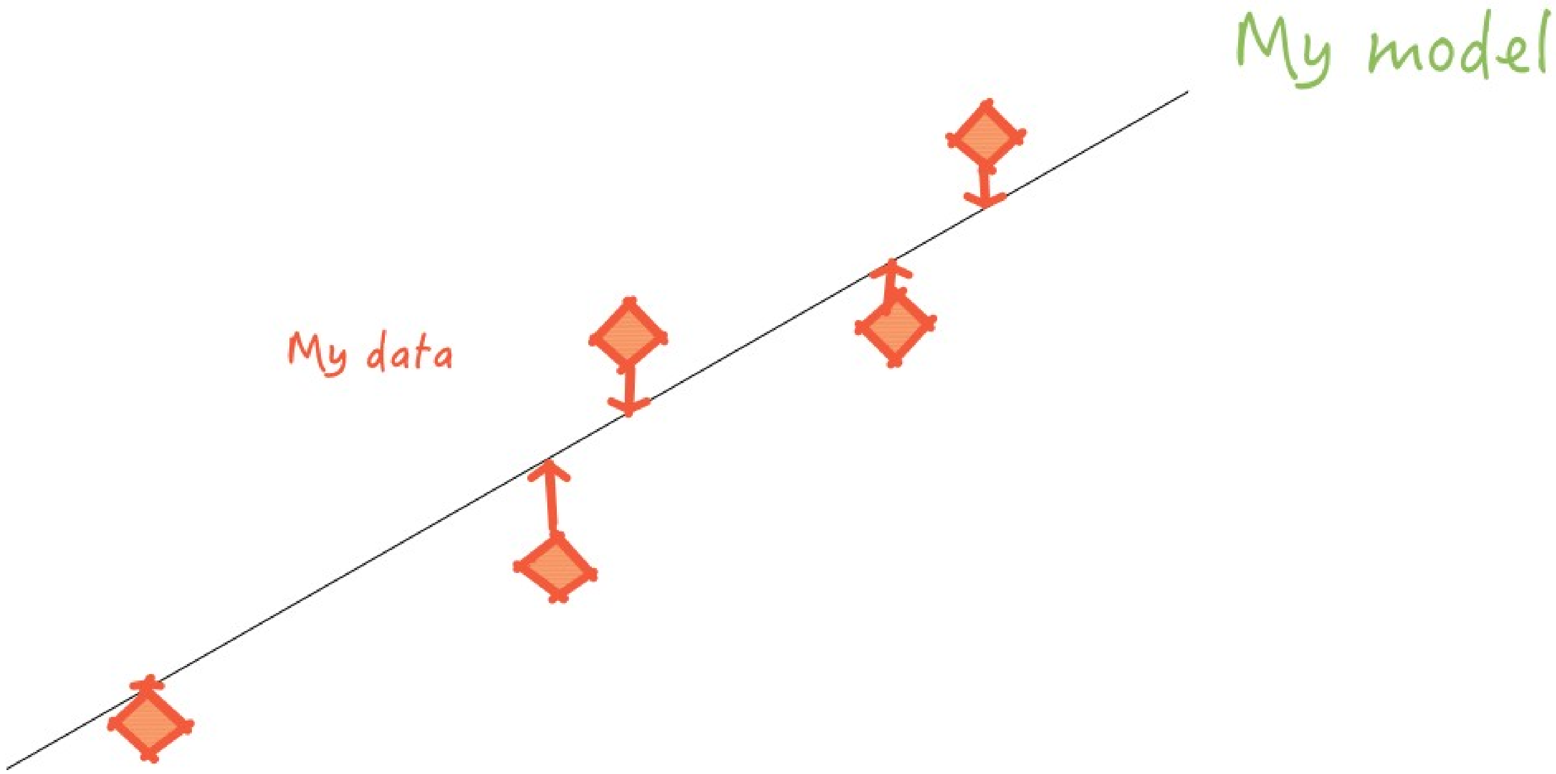
0

$$C(T_0, T_1) = 0$$



Cost Function Illuminated

- We want to find the best possible line to approximate the data
- Each line will have a cost
- We want to find a line that will minimize the cost



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Gradient Descent

One Variable

➔ **Gradient Descent**

Many Variables

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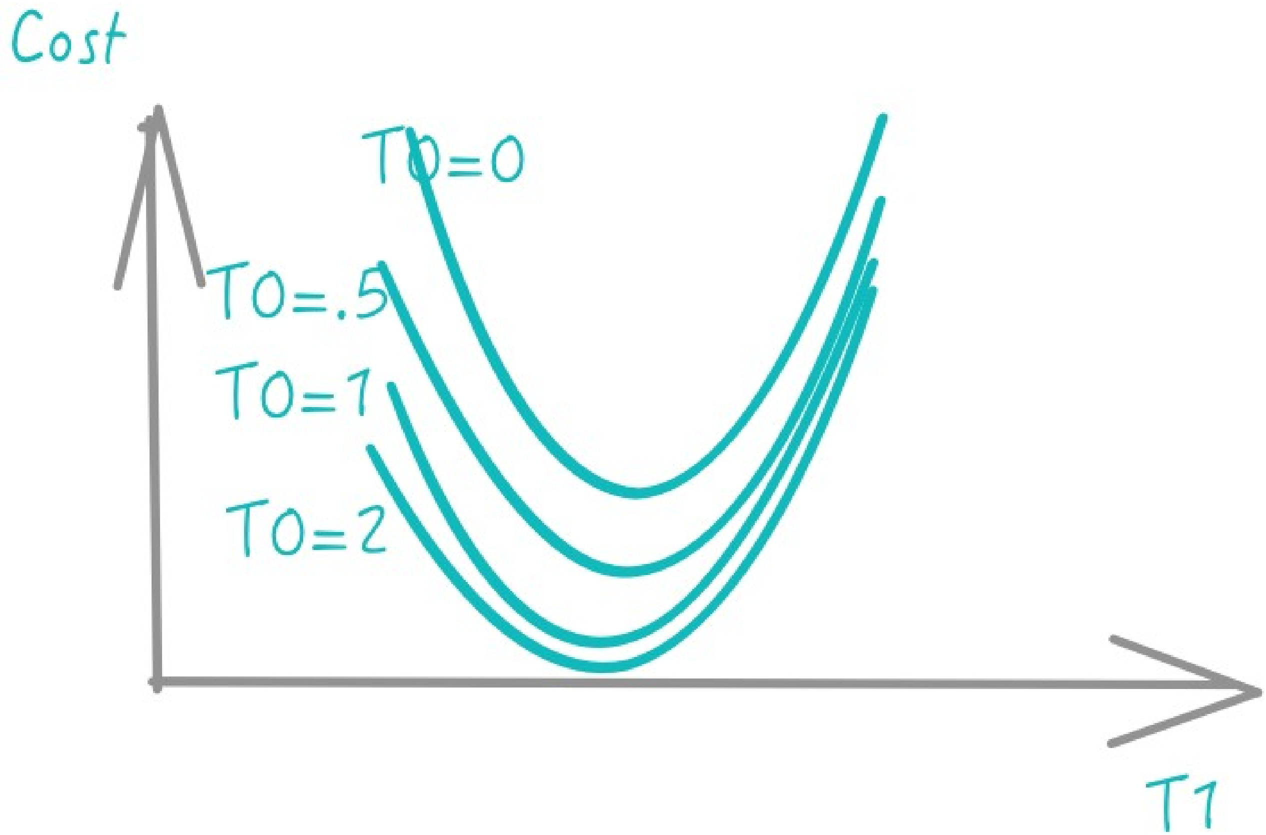
Optimization Problem

- We have the cost formula
- We can calculate the cost for every T_0 and T_1
- We want to find the best values of T_0 and T_1



