Simplifying the integrand by a polynomial:

In[1]:= `n1 = Series[Sin[x], {x, 0, 6}]`

Out[1]= $x - \frac{x^3}{6} + \frac{x^5}{120} + O[x]^7$

In[2]:= `n2 = Series[Cos[x], {x, 0, 6}]`

Out[2]= $1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + O[x]^7$

In[10]:= `n3 = Series[Exp[-1/x], {x, 0, 6}]`

Out[10]= $e^{-\frac{1}{x}+O[x]^8}$

In[32]:= `n4 = Series[Exp[-r/Cos[x]], {x, 0, 6}]`

Out[32]= $e^{-r} - \frac{1}{2}\left(e^{-r} r\right) x^2 + \frac{1}{24} e^{-r} r \left(-5 + 3 r\right) x^4 - \frac{1}{720}\left(e^{-r} r \left(61 - 75 r + 15 r^2\right)\right) x^6 + O[x]^7$

Here, r is the optical density

Out[25]= $e^{-r} - \frac{1}{2}\left(e^{-r} r\right) x^2 + \frac{1}{24} e^{-r} r \left(-5 + 3 r\right) x^4 - \frac{1}{720}\left(e^{-r} r \left(61 - 75 r + 15 r^2\right)\right) x^6 + O[x]^7$

In[14]:= `n5 = Series[Sin[x] * Cos[x], {x, 0, 11}]`

Out[14]= $x - \frac{2 x^3}{3} + \frac{2 x^5}{15} - \frac{4 x^7}{315} + \frac{2 x^9}{2835} - \frac{4 x^{11}}{155925} + O[x]^{12}$

In[16]:= `numerator = 1.334 * π * n5`

Out[16]= $4.19088\, x - 2.79392\, x^3 + 0.558785\, x^5 - 0.0532176\, x^7 + 0.00295653\, x^9 - 0.00010751\, x^{11} + O[x]^{12}$

In[17]:= `n6 = Series[(1.334 * Sin[x])^2, {x, 0, 12}]`

Out[17]= $1.77956\, x^2 - 0.593185\, x^4 + 0.0790914\, x^6 - 0.00564938\, x^8 + 0.000251084\, x^{10} - 7.6086 \times 10^{-6}\, x^{12} + O[x]^{13}$

In[18]:= `denominator = (1 - n6)^5`

Out[18]= $1 - 8.89778\, x^2 + 34.6341\, x^4 - 77.8629\, x^6 + 112.861\, x^8 - 112.145\, x^{10} + 79.86\, x^{12} + O[x]^{13}$

In[19]:= `frac = numerator / denominator`

Out[19]= $4.19088\, x + 34.4956\, x^3 + 162.346\, x^5 + 576.052\, x^7 + 1715.83\, x^9 + 4533.5\, x^{11} + O[x]^{12}$

In[33]:= `eqn = frac * n4`

Out[33]= $4.19088\, e^{-r} x + \left(34.4956\, e^{-r} - 2.09544\, e^{-r} r\right) x^3 + \left(162.346\, e^{-r} - 17.2478\, e^{-r} r + 0.17462\, e^{-r} r \left(-5 + 3 r\right)\right) x^5 + \left(576.052\, e^{-r} - 81.1729\, e^{-r} r + 1.43732\, e^{-r} r \left(-5 + 3 r\right) - 0.00582067\, e^{-r} r \left(61 - 75 r + 15 r^2\right)\right) x^7 + O[x]^8$

Eqn is our simplified polynomial approximation to our integral

Analyzing the accuracy of the polynomial approximation and the actual integral:

We are analysing the situation at the an arbitrary optical density (r value) of 1.

In[35]:=
```
approx = 1.5417402846210897` x + 11.919368870971237` x^3 +
  53.2501009183317` x^5 + 180.99634734711293` x^7 + O[x]^8
```

Out[35]= $1.54174\,x + 11.9194\,x^3 + 53.2501\,x^5 + 180.996\,x^7 + O[x]^8$

In[36]:=
```
actual = (1.334 * Pi * Sin[x] * Cos[x] * Exp[-1 / Cos[x]]) / (1 - (1.334 * Sin[x])^2)^(1/2)
```

Out[36]= $$\frac{4.19088\, e^{-\mathrm{Sec}[x]}\, \mathrm{Cos}[x]\, \mathrm{Sin}[x]}{\sqrt{1 - 1.77956\, \mathrm{Sin}[x]^2}}$$

