1 Big G Measurement

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1.1 Table of Contents:

2 Background and Theory:

2.1 Law of Universal Gravitation

Isaac Newton's law of universal gravitation proposed that the gravitational attraction between any two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers **[1]**. In equation form, this is often expressed as follows:

$$F_g = G\frac{m_1 m_2}{r^2} \tag{1}$$

The constant of proportionality in this equation is G - the universal gravitation constant - and is $6.67384x10^{-11} m^3 kg^{-1} s^{-2}$. The value of G was not experimentally determined until nearly a century later (1798) by Lord Henry Cavendish using a torsion balance.

2.2 Cavendish Experiment

Cavendish's apparatus **[2]** for experimentally determining the value of G involved a light, rigid rod about 2-feet long. Two small lead spheres were attached to the ends of the rod and the rod was suspended by a thin wire. When the rod becomes twisted, the torsion of the wire begins to exert a torsional force that is proportional to the angle of rotation of the rod. The more twist of the wire, the more the system pushes backwards to restore itself towards the original position. Cavendish had calibrated his instrument to determine the relationship between the angle of rotation and the amount of torsional force. A diagram of the apparatus is shown below.

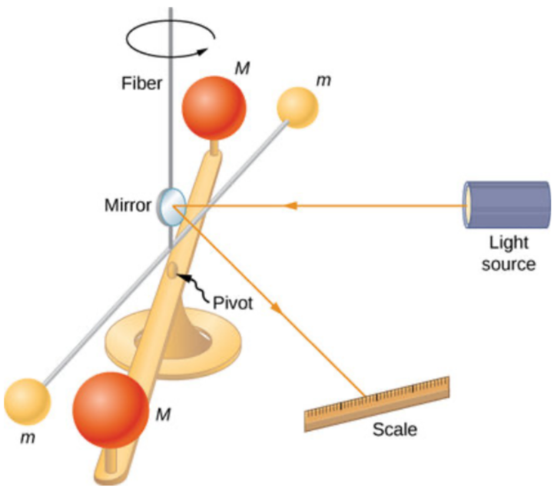

Fig. 1: Schematic of Cavendish Apparatus (Credit: GOOGLE)

Cavendish then brought two large lead spheres near the smaller spheres attached to the rod. Since all masses attract, the large spheres exerted a gravitational force upon the smaller spheres and twisted the rod a measurable amount. Once the torsional force balanced the gravitational force, the rod and spheres came to rest and Cavendish was able to determine the gravitational force of attraction between the masses. By measuring m_1, m_2, d and F_g, the value of G could be determined. Cavendish's measurements resulted in an experimentally determined value of $6.75x10^{-11} Nm^2/kg^2$. Today, the currently accepted value is $6.67259x10^{-11} Nm^2/kg^2$.

2.3 Big G Measurement

2.3.1 Experimental Set Up

We have devised a set up similar to a Cavendish Apparatus with the use of a TeachSpin Torsional oscillator, a mirror, measuring rule, and a laser (see **Section 3**)

The laser is reflected off a mirror, which rotates with the pendulum (the hanging masses in our set up). This causes a laser pointer dot to slowly track back and forth across the opposing wall of our lab. The dynamics of these oscillations are used to determine G.

Once the laser was aligned, measurements were taken, every 30s, from the centre of the stripe along the tape measure scale. A measuring tape was affixed to the wall such that the stripe tracks along it.

The key feature of the experiment is to measure the difference in equilibrium positions of the laser spot when the large mass is in Position A (I) or B (II).

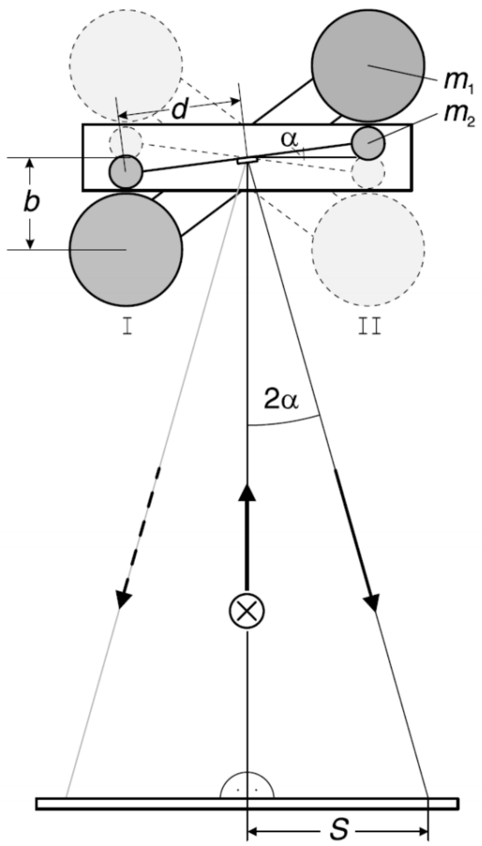

Fig. 2: Schematic showing our set up with the Positions A and B scheme (credit: TeachSpin)

2.3.2 Methodology

Balance the torsion force (the twist) on the wire with the torque resulting from the gravitational attraction of masses M and m.

$$\tau\theta = D \times G\frac{m_1 m_2}{r^2}$$

$$\therefore G = \frac{\tau r^2 \theta}{D m_1 m_2}$$

The torsion constant τ can be found by measuring the period T of small oscillations of the Torsional Oscillator.

$$\tau = \frac{2\pi r^2 m D}{T^2}$$

We can now combine the expressions to find G

$$G = \frac{2\pi r^2 m D}{T^2} \times \frac{r^2 \theta}{D m_1 m_2}$$

$$\therefore G = \frac{2\pi^2 r^2 D\theta}{MT^2}$$

Since the mirror rotation is twice the pendulum rotation, and between positions A and B we double θ.

$$\theta = \frac{\theta_A - \theta_B}{4}$$

$$\therefore G = \frac{\pi^2 r^2 D}{2MT^2}(\theta_A - \theta_B) \tag{2}$$

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3 Experimental Progress Report

3.1 Day 1:

We played around with the TO apparatus trying to understand how it works. We found a wire that was good for our project, however, it didn't have hoops for attaching the masses. So we looped it to make it work. The length was too long so with permission from Dr. Akchurin we cut it.

Fig. 3: Experimental Set Up Idea (credit: TeachSpin)

We realized upon further investigation that this set up cannot be used to do calculate G because the forces are vertical aligning with the influence of Earth. This means that our experimental set up should be such that the gravitational force between the stationary and hanging masses must be perpendicular to that of their weights.

We are now thinking about using the Torsional Balance set up for conducting our experiment.

