

Background

Faraday's law of EM induction

$$\mathcal{E} = \frac{-d\phi}{dt}$$

where \mathcal{E} - emf
 ϕ - mag flux

$$\phi = \oint_S \vec{B} \cdot d\vec{A}$$

$$\nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\text{or } \phi = \oint_S \vec{E} \cdot d\vec{l} = \frac{-d}{dt} \oint_S \vec{A} \cdot d\vec{A}$$

We know emf can be induced by changing \vec{B} field
 if $\frac{d\vec{B}}{dt} \neq 0 \rightarrow \mathcal{E}$

Questions

5. Using:

$$\mathcal{E} = \int_0^r (\underline{v} \times \vec{B}) \cdot d\vec{s}$$

where $\underline{v} = r \times \omega$

$$\rightarrow \mathcal{E} = \frac{r^2 \omega B}{2}$$

$$I = \frac{|\mathcal{E}|}{R} = \frac{r^2 \omega B}{2R}$$

resistance

6. Using Biot-Savart

$$B = \frac{\mu_0 I r^2}{2(r^2 + x^2)^{3/2}} \rightarrow \frac{\mu_0 I n r^2}{2(r^2 + x^2)^{3/2}} \text{ for } n \text{ turns in coil}$$

For halfway point in HC $x = r/2$

$$\rightarrow B = \frac{\mu_0 I n r^2}{2[r^2 + (\frac{r}{2})^2]^{3/2}}$$

$$B_T = 2B = \frac{2\mu_0 I n r^2}{2[r^2 + (\frac{r}{2})^2]^{3/2}} = \frac{\mu_0 I n r^2}{[r^2 + \frac{r^2}{4}]^{3/2}}$$

$$\rightarrow B_T = \frac{\mu_0 I n r^2}{(\frac{5r^2}{4})^{3/2}} = \frac{\mu_0 I n}{r} \left(\frac{4}{5}\right)^{3/2}$$

$$\therefore B_T = \left(\frac{8}{5\sqrt{5}}\right) \frac{\mu_0 I n}{r}$$

7. We know \mathcal{E} is induced from change in flux of \vec{B} field, which is indirectly dependent on time.

8. The direction of \vec{B} will rotate with the coils