On the Focusing of Sunlight by Ocean Waves

HILBERT SCHENCK, JR.
Clarkson College of Technology, Potsdam, New York

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A common optical phenomenon observed in shallow water is the presence of bright lines or bands of light moving across the sea bottom at surface-wave velocity. It is shown that these regions of increased intensity are mainly produced by the refraction of sunlight at the wavy surface. The depth at which intensity peaks reach a maximum is shown to be a function of the wave shape, trochoidal or sinusoidal, and the wave dimensions. This depth increases as the wave becomes longer and flatter. Intensity peaks of up to six times the average intensity level are predicted to be theoretically possible.

INTRODUCTION

ONE of the most unusual optical effects observable in clear, shallow water are periodic bright bands of light that sweep across the bottom, sharply illuminating a swath of sand or coral as they move. In some cases, these moving light bands are so sharply defined that their edges are clearly seen by a diver. That part of the bottom momentarily illuminated is so bright that photographs correctly exposed for average bottom illumination are over-exposed in the lighted region. It is soon apparent from observation that these moving bands of illumination are produced by surface waves, and that their brightest portion lies generally beneath a wave's crest.

In a confused sea, such as that produced by the meeting of wind waves and tidal currents, the bands degenerate into bright, irregularly-shaped patches, each patch having a complex pattern of bright lines and spots. It soon became apparent that changes in the optical path due to wave-produced depth variations could not possibly account for the wide variations in intensity. A more acceptable explanation of the intensity variation can be set forth if we consider that collimated light is incident on a wavy surface, whose curvature varies from point to point. Thus the light passing through different parts of a wave is refracted in different directions at the air-water interface. Rays beneath the surface may thus be spread out under some areas of the wave and bunched under others, producing intensity variations on a plane surface (the bottom) parallel to the surface of the ocean. It will be shown in the following analysis that this focusing effect may be sufficient to produce large intensity variations along a sea bottom.

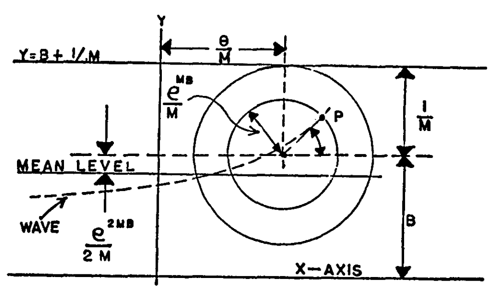

FIG. 1(a). The trochoidal curve is generated by a point on a circle concentric with a larger circle that rolls on the line having the Y coordinate shown. Point P is the generating point. The "mean level" line is the profile the wave would assume if allowed to assume a quiescent state.

NOON SUNLIGHT FOCUSING BY A TROCHOIDAL WAVE

In 1802, Professor Gerstner of Germany showed that a trochoidal-shaped wave would exactly satisfy the wave equations of classical hydrodynamics with proper boundary conditions imposed.[1] Later, Stokes pointed out that Gerstner's theory was not compatible with irrotational motion, as generally assumed in classical hydrodynamics and photogrammetric pictures of ocean waves showed large deviations from the trochoidal form.[2] Since no other theoretical form is known, let us consider the effect of a trochoidal shaped interface on incident sunlight.

Figure 1 shows the nomenclature of the trochoidal curve and the passage of a typical ray through the curved interface and downward to a horizontal bottom.

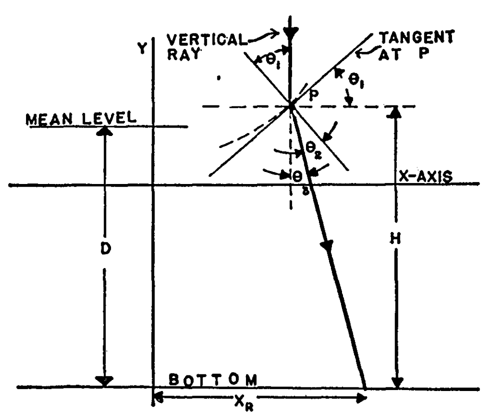

FIG. 1(b). A vertical ray, incident on the trochoidal surface at point P, is refracted to follow the heavy line as shown. The depth of the bottom plane is taken as the distance from the bottom to the "mean level" line (D).

[1] H. Lamb, *Hydrodynamics* (Cambridge University Press, New York), 1932.

[2] Sverdrup, Johnson, and Fleming, *The Oceans* (Prentice-Hall, Inc., New York), 1942.

Perpendicular, collimated light is incident on the trochoidal interface separating two media having indices of refraction η_1 and η_2. The intensity of this light over a flat, horizontal plane is uniform, as would be the case with bright sunlight at noon over a tropic area.