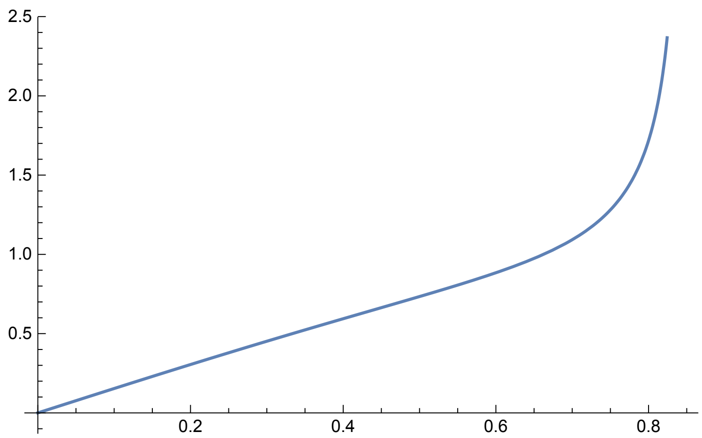
In[43]:=
```
ratio = actual / approx
```

Out[43]= $1. - 8.008\,x^2 + 27.608\,x^4 - 53.9649\,x^6 + O[x]^7$

Plan is to see if the actual and approx functions are well aligned or not. For some reason mathematica doesn’t let me plot approx

In[50]:=
```
p1 = Plot[actual, {x, 0, ArcSin[1 / 1.334]}]
```

Out[50]=



In[49]:=

In[42]:=
```
p1 = Plot[approx, {x, 0, ArcSin[1/1.334]}]
```

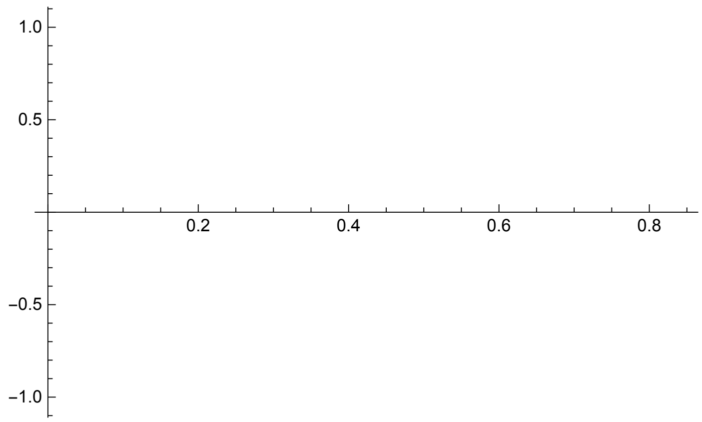
SeriesData: Attempt to evaluate a series at the number 0.000017313124349077037`. Returning Indeterminate.

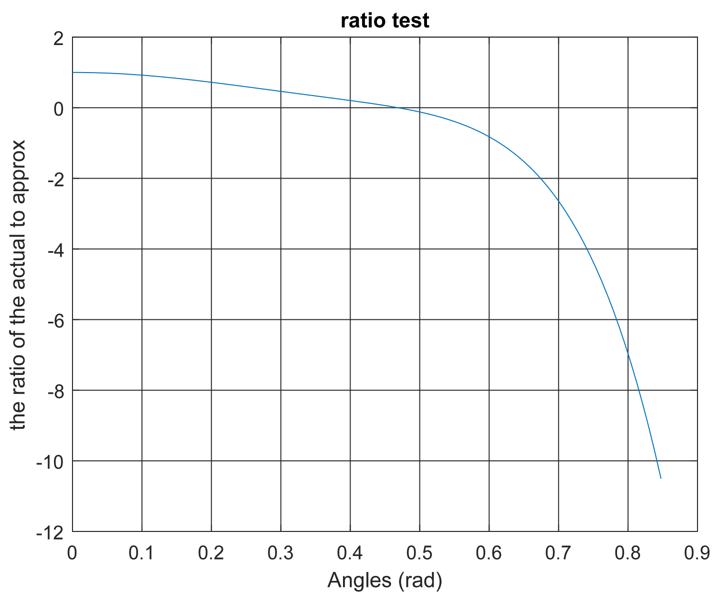
SeriesData: Attempt to evaluate a series at the number 0.017313141644905557`. Returning Indeterminate.

SeriesData: Attempt to evaluate a series at the number 0.034608970165462036`. Returning Indeterminate.

General: Further output of SeriesData::ssdn will be suppressed during this calculation.

Out[42]=





}

However, MATLAB plotted the ratio function. From the plot we can see that our approximation is not very good as it only valid till 0.2 rads.

Further plan is to use higher order terms and see if there is better approximation polynomial available.

Another plan is to use analytical methods to simplify our integral. (Assuming there is any)