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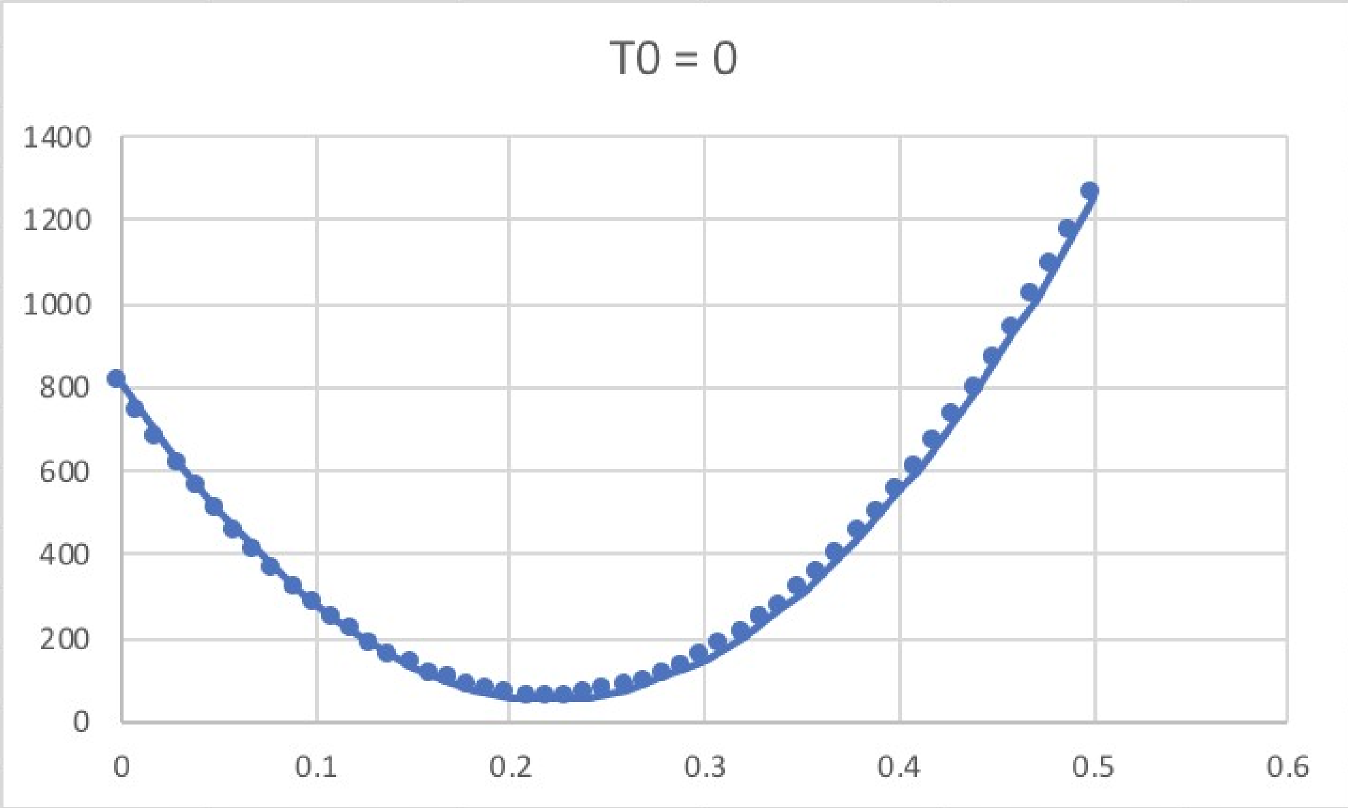
MSE for T_0=0.0

1	0	804
2	0.01	737.54
3	0.02	674.16
4	0.03	613.86
5	0.04	556.64
6	0.05	502.5
7	0.06	451.44
8	0.07	403.46
9	0.08	358.56
10	0.09	316.74
11	0.1	278
12	0.11	242.34
13	0.12	209.76
14	0.13	180.26
15	0.14	153.84
16	0.15	130.5
17	0.16	110.24
18	0.17	93.06
19	0.18	78.96



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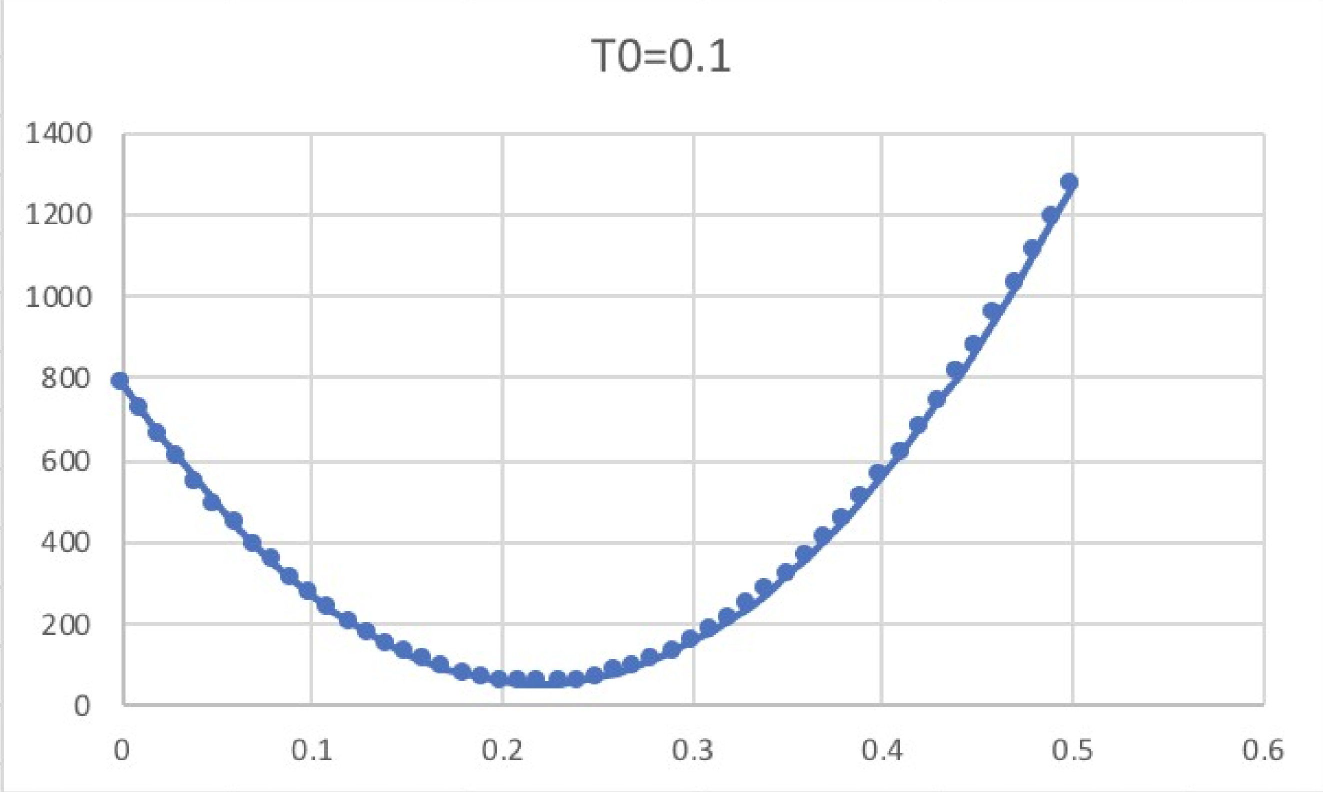
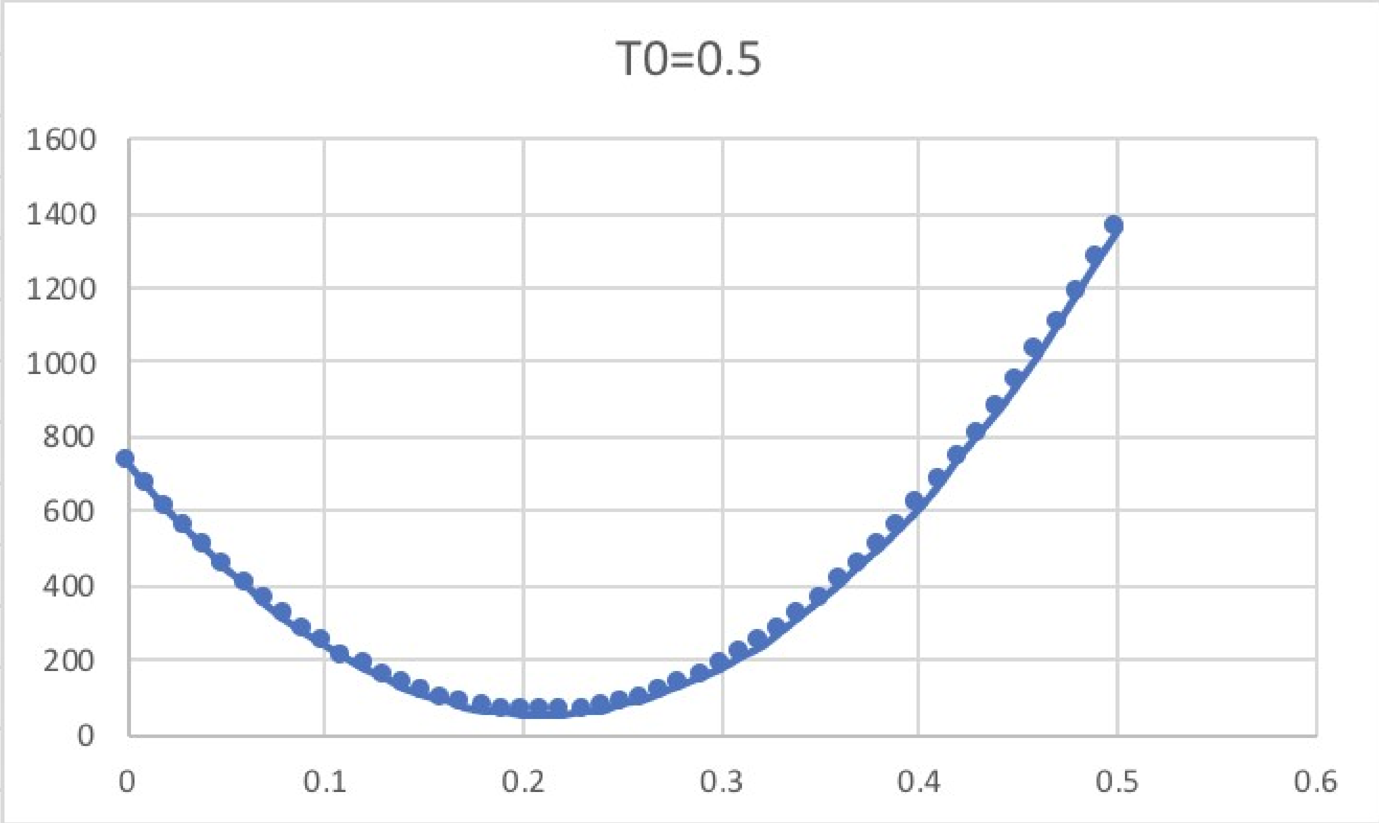
MSE for T_0=0.1