With the x and y axes as shown in Fig. 1, the equations for the trochoid are

$$x=\frac{\theta}{M}+\frac{e^{MB}}{M}\cos\theta \tag{1}$$

and

$$y=B+\frac{e^{MB}}{M}\sin\theta. \tag{2}$$

Examining Eqs. (1) and (2), it is apparent that a full trochoidal wave will have a length of $2\pi/M$ and a height from crest to hollow of $2e^{MB}/M$. Thus, the ratio of wavelength to wave height (K), is given by

$$K=\frac{\pi}{e^{MB}}. \tag{3}$$

In order to establish the direction of any light ray after its passage through the interface, we must find the slope of the wave at every point. Differentiating Eqs. (1) and (2) and combining the results gives

$$\frac{dY}{dX}=\frac{e^{MB}\cos\theta}{(1-e^{MB}\sin\theta)}, \tag{4}$$

the slope of the interface as a function of the parametric angle, θ. Examining Fig. 1(b), we see that the angle of incidence, θ_1, of a vertical ray is equal to the slope of the wave and thus given by

$$\theta_1=\tan^{-1}\frac{dY}{dX}. \tag{5}$$

The angle of refraction, θ_2, is then given by

$$\theta_2=\sin^{-1}\left(\frac{\eta_1}{\eta_2}\sin\theta_1\right). \tag{6}$$

Then the angle this downward ray makes with a vertical line passing through point P, the point of entry, θ_3, is found from

$$\theta_3=\theta_1-\theta_2. \tag{7}$$

Let H be the vertical distance from the point P to the horizontal plane underwater at which the ray terminates, and D be the mean depth to the underwater plane from the water surface if it were undisturbed. The location of a mean (quiescent) surface produced by fluid in a trochoidal form in our coordinate system is given by[1]

$$Y_{\text{mean}}=B-\frac{e^{2MB}}{2M}. \tag{8}$$

Then the vertical distance from P to the bottom plane (H) can be given in terms of the mean depth, D, which we may hold constant in a case involving a level bottom.

$$H=D+Y_{\text{at }P}-\left(B-\frac{e^{2MB}}{2M}\right). \tag{9}$$

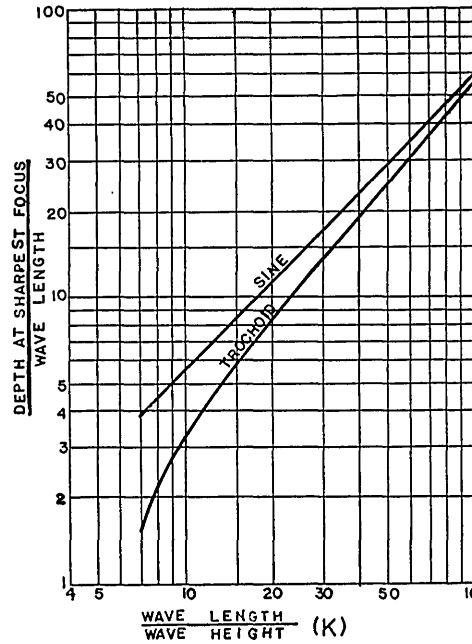

FIG. 3. The depth at which maximum intensity occurs is shown to be a function of the wave steepness, if the wavelength remains constant. The trochoidal wave produces sharpest focus closer to the surface than the sinusoidal wave.

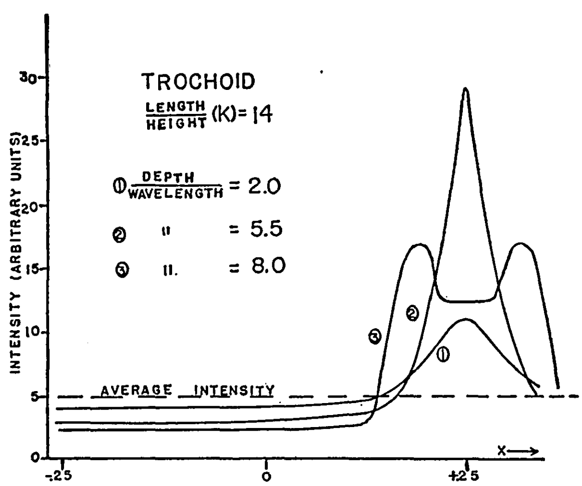

FIG. 2. A steep trochoidal wave having a length-to-height ratio of 14 is shown to produce a peak of maximum intensity at a depth equal to 5.5 times the wavelength, which is taken as 1. Notice that the wave crest lies at an X value of $+0.25$, and the wave trough lies at -0.25, so that approximately one-half wavelength is covered in this plot.

