# Calculating the Gravitational Constant

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In this experiment we measure the gravitational constant $G$ (m$^3$ kg$^{-1}$ s$^2$) by utilizing a variation of the Cavendish apparatus. Our calculated value is $(1.19 \pm 0.6) \times 10^{-8}$, which is roughly 180 times greater than the accepted value of $6.67\times10^{-11}$. Our measurements could have came closer to the real value had we been able to eliminate vibrations and external factors in the building where the experiment was performed.

## I. INTRODUCTION

In 1798, the Cavendish apparatus, named after Lord Henry Cavendsih, was the device used to measure the gravitational constant that Newton had derived in his inverse square law of gravity,

$$F = -\frac{GmM}{r^2} \text{ Eq. (1)}$$

where $m$ and $M$ are two different masses, $r$ is the distance between them, and $G$ is noted as the gravitational constant. Since that time, over 200 experiments have been carried out to calculate this constant, and not all of them have had much success. In fact, one of the latest measurements for $G$ that CODATA (2018) has published gives an uncertainty in G of $2.2\times10^{-5}$ (220 ppm), which is relatively high. This makes $G$ one of the worst calculated constants. Cavendish referred to his experiment as the experiment that would measure the weight of the world. He took into account this equation and used his apparatus to begin his calculations.

## II. METHOD

Cavendishs' set-up consisted of a torsional pendulum made of a rod horizontally suspended from a wire. Said rod has a two inch diameter lead sphere on each end. Additionally, it also has two 12 inch lead spheres placed near the smaller spheres. These two large balls are positioned on alternate sides of the rod as well. The gravitational attraction between the pairs of spheres causes the rod to rotate, thus twisting the wire supporting the rod. The rod will then stop rotating when it reaches an angle when the twisting force of the wire balances with the combined gravitational force between both pairs of spheres. By measuring this angle of the rod and the torque of the wire at this angle, Cavendish was able to determine the force between the spheres and thus the gravitational constant as well.

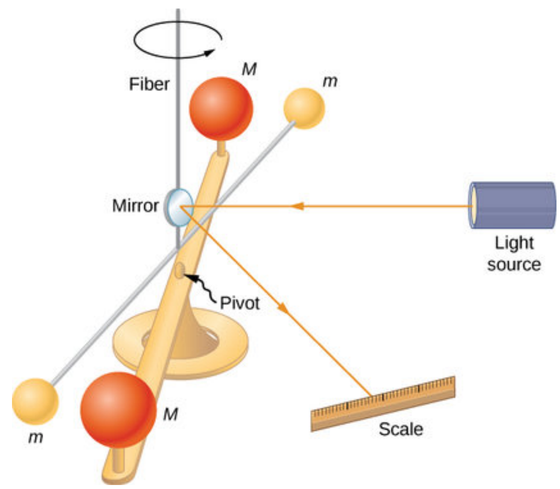


Fig. (1)

When the system is in equilibrium, we can equate the torque on the torsion wire, $k\theta$, to the torque created by the gravitational pull of the masses, $LF$, where L is the length of the rod, to derive an equation for $G$.

$$k\theta = LF = \frac{LGmM}{r^2} \text{ Eq.(2)}$$

To find the torsion coefficent, $k$, we use the resonant oscillation period $T = 2\pi\sqrt{\frac{I}{k}}$

where moment of inertia $I = \frac{mL^2}{2}$, due to 2 spheres and thus $T = 2\pi\sqrt{\frac{mL^2}{2k}}$.

Our new value for $k$ is then $k = \frac{2\pi^2 mL^2}{T^2}$

Substituting the new $k$ value into Eq. (2), we can derive that

$$G = \frac{2\pi^2 L r^2 \theta}{MT^2} \text{ Eq.(3)}$$

Our set-up is different but based off of this apparatus. In our system we have a laser beam bouncing off of a mirror onto a screen. The mirror is placed on top of the disk of the torsional oscillator that we're using. The center of this disk has a hole where a metal cylinder goes through. This cylinder also has a small hole at its center where we stuck a pole through. Said pole has a mass $m$ attached at its opposite end. What we're attempting to do with this set-up is place a much larger mass $M$ next to $m$. If theres an attracation between them, then the disk will move ever so slightly, and so will the mirror on top of it and thus the laser dot on the screen will move as well.

