



IES MASTER

Institute for Engineers (IES/GATE/PSUs)

**GATE
2022**

**COMPUTER
SCIENCE**

Detailed Solution

EXAM DATE: 05-02-2022

FORENOON SESSION (09:00 AM-12:00 PM)

Office Address

DELHI: F-126, Katwaria Sarai, New Delhi - 110 016

Ph: 011-41013406, Mobile: 8130909220, 9711853908

Web: www.iesmaster.org

E-mail: info@iesmaster.org

APTITUDE

1. Let r be a root of the equation $x^2 + 2x + 6 = 0$. Then the value of the expression $(r + 2)(r + 3)(r + 4)(r + 5)$ is
- (a) 126 (b) -51
(c) 51 (d) -126

Sol: (d)

r be the root of the equation $x^2 + 2x + 6 = 0$ so it will satisfy

$$r^2 + 2r + 6 = 0 \quad \dots(i)$$

Now, $(r + 2)(r + 3)(r + 4)(r + 5)$

$$\Rightarrow (r^2 + 5r + 6)(r^2 + 9r + 20)$$

$$\Rightarrow (r^2 + 2r + 6 + 3r)(r^2 + 2r + 6 + 7r + 14)$$

$$\Rightarrow (0 + 3r)(0 + 7r + 14)$$

$$\Rightarrow 3r(7r + 14)$$

$$\Rightarrow 21(r^2 + 2r) = 21(-6)$$

$$\Rightarrow -126$$

Option (d) is correct.

2. Some people believe that "what gets measured, improves". Some others believe that "what gets measured, gets gamed". One possible reason for the difference in the beliefs is the work culture in organizations. In organizations with good work culture, metrics help improve outcomes. However, the same metrics are counterproductive in organizations with poor work culture. Which one of the following is the CORRECT logical inference based on the information in the above passage?
- (a) Metrics are always counterproductive in organizations with good work culture
(b) Metrics are useful in organizations with good work culture.
(c) Metrics are useful in organizations with poor work culture.
(d) Metrics are never useful in organizations with good work culture.

Sol: (b)













Metrics are useful in organizations with good work culture.

3. A palindrome is a word that reads the same forwards and backwards. In a game of words, a

player has the following two plates painted with letters.



From the additional plates given in the options, which one of the combinations of additional plates would allow the player to construct a five-letter palindrome. The player should use all the five plates exactly once. The plates can be rotated in their plane.

- (a)   
(b)   
(c)   
(d)   

Sol: (c)




A word, sentence or a number that reads the same backward or forward.

Ex. RADAR

MADAM

REFER

12321

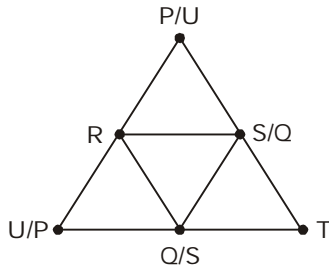
So, option (c)    (after rotating 2nd and 3rd plates) is a palindrome.

Option (c) is correct.

4. The corners and mid-points of the sides of a triangle are named using the distinct letters, P, Q, R, S, T and U, but not necessarily in the same order. Consider the following statements :
- The line joining P and R is parallel to the line joining Q and S.
 - P is placed on the side opposite to the corner T.
 - S and U cannot be placed on the same side.
- Which one of the following statements is correct based on the above information?
- (a) P cannot be placed at a corner
(b) R cannot be placed at a corner
(c) S cannot be placed at a corner
(d) U cannot be placed at a mid-point

Sol: (c)

Using above information we can draw,



S can't be place at a corners because PR is parallel with QS.

Option (c) is correct.

5. A box contains five balls of same size and shape. Three of them are green coloured balls and two of them are orange coloured balls. Balls are drawn from the box one at a time. If a green ball is drawn, it is not replaced. If an orange ball is drawn, it is replaced with another orange ball.

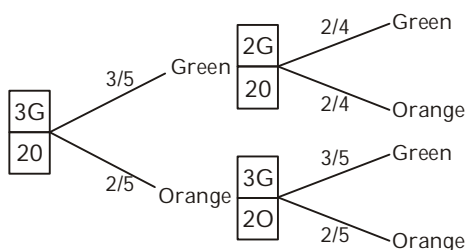
First ball is drawn. What is the probability of getting an orange ball in the next draw ?

- (a) $\frac{23}{50}$ (b) $\frac{1}{2}$
(c) $\frac{19}{50}$ (d) $\frac{8}{25}$

Sol: (a)

G → green

O → Orange



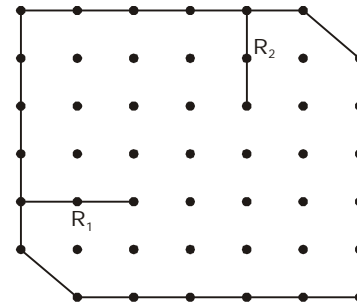
$$\begin{aligned} P(E) &= \frac{3}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{2}{5} \\ &= \frac{3}{10} + \frac{4}{25} \\ &= \frac{23}{50} \end{aligned}$$

Option (a) is correct.

6. A plot of land must be divided between four families. They want their individual plots to be similar in shape, not necessarily equal in area. The land has equally spaced poles, marked as

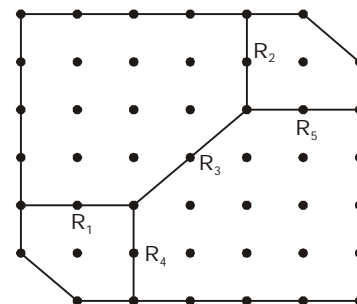
dots in the below figure. Two ropes, R1 and R2, are already present and cannot be moved.

What is the least number of additional straight ropes needed to create the desired plots ? A single rope can pass through three poles that are aligned in a straight line.



- (a) 2 (b) 3
(c) 4 (d) 5

Sol: (b)



R₃ → First additional Rope

R₄ → Second additional Rope

R₅ → Third additional Rope

So, using 3 additional ropes. We are able to divide into 4 similar shape plots.

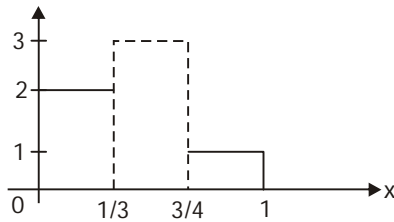
7. A function $y(x)$ is defined in the interval $[0, 1]$ on the x-axis as

$$y(x) = \begin{cases} 2 & \text{if } 0 \leq x < \frac{1}{3} \\ 3 & \text{if } \frac{1}{3} \leq x < \frac{3}{4} \\ 1 & \text{if } \frac{3}{4} \leq x < 1 \end{cases}$$

Which one of the following is the area under the curve for the interval $[0, 1]$ on the x-axis.

- (a) $\frac{13}{6}$ (b) $\frac{6}{5}$
(c) $\frac{5}{6}$ (d) $\frac{6}{13}$

Sol: (a)



$$\begin{aligned} \text{Area} &= 2 \times \frac{1}{3} + 3 \times \left(\frac{3}{4} - \frac{1}{3} \right) + 1 \times \left(1 - \frac{3}{4} \right) \\ &= \frac{2}{3} + 3 \times \frac{5}{12} + 1 \times \frac{1}{4} \\ &= \frac{2}{3} + \frac{15}{12} + \frac{1}{4} = \frac{8+15+3}{12} = \frac{26}{12} = \frac{13}{6} \end{aligned}$$

Another Solution :

$$y(x) = \begin{cases} 2 & \text{if } 0 \leq x < \frac{1}{3} \\ 3 & \text{if } \frac{1}{3} \leq x < \frac{3}{4} \\ 1 & \text{if } \frac{3}{4} \leq x \leq 1 \end{cases}$$

$$\begin{aligned} \text{Area} &= \int_0^1 y(x) dx \\ &\Rightarrow \int_0^{1/3} 2 dx + \int_{1/3}^{3/4} 3 dx + \int_{3/4}^1 1 dx \\ &\Rightarrow 2[x]_0^{1/3} + 3[x]_{1/3}^{3/4} + [x]_{3/4}^1 \\ &\Rightarrow \frac{2}{3} + 3\left(\frac{3}{4} - \frac{1}{3}\right) + \frac{1}{4} \\ &\Rightarrow \frac{8+15+3}{12} = \frac{26}{12} = \frac{13}{6} \end{aligned}$$

8. In a recently conducted national entrance test, boys constituted 65% of those who appeared for the test. Girls constituted the remaining candidates and they accounted for 60% of the qualified candidates.

Which one of the following is the correct logical inference based on the information provided in the above passage ?

- The number of boys who appeared for the test is less than the number of girls who appeared
- The number of boys who qualified the test is less than the number of girls who qualified.
- Equal number of boys and girls appeared for the test

- Equal number of boys and girls qualified.

Sol: (b)

Let total candidates appeared = x

$$\text{Appeared boys} = 65\%x = 0.65x$$

$$\text{Appeared girls} = 35\%x = 0.35x$$

Let total qualified = y

$$\text{qualified boys} = 40\%y = 0.4y$$

$$\text{qualified girls} = 60\%y = 0.6y$$

Option (b) is correct because $0.6y > 0.4y$.

Option (b) is correct.

9. The _____ is too high for it to be considered _____.

- fair/fare
- fare/fair
- fare /fare
- faer /fair

Sol: (b)

The fare is too high for it to be considered fair.

10. Given below are four statements.

Statement 1 : All students are inquisitive

Statement 2 : Some students are inquisitive

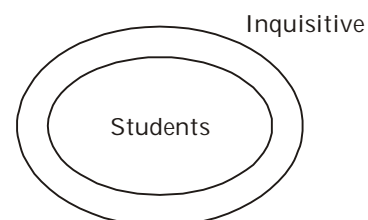
Statement 3 : No student is inquisitive

Statement 4 : Some students are not inquisitive

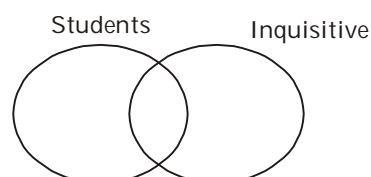
From the given four statements, find the two statements that CANNOT BE TRUE simultaneously, assuming that there is at least one student in the class.