We wish to locate the position of any ray on the horizontal bottom plane. From Fig. 1(b) it is apparent that

$$X_R = X_{\text{at }P} + (H \tan\theta_3), \tag{10}$$

where X_R is the terminating point of the ray striking the surface at P.

It is now possible to solve this system of equations for a series of equally-spaced vertical rays incident on the wavy surface. To do this, let us elect several values of K, the ratio of length-to-height. Further, let us assign a value of one to the length of a full wave, $2\pi/M$. Then M will equal 2π, and knowing K we can arrive at a value of B for the trochoidal equations. Further we will assume that the bottom is parallel to the undisturbed surface (D is constant).

Since the wave in this coordinate system is symmetric about the point $X=+0.25$, let us consider a system of forty rays striking the half-wave from $X=+0.25$ to $X=-0.25$. D can be expressed as multiples of a full wavelength, and the foregoing series of equations can be applied to each ray. The bottom plane can be divided into a series of equal increments and the number of rays terminating in each increment will give an approximate measure of the light intensity at the center of the increment. With forty rays distributed over half the wave, it was found that taking eight increments over this interval would yield good intensity curves. The limiting condition of a flat sea would then result in each increment having an "intensity" of five rays, and the results of the calculations can be compared to this base level.

Considering Eqs. (1) through (10) it is apparent that there are three variables in the problem, the length-to-height ratio (K), the wavelength (taken as unity throughout this paper), and the mean depth to the bottom plane (D). Five values of K were taken for the trochoidal case, 7, 14, 28, 56, and 112. Multiples of seven were arbitrarily chosen because current thinking on wave structure[2] suggests that a K value of 7 is the minimum that can be supported at sea without cresting. It is doubtful, however, whether this steepest possible wave is a true trochoid.

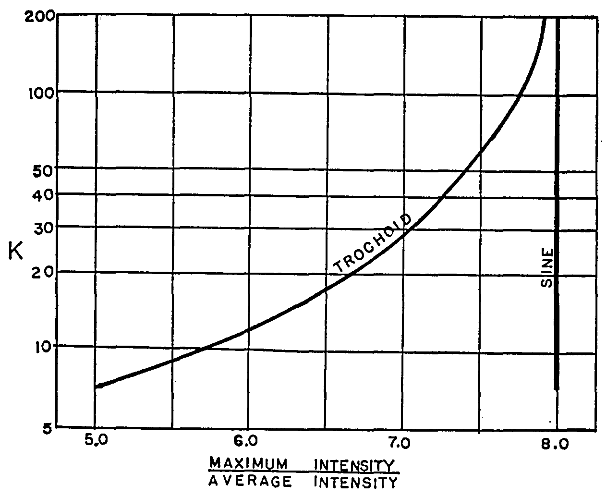

FIG. 4. The ratio of maximum intensity at the critical depth to average intensity with no wave present is a function of the steepness of the wave in the case of a trochoid, but not in the case of a sine wave.

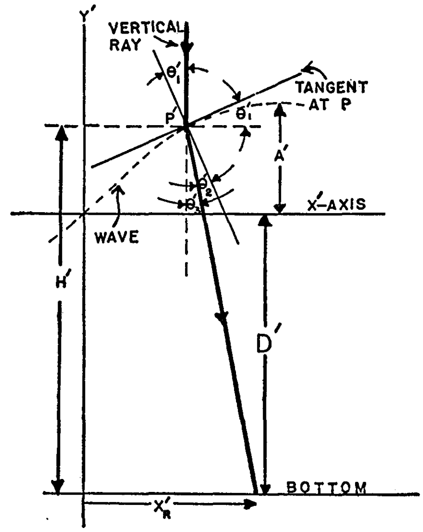

FIG. 5. The nomenclature of a sinusoidal wave passing through the origin and its effect on a vertical ray of light is demonstrated. Notice that A is the maximum rise of the wave above the x axis and that the mean level of a sine wave is the x axis itself.

For each K value the intensity pattern of a full system of rays was calculated for several values of D, in an attempt to locate that depth that would give the sharpest focusing effect. Figure 2 shows a typical series of curves for a K value of 14, with D as parameter. For this K it appeared that a D value of 5.5 times the wavelength (value 1) produced the maximum light intensity beneath the wave crest. Notice that even at this critical depth the intensity level beneath most of the wave is not much lower than the mean value for a calm surface, but that the intensity beneath the crest is sharply peaked to almost six times the calm-sea value (5.0). Continuing the calculations in this manner resulted in Fig. 3, which relates the depth at which sharpest focusing occurs to the value of K for the wave in question. Figure 4 shows the ratio of the number of rays at the maximum intensity point to the number of rays for a calm sea as a function of the length-to-height ratio (K). Figures 3 and 4 demonstrate that short, steep waves focus light relatively nearer the surface, but that the focus is slightly more diffuse.