3.2 Day 2:

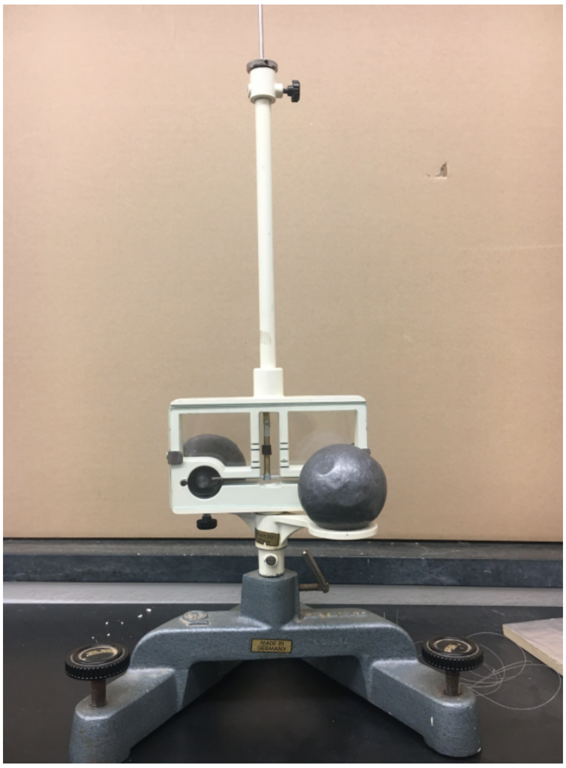


Fig. 4: Torsional Balance Apparatus

After we reported the issues to the instructors and shared our resolutions. We were asked by Dr. Akchurin to look for a torsion wire (or the closest thing) from the Machine shop and to carry out our hobbyist version of the experiment since it may not be possible to fix the Torsional Balance apparatus.

In the meanwhile, we started to look into a theoretical prediction for the value of G by connecting ideas from Higg's mechanism and loop quantum theory of gravity. This excursion was a pleasant yet futile use of time.

3.3 Day 3:

While waiting on the machine shop experts to return from lunch break, I looked around the lab rooms to seek any wires that could be of use to our experiment. We did not find any.

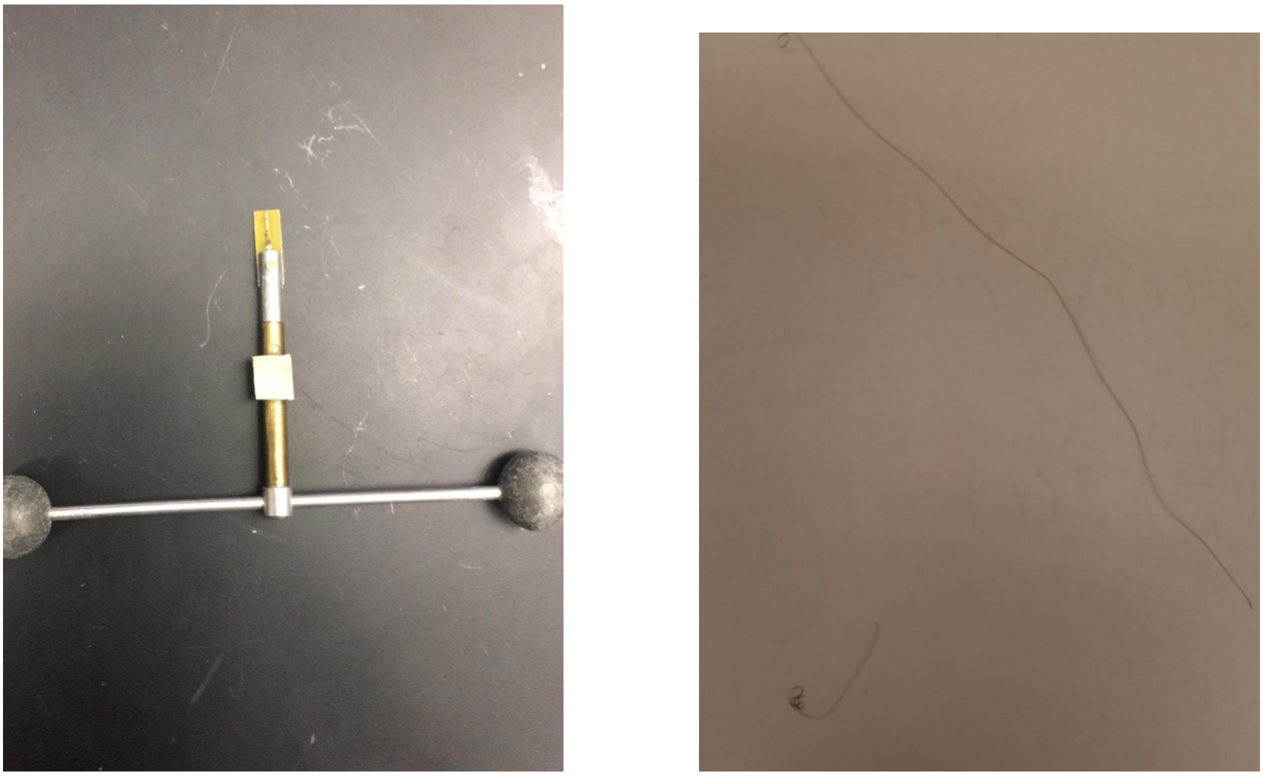


Fig. 5: Issues with the Torsional Balance Set Up (broken balancing rod and snapped torsion wire)

After talking to Kim about our issue, he said that he could possibly fix the Torsional Balance apparatus. He found an extra torsion wire in his supplies and he took the apparatus in the shop to work on it. He insisted that he perform the procedure himself rather than us since there is only one wire left and there is no room for error. Kim returned to room 030 and was asking if we saw a "small copper holder screw" that he accidentally dropped while transporting the apparatus. We searched for an hour and it is nowhere to be found (it is really thin and light). Kim assured us that it he probably had a replacement and left.