	A	B
1	0	788.9
2	0.01	723.14
3	0.02	660.46
4	0.03	600.86
5	0.04	544.34
6	0.05	490.9
7	0.06	440.54
8	0.07	393.26
9	0.08	349.06
10	0.09	307.94
11	0.1	269.9
12	0.11	234.94
13	0.12	203.06
14	0.13	174.26
15	0.14	148.54
16	0.15	125.9
17	0.16	106.34
18	0.17	89.86
19	0.18	76.46



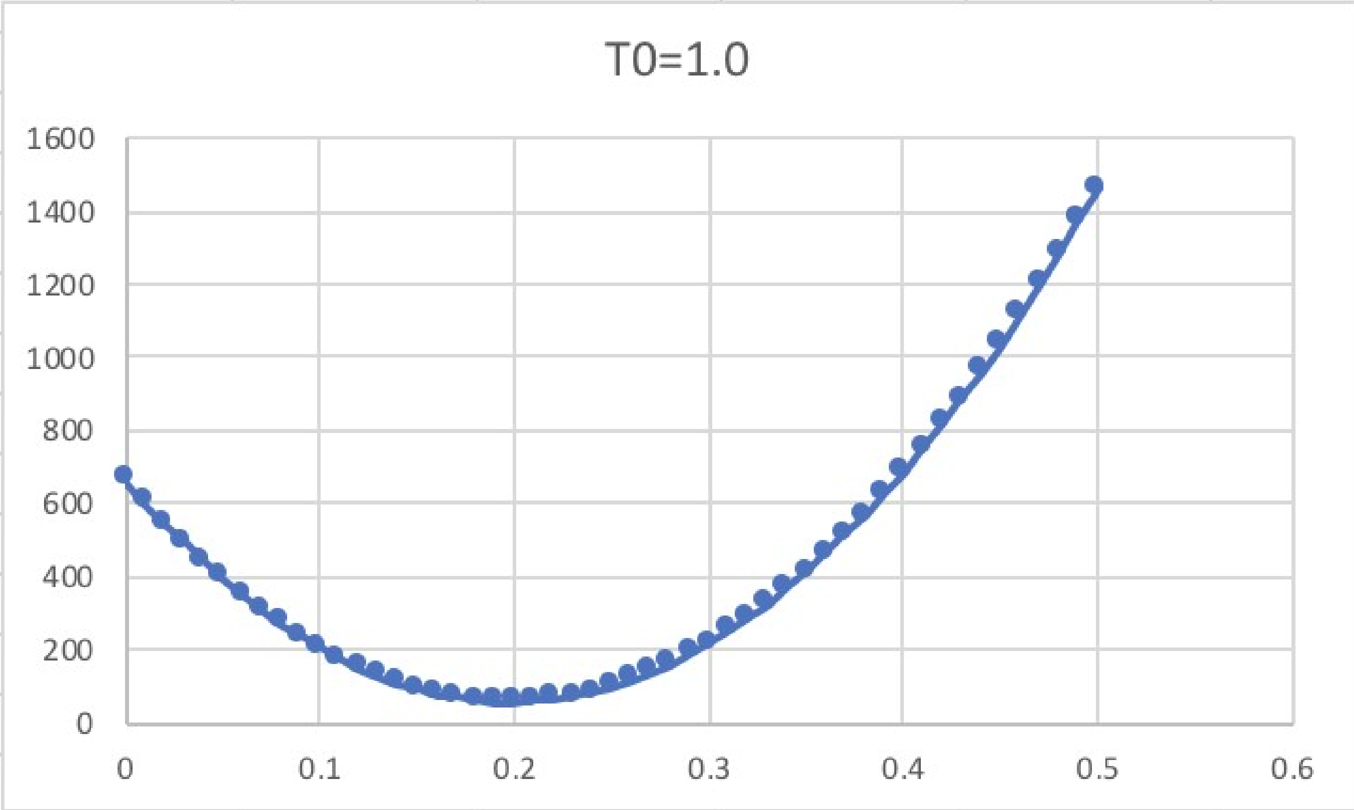
MSE for T_0=0.5

1	0	730.5
2	0.01	667.54
3	0.02	607.66
4	0.03	550.86
5	0.04	497.14
6	0.05	446.5
7	0.06	398.94
8	0.07	354.46
9	0.08	313.06
10	0.09	274.74
11	0.1	239.5
12	0.11	207.34
13	0.12	178.26
14	0.13	152.26
15	0.14	129.34
16	0.15	109.5
17	0.16	92.74
18	0.17	79.06
19	0.18	68.46