- Statement 1 and Statement 3
- Statement 3 and Statement 4
- Statement 1 and Statement 2
- Statement 2 and Statement 4

Sol: (a)

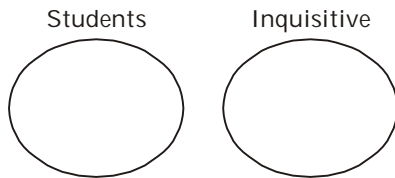


1. All students are inquisitive.



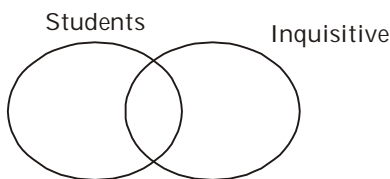
2. Some students are inquisitive.

If all are true then some also true so first and second can be true simultaneously.



3. No student is inquisitive.

4. Some students are not inquisitive.



If some students are inquisitive true then some students are not inquisitive is also true. Second and fourth can be true simultaneously.

So, option (a) is correct.

TECHNICAL

11. Let WB and WT be two sets associate cache organizations that use LRU algorithm for cache block replacement. WB is a write back cache and WT is a write through cache. Which of the following statements is FALSE ?

- (a) A read miss in WB will never lead to eviction of a dirty block from WB.
- (b) Each cache block in WB and WT has a dirty bit.
- (c) Eviction of a block from WT will not lead to data transfer from cache to main memory.
- (d) Every write hit in WB leads to a data transfer from cache to main memory.

Sol: (a, b, d)

In write through policy, all the write operation is made in main memory and cache memory simultaneously, ensure that main memory is valid.

In write back policy, at the time of block replacement when dirty bit is set on the line changes is written back into the memory.

The cache eviction is a strategy in which the data is removed from the cache.

- (i) To make room for more relevant cache entries.

- (ii) To shrink the cache to make available more RAM for other users.

- (a) For read/write misses in write back, a line needed to be evicted for the newly fetched block. Hence, option (a) is false.

- (b) In write back, dirty bit is set for those lines which are updated.

In write through, no dirty bit is required. Hence option (b) is FALSE.

- (c) In write through, no need to do eviction of a block from cache. So there is no data transfer required from cache to main memory. Hence, option (c) is TRUE.

- (d) In write back, data transfer from cache to memory is required at the time of block replacement, i.e. when eviction required. Hence, option (d) is FALSE.

12. In a relational data model, which one of the following statements is TRUE ?

- (a) A relation with only two attributes is always in BCNF.
- (b) BCNF decomposition preserve functional dependencies.
- (c) Every relation has at least one non-prime attribute.
- (d) If all attributes of a relation are prime attributes, then the relation is in BCNF.

Sol: (a)

At last one of the following holds in BCNF.

- (i) $\alpha \rightarrow \beta$ is a trivial functional dependency i.e. $\beta \subseteq \alpha$.

- (ii) α is a superkey.

Thus, a relation with only two attribute must be in BCNF.

BCNF decomposition doesn't preserve functional dependencies.

It is not mandatory that every relation has at least one non-prime attribute.

If all attributes of relation are prime attribute, then the relation is always in 3NF.

13. Consider the following languages :

$$L_1 = \{ww \mid w \in \{a,b\}^*\}$$

$$L_2 = \{a^n b^n c^m \mid m, n \geq 0\}$$

$$L_3 = \{a^m b^n c^n \mid m, n \geq 0\}$$

Which of the following statements is/are FALSE ?

- (a) Neither L_1 nor L_2 is context-free.
- (b) L_2 , L_3 and $L_2 \cap L_3$ all are context-free
- (c) Neither L_1 nor L_2 is complement is context-free
- (d) L_1 is not context-free but L_2 and L_3 are deterministic context-free.

Sol: (a, b, c)

Given languages,

$$L_1 = \{ww \mid w \in \{a, b\}^*\}$$

$$L_2 = \{a^n b^n c^m \mid m, n \geq 0\}$$

$$L_3 = \{a^m b^n c^n \mid m, n \geq 0\}$$

Language L_1 is not accepted by PDA, because we can't figure out middle element of string. Hence it is not context free language.

Language L_2 is accepted by PDA, because each element 'a' is pushed in the stack and for each element 'b' pop operation is performed, and finally any number of input symbol 'c' is possible. Hence language L_2 is context free language.

Language L_3 is accepted by PDA, because after any number of input element 'a', for element 'b' push operation is performed and for element 'c' pop operation is performed & stack becomes empty.

Hence, language L_3 is context free language.

Thus, L_1 - Not context free

L_2 - Context free.

Option (a) is FALSE.

L_2 - Context free

L_3 - Context free

$$L_2 \cap L_3 = \{a^n b^n c^n \text{ or } a^m b^m c^m \mid m, n \geq 0\}$$

This language is context sensitive language.

Option (b) is FALSE.

L_1 - Not context free

\bar{L}_1 - Context free

Option (c) is FALSE.

L_1 - Not context free

L_2 - Deterministic context free

L_3 - Deterministic context free

Option (d) is TRUE.

14. The value of the following limit is ____.

$$\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{1 - e^{2\sqrt{x}}}$$

Sol: (-0.5)

$$\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{1 - e^{2\sqrt{x}}}$$

$$\Rightarrow \text{put } 0 \text{ in equation } \Rightarrow \frac{0}{1-1} = \frac{0}{0}$$

\Rightarrow Apply L' hospital rule, we get (differentiate numerator and denominator)

$$\Rightarrow \lim_{x \rightarrow 0^+} \frac{\frac{1}{2\sqrt{x}}}{0 - e^{2\sqrt{x}} \cdot \frac{2}{2\sqrt{x}}}$$

$$\Rightarrow \lim_{x \rightarrow 0^+} \frac{\frac{1}{2} \sqrt{x}}{-2e^{2\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}}$$

$$\Rightarrow \lim_{x \rightarrow 0^+} \frac{1}{-2e^{2\sqrt{x}}}$$

$$\Rightarrow -\frac{1}{2} = -0.5$$

15. Which one of the following is the closed form for the generating function of the sequence $\{a_n\}_{n \geq 0}$ defined below ?

$$a_n = \begin{cases} n+1, & n \text{ is odd} \\ 1, & \text{otherwise} \end{cases}$$

$$(a) \frac{x(1+x^2)}{(1-x^2)^2} + \frac{1}{1-x} \quad (b) \frac{x(3-x^2)}{(1-x^2)^2} + \frac{1}{1-x}$$

$$(c) \frac{2x}{(1-x^2)^2} + \frac{1}{1-x} \quad (d) \frac{x}{(1-x^2)^2} + \frac{1}{1-x}$$

Sol: (a)

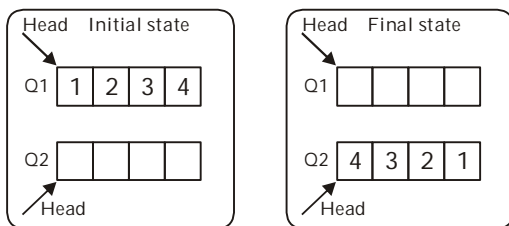
$$a_n = \begin{cases} n+1 & \text{if } n \text{ is odd} \\ 1 & \text{otherwise} \end{cases}$$

$$\frac{(n-1)(n-K+1)}{2} = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \dots$$

$$= 1 + 2x + x^2 + 4x^3 + x^4 + 6x^5 + \dots$$

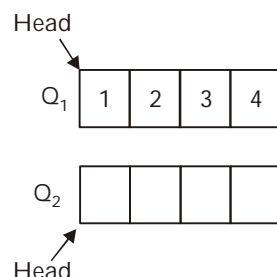
$$\begin{aligned}
 &= (1 + x + x^2 + x^3 + \dots) + x(1 + 3x^2 + 5x^4 + \dots) \\
 &= \frac{1}{(1-x)} + x \frac{d}{dx} (x + x^3 + x^5 + \dots) \\
 &= \frac{1}{(1-x)} + x \frac{d}{dx} \left[\frac{x}{(1-x^2)} \right] \\
 &= \frac{1}{(1-x)} + \frac{x(1+x^2)}{(1-x^2)^2}
 \end{aligned}$$

16. Consider the queues Q_1 containing four elements and Q_2 containing none (shown as the initial state in the figure). The only operations allowed on these two queues are Enqueue (Q , element) and Dequeue (Q). The minimum number of Enqueue operations on Q_1 required to place the elements of Q_1 in Q_2 in reverse order (shown as the final state in the figure) without using any additional storage is _____.

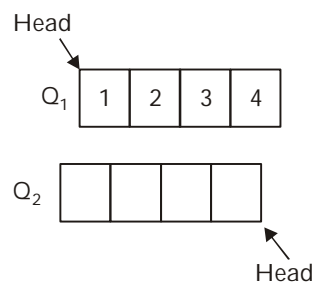


Sol: (0)

Given queues Q_1 and Q_2

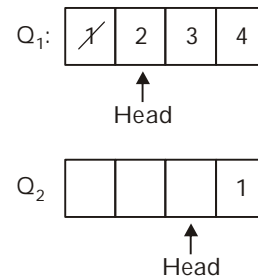


Traverse head of Q_2 at the end of queue so Queue format becomes,

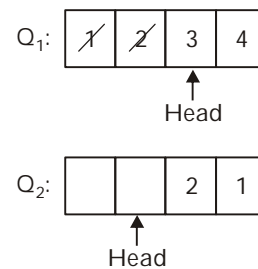


Now perform dequeue operation in Q_1 at head position and perform enqueue operation in Q_2 at head position.

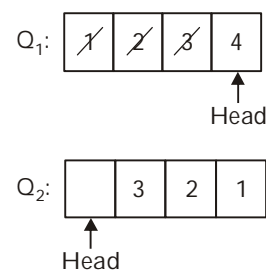
Step (1)



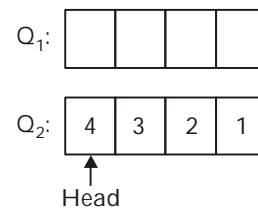
Step (2)



Step (3)

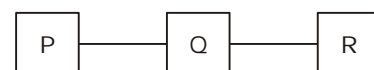


Step (4)



Thus, there are zero enqueue operation performed on queue Q_1 .