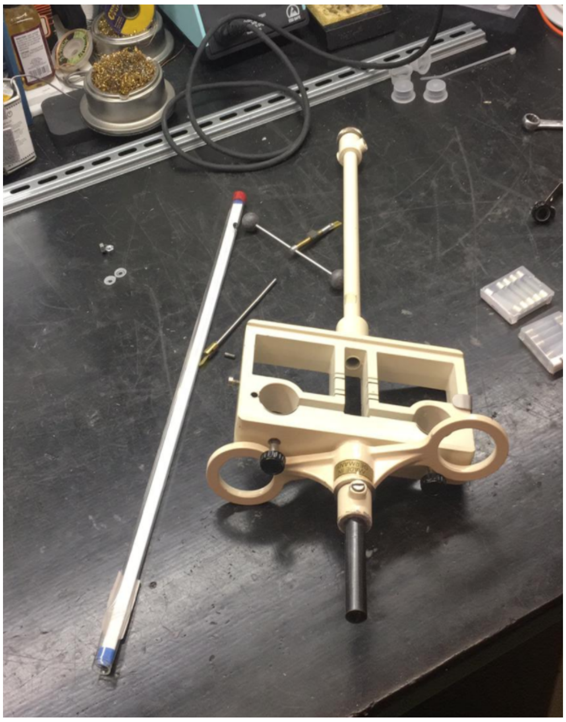


Fig. 6: The Torsional Balance Apparatus on Kim's repair desk

Four hours later, Kim brought back the apparatus. He, sadly, had snapped the torsion wire since it was a very tricky procedure so he decided to use a fine **0.2337 mm** copper wire instead. Due to the ductile nature of the copper wire, it will stretch due to the weight of the hanging masses and he instructed us how to restore the hanging mass system to default if it does so by loosening and pulling the top screw.

He further showed us how to manipulate the equipment to conduct our experiment. He has advised us to **handle the apparatus very gently** and **to always lock the masses before moving the apparatus.** This is crucial to make sure the apparatus is functioning and not out of order since torque of the hanging masses will snap the wire. He also asked us to report our best finding to him so that he can assess the viability of using copper wires instead of the standard *torsion wire* to carry out this experiment.

He has also given us the laser needed to make the angle displacement measurements.

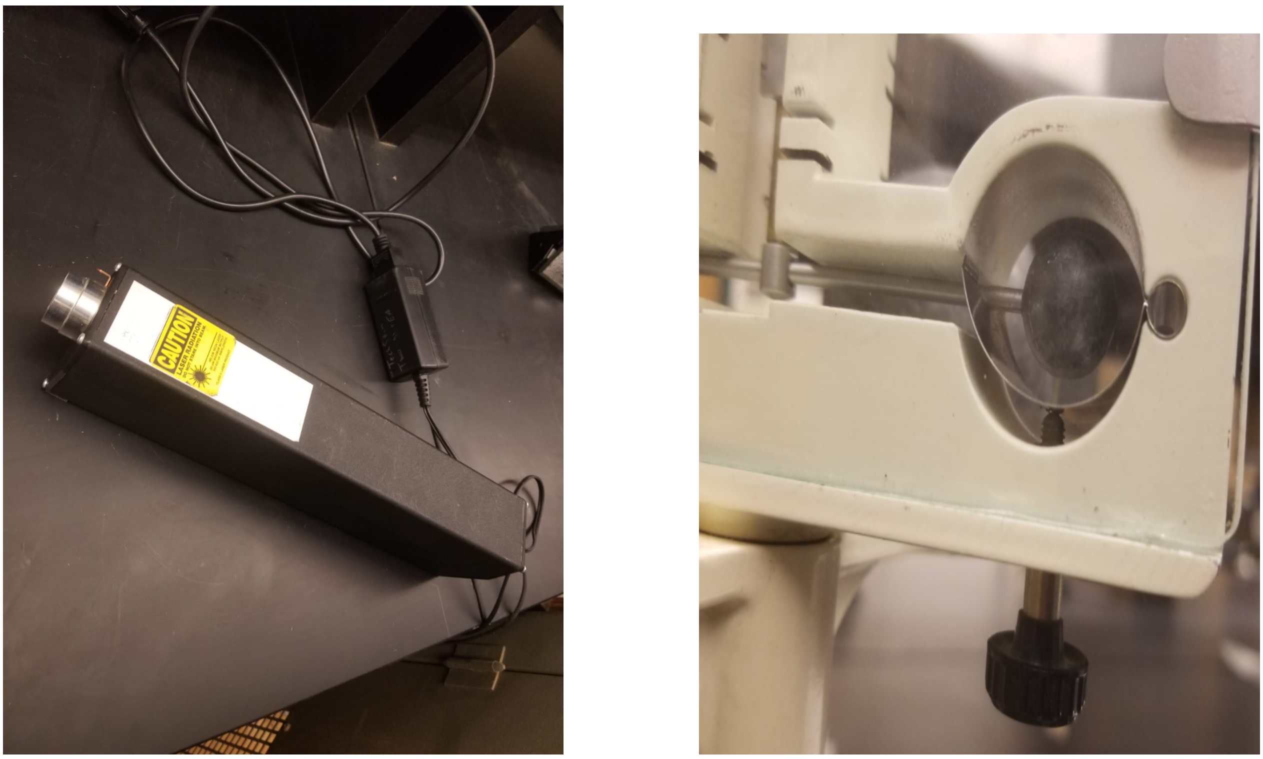


Fig. 7: The laser we used in the experiment and a picture showing locked mass

3.4 Day 4:

Today, we set up the laser and devised a measurement screen to record the change in mirror movements through change of laser pointer.

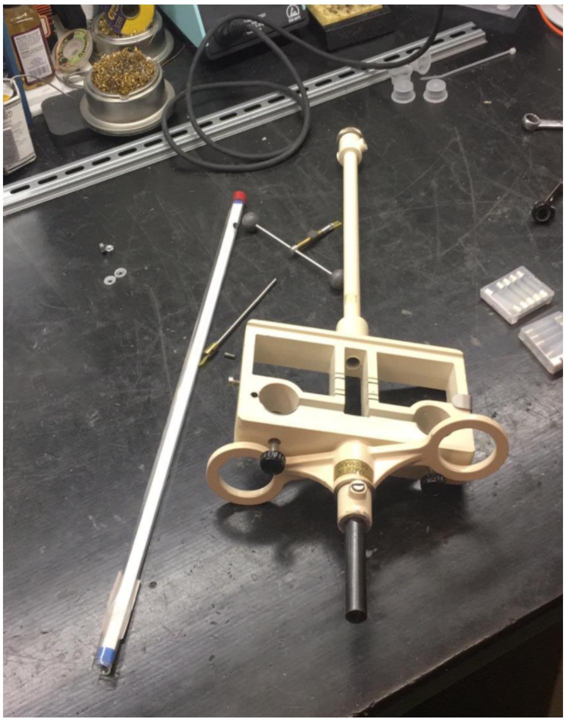


Fig. 8: Set up to calibrate laser beam path and record data

Before, starting the experiment, we ensured the system had no "zero error". This was done by having the masses move freely over the circular pad by dialing the knobs after it was kept on flat surface. We also adjusted the length of the copper wire as we saw it give over the last day.

Some of the measurements we made are:

Distance from balance to laser: 80.01 ± 0.01 cm Distance from balance to wall: 2.27 ± 0.01 m

We took data for two half an hour runs and recorded the screen the entire time at a 30 second fidelity.