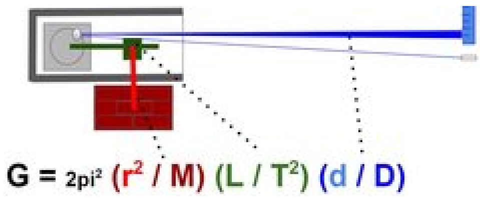

Fig. (2)

This changes our value of $I$ to $I = mL^2$ and running through all the substitutions again we arrive at

$$G = \frac{4\pi^2 L r^2 \theta}{MT^2} \text{ Eq. (4)}$$

Instead of using $\theta$, if we let $d =$ the displacement of our laser dot and $D =$ the distance from the laser dot to the light source, then we can say,

$$G = \frac{2\pi^2 L r^2 d}{MT^2 D} \text{ Eq. (5)}$$

This is the equation we will use to calculate $G$.

## III. RESULTS AND DISCUSSION

Seen below are the set up parameters for our experiment design.

$$D = 64 \pm 1(m)$$
$$L = 0.30 \pm 0.01(m)$$
$$T = 6 \pm 0.2(s)$$
$$r = 0.32 \pm 0.01(m)$$
$$M = 220 \pm 5(kg)$$

These values yielded a displacement value of

$$\mathrm{d} = 0.01 \pm 0.005(m).$$

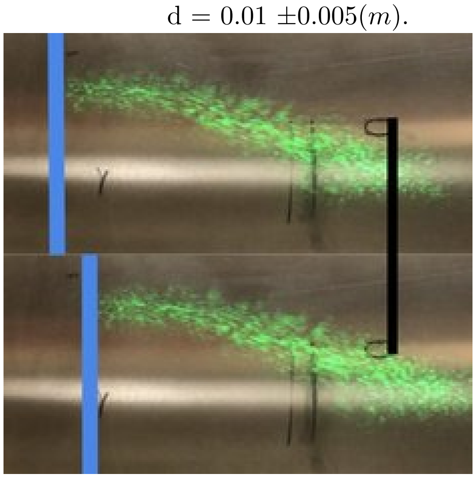
Fig. (3)
Illustration of laser dot moving (blue) against reference lines (black)

Taking these values and plugging them into our Eq. (5), we get that

$$G = (1.19 \pm 0.6) \times 10^{-8} \ (\mathrm{m^3 \ kg^{-1} \ s^2}).$$

This value is approximately 180 times larger than the accepted value mentioned above. If we were to take the smallest values that our uncertainties allow, we would have a $G$ value which only is nineteen times greater than the real value. Considering our set-up and lack of use of electronic devices that could measure parameters better than we can, I'd say we came fairly close to the real value.

Majority of our errors from our system came from the fact that said system is not in complete isolation. Our system is subject to uncontrollable vibrations from the building we're in, people talking or walking next to it, and also the vibrations from the floor caused by placing our large mass $M$ next to $m$. In an ideal situation, we would need a remote controlled set-up placed inside a vacuum chamber that would eliminate any type of vibration that could move or displace said set-up. Furthermore, the use of electronics or programs that could measure values with a much greater accuracy than we can would be beneficial.

## IV. CONCLUSION

Although we measured a value for $G$ that came fairly close to the real one, we still could've came to a closer value. One way to do this would be to take a look at our Eq. (5) again, remove the constants, and break it down.

$$G = \frac{L}{T} \times \frac{r}{M} \times \frac{d}{D}$$

Here we can see that to make our equipment more sensitive, we could increase the rod length and have a bigger mass hanging from it. Additionally, we can also decrease the distance $r$ between the two masses and have our big mass $M$ be larger. From here we could increase the distance $D$ that the laser travels to create a set-up that's more sensitive to calculating the laser displacement $d$. Basically, by having a set-up much larger we can more accuratley define our $G$ value.

One thing that we couldn't eliminate was vibrations. The building where the experiment was performed shakes very slightly sometimes and this causes a displacement in our set-up that we don't want. A good portion of our error came from this simple fact. If there was a way to remove this factor, and enlarge our set-up, then we could very well have a more isolated system that could measure $G$ more precisely.

## V. CITATIONS

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