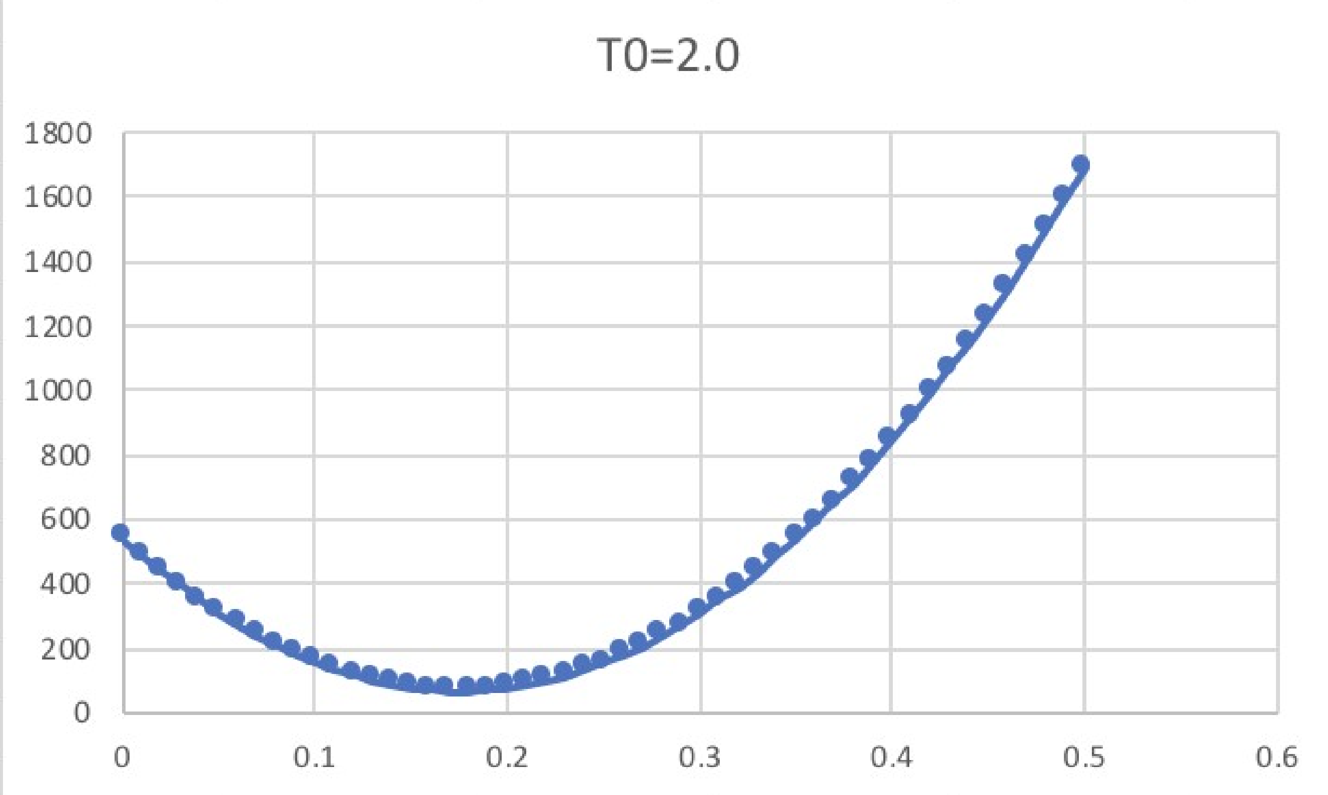
MSE for T_0=1.0

1	0	662
2	0.01	602.54
3	0.02	546.16
4	0.03	492.86
5	0.04	442.64
6	0.05	395.5
7	0.06	351.44
8	0.07	310.46
9	0.08	272.56
10	0.09	237.74
11	0.1	206
12	0.11	177.34
13	0.12	151.76
14	0.13	129.26
15	0.14	109.84
16	0.15	93.5
17	0.16	80.24
18	0.17	70.06
19	0.18	62.96

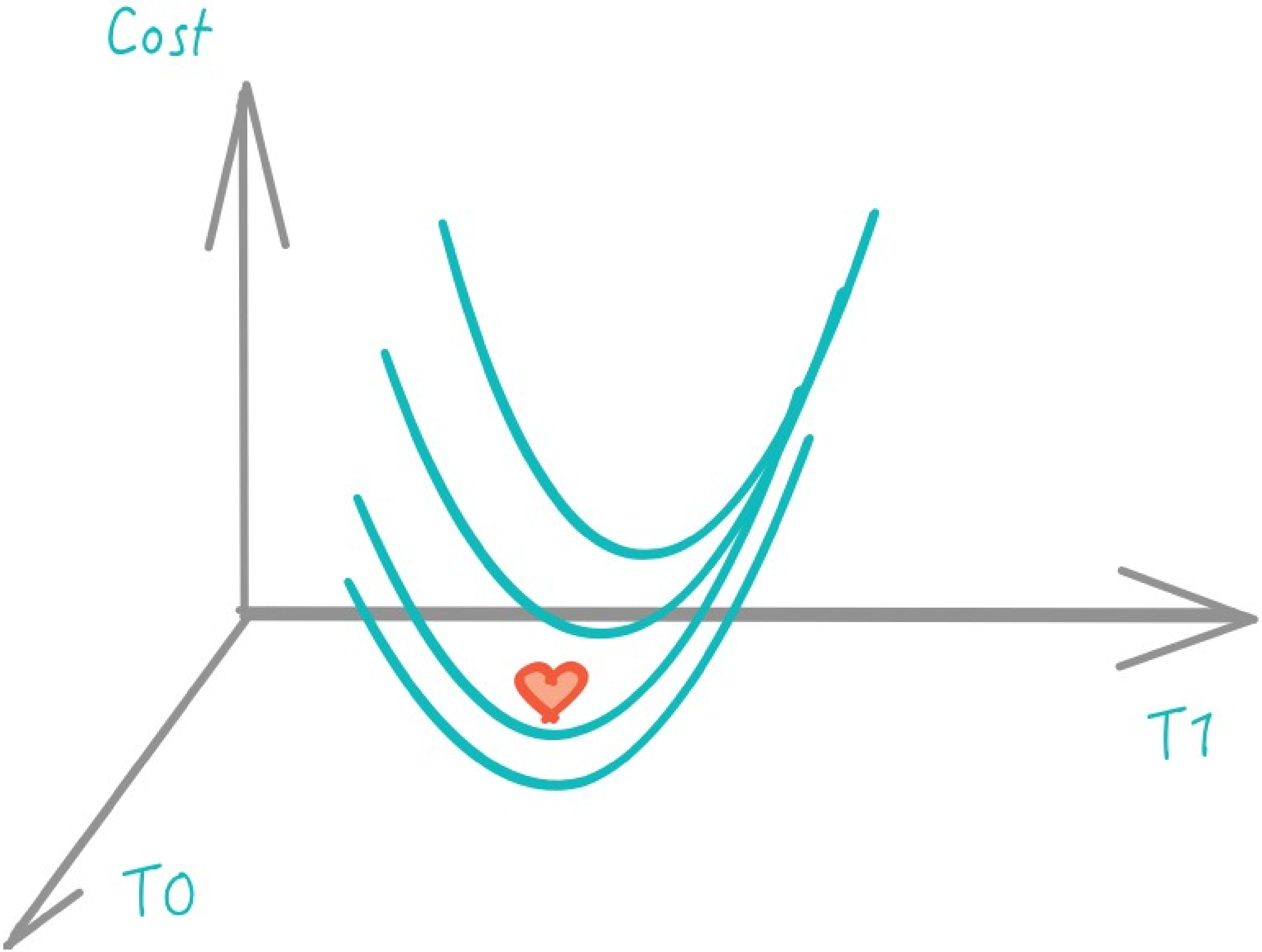


MSE for T_0=2.0

1	0	540
2	0.01	487.54
3	0.02	438.16
4	0.03	391.86
5	0.04	348.64
6	0.05	308.5
7	0.06	271.44
8	0.07	237.46
9	0.08	206.56
10	0.09	178.74
11	0.1	154
12	0.11	132.34
13	0.12	113.76
14	0.13	98.26
15	0.14	85.84
16	0.15	76.5
17	0.16	70.24
18	0.17	67.06
19	0.18	66.96