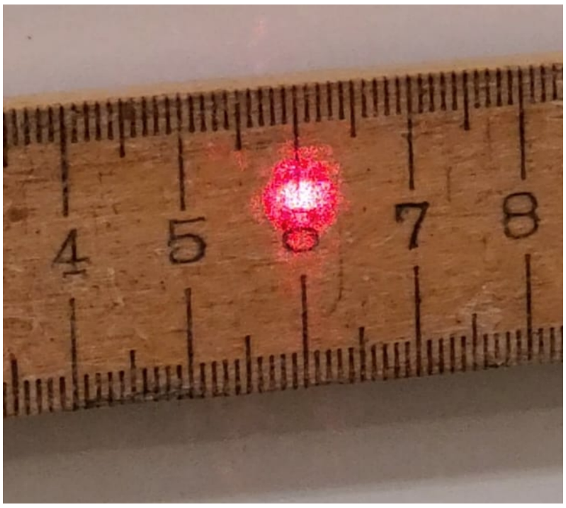


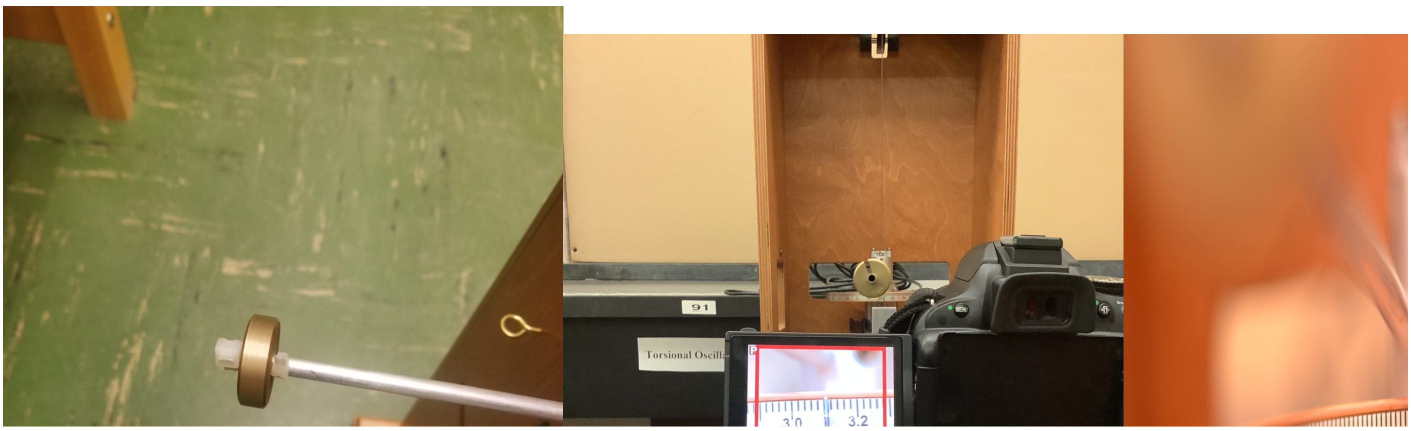
Fig. 9: Sample picture showing laser position

The measurements show no real change of position of the laser pointer, this may be due to the short length between the mirror and screen. However, upon investigation we conclude that it is the copper wire and its properties that was the culprit and not the distance. We hypothesize that it is the ductility of copper and its low κ value and possible non linear relationship between torque that makes it not have the same properties as a torsion wire.

3.5 Day 5:

Fig. 10: Initial Set Up using Torsional Oscillator

We pivoted our experimental design yet again and followed the one outlined by **[3]** with modifications. We used the Torsional Oscillator set up, dismounted the permanent magnets to allow the disc to rotate freely and used a 50.5 ± 0.01 cm rod. We hung two 0.25 kg masses with a separation distance of 0.485 ± 0.0001 m between them. We also had two sets of 10 lead bricks weighing 120 kg as our stationary mass 0.15 ± 0.01 m away from the stationary masses.



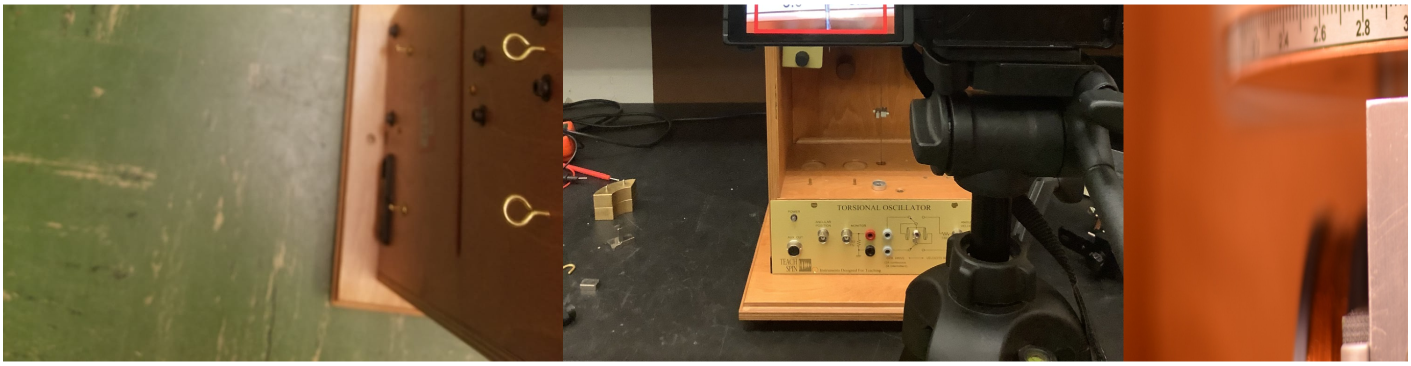


Fig. 11: (a) Zip lock clamps holding mass in place (b) Data Acquisition Set Up using Camera (c) Picture taken by Camera

After observing the oscillations, we realized two pitfalls of our apparatus.

1. The oscillations recorded are on the order of the uncertainty of our readout system (i.e. the tape measure on the disc)
2. The center of mass of the lead brick system is not on the same plane as the hanging masses.

Thus, we decided to changed the layout to address these issues. Moreover, we took Dr. Akchurin's advise from our class presentation and doubled the length of our rod to now have a separation distance between the two masses to be 0.97 ± 0.01 m.

We scoured the physics building to find something that could hold the weight of the lead bricks and help elevate them so that the center of the mass is on the same plane as the hanging masses. Cris Perez heroically solved our problems and made a wooden stool for us. We used this stool and added some extra lead bricks such that the center of mass is on the same plane and the gravitational attraction between them and the hanging mass is greater.