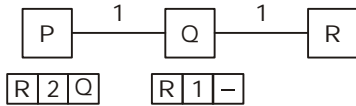
17. Consider a network with three routers P, Q, R shown in the figure below. All the links have cost of unity.



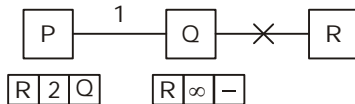
The routers exchange distance vector routing information and have converged on the routing tables, after which the link Q-R fails. Assume that P and Q send out routing updates at random times, each at the same average rate. The probability of a routing loop formation

(rounded off to one decimal place) between P and Q, leading to count-to-infinity problem, is _____.

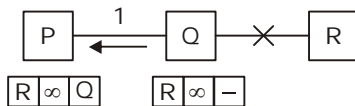
Sol: (0.5)



When link between Q and R is broken.

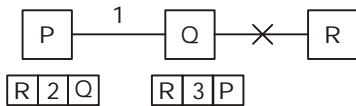
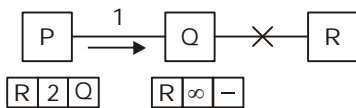


When Node P receives triggered update first from node Q, then P successfully updated.



In this case no issue observed and count-infinity problem doesn't arise.

If node Q receives update first from node P.



So, routing table of Q is updated and it causes count-infinity problem.

So, out of two node, one node is responsible for count-to-infinity issue.

So, the probability of a routing loop formation between P and Q is 0.5.

- 18.** Consider the following threads, T_1 , T_2 and T_3 executing on a single processor, synchronized using three binary semaphore variables, S_1 , S_2 and S_3 , operated upon using standard wait () and signal (). The threads can be context switched in any order and at any time.

T_1	T_2	T_3
while (true) { wait (S_3); print ("C"); signal (S_2); }	while (true) { wait (S_1); print ("B"); signal (S_3); }	while (true) { wait (S_2); print ("A"); signal (S_1); }

Which initialization of the semaphores would print the sequence BCABCA BCA ... ?

- (a) $S_1 = 1$; $S_2 = 0$; $S_3 = 0$
 (b) $S_1 = 1$; $S_2 = 1$; $S_3 = 1$
 (c) $S_1 = 1$; $S_2 = 1$; $S_3 = 0$
 (d) $S_1 = 0$; $S_2 = 1$; $S_3 = 1$

Sol: (a)

Given threads are T_1 , T_2 and T_3 and three binary semaphore variable is used for synchronization S_1 , S_2 and S_3 .

T_1	T_2	T_3
while(true){ wait(S_3); pr int("c"); signal(S_2);	while(true){ wait(S_1); pr int("B"); signal(S_3);	while(true){ wait(S_2); pr int("A"); signal(S_1);

Given sequence need to print,

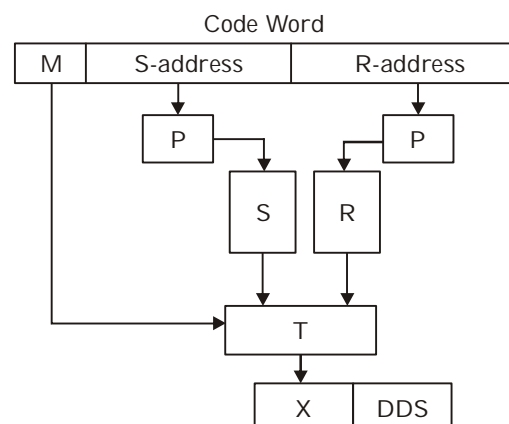
BCABCABCA...

First element in this sequence is 'B'. It means thread T_2 should execute first.

Thus, at this moment.

$S_1 = 1$, $S_2 = 0$, $S_3 = 0$

- 19.** Consider a digital display system (DDS) shown in the figure that displays the contents of register X. A 16-bit code word is used to load a word in X, either from S or from R. S is a 1024-word memory segment and R is a 32-word register file. Based on the value of mode bit M, T selects an input word to load in X. P and Q interface with the corresponding bits in the code word to choose the addressed word. Which one of the following represents the functionality of P, Q, and T ?



- (a) P is 10:1 multiplexer; Q is 5:1 multiplexer; T is 2:1 multiplexer
- (b) P is $10:2^{10}$ decoder; Q is $5:2^5$ decoder; T is 2:1 multiplexer.
- (c) P is $10:2^{10}$ decoder; Q is $5:2^5$ decoder; T is 2:1 encoder
- (d) P is 1:10 de-multiplexer; Q is 1:5 de-multiplexer; T is 2:1 multiplexer

Sol: (b)

Given digital display system, in which 16-bit code word is used.

S is a 1024-word memory segment.

R is a 32-word register file.

So, there are 10-input required for decoder P and output of decoder P is 2^{10} .

Similarly, there are 5-input required for decoder Q and output of decoder Q is 2^5 .

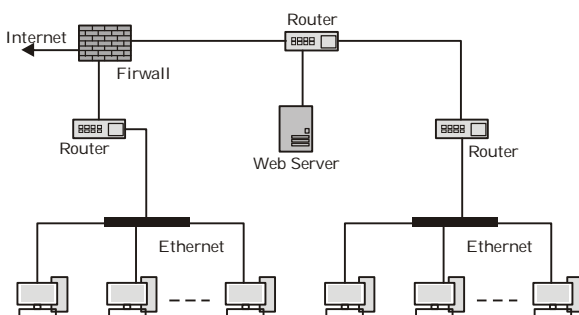
And T is a multiplexer, which takes 2 input and gives 1 output.

$P \equiv 10:2^{10}$ decoder

$R \equiv 5:2^5$ decoder

$T \equiv 2:1$ Multiplexer

20. Consider an enterprise network with two Ethernet segments, a web server and a firewall, connected via three routers as shown below:



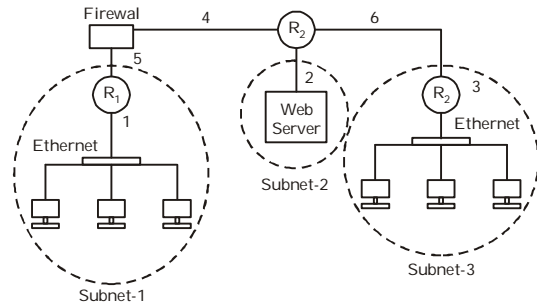
What is the number of subnets inside the enterprise network ?

- (a) 3 (b) 6
(c) 8 (d) 12

Sol: (b)

Given an enterprise network with two ethernet segments, a web server and a firewall connected

via three routers. Each interface of router formed a subnet.



Number of subnet = 6

21. Consider four processes P, Q, R and S scheduled on a CPU as per round robin algorithm with a time quantum of 4 units. The processes arrive in the order P, Q, R, S, all at time $t = 0$. There is exactly one context switch from S to Q, exactly one context switch from R to Q and exactly two context switch from Q to R. There is no context switch from S to P. Switching to a ready process after the termination of another process is also considered a context switch. Which one of the following is NOT possible as CPU burst time (in time units) of these processes?

- (a) $P = 4, Q = 12, R = 5, S = 4$
(b) $P = 2, Q = 9, R = 5, S = 1$
(c) $P = 3, Q = 7, R = 7, S = 3$
(d) $P = 4, Q = 10, R = 6, S = 2$

Sol: (c)

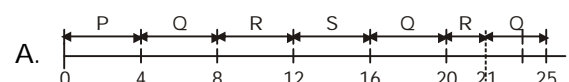
Given four processes P, Q, R and S.

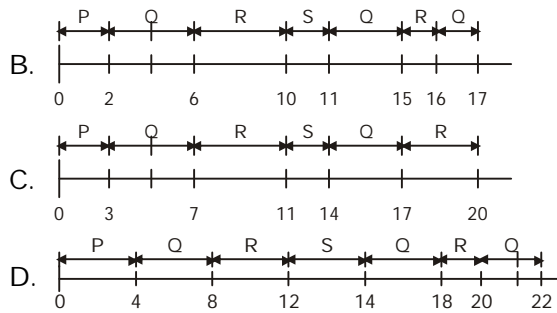
Scheduling algorithm : Round Robin

Time quanta = 4 time unit.

All the processes arrives at time $t = 0$.

- (i) Exactly one context switch from S to Q.
(ii) Exactly one context switch from R to Q.
(iii) Exactly two context switch from Q to R.
(iv) No context switch from S to P.
(v) Switching to a ready process after the termination of another process is also considered a context switch.





In option (c), there is no context switching exist from R to Q.

22. Consider a relation $R(A, B, C, D, E)$ with the following three functional dependencies.

$AB \rightarrow C$; $BC \rightarrow D$; $C \rightarrow E$;

The number of superkeys in the relation R is _____.

Sol: (8)

Given three functional dependencies,

$AB \rightarrow C$

$BC \rightarrow D$

$C \rightarrow E$

Compute closure of attribute,

$$(AB)^+ = \{A, B, C, D, E\}$$

$$(BC)^+ = \{B, C, D, E\}$$

$$C^+ = \{C, E\}$$

Only one candidate key is possible for given relation : $\{A, B\}$

Superkey can be found by adding any combination of attribute $\{C, D, E\}$ with candidate key (AB) .

Number of possible combination with candidate key required = $2^3 = 8$.

23. Which of the following statements is/are TRUE for a group G?

- (a) If G is commutative, then a subgroup of G need not be commutative
- (b) If the order of G is 2, then G is commutative
- (c) If for all $x, y \in G$, $(xy)^2 = x^2y^2$, then G is commutative.
- (d) If for all $x \in G$, $x^2 = 1$, then G is commutative. Here, 1 is the identity element of G.

Sol: (b, c, d)

- (a) If G is commutative then the subgroup is G is also commutative.

Let take I is a subgroup of G, if $e, f \in I$, we have $e, f \in G$, then $ef = fe$. So, I is commutative.