Cost vs T_0 and T_1



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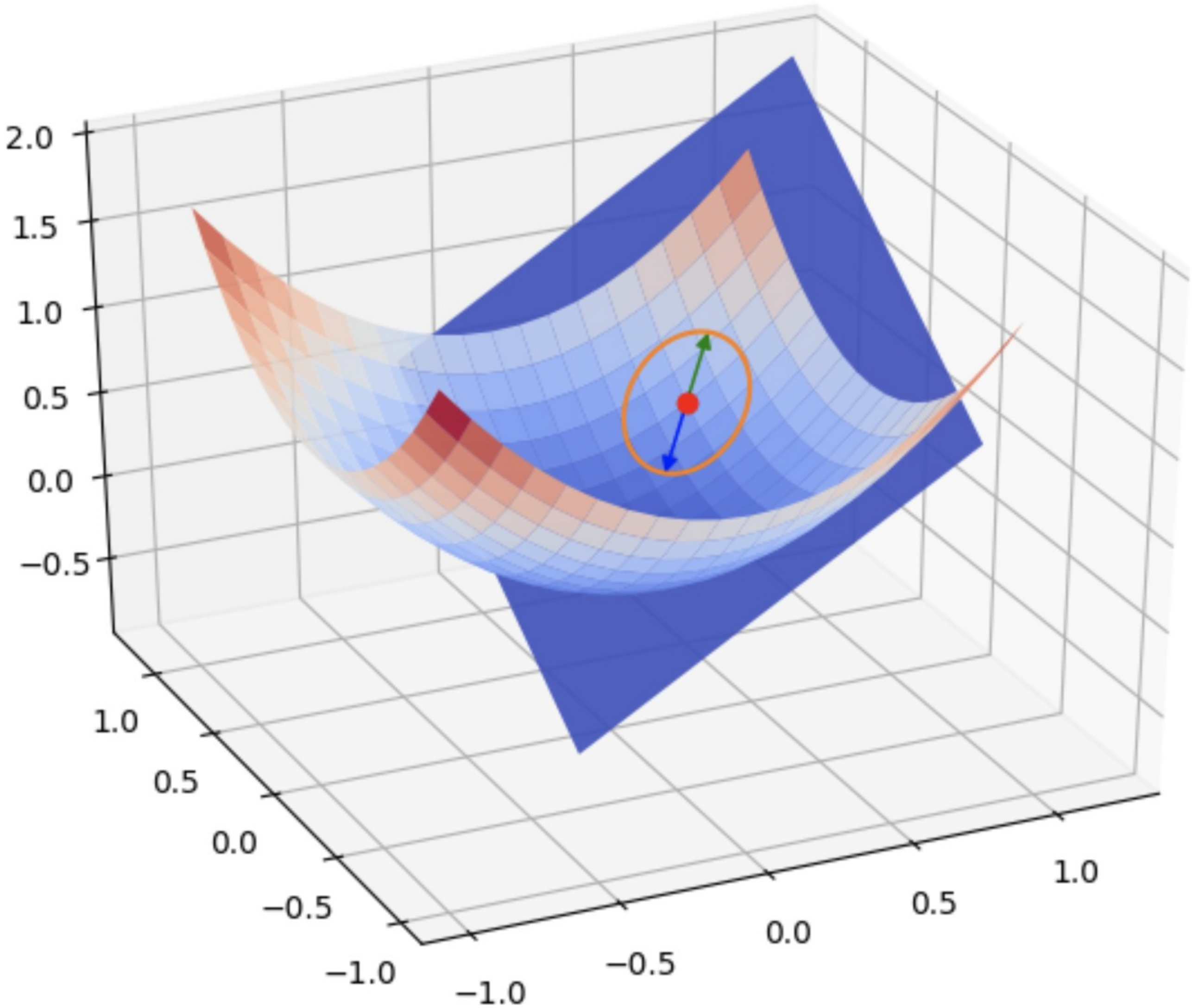
Gradient Descent Idea



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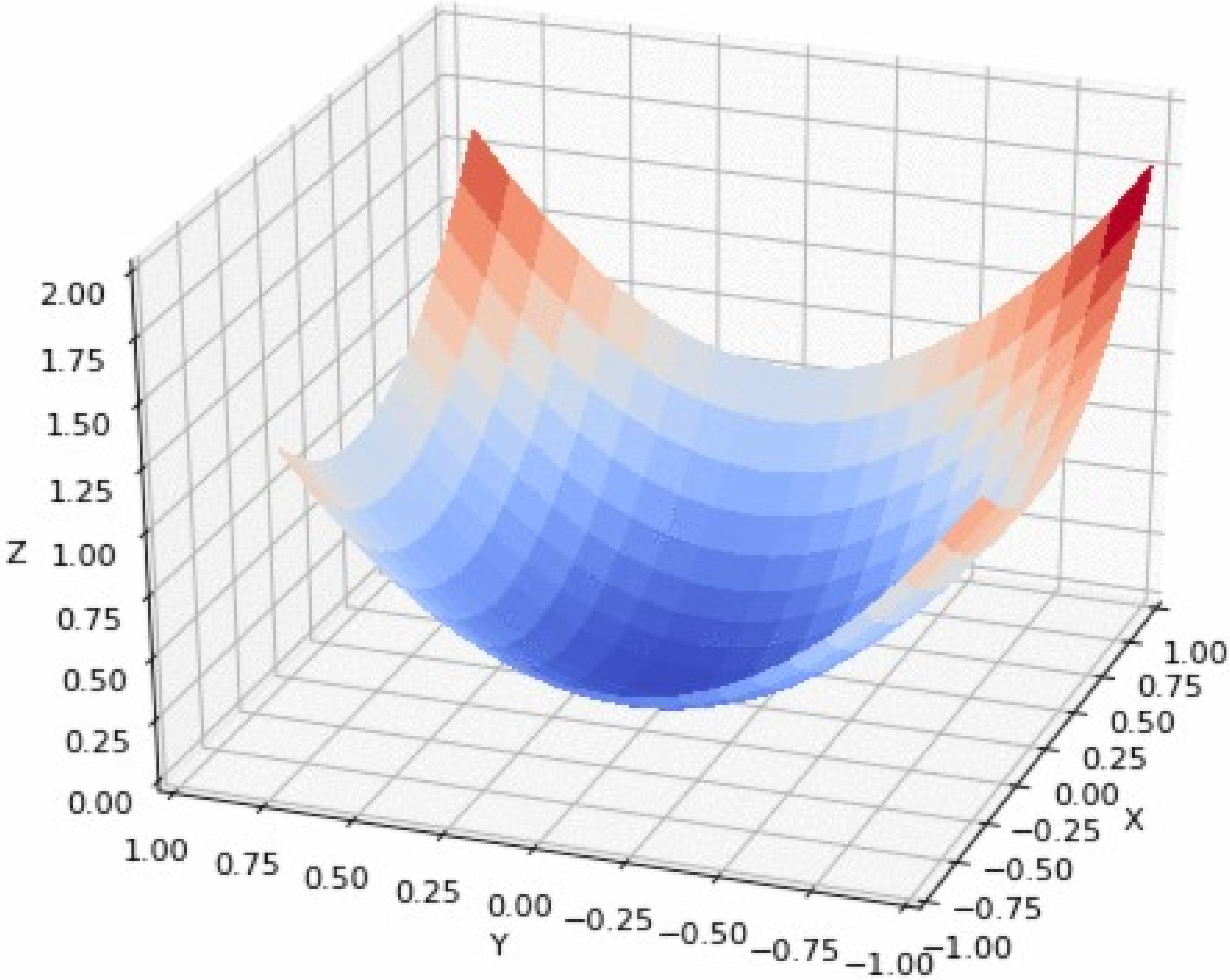
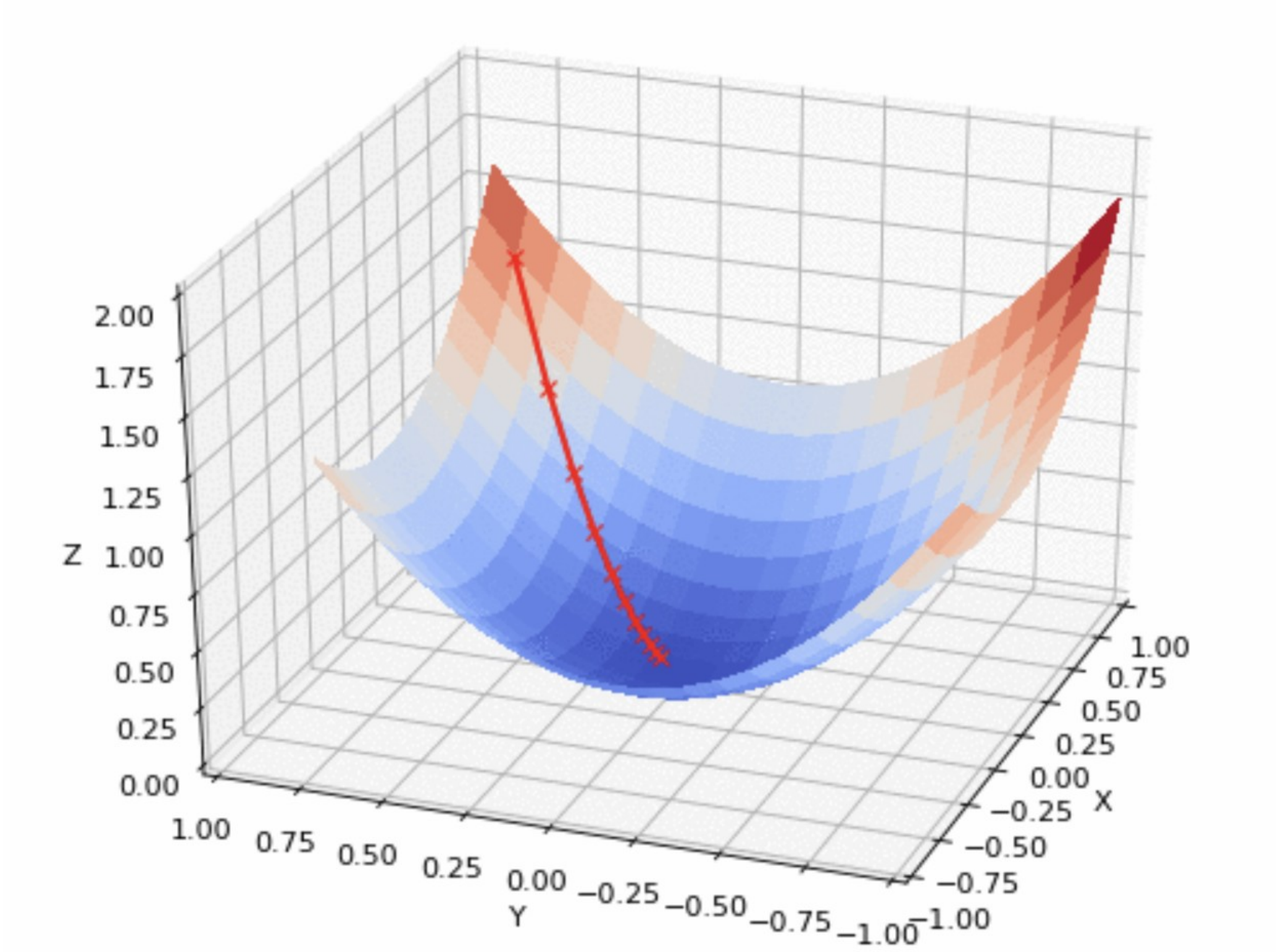
Gradient Descent Demo