The new set up utilized 17 lead bricks arranged in the following manner: 2 by 8 bricks with one brick placed along the central axis perpendicular to that of the hanging masses.

Fig. 12: New Lead Brick Configuration using the wooden stool

We also replaced the Data Acquisition Scheme from simply being camera based to a laser and mirror based (with camera). We collected the smallest mirror and a polarizer we could find from the optics lab. The decision behind which polarizer to use was made experimentally with the criteria being which polarizer made the laser spot radius the smallest.

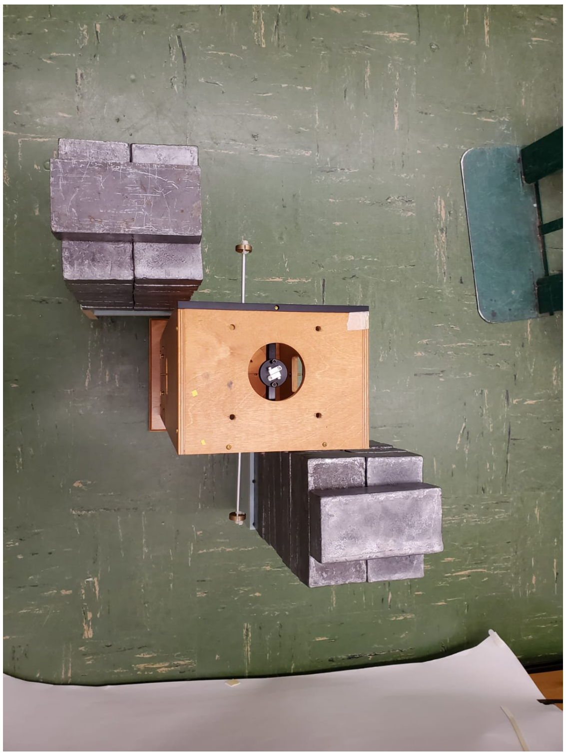
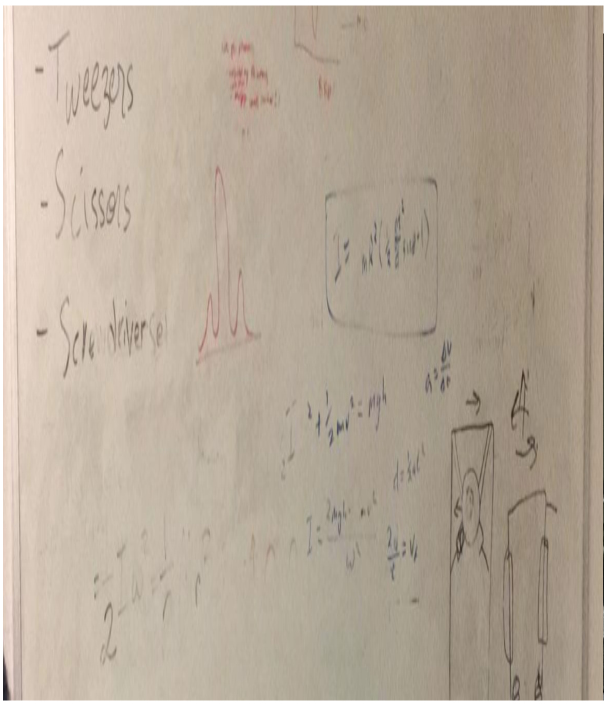
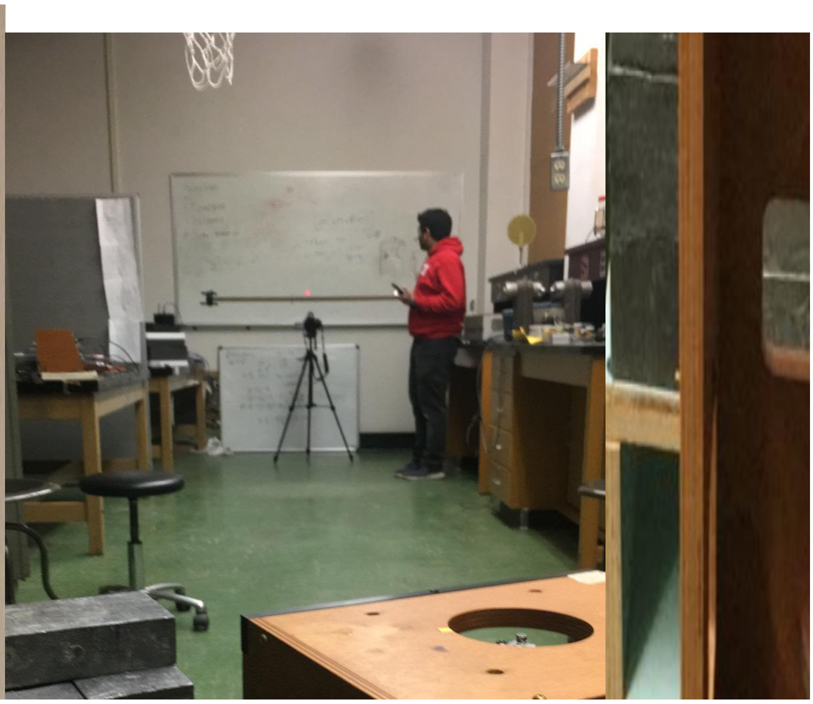


Fig. 13: Bird's Eye View of our final Set Up

We also changed the layout of the room (moved the tables away, pushed the gray divider to near the white board and taped the back of a poster to it) so that we can ensure our laser pointer set up can pick up on the faintest oscillation based on the room we were in.

After an embarrassingly long time spent in trial and error, we finally figured out how to place the mirror onto the disc without damping its motion (solution - let it stand on the surface of the disc) and also recorded the distance and angle at which the laser needs to point at the mirror so as to see the laser pointer on our screen. At this point, we also taped a 2 meter rule stick to the white board and drew out the gradings on it for better measurement through the camera.