- (b) If the order of 'G' is 2 means it is order of prime, so 'G' is commutative.

$$(c) (xy)^2 = x^2y^2$$

$$\Rightarrow xyxy = xx yy$$

Take x^{-1} and y^{-1} on both side.

$$x^{-1}xyxyy^{-1} = x^{-1}xxyyy^{-1}$$

$yx = xy$, so it is commutative.

- (d) $\forall x \in G, x^2 = 1$ then G is

$$xx = 1 \Rightarrow x = x^{-1}$$

If every element has its own inverse in a graph, then graph is commutative.

24. Consider a simple undirected unweighted graph with at least three vertices. If A is the adjacency matrix of the graph, then the number of 3-cycles in the graph is given by the trace of

- (a) A^3 divided by 6
- (b) A^3
- (c) A^3 divided by 3
- (d) A^3 divided by 2

Sol: (a)

All pair shortest paths for adjacency matrix = A_{ij}^n .

A_{ij}^n = n vertices can walk from i - j.

Since, the cycle has 3 vertices and it is counted for every vertex, we need to divide by 3. To get

the three vertex loop for directed graph = $\frac{A_{ij}^3}{3}$.

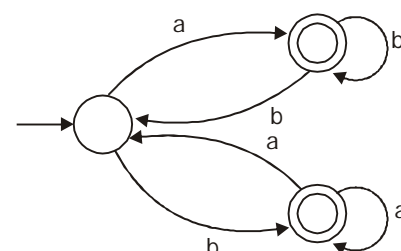
For undirected graph, A-B-C-A is same as A-C-B-A cycle. So, 2 possibility will be formed.

The number of 3-cycle for undirected graph is

$$\frac{A_{ij}^3}{3 \times 2}$$

$$= \frac{A_{ij}^3}{6}$$

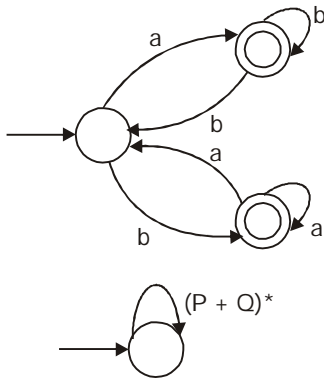
25. Which one of the following regular expressions correctly represents the language of the finite automation given below ?



- (a) $(ab^*b)^*ab^* + (ba^*a)^*ba^*$
 (b) $ab^*bab^* + ba^*aba^*$
 (c) $(ab^*b + ba^*a)^*(a^* + b^*)$
 (d) $(ba^*a + ab^*b)^*(ab^* + ba^*)$

Sol: (d)

Given finite automation,



$$(P + Q)^*(ab^* + ba^*)$$

$$(ab^*b + ba^*a)^*(ab^* + ba^*)$$

26. Consider the augmented grammar with $\{+, *, (,), id\}$ as the set of terminals.

$$S' \rightarrow S$$

$$S \rightarrow S + R \mid R$$

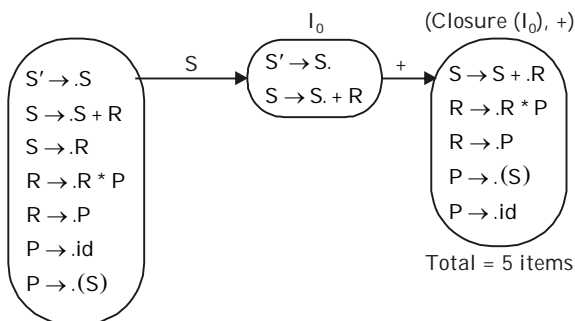
$$R \rightarrow R * P \mid P$$

$$P \rightarrow (S) \mid id$$

If I_0 is the set of two LR (0) items $\{[S' \rightarrow S], [S \rightarrow S, + R]\}$, then goto ($\text{closure}(I_0), +$) contains exactly _____ items.

Sol: (5)

Apply augmented grammar is :



27. A cache memory that has a hit rate of 0.8 has an access latency 10 ns and miss penalty 100 ns. An optimization is done on the cache to reduce the miss rate. However, the optimization

results in an increase of cache access latency to 15 ns, whereas the miss penalty is not affected. The minimum hit rate (rounded off to two decimal places) needed after the optimization such that it should not increase the average memory access time is _____.

Sol: (85%)

Cache memory hit rate = 0.8

Access Latency = 10 nsec.

Miss penalty = 100 nsec.

Optimization increases cache access latency to 15 nsec.

Miss penalty is not affected.

Average memory access time in first case

$$= 0.8 \times 10 + (1 - 0.8) \times 100 = 28 \text{ nsec}$$

Average memory access time in second case

$$= x \times 15 + (1 - x) \times 100 = 28$$

$$15x + 100 - 100x = 28$$

$$85x = 72$$

$$x = \frac{72}{85} = 0.85$$

28. Which of the following statements is/are TRUE with respect to deadlocks ?

- (a) If the current allocation of resources to processes leads the system to unsafe state, then deadlock will necessarily occur.
 (b) Circular wait is a necessary condition for the formation of deadlock.
 (c) In the resource-allocation graph of a system, if every edge is an assignment edge, then the system is not in deadlock state.
 (d) In a system where each resource has more than one instance, a cycle in its wait-for graph indicates the presence of a deadlock.

Sol: (b, c)

- (a) An unsafe state doesn't leads to deadlock.
 (b) Circular wait is a necessary condition for the formation of deadlock.
 (c) If there is no request edge then no deadlock.
 (d) No deadlock occurred when more than one instance of resource exist.

29. Consider the following three relations in a relational database.

Employee(eld, Name), Brand(bld, bName),
Own(eld, bld)

Which of the following relational algebra expressions return the set of elds who own all the brands ?

- (a) $\Pi_{\text{eld}}((\Pi_{\text{eld}}(\text{Own}) \times \Pi_{\text{bld}}(\text{Own})) / \Pi_{\text{bld}}(\text{Brand}))$
- (b) $\Pi_{\text{eld}}(\Pi_{\text{eld bld}}(\text{Own}) / \Pi_{\text{bld}}(\text{Brand}))$
- (c) $\Pi_{\text{eld}}(\text{Own}) - \Pi_{\text{eld}}((\Pi_{\text{eld}}(\text{Own}) \times \Pi_{\text{bld}}(\text{Brand})) - \Pi_{\text{eld bld}}(\text{Own}))$
- (d) $\Pi_{\text{eld}}(\Pi_{\text{eld bld}}(\text{Own}) / \Pi_{\text{bld}}(\text{Own}))$

Sol: (b, c)

Given three relation,

Employee(eld, Name)

Brand(bld, bName)

Own(eld, bld)

Need to find "set of elds who own all the brands".

Required answer,

$$\pi_{\text{eld}}(\pi_{\text{eld, bld}}(\text{own}) / \pi_{\text{bld}}(\text{Brand}))$$

$$\pi_{\text{eld}}(\text{own}) - \pi_{\text{eld}}((\pi_{\text{eld}}(\text{own}) \times \pi_{\text{bld}}(\text{Brand})) - \pi_{\text{eld, bld}}(\text{own}))$$

30. Consider the data transfer using TCP over a 1 Gbps link. Assuming that the maximum segment lifetime (MSL) is set to 60 seconds, the minimum number of bits required for the sequence number field of the TCP header, to prevent the sequence number space from wrapping around during the MSL is _____.

Sol: (33)

$$\text{Bandwidth} = 1 \text{ Gbps} = \frac{2^{30}}{8} \text{ bytes/sec}$$

Maximum segment lifetime (MSL) = 60 seconds

$$\text{Wrap around time} = \frac{\text{Total sequence number}}{\text{Bandwidth}}$$

Wrap around time > Maximum segment lifetime

$$\frac{\text{Total sequence number}}{\text{Bandwidth}} > \text{Maximum segment life time.}$$

Suppose, minimum number of bits required for the sequence number field = x.

$$\frac{2^x}{1 \times \frac{2^{30}}{8}} > 60$$

$$2^x = \frac{60 \times 2^{30}}{8}$$

$$\log 2^x > \log \left(\frac{60 \times 2^{30}}{8} \right)$$

$$x > \log 60 + 30 - \log 8$$

$$\text{Minimum value of } x = 30 + 5.9 - 3 \approx 33$$

31. The number of arrangements of six identical balls in three identical bins is _____.

Sol: (7)

So, balls and bins are identical, the possible ways are :

We have three bins.

B ₁	B ₂	B ₃
6	0	0
5	1	0
4	1	1
4	2	0
3	2	1
3	3	0
2	2	2

Total ways is 7.

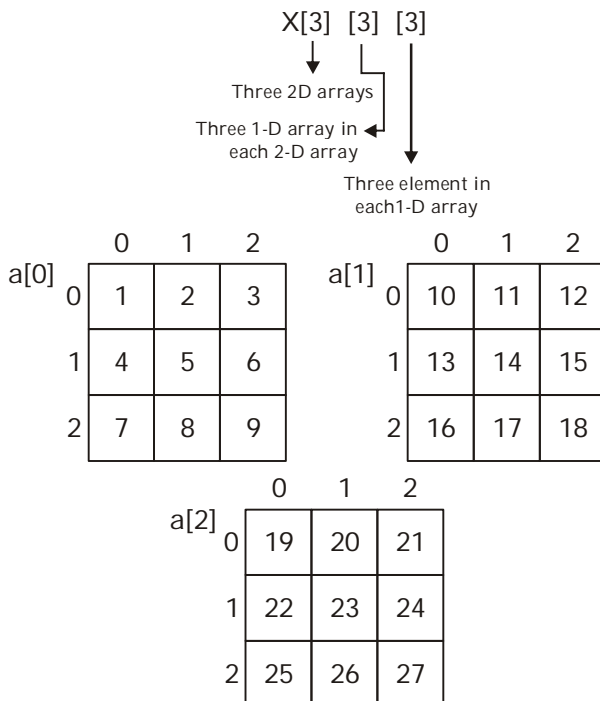
32. What is printed by the following ANSI C program?

```
#include<stdio.h>

int main(int argc, char *argv[ ])
{
    int a[3][3][3] =
    { { 1, 2, 3, 4, 5, 6, 7, 8, 9 },
      {10, 11, 12, 13, 14, 15, 16, 17, 18},
      {19, 20, 21, 22, 23, 24, 25, 26, 27}};
    int i = 0, j = 0, k = 0;
    for (i = 0; i < 3; i++) {
        for (k = 0; k < 3; k++) {
            printf("%d", a[i][j][k]);
            printf("\n");
        }
    }
    return 0;
}
```

- 1 2 3
(a) 10 11 12
19 20 21
1 2 3
(b) 4 5 6
7 8 9
1 4 7
(c) 13 14 15
25 26 27
1 4 7
(d) 10 13 16
19 22 25

Sol: (a)



- a[000] = 1
a[001] = 2
a[002] = 3
a[100] = 10
a[101] = 11
a[102] = 12
a[200] = 19
a[201] = 20
a[202] = 21

33. Consider a 100 Mbps link between an earth station (sender) and a satellite (receiver) at an altitude of 2100 km. The signal propagates at a

speed of 3×10^8 m/s. The time taken (in milliseconds, rounded off to two decimal places) for the receiver to completely receive a packet of 1000 bytes transmitted by the sender is _____.