Gradient Descent Result



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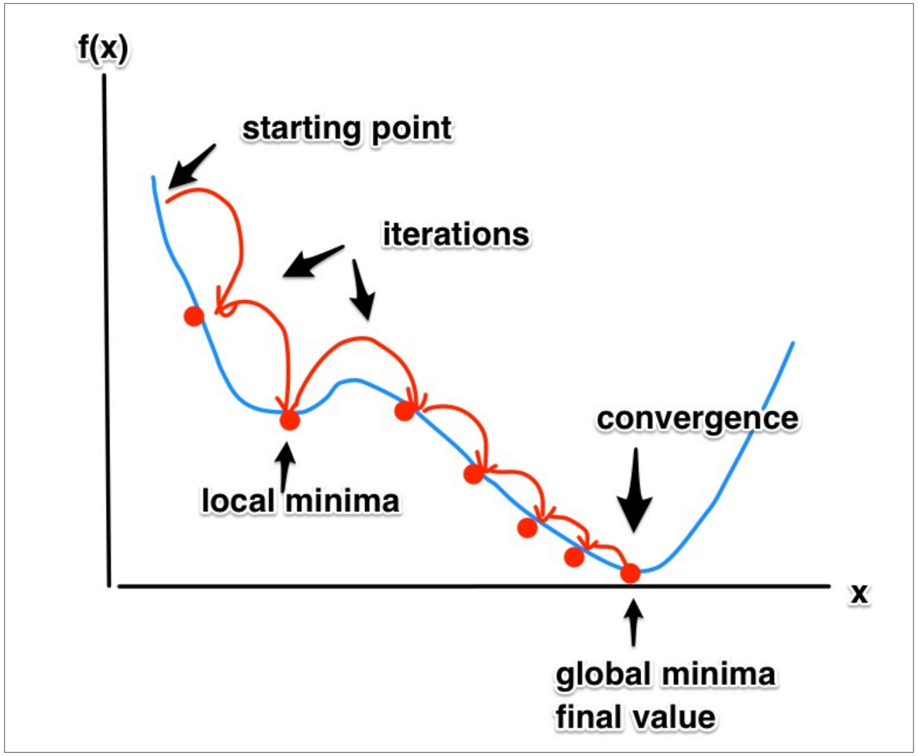
Algorithm for Gradient Descent

- $T = (T_0, T_1)$
  - Generally, $T = (T_0, T_1, T_2, \ldots, T_n)$

1. $T_{next} = Tcurrent + Direction * Step$
2. $Direction = \frac{dCost}{dT}$
3. $Step = Learning\ Step$

- Repeat until convergence
  - Next values are worse, or
  - The improvement is too small

- Repeat until convergence
  - Next values are worse, or
  - The improvement is too small



Batch Gradient Descent

- We did "**batch**" gradient descent
  - We used all training samples
  - There are versions that will use some groups of samples
- There exists a precise solution in linear algebra
  - But gradient descent scales better
- Gradient descent will be used everywhere

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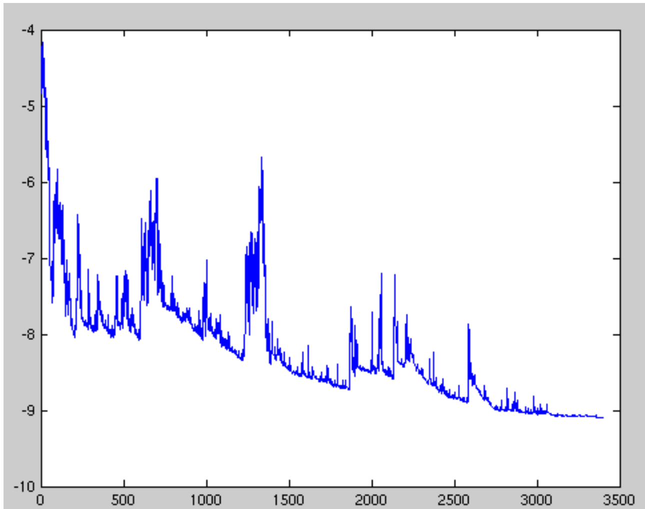
Stochastic Gradient Descent

- For each step of the Gradient Descent, we need a derivative

$$Direction = \frac{dCost}{dT}$$

- How many calculations is that?
- How many calculations is that?
  - Proportionate to m (cost)
  - Proportionate to m (cost)
  - Proportionate to n (predictors)
  - Proportionate to n (predictors)
- Stochastic Gradient Descent
- Stochastic Gradient Descent
  - samples data at each step
  - samples data at each step



Lab: Linear Regressions

Lab

- **Overview**:
  Practice Linear Regression

- **Approximate Time**:
  30 mins

- **Instructions**:
  - **Linear-regression / 1-lr**
  - Follow appropriate Python / R / Spark instructions
  - (1T-lr-tips.ipynb, 1K-lr-tips.ipynb)

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Many Variables

One Variable

Gradient Descent

➔**Many Variables**

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Problem: House Prices

Sale Price $	Bedrooms	Bathrooms	Sq.ft Living	Sq.ft Lot
280,000	6	3	2,400	9,373
1,000,000	4	3.75	3,764	20,156
745,000	4	1.75	2.060	26,036
425,000	5	3.75	3,200	8,618
240,000	4	1.75	1,720	8,620
327,000	3	1.5	1,750	34,465
347,000	4	1.75	1,860	14,650

- Multiple factors are needed to predict house prices
- This is called **multiple linear regression**
- Terminology note
  - "Multivariable" would mean a vector output, not scalar