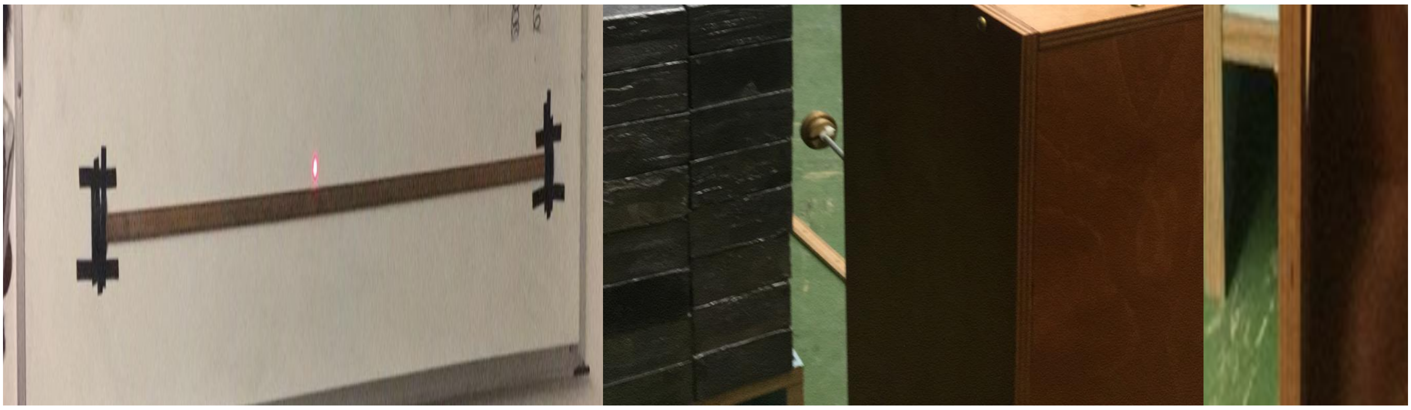


Fig. 14: (a) Measuring Tape on Screen (b) New room layout with a screen for making measurements (c) Mirror installed on the disc

3.6 Day 6:

We returned to the lab around midnight and did the last touches required to make sure we can start our first good run. We increased the hanging masses to 0.250 kg each and decreased the separation between hanging and stationary masses to m to ensure a strong Gravitational attraction that can be registered by our set up. We also set our camera such that it takes a picture every 30 second for a 2.5 hour period. The new set of measurements made:

distance from mirror to screen = 9.3 ± 0.1 m

distance from hanging mass to stationary mass = 0.076 ± 0.001 m (calculated using the center of masses)

mass of the lead brick mass = 204 ± 1 kg

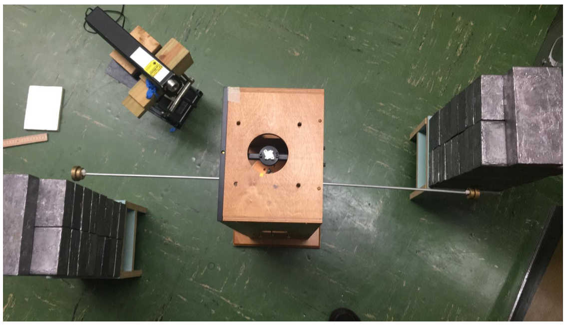


Fig. 15: Bird's Eye View of the set up before taking data (Position A Set up)

We came back at 9 in the morning and took the camera back home to save the pictures and charge it. Before leaving, we changed the lead brick masses so that now they are in the opposite half of the rod (Position B Set up). We did not take any data during the daytime since there was renovation work going next to our room.

3.7 Day 7:

We returned again around midnight and started recording data with the Position B Set up for 2 hours.

3.8 Day 8:

We rearranged the room back to how it was, put the lead bricks away and restored the Torsional Oscillator to its original state.

4 Conclusions and Summary

Using the images that we recorded while taking data in Position A and B set up, we recorded the position of the laser pointer dot on the screen from the origin. All credit for this task is given to Sam who painstakingly went through 640 images to tabulate these values.

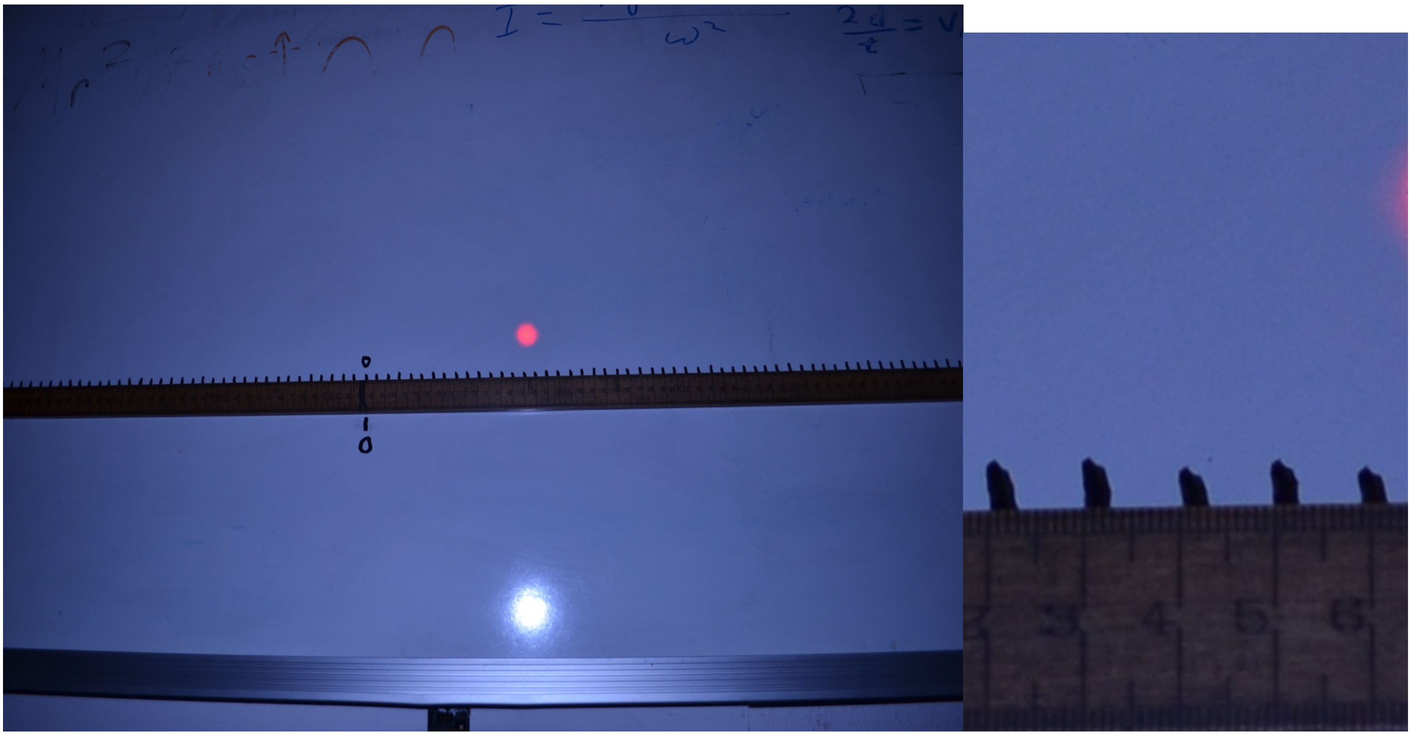


Fig. 16: (a) Raw picture from Camera (b) Zoomed in picture to record the distance

We created a distance versus time curve of this oscillation data and filtered away entries that were entirely systematic noise using Butterpass filter of 4th order with a cutoff frequency of $0.2Hz$ (determined through trial and error). We also disregarded entries were random noise due to random events were registered. For example, in Fig. 17, we can see that around at the 2 hour mark, our system gets disturbed due to a random external event (we suspect the HVAC system pressure change), so we trim our data set to ignore these entries.