FOCUSING BY A SINUSOIDAL WAVE

For comparative purposes, we may consider the focusing produced by a pure sinusoidal wave. Following the same nomenclature and a similar diagram (Fig. 5), but using the prime (′) symbol to identify variables in the sine case, we can set up a similar but more easily handled set of equations.

The position of any point (P') on the wave's surface can be found from

$$Y' = A \sin\frac{\pi X'}{L'}, \qquad (11)$$

and the ratio of wavelength to wave height (K') will be

$$K' = \frac{L'}{2A}. \qquad (12)$$

Differentiating Eq. (11) gives

$$\frac{dY'}{dX'} = \frac{\pi A}{L'} \cos\frac{\pi X'}{L'}, \qquad (13)$$

and Eqs. (5), (6), and (7) can now be used (with angles now bearing the prime symbol) to find the angle a ray makes with a vertical line passing through its entry point. The median level of the water in a sinusoidal shape is simply the x axis, so that

$$H' = D' + Y_{\mathrm{at}\ P} \qquad (14)$$

will give the distance from point P' to the horizontal bottom plane, located at a mean depth, D'. Then the displacement of any ray at its terminal point is found from Eq. (10), again using prime symbols.

Solving this new system of equations for K' values of (7), (14), and (28) gives the curves shown on Figs. 3 and 4 for the sine wave. It can be seen that the sine wave will cause maximum focusing at a greater depth than the trochoid and its focus will be sharper. Approaching the K value of 100, the trochoid closely approaches the sine wave as regards focusing behavior, just as its shape approaches the simpler sinusoidal shape as the wave steepness decreases.

CONCLUSIONS

It has been shown that either a trochoidal or sinusoidal wave will focus collimated light from a vertical direction on a level bottom. As the plane beneath the wave on which the light is focused increases in depth, the focus becomes sharper until a large proportion of the energy incident on the wave is concentrated beneath the crest. As the depth of the bottom continues to increase, the focus grows more diffuse and two or more intensity peaks may be found under a single wave. Short, steep waves of either type have their point of maximum focus nearer the surface than shallow waves, and the trochoid shape gives a more diffuse focus than the sine shape.

It should be noted that the water beneath the surface is taken here to be perfectly transparent to light, a condition that is only approached in offshore, tropic water. In northern and inshore water, light beneath the surface suffers large decreases in intensity as it travels downward. If the loss is primarily due to absorption, then a focusing effect will still be observed since the difference in path distance of the various rays through the absorbing media is not great enough to mask the focusing due to refraction. Most turbid water, however, has many suspended scattering centers, i.e., sand, mud, plankton. In this water, the focusing effect is rapidly eliminated with depth, since that portion of the water most sharply illuminated will also show maximum scattering. The redirection of light from the highly illuminated water volume into less well-lighted regions soon destroys the focusing effect.

As an example, it is difficult to see any light patterns due to wave passage in five to ten foot depths around the turbid waters of Cape Cod, while in the clear waters of the Florida Keys, wave-produced intensity variations are readily detected in depths greater than forty feet.

Figure 3 shows that long, shallow waves will produce a theoretical sharp focus at very great depths. For example, a two-foot wave with a length of 200 ft will focus light at a depth of 55 times its length, or 11 000 ft below the surface. Since even the clearest water will not allow light passage to such extreme depths, it can be seen that only short, steep waves are of practical interest in the focusing problem. Such waves are most often found inshore on shelving beaches and over reefs and shoals.

DISCUSSION

There are a number of areas of scientific interest that might profit from a further study of wave-produced intensity variations underwater. A few possibilities will be noted here and the reader can no doubt add others.

Marine Biology

A suggestion has been made[3] that migratory fish may gain navigational help from the polarization patterns underwater. If this is so, navigational information might also be used by fish in the form of light patterns produced by prevailing surface waves. The velocity effects of surface waves are almost completely damped at a depth equal to half a wavelength.[2] Thus the visual cues of intensity changes are a deeper-reaching phenomena in clear water and might actually serve to locate special areas or point the direction of prevailing winds. In clear lakes and streams the observant fisherman can see intricate light patterns rippling along the bottom, created by surface disturbances. The familiar ring of ripples produced by a stone or lure thrown into the water is echoed below the surface by ever widening rings of abnormal light intensity. It would seem reasonable that such cues would serve to attract or repel fish to the focal point of the disturbance, since they remain long after the sound of the splash has faded.