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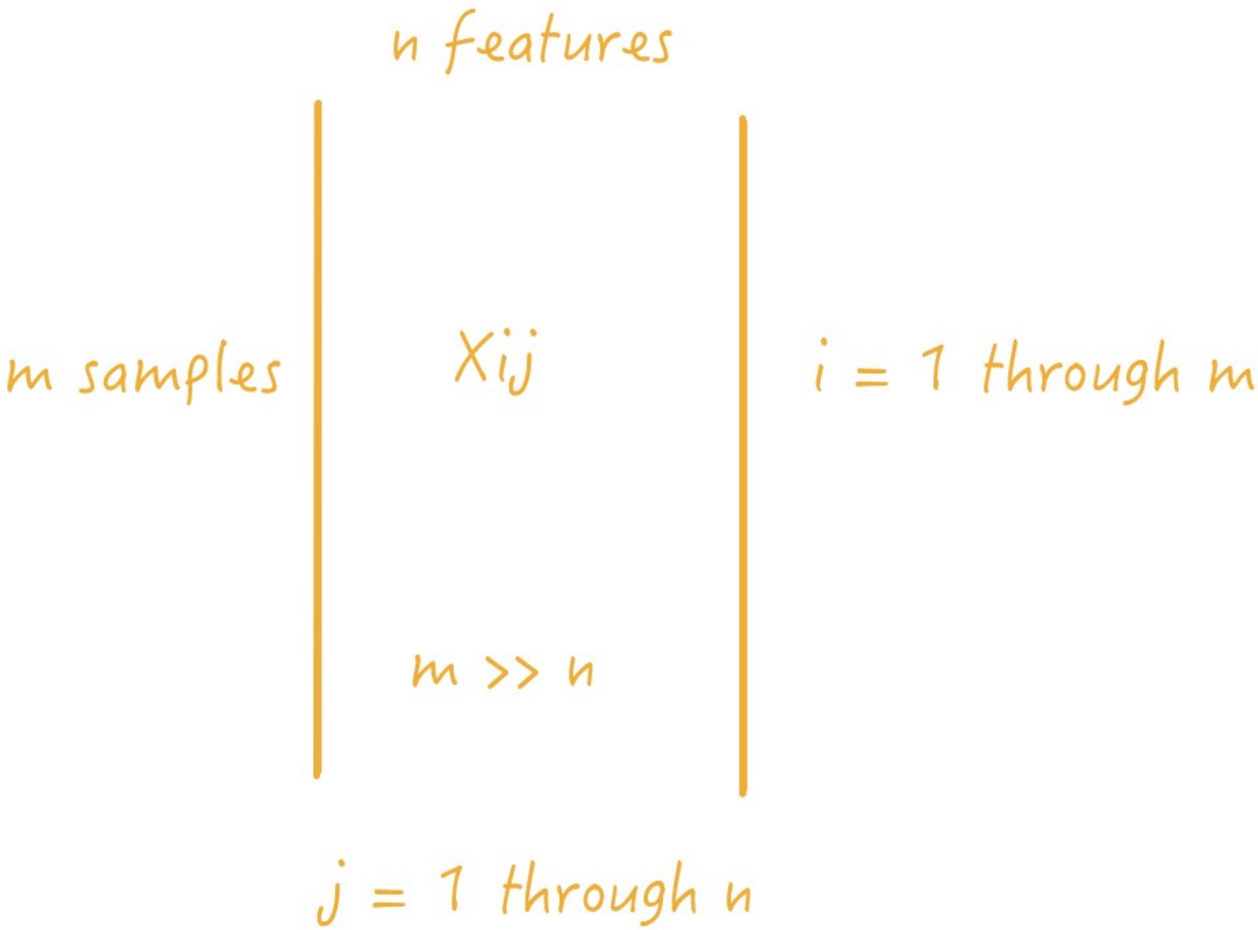
Our Data is a Matrix

- What are the m and n for the previous slides?



Linear Regression With Many Variables

- **Multiple linear regression**

- Many inputs ➔ One output
- $(x_1, x_2, x_3, \ldots, x_n)$ ➔ y
- Our hypothesis (or model)

$$y = T_0 + T_1 x_1 + T_2 x_2 + T_3 x_3 + \ldots + T_n x_n$$

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Cost Function

- Cost of replacing data with our model
- Measures the accuracy of our hypothesis (model)

$$C(T_0, T_1, , , Tn) = \frac{1}{2m} \sum_{1}^{m} (y'_i - y_i)^2$$

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Cost Function Breakdown

- Better, use function h_T instead of

$$C(T_0, T_1, , , Tn) = \frac{1}{2m}\sum_{1}^{m}(y'_i - y_i)^2$$

- Where
- Better, use function h_T instead of y'_i

$$C(T_0, T_1, , , Tn) = \frac{1}{2m}\sum_{1}^{m}(h_T(x_i) - y_i)^2$$

- Where

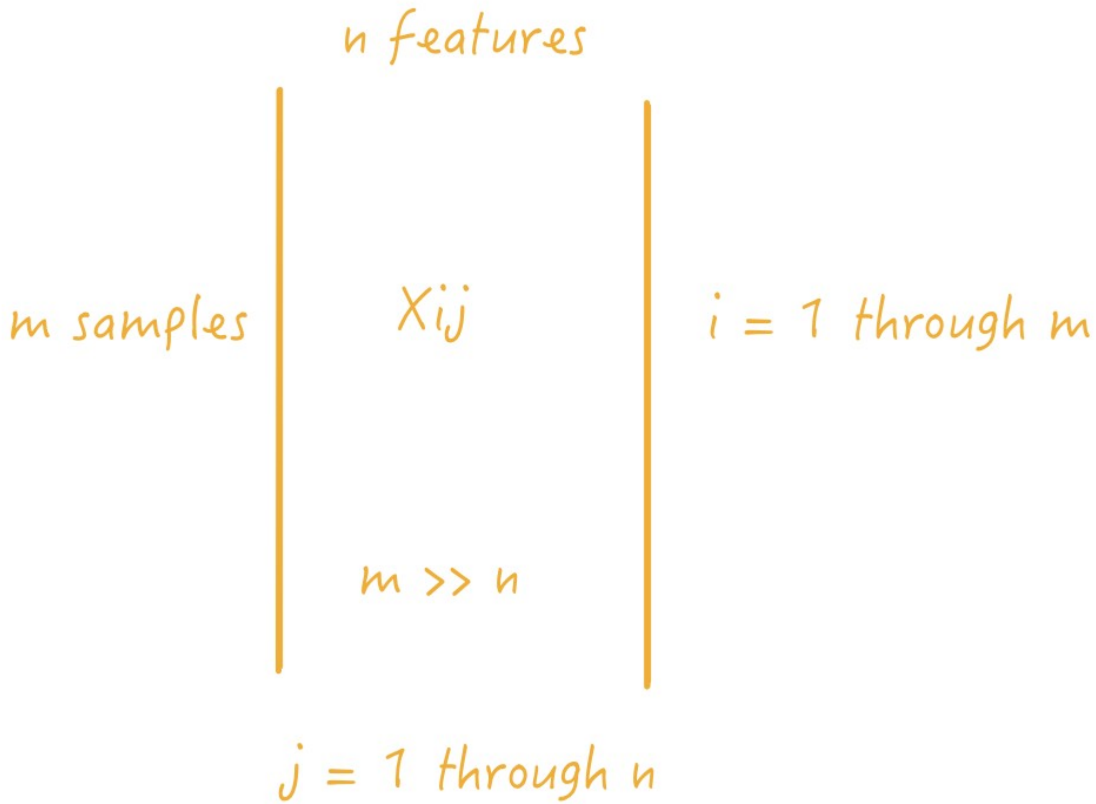
$$h_{T(x_i)} = T_0 + T_1x_1 + T_2x_2 + \ \dots + T_nx_n$$

Solution Advice

- Verify the dimensions
- Feature scaling
- Choose learning rate

Verify the Dimensions

- Number of features: n
- Feature index j = 1 to n
- Number of data points: m
- Data index i = 1 to m