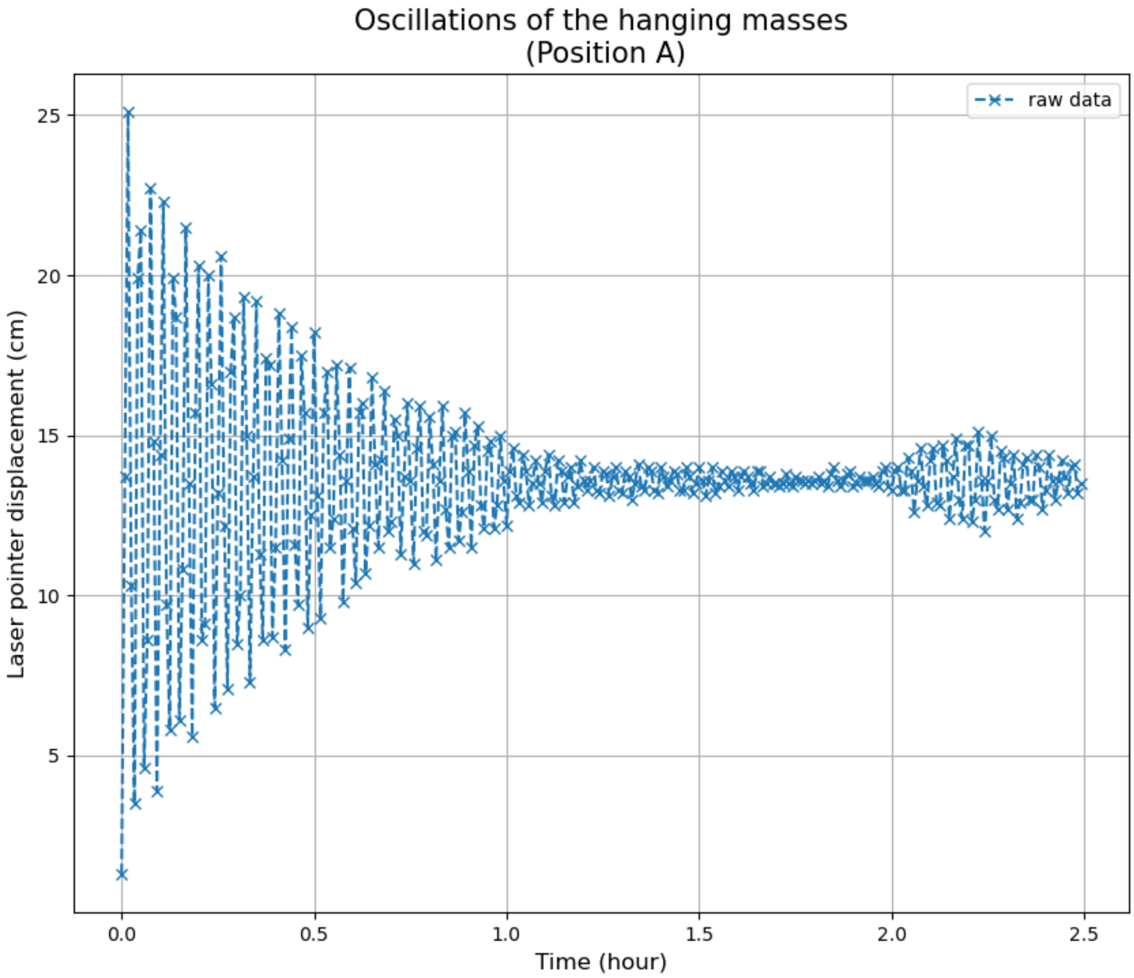


Fig. 17: Oscillations of the hanging masses (Position A) measured at the screen

Fig. 18 shows the calibrated dataset that we use to make our measurements for G.