Sol: (7.08)

Given, Bandwidth = 100 Mbps

Distance = 2100 Km.

Propagation speed = 3×10^8 m/sec

Time taken to receive the packet

= Transmission time + Propagation time

$$\begin{aligned} \text{Transmission time} &= \frac{\text{Packet size}}{\text{Bandwidth}} = \frac{1000 \text{ bytes}}{100 \text{ Mbps}} \\ &= \frac{1000 \times 8 \text{ bits}}{100 \times 10^6 \text{ bits/sec}} \\ &= 0.08 \text{ Msec.} \end{aligned}$$

$$\text{Propagation time} = \frac{\text{Distance}}{\text{Propagation speed}}$$

$$\begin{aligned} &= \frac{2100 \text{ Km}}{3 \times 10^8 \text{ m/sec}} \\ &= \frac{2100 \times 10^3 \text{ m}}{3 \times 10^8 \text{ m/sec}} \\ &= 7 \times 10^{-3} \text{ sec} \\ &= 7 \text{ msec.} \end{aligned}$$

Required time = $7 + 0.08 = 7.08$ msec.

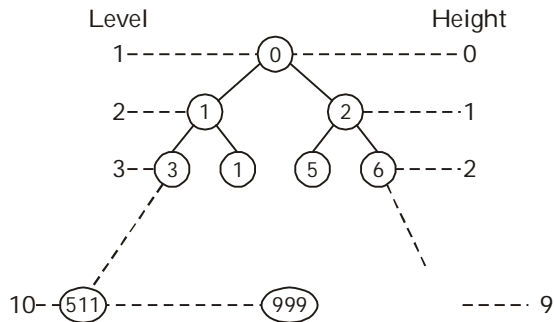
34. Suppose a binary search tree with 1000 distinct elements is also a complete binary tree. The tree is stored using the array representation of binary heap trees. Assuming that the array indices start with 0, the 3rd largest element of the tree is stored at index _____.

Sol: (509)

Given binary search tree with 1000 distinct elements. And the tree is stored using the array representation of binary heap trees. Array indices start with 0.

Index number of first node at each level can be found by $(2^h - 1)$, where h is the height of the tree.

And, Number of node at each level can be find out by $(2^n - 1)$, where n is the number of level.



At 10th level number of nodes = $2^{10-1} = 512$.

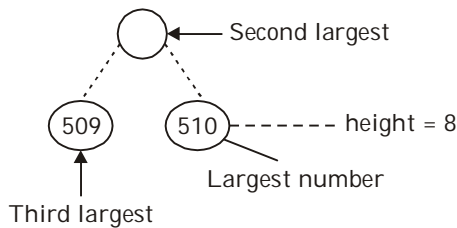
At height 9, index number of first node
 $= 2^9 - 1 = 511$

Since, total number of node is 1000, so we need to check upper level. Because rightmost number in binary seachtree is maximum.

At 9th level, no. of nodes = $2^{9-1} = 256$.

At high 8, index no. of first node = $2^8 - 1 = 255$.

Index number of last node in 9th level,
 $= 255 \times 2$
 $= 510$



Required answer = 509

35. What is printed by the following ANSI C program ?

```
#include<stdio.h>
int main (int argc, char *argv [] )
{
    int x = 1, z [2] = [10, 11];
    int *p = NULL;
    p = &x;
    *p = 10;
    p = &z [1];
    * (&z[0] + 1) += 3;
    printf("%d, %d, %d\n", x, z[0], z[1] );
    return 0;
}
```

- (a) 10, 10, 14 (b) 1, 10, 11
 (c) 10, 14, 11 (d) 1, 10, 14

Sol: (a)

\Rightarrow $\begin{matrix} 2000 \\ \boxed{1} \\ x \end{matrix}$, $z \begin{matrix} 0 & 1 \\ \boxed{10} & \boxed{11} \\ 3000 & 3002 \end{matrix}$, $\begin{matrix} 4000 \\ \boxed{\text{Null}} \\ p \end{matrix}$ \rightarrow Compute (Line 1)

\Rightarrow $\begin{matrix} x \\ \boxed{1} \\ 2000 \end{matrix}$, $z \begin{matrix} 0 & 1 \\ \boxed{10} & \boxed{11} \\ 3000 & 3002 \end{matrix}$, $\begin{matrix} p \\ \boxed{\text{Null}} \\ 4000 \end{matrix}$ \rightarrow Compute (Line 2)

\Rightarrow $\begin{matrix} x \\ \boxed{1} \\ 2000 \end{matrix}$ 10, $z \begin{matrix} 0 & 1 \\ \boxed{10} & \boxed{11} \\ 3000 & 3002 \end{matrix}$, $\begin{matrix} p \\ \boxed{2000} \\ 4000 \end{matrix}$ \rightarrow Compute (Line 3)

\Rightarrow $\begin{matrix} x \\ \boxed{10} \\ 2000 \end{matrix}$, $z \begin{matrix} 0 & 1 \\ \boxed{10} & \boxed{11} \\ 3000 & 3002 \end{matrix}$, $\begin{matrix} p \\ \boxed{3002} \\ 4000 \end{matrix}$ \rightarrow Compute (Line 4)

$\Rightarrow *(&z[0] + 1) += 3$

$\Rightarrow *(3000 + 1) += 3$

$\Rightarrow *(3002) += 3$

$\Rightarrow (11) += 3$

$\Rightarrow 14$

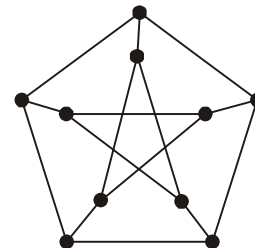
\Rightarrow $\begin{matrix} x \\ \boxed{10} \\ 2000 \end{matrix}$, $\begin{matrix} z[0] & z[1] \\ \boxed{10} & \boxed{14} \\ 3000 & 3002 \end{matrix}$, $\begin{matrix} p \\ \boxed{3002} \\ 4000 \end{matrix}$

$\Rightarrow x = 10$

$z[0] = 10$

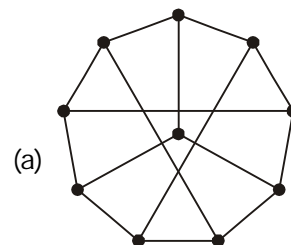
$z[1] = 14$

36. The following simple undirected graph is referred to as the Peterson graph.



Which of the following statements is/are TRUE?

The following graph is isomorphic to the Peterson graph.



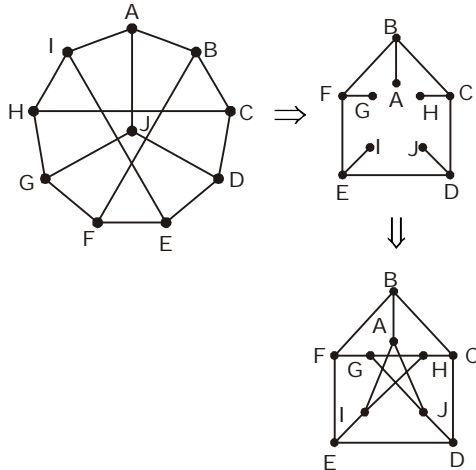
(a) The size of the largest independent set of the given graph is 3. (A subset of vertices of a graph form an independent set if no two vertices of the subset are adjacent.)

- (c) The graph has a Hamiltonian path.
(d) The chromatic number of the graph is 3.

Sol: (a, c, d)

- (a) Yes, it is isomorphic to the given graph.

Step-I

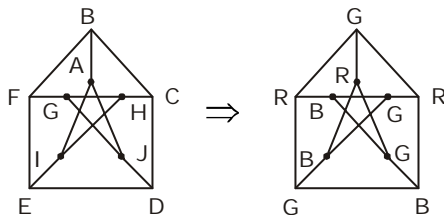


- (b) Vertex independent set is a set of vertices which are not adjacent.

Maximal vertex independent set is a set in which we cannot add one more vertex to it. So, largest independent set of Peterson graph is 4.

- (c) Peterson graph has Hamiltonian path but not Hamiltonian cycle.

- (d) Yes, it is true.

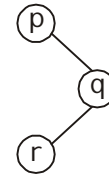


37. Which of the properties hold for the adjacency matrix A of a simple undirected unweighted graph having n vertices ?

- (a) If the sum of all the elements of A is at most $2(n-1)$, then the graph must be acyclic.
(b) If the graph is connected, then none of the entries of $A^{n-1} + I_n$ can be zero.
(c) The diagonal entries of A^2 are the degrees of the vertices of the graph.
(d) If there is at least a 1 in each of A 's rows and columns, then the graph must be connected.

Sol: (c)

- (c) Consider a graph \Rightarrow



$$K \text{ is adjacency matrix} = \begin{matrix} & \begin{matrix} p & q & r \end{matrix} \\ \begin{matrix} p \\ q \\ r \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \end{matrix}$$

$$K^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Degree sequence of $(p, q, r) = (1, 2, 1)$

Diagonal elements $= (1, 2, 1)$

38. Consider a simple undirected graph of 10 vertices. If the graph is disconnected, then the maximum number of edges it can have is _____.