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Feature Scaling

- Approximately

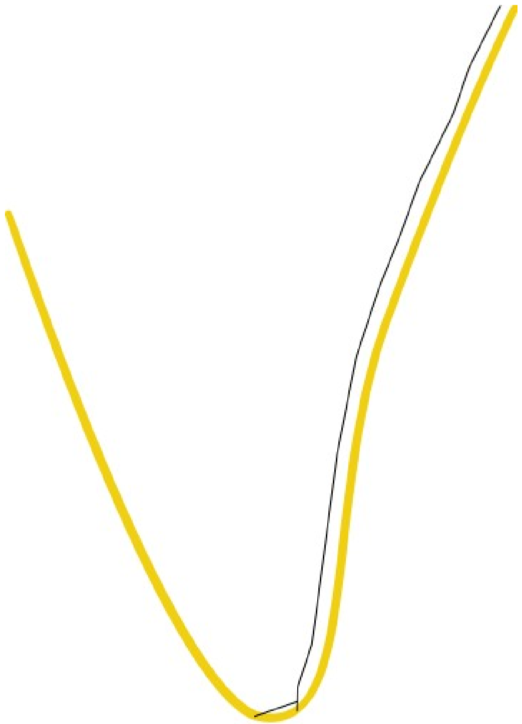
$$-0.5 \leq x_j \leq 0.5$$

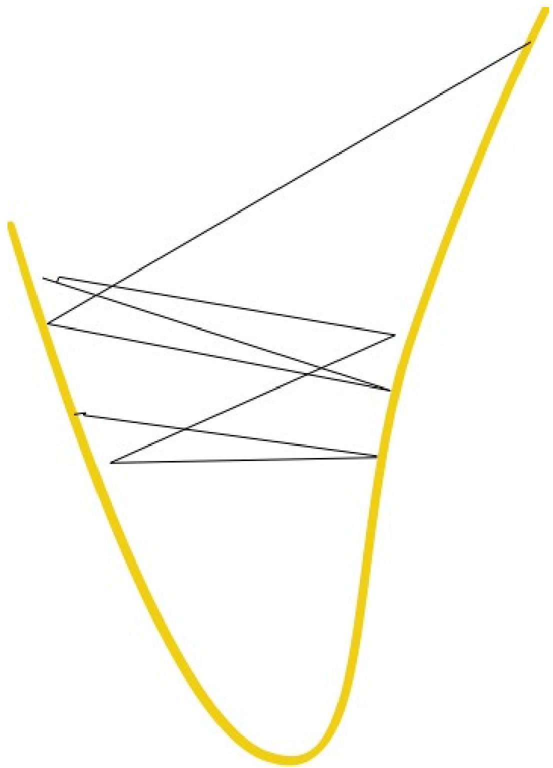
- For example, floor area

$$x_3 = \frac{sq.ft - 2000}{4000}$$

Chose Learning Rate

- Sufficiently small learning rate gives always improving cost
- Avoid jumps up and down
- Good Bad





Lab: Multiple Linear Regression

Lab

- **Overview**:
  Practice Multiple Linear Regressions
- **Approximate Time**:
  30 mins
- **Instructions**:
  Follow appropriate Python / R / Spark instructions
  - LIR-2: House prices
  - (2T-mlr-house-prices.ipynb, 2K-mlr-house-prices.ipynb)

Regularization

- Regularization seeks to
  - Minimize RSS of model
  - And to reduce the complexity of the model
- How to reduce the complexity of model
  - Removing unnecessary coefficients (b_1, b_2 ..etc)
  - Keeping coefficient values getting too large (**parameter shrinkage**)
    - Large coefficients amplify certain parameters
- **Minimize = RSS + ƛ * penalty on the parameters**
- Two types of regularizations
  - **Lasso Regression** (**L1 regularization**): Minimizes the absolute sum of the coefficients
  - **Ridge Regression (L2 regularization)** : Minimizes the squared absolute sum of the coefficients

$$Y = b_0 + b_1X_1 + b_2X_2 + \ \dots + b_pX_p + e$$

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Ridge vs. Lasso

- Ridge regression will **'minimize' coefficients** but not to zero
  - Called **parameter shrinkage**
- Lasso regression **can shrink parameters can also set them zero**!
  - By setting some coefficients to zero, it eliminates certain features
  - Called **variable/feature selection**
- Lambda (ƛ) can be calculated using cross validation

$$Y = b_0 + b_1 X_1 + b_2 X_2 + \ldots + b_p X_p + e$$

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Lab: Multiple Linear Regression

Lab

- **Overview**:
  Practice Multiple Linear Regressions

- **Approximate Time**:
  30 mins

- **Instructions**:
  Follow appropriate Python / R / Spark instructions
  - LIR-2: House prices
  - LIR-3: AIC

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Preparing Data for Linear Regressions

- **Linear Assumption:**
  Linear Regression assumes linear relationship between input and output.
  May be need to transform data (e.g. log transform) to make the relationship linear
- **Remove Noise:**
  Remove outlier data
- **Remove Collinearity**:
  Linear regression will over-fit your data when you have highly correlated input variables
- **Gaussian Distributions**
  Linear regression will make more reliable predictions if your input and output variables have a Gaussian distribution.
  Transform data (e.g. logarithmic) make their distribution more Gaussian looking for better results

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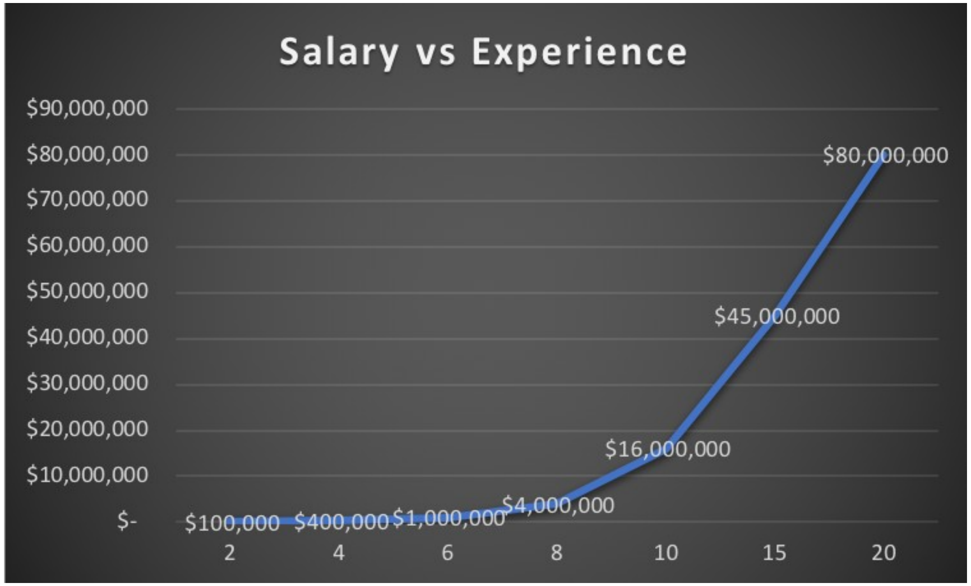
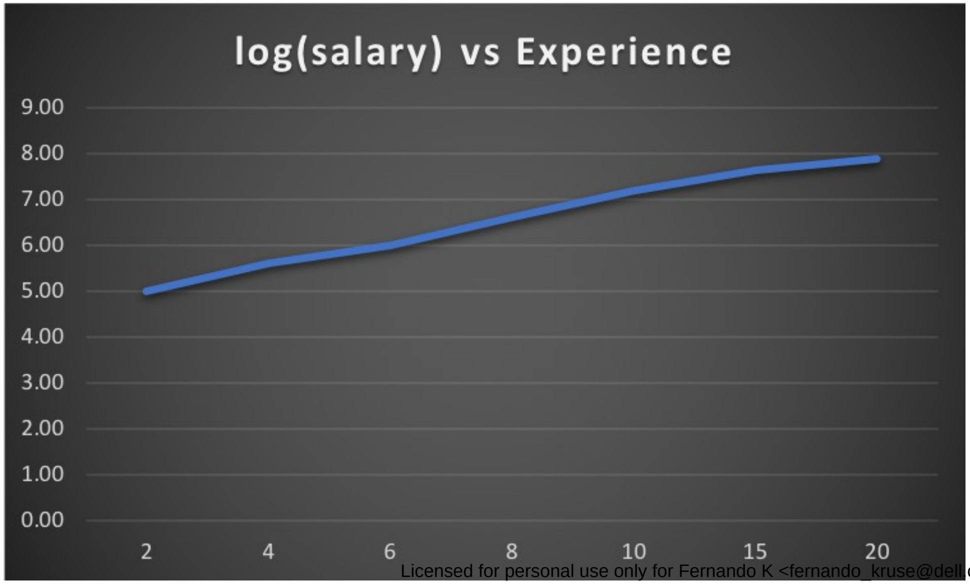
Preparing Data for Linear Regression

experience (years)	salary	log(salary)
2	$ 100,000	5.00
4	$ 400,000	5.60
6	$ 1,000,000	6.00
8	$ 4,000,000	6.60
10	$ 16,000,000	7.20
15	$ 45,000,000	7.65
20	$ 80,000,000	7.90



Actual data.
Not Linear



Data scaled at logarithmic scale.
Linear!

Review Questions

- What is Linear Regression?

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