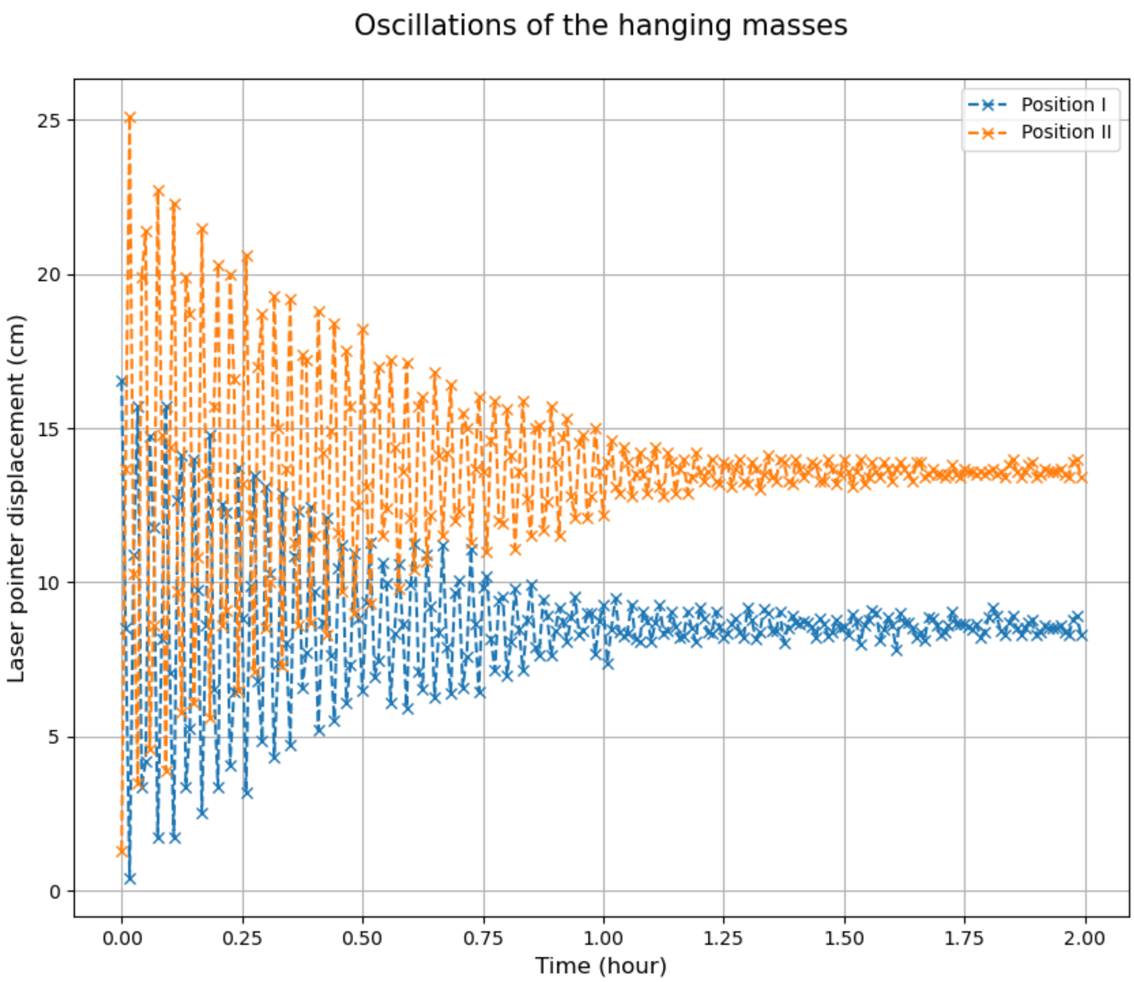


Fig. 18: Oscillations of the hanging masses (Position A and B) measured at the screen

Curve fitting regimes yield the equilibrium displacement for Position A and B to be 0.125 m and 0.062 m respectively and finds the period of oscillation to be 150 s. Using equation 2 from **Section 2.3**, and all the measurements we recorded in this logbook for this calculation we find

$$G = \frac{\pi^2 r^2 D}{2MT^2}(\theta_A - \theta_B)$$

$$G = \frac{\pi^2 (0.076)^2 (0.970)}{2(204)(150)^2}(0.013440050908854227 - 0.006665679038682285)$$

$$G \approx 4.08 \times 10^{-11} m^3 kg^{-1} s^{-2}$$

Using error propagation to calculate the quantitative uncertainty in our calculation, we find

$$\delta G = \pm 3.97 \times 10^{-11} m^3 kg^{-1} s^{-2}$$

However, there are other significant qualitative errors in our method. For example, in addition to the moment of momentum caused by the attractive force of the respective opposing lead brick set up, the attractive force of the respectively more distant lead brick set up gives rise to an antitorque moment which acts perpendicular to the transverse beam. Thus, the gravitational constant calculated would be off by some correction factor that accounts for this phenomenon. There is also uncertainties in the fit values as well.

Nevertheless, with our measurements, methodology and calculations, we have measured G to be $(4.08 \pm 3.97) \times 10^{-11}$ which is 39% within the expected value of $6.67384 \times 10^{-11} m^3 kg^{-1} s^{-2}$ in the average case scenario. We have triumphed in our personal agenda of measuring this constant better than any previous groups in this lab **[4]**.

5 Questions

1. Derive big G and identify the important variables. Explain how your experiment limits these variables.

We know,

$$F_g = G\frac{m_1 m_2}{r^2}$$

from Eqn 1 and

$$\tau = \vec{d} \times \vec{F}_g$$

where τ is the torque of the force $\vec{F}_g$ applied perpendicular to displacement $\vec{d}$. We also know that angular displacement of torsion wire, θ, results in a torque directly proportional to it.

$$\tau_t = \kappa\theta$$

We also know the period (T) -torsion constant (κ) relationship as follows:

$$T = 2\pi\sqrt{\frac{I}{\kappa}}$$

here I is the moment of intertia of the hanging mass

At equilibrium, the gravitational force is balanced by the torque due to the torsion wire, τ_t

$$\tau_t = F_g$$

Making relevant substitutions.

$$\frac{\kappa\theta}{D} = G\frac{m_1 m_2}{r^2}$$

$$\frac{4\pi^2 I\theta}{DT^2} = G\frac{m_1 m_2}{r^2}$$

$$G = \frac{4\pi^2 I\theta r^2}{DT^2 m_1 m_2}$$

In our experiment, we replace I by its formula for our masses such that our only dynamic (dependent) variables are T and θ with all other variables being static (needing to be measured once).

2. What are the possible errors in your apparatus and how could they be removed?

Systematic errors such as measurement uncertainties in lengths, masses, etc. Moreover, there are random sources such constant people walking, wind currents, HVAC pressure differentials. There are also qualitative errors such as the effect of the antitorque and fit parameters as discussed in **Section 4**.

3. Why is the gravity ignored when finding the force between two charged particles? (Hint: take the ratio between the electric force and the gravitational force.)

For charged particles such as electrons, the gravitational force is much smaller than the electric forces.

$$F_g = \frac{Gm_1m_2}{r^2}, \; F_e = \frac{kq_1q_2}{r^2}$$

$$\frac{F_g}{F_e} = \frac{Gm_1m_2}{kq_1q_2}$$

Notice, the distance separating the electrons does not matter when comparing the forces. Since we know these constants, we find that the gravitational force is $2.4x10^{-43}$ the size of the electric force. Since we know nothing in the universe to within two parts in 10^{43}, we may safely ignore gravity.

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4. Describe the Cavendish Apparatus.

See **Section 2.2**.

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5. If you were to use a Cavendish Apparatus to calculate G, what errors would you expect to see and how would you eliminate them?

The errors we would see in this set up is almost identical to the errors we tackle in our experiment with two major changes - air currents our set up and not the Cavendish Apparatus and owing to the low masses the devise is much more sensitive to environmental noise such as people walking, cars, etc. To eliminate these errors, ideally the experiment should be done in an isolated remote place with minimal environmental noise, and the hanging masses in the box being in a vacuum.

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6 References

[1] I. Newton, "Principia book III The system of the world Proposition 10, Theorem 10," 1687; see I. B. Cohen, Isaac Newton: The Principia (Univ. California Press, Berkeley (1999), p. 815.

[2] H. Cavendish, "Experiments to determine the density of the earth," Philos. Trans. London 88, 469–526 (1798). [https://doi.org/10.1098/rstl.1798.0022 (https://doi.org/10.1098/rstl.1798.0022)](https://doi.org/10.1098/rstl.1798.0022), Google ScholarCrossref

[3] Valdes, "Calculating the Gravitational Constant", Intermediate Lab Paper, Department of Physics and Astronomy, Texas Tech University, 2019

[4] Nural Akchurin, Intermediate Physics Lab Manual, Department of Physics and Astronomy, Lubbock, TX, fall ed. (2021).