Sol: (36)

We are given that vertices $= n$

The maximum number of possible edges in an undirected graph with ' n ' vertices and ' k ' components is :

Example :

To get maximum edges, take one vertex each for each component, except last component.

Now, $(K-1)$ components have 1 vertex each and last component has $n-(K-1)$ vertices. Make the last component complete i.e. it has

$${}_{n-(k-1)}C_2 = \frac{(n-K)(n-K+1)}{2}$$

$$\text{Number of edges (e)} = \frac{(n-K)(n-K+1)}{2}$$

$$\text{Complement (K)} = 2$$

$$= \frac{(10-2)(10-2+1)}{2}$$

$$= \frac{8 \times 9}{2} = 36$$

39. Which one of the following statements is TRUE for all positive functions $f(n)$?

- (a) $f(n^2) = \Omega(f(n)^2)$
- (b) $f(n^2) = o(f(n)^2)$
- (c) $f(n^2) = O(f(n)^2)$, when $f(n)$ is an exponential function
- (d) $f(n^2) = \theta(f(n)^2)$, when $f(n)$ is a polynomial

Sol: (d)

Growth rate of exponential function is greater than polynomial function.

Thus, $f(n^2) = \theta(f(n)^2)$, when $f(n)$ is a polynomial.

40. Consider a simple undirected weighted graph G , all of whose edge weights are distinct. Which of the following statements about the minimum spanning trees of G is/are TRUE ?

- (a) The edge with the second smallest weight is always part of any minimum spanning tree of G .
- (b) Suppose $S \subseteq V$ be such that $S \neq \phi$ and $S \neq V$. Consider the edge with the minimum weight such that one of its vertices is in S and the other in V/S . Such an edge will always be part of any minimum spanning tree of G .
- (c) G can have multiple minimum spanning trees.
- (d) One or both of the edges with the third smallest and the fourth smallest weights are part of any minimum spanning tree of G .

Sol: (a, b, d)

- (a) TRUE
- (b) TRUE
- (c) No, G can not have multiple minimum spanning trees when all the edges are distinct.
- (d) TRUE

41. Which of the following is/are undecidable ?

- (a) Given a Turing machine M , decide if $L(M)$ is regular.

- (b) Given a Turing machine M , decide if M accepts all strings.
- (c) Given a Turing machine M , decide if M takes more than 1073 steps on every string.
- (d) Given two Turing machine M_1 and M_2 , decide if $L(M_1) = L(M_2)$.

Sol: (a, b, d)

- (a) Undecidable
- (b) Undecidable
- (c) Decidable, Turing Machine M decide if language L takes more than 1073 steps. Language \bar{L} takes almost 1073 steps. So, it is decidable. Then its complement is also decidable. Hence, L is decidable.
- (b) Undecidable

42. Let $G(V, E)$ be a directed graph, where $V = \{1, 2, 3, 4, 5\}$ is the set of vertices and E is the set of directed edges, as defined by the following adjacency matrix A .

$$A[i][j] = \begin{cases} 1, & 1 \leq j \leq i \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

$A[i][j] = 1$ indicates a directed edge from node i to node j . A directed spanning tree of G , rooted at $r \in V$, is defined as a subgraph T of G such that the undirected version of t is a tree, and T contains a directed path from r to every other vertex in V . The number of such directed spanning trees rooted at vertex 5 is _____.

Sol: (24)

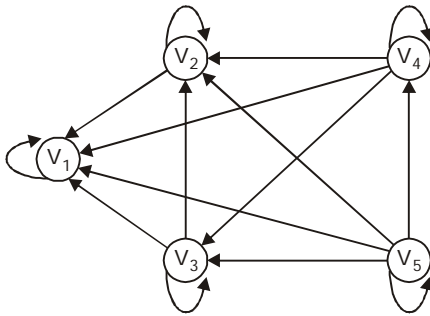
Given vertex,

$$V = \{1, 2, 3, 4, 5\}$$

A matrix 'A' is given in such a way that,

$$A[i][j] = \begin{cases} 1 & 1 \leq j \leq i \leq 5 \\ 0 & \text{Otherwise} \end{cases}$$

$$A = \begin{matrix} & \xrightarrow{i} & 1 & 2 & 3 & 4 & 5 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \downarrow j & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{matrix}$$



Directed spanning tree rooted at vertex 5 is
 $4 \times 3 \times 2 \times 1 = 24$

43. Consider routing table of an organization's router shown below:

Subnet Number	Subnet Mask	Next Hop
12.20.164.0	255.255.252.0	R1
12.20.170.0	255.255.254.0	R2
12.20.168.0	255.255.254.0	Interface 0
12.20.166.0	255.255.254.0	Interface 1
default		R3

Which of the following prefixes in CIDR notation can be collectively used to correctly aggregate all of the subnets in the routing table ?

- (a) 12.20.168.0/22 (b) 12.20.164.0/21
 (c) 12.20.164.0/22 (d) 12.20.164.0/20

Sol: (a, c)

Given routing table,

- 12.20.164.0 255.255.252.0
 12.20.10100100.00000000
- 12.20.170.0 255.255.254.0
 12.20.10101010.00000000
- 12.20.168.0 255.255.254.0
 12.20.10101000.00000000
- 12.20.166.0 255.255.254.0
 12.20.10100110.00000000

Subnet (2) and (3) can be aggregated.

12.20.10101010.00000000
 12.20.10101000.00000000
 Network id Host id

12.20.168.0/22

Subnet (1) and (4) can be aggregated

12.20.10100100.00000000
 12.20.10100110.00000000
 Network id Host id

12.20.164.0/22

44. Suppose we are given n keys, m hash table slots, and two simple uniform hash functions h_1 and h_2 . Further suppose our hashing scheme uses h_1 for the odd keys and h_2 for the even keys. What is the expected number of keys in a slot ?

- (a) $2n/m$ (b) n/m
 (c) m/n (d) $n/2m$

Sol: (b)

Number of keys = n

Number of slots = m

As per question, we need to find out number of expected keys in a slot, i.e. each slot how many keys are possible.

So, each slot expected key should be n/m .

45. Which one of the following statements is FALSE?

- (a) The memory access time using a given inverted page table is always same for all incoming virtual addresses.
 (b) If the virtual address of a word given by CPU has a TLB hit, but the subsequent search for the word results in a cache miss, then the word will always be present in the main memory.
 (c) The TLB performs an associative search in parallel on all its valid entries using page number of incoming virtual address.
 (d) In a system that uses hashed page tables, if two distinct virtual addresses V_1 and V_2 map to the same value while hashing, then the memory access time of these addresses will not be the same.

Sol: (a)

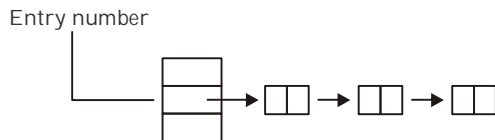
(a)

Inverted Page table

frame 0	Process	Page
Frame 1	Process	Page
	⋮	⋮

Memory access time using inverted page table is not same because in inverted page table no indexing is applied and there is no equal linear searching. So, this statement is false.

- (b) TLB hit means word will always present in main memory.
- (c) TLB perform parallel search.
- (d) Virtual address \Rightarrow entry number.



If they map to same value while hashing, there memory access time of addresses will not same because there is a chance that some elements are present at the end of linked list.

46. Consider the following two statements with respect to the matrices $A_{m \times n}$, $B_{n \times m}$, $C_{n \times n}$ and $D_{n \times n}$.

Statement 1: $\text{tr}(AB) = \text{tr}(BA)$

Statement 2: $\text{tr}(CD) = \text{tr}(DC)$

where $\text{tr}()$ represents the trace of a matrix. Which one of the following holds?

- (a) Statement 1 is wrong and Statement 2 is correct.
- (b) Statement 1 is correct and Statement 2 is wrong.
- (c) Both Statement 1 and Statement 2 are correct
- (d) Both Statement 1 and Statement 2 are wrong.

Sol: (c)

The trace of a matrix is the sum of the diagonal elements of the matrix.

- In this question, property of trace is used that is trace of product $(AB) = \text{trace of product } (BA)$
- Statement 1 \rightarrow it said $\text{tr}(A_{m \times n} \times B_{n \times m}) = \text{tr}(B_{n \times m} \times A_{m \times n})$.

Let take example.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}_{2 \times 3}, B = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}_{3 \times 2}$$

$$\text{tr}(AB) = \begin{bmatrix} 14 & 22 \\ 32 & 77 \end{bmatrix} = 14 + 77 = 91$$

$$\text{tr}(BA) = \begin{bmatrix} 17 & 22 & 27 \\ 22 & 29 & 36 \\ 27 & 36 & 45 \end{bmatrix} = 17 + 19 + 45 = 91$$

$$91 = 91$$

Statement I is correct.

Statement-II :

$$\text{tr}(C_{n \times n} \times D_{n \times n}) = \text{tr}(D_{n \times n} \times C_{n \times n})$$

Take example,

$$C = \begin{bmatrix} 2 & 2 \\ 3 & 5 \end{bmatrix}_{2 \times 2}, D = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$$

$$\text{tr}(CD) = \begin{bmatrix} 8 & 12 \\ 18 & 26 \end{bmatrix} = 8 + 26 = 34$$

$$\text{tr}(DC) = \begin{bmatrix} 8 & 12 \\ 18 & 26 \end{bmatrix} = 26 + 8 = 34$$

Statement II is also true.

47. Which of the following statements is/are TRUE?

- (a) Every subset of a recursively enumerable language is recursive.
- (b) If a language L and its complement \bar{L} are both recursively enumerable, then L must be recursive.
- (c) Complement of a context-free language must be recursive.
- (d) If L_1 and L_2 are regular, then $L_1 \cap L_2$ must be deterministic context-free.

Sol: (b, c, d)

- (a) It is not necessary that every subset of a recursively enumerable language is recursive.
- (b) If language L and its complement (\bar{L}) both are recursively enumerable then L must be recursive.
- (c) Context free language is recursive so its complement is also recursive.
- (d) Regular languages are closed under intersection, so their intersection must be deterministic context free.