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[3] T. H. Waterman, Science **120**, 927–932 (1954).

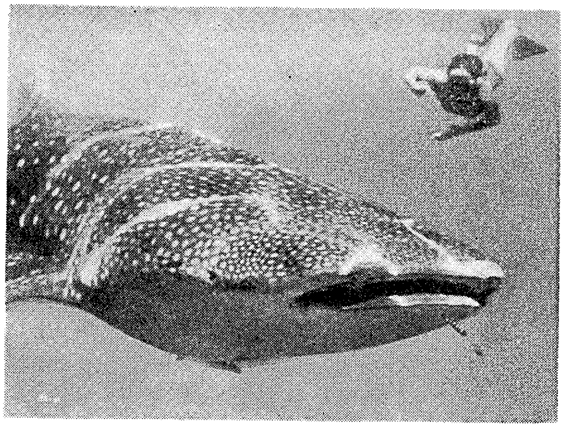

FIG. 6. This underwater photograph made in the Red Sea shows a huge whale shark at an unknown depth. Notice the brilliant light bands lacing the sea creature's back which are undoubtedly produced by small surface waves immediately overhead.

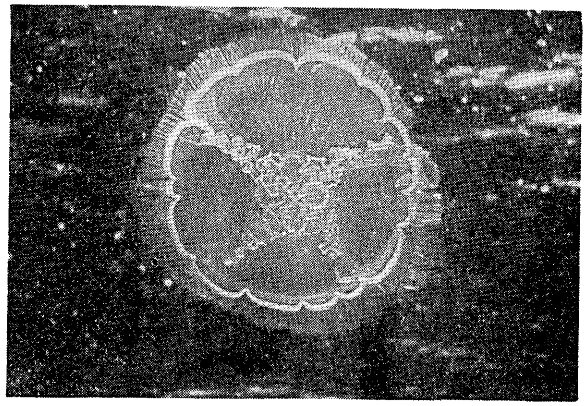

FIG. 7. This underwater photograph of a large moon jelly was taken with the camera pointed upward at about a forty-five degree angle. Trains of waves on the surface are clearly shown by light refracted at their sloping sides toward the camera.

Physical Oceanography

The periods and lengths of ocean waves are usually studied by pressure-sensing devices located in shallow depths.[4] Some actual wave shapes have been obtained by stereo photographs taken from ships.[2] By locating a series of photocells in clear water at various depths, it would be possible to obtain not only wave periods, but wave shapes as well, applying mathematical methods similar to those presented in this paper. Unfortunately the calculations are so tedious that fast computing equipment would probably be required for reduction of the photocell data.

Fluid Mechanics

The fact that open water waves will produce a sharp focus at the proper depth suggests a rather interesting method of obtaining information on the wave patterns around a towed, hydrodynamic model. A large sheet of photographic paper located at the correct depth in the towing tank would be subjected to a short burst of collimated light as the model passed overhead. When developed the paper would display dark lines that would correspond to the wave crests around the model. Such an experiment would have to be done with care, however, since double intensity peaks can result from a single wave if the water is too deep (see Fig. 2). The commonly used hydraulic analog of supersonic flow can be made more effective by proper lighting and selection of water depth. Operating such equipment with a translucent bottom would result in bright lines indicating the wave patterns in the analog.

[4] G. E. R. Deacon, *Science News 4* (Penguin Books, Harmondsworth, 1947), pp. 77–103.

Underwater Photography

Figures 6 and 7 show two underwater photographs which illustrate two aspects of the effect of waves on surface light. Figure 6, taken by Dr. Hans Hass in the Red Sea, shows the characteristic bright lines caused by surface waves as displayed on the back of a giant whale shark. Although this picture has been retouched, it can easily be seen that these bright lines are overexposed and actually mask the fish's markings where they occur. Figure 7 shows a moon jelly or *Aurelia aurita* in Florida Key water, taken from below. Notice the bright patterns on the surface indicating regions where the slope of the water's surface is such that skylight breaks through. It is apparent from this photograph that in the confused seas found over shallow reefs, more than one wave can contribute to a single line, so that extreme intensity variations might be possible if conditions were exactly right. Focused light may be of assistance to the photographer if he wishes to illuminate some dark cranny or add contrast to the generally flat lighting found underwater. Underwater conditions can be judged by applying observed wavelengths and heights to Fig. 3.