48. What is printed by the following ANSI C program?

```
#include<stdio.h>

int main (int argc, char *argv [ ] ) {
```

```
char a = 'P' ;
char b = 'X' ;
char c = (a & b) + '*' ;
char d = (a | b) - '-' ;
char e = (a ^ b) + '+' ;
printf("%c %c %c/n", c, d, e);
return 0;
}
```

ASCII encoding for relevant characters is given below

A	B	C	...	Z
65	66	67	...	90

a	b	c	...	z
97	98	99	...	122

*	+	-
42	43	45

- (a) P x + (b) z K S
(c) 122 75 83 (d) * - +

Sol: (b)

Step-I \Rightarrow $\boxed{80}$ $\boxed{120}$ because we take small 'p'
a b

and capital 'X'.

char c \Rightarrow (a & b) + '*'
 \Rightarrow Bitwise & = (80 & 120) + 42
 \Rightarrow 80 = 1010000
120 = 1111000
a&b = 1010000 = 80+42=12 = z

char d \Rightarrow (a | b) + '-'
 \Rightarrow Bitwise | = (80 | 120) + 45
 \Rightarrow 80 = 1010000
120 = 1111000
a|b = 1111000 = 120+45=75 = K

Char e \Rightarrow (a ^ b) + '+'
 \Rightarrow Bitwise ^ = (80 ^ 120) + 43
 \Rightarrow a^b = 1010000
1111000
a^b = 0101000 = 40+43=83 = S
print character = (z, K, S)

49. Consider three floating point numbers A, B and C stored in registers R_A , R_B and R_C , respectively as per IEEE-754 single precision floating point format. The 32-bit content stored in these registers (in hexadecimal form) are follows.

$R_A = 0 \times C1400000$
$R_B = 0 \times 42100000$
$R_C = 0 \times 41400000$

Which one of the following is FALSE ?

- (a) $B = 3C$ (b) $A + C = 0$
(c) $(B - C) > 0$ (d) $C = A + B$

Sol: (d)

$$C = A + B$$

$$R_A = 0 \times C1400000 \Rightarrow 1100 \ 0001 \ 0100 \ 00000$$

$$R_B = 0 \times 42100000 \Rightarrow 0100 \ 0010 \ 0001 \ 00000$$

$$R_C = 0 \times 41400000 \Rightarrow 0100 \ 0001 \ 0100 \ 00000$$

If you see carefully A and C are changes only in sign bit.

$$\text{IEEE-754 single precision} = (-1)^S * 1.M * 2^{E-127}$$

In every register, first bit is sign bit, next 8 bit is for exponent, and after that remaining bit is mantissa.

$$R_A = 1100 \ 0001 \ 01000 \ 00000$$

$$= \text{Sign} = (-)$$

$$= \text{Exponent} = 130$$

$$= (-1)^{-1} 1.1 \times 2^{130-127}$$

$$= -1.1 \times 2^3$$

$$= (-1100)_2 = -12$$

R_C = Similarly $R_C = +12$ because only difference is sign bit.

$$R_B = 0100 \ 0010 \ 0001 \ 00000$$

$$= \text{Sign} = (+)$$

$$= \text{Exponent} = 132$$

$$= +1.001 \times 2^{132-127}$$

$$= +1.001 \times 2^5$$

$$= +(100100) = +36$$

So, false option is $C = A + B$

50. Which one of the following facilitates transfer of bulk data from hard disk to main memory with the highest throughput ?
(a) Programmed I/O transfer
(b) Polling based I/O transfer
(c) Interrupt drive I/O transfer
(d) DMA based I/O transfer

Sol: (d)

In programmed I/O CPU time is wastes.

In interrupt driven I/O at particular time interval CPU check the interrupt and CPU transfer the data to memory as per interrupt requirement. But it doesn't transfer the data with maximum throughput.

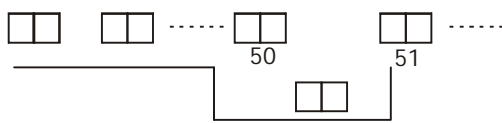
In polling based I/O, the I/O device is continuously poll by asking whether it needs CPU or not. So it also transfer data to main memory with low throughput.

DMA based I/O transfer doesn't involved CPU for transferring data to main memory and its throughput is maximum.

51. Consider two files systems A and B, that use contiguous allocation and linked allocation, respectively. A file of size 100 block is already stored in A and also in B. Now, consider inserting a new block in the middle of the file (between 50th and 51st block), whose data is already available in the memory. Assume that there are enough free blocks at the end of the file and that the file control blocks are already in memory. Let the number of disk accesses required to insert a block in the middle of the file in A and B are n_A and n_B , respectively, then the value of $n_A + n_B$ is _____.

Sol: (153)

For linked allocation, we need to access the block from 1 to 50, new block access that should insert between 50 and 51 block, and access 51 block. So, the total access is 52.



Total 52 access

For contiguous allocation, we have 100 block, in contiguous allocation first we push the 100th block to 101th block (we need two access), then we push 99th block to 100th block (we need two access again) and so on upto 51th block. Now, 51 block is empty, then we push new block to 51 position list need one access). So, total access are

$$\underbrace{50 \times 2}_{\text{(For 51 to 100 block)}} + \underbrace{1}_{\text{for new block}} = 100 + 1 = 101$$

Total required access are $101 + 52 = 153$.

52. Which of the following is/are the eigen vector(s) for the matrix given below ?

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix}$$

(a) $\begin{bmatrix} -1 \\ 0 \\ 2 \\ 2 \end{bmatrix}$

(b) $\begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

(d) $\begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

Sol: (a, b, d)

This question is solved with verification of option. This will help us to take less time.

\Rightarrow Use $AX = \lambda X$ in each options.

λ = Scalar quantity.

(a) $AX = \lambda X$

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 2 \\ 2 \end{bmatrix} = 3 \begin{bmatrix} -1 \\ 0 \\ 2 \\ 2 \end{bmatrix} \Rightarrow AX = 3X$$

(b) $AX = \lambda X$

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix} = 3 \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \end{bmatrix} \Rightarrow AX = 3X$$

(c) $AX = \lambda X$

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \\ 25 \end{bmatrix} \Rightarrow AX \neq \lambda X$$

(d) $AX = \lambda X$

$$\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow AX = 1X$$

53. Consider a system with 2 KB direct mapped data cache with a block size of 64 bytes. The system has a physical address space of 64 KB

and a word length of 16 bits. During the execution of a program, four data words P, Q, R and S are accessed in that order 10 times (i.e. PQRSPQRS). Hence, there are 40 accesses to data cache altogether. Assume that the data cache is initially empty and no other data words are accessed by the program. The addresses of the first bytes of P, Q, R and S are 0xA248, 0xC28A, 0xCASA, and 0xA262, respectively. For the execution of the above program, which of the following statements is/are TRUE with respect to the data cache ?

- (a) Every access to S is a hit
- (b) At the end of the execution only R and S reside in the cache.
- (c) Once P is brought to the cache it is never evicted
- (d) Every access to R evicts Q from the cache.

Sol: (a, c, d)

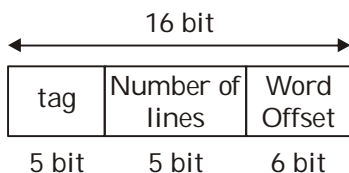
Cache Memory Size = 2 KB

Main Memory Size = 64 KB

Block Size = 64 B

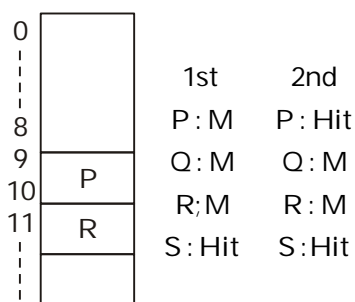
$$\text{Number of Lines} = \frac{\text{Cache Memory Size}}{\text{Block Size}}$$

$$= \frac{2K}{64} = \frac{2^{11}}{2^6} = 2^5$$



P:(A248)H 1010 0010 0100 1000; 9th block
 Q:(C284)H 1100 0010 1000 0100; 10th block
 R:(CA8A)H 1100 1010 1000 1010; 10th block
 S:(A262)H 1010 0010 0110 0010; 9th block

But P&S are from same memory block (10100).



- Every access of S is hit.
- Once P is brought to the cache it is never evicted.
- Every access to R evicts Q from cache.

54. Consider the following languages :

$$L_1 = \{a^n w a^n \mid w \in \{a, b\}^*\}$$

$$L_2 = \{w x w^R \mid w, x \in \{a, b\}^*, |w|, |x| > 0\}$$

Note that w^R is the reversal of the string w . Which of the following is/are TRUE ?

- (a) L_1 and L_2 are context-free
- (b) L_1 and L_2 are regular
- (c) L_1 is regular and L_2 is context-free.
- (d) L_1 and L_2 are context-free but not regular.

Sol: (a, b, c)

Given language,

$$L_1 = \{a^n w a^n \mid w \in \{a, b\}^*\}$$

$$L_2 = \{w x w^R \mid w, x \in \{a, b\}^*, |w|, |x| > 0\}$$

Regular expression can be written for the language L_1 and L_2 .

For L_1 ,

$$(a + b)^*$$

For L_2 ,

$$a(a + b)^+ a + b(a + b)^+ b$$

Hence, both language are regular. And regular language are context free.

55. Consider the relational database with the following four schemes and their respective instances.

Student(sNo, sName, dNo) **Dept**(dNo, dName)
Course(cNo, cName, dNo) **Register** (sNo, cNo)

Student		
<u>sNo</u>	sName	dNo
S01	James	D01
S02	Rocky	D01
S03	Jackson	D02
S04	Jane	D01
S05	Milli	D02

Dept	
<u>dNo</u>	dName
D01	CSE
D02	ESE

Course		
cNo	cName	dNo
C11	DS	D01
C12	OS	D01
C21	DE	D02
C22	PT	D02
C23	CV	D03

Register	
sNo	cNo
S01	C11
S01	C12
S02	C11
S03	C21
S03	C22
S03	C23
S04	C11
S04	C12
S05	C11
S05	C21

SQL query :

SELECT * FROM Student As a WHERE NOT EXIST

(select cNo from Course where dNo = "D01" EXCEPT.

SELECT cNo FROM Register WHERE sNo = S.sNo)

The number of rows returned by the above SQL, query is ____.

Sol: (2)

Given schemes,

Student(SNO, SName, dNo)

Dept(dNo, dName)

Course(CNO, CName, dNo)

Register (SNo, CNo)

Select * from student as S where NOT EXIST (select CNO from course where dNo = "D01" EXCEPT

Select CNO FROM Register WHERE SNO=S.SNO)

In above case, first inner query return is independent and return <{C11, C12}>.

Second inner query is dependent on student relation. The condition is sNo = S.sNo.

So, for sNo = S01, second inner query returns <C11, C12>. And the except operation between two inner query gives empty result.

Except operation : $R_1 \text{ EXCEPT } R_2$

$$= R_1 - (R_1 \cap R_2).$$

Now, NOT EXIST returns TRUE and outer query gives <S01, James, D01> as output.

When second inner query is checked for sNo = S02. It return <C11>. And the except operation between two inner query gives non-empty result. AND NOT EXISTS gives FALSE result. And outer query doesn't give any result.

Similarly for sNo = S04, outer query gives output <S04, jane, D01>.

Hence 2 rows found.

56. Consider the following grammar along with translation rules.

$$S \rightarrow S_1 \# T \quad \{S_{val} = S_{1.val} * T_{avl}\}$$

$$S \rightarrow T \quad \{S_{val} = T_{val}\}$$

$$T \rightarrow T_1 \% R \quad \{T_{val} = T_{1.val} \div R_{val}\}$$

$$T \rightarrow R \quad \{T_{val} = R_{val}\}$$

$$R \rightarrow id \quad \{R_{val} = id_{val}\}$$

Here # and % are operators and id is a token that represents an integer and id_{val} represents the corresponding integer value. The set of non-terminals is {S, T, R, P} and a subscripted non-terminal indicates an instance of the non-terminal.

Using this translation scheme, the computed value of S_{val} for root of the parse tree for the expression $20\#10\%5\#8\%2\%2$ is ____.

Sol: (80)

$$S \rightarrow S_1 \# T \quad \{S_{val} = S_1 \text{ val } * T_{val}\}$$

$$S \rightarrow T \quad \{S_{val} = T_{val}\}$$

$$T \rightarrow T_1 \% R \quad \{T_{val} = T_{1.val} \div R_{val}\}$$

$$T \rightarrow R \quad \{T_{val} = R_{val}\}$$

$$R \rightarrow id \quad \{R_{val} = id_{val}\}$$

$$\Rightarrow 20 \times 10 \div 5 \times 8 \div 2 \div 2$$

Rule 1 : has higher priority then * because it is away from starting symbol.

Rule 2 : Both (\div and $*$) are left associative.

$$\Rightarrow 20 \times 10 \div 5 \times 8 \div 2 \div 2$$

$$\Rightarrow 20 \times 2 \times 4 \div 2$$

$$\Rightarrow 20 \times 2 \times 2$$

$$\Rightarrow 40 \times 2 = 80$$

57. Consider solving the following system of simultaneous equation using LU decomposition.

$$x_1 + x_2 - 2x_3 = 4$$

$$x_1 + 3x_2 - x_3 = 7$$

$$2x_1 + x_2 - 5x_3 = 7$$

where L and U are denoted as

$$L = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & I_{22} & 0 \\ L_{31} & I_{32} & I_{33} \end{bmatrix}, U = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & 0 & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

Which of the following is the correct combination of values for L_{32} , U_{33} , and x_1 ?

- (a) $L_{32} = 2, U_{33} = 2, x_1 = -1$
- (b) $L_{32} = -\frac{1}{2}, U_{33} = -\frac{1}{2}, x_1 = 0$
- (c) $L_{32} = 2, U_{33} = -\frac{1}{2}, x_1 = -1$
- (d) $L_{32} = -\frac{1}{2}, U_{33} = 3, x_1 = -0$

Sol: (b)

Let take coefficient matrix.

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 1 & 3 & -1 \\ 2 & 1 & -5 \end{bmatrix}$$

Perform row-column operation.

$$R_2 \rightarrow R_2 - R_1$$

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & 2 & 1 \\ 2 & 1 & -5 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - 2R_1$$

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & 2 & 1 \\ 0 & -1 & -1 \end{bmatrix}$$

$$R_3 \rightarrow R_3 - \left(-\frac{1}{2}\right)R_2$$

$$U = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

$$A = L * U$$

$$\begin{bmatrix} 1 & 1 & -2 \\ 1 & 3 & -1 \\ 2 & 1 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 2 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 \\ 0 & 2 & 1 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$$

$$U_{33} = -\frac{1}{2}$$

$$L_{32} = -\frac{1}{2}$$

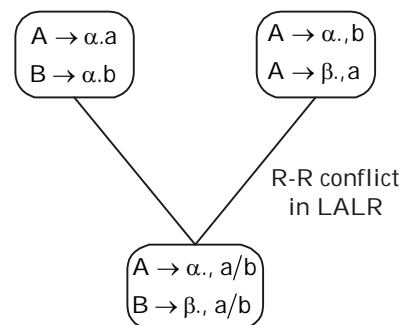
58. Which one of the following statements is TRUE?

- (a) LR (1) parsing is sufficient for deterministic context-free language.
- (b) Symbol table is accessed only during the lexical analysis phase.
- (c) Data flow analysis is necessary for run-time memory management.
- (d) The LAIR(1) parse for a grammar G cannot have reduce-order conflict if the LR(1) parser for G does not have a reduce-reduce conflict.

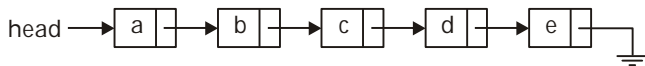
Sol: (a)

We can create a LR parser to pass a DCFL language. Sometime LR parser cannot parse context free language (CFL) because CFL are sometime inherently ambiguous and we cannot design a LR parser for ambiguous grammar. So, LR(1) is sufficient for DCFL.

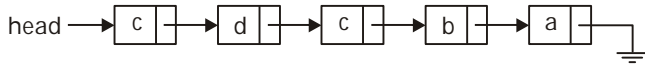
- Symbol table can be accessed in all phases of compiler.
- Data flow analysis is necessary in code-optimization.
- LALR may have reduce-reduce conflict even if CLR don't have any RR conflict (conflict arising due to merging of states where lookaheads are same).



59. Consider the problem of reversing a singly linked list. To take an example, given the linked list below,



the reversal linked list should look like.



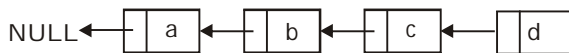
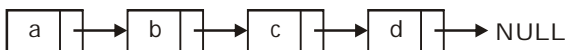
Which one of the following statements is TRUE about the time complexity of algorithms that solve the above problem in $O(1)$ space?

- (a) The best algorithm for the problem takes $\theta(n)$ time in the worst case.
- (b) It is not possible to reverse a singly linked list in $O(1)$ space.
- (c) The best algorithm for the problem takes $\theta(n \log n)$ time in the worst case.
- (d) The best algorithm for the problem takes $\theta(n^2)$ time in the worst case.

Sol: (a)

Three pointers are required to reverse the linked list.

So, in worst case $\theta(n)$ time required to reverse the linked list.



60. Let $R_1(z)$ and $W_1(z)$ denote read and write operations on a data element z by a transaction T_i , respectively. Consider the schedules S with four transactions.

$R_1(x)R_2(x)R_3(x)R_1(y)W_1(y)W_2(x)W_3(y)R_4(y)$

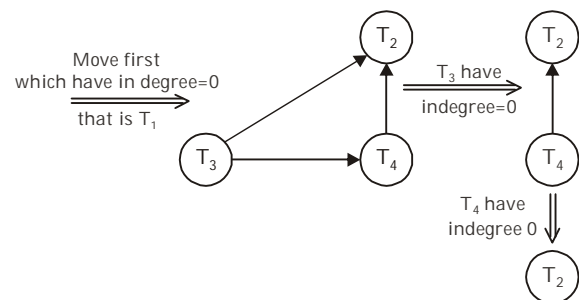
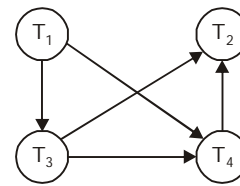
Which one of the following serial schedules is conflict equivalent to S ?

- (a) $T_3 \rightarrow T_1 \rightarrow T_4 \rightarrow T_2$
- (b) $T_1 \rightarrow T_4 \rightarrow T_3 \rightarrow T_2$
- (c) $T_1 \rightarrow T_3 \rightarrow T_4 \rightarrow T_2$
- (d) $T_4 \rightarrow T_1 \rightarrow T_3 \rightarrow T_2$

Sol: (c)

Given schedule S with four transactions,

T_1	T_2	T_3	T_4
			$R_4(x)$
	$R_2(x)$		
$R_1(y)$		$R_3(x)$	
$W_1(y)$			
	$W_2(x)$		
		$W_3(y)$	
			$R_4(y)$



Thus, $T_1 \rightarrow T_3 \rightarrow T_4 \rightarrow T_2$

61. Let R_1 and R_2 be two 4-bit register that store numbers in 2's complement form. For the operation $R_1 + R_2$, which one of the following values of R_1 and R_2 gives an arithmetic overflow?

- (a) $R_1 = 1001$ and $R_2 = 1111$
- (b) $R_1 = 0011$ and $R_2 = 0100$
- (c) $R_1 = 1100$ and $R_2 = 1010$
- (d) $R_1 = 1011$ and $R_2 = 1110$

Sol: (c)

Given two four bit registers R_1 and R_2 .

(a) $R_1 = 1001$

$R_2 = 1111$

$$\begin{array}{r}
 R_1 + R_2 = \quad 1 \ 0 \ 0 \ 1 \\
 \quad \quad \quad 1 \ 1 \ 1 \ 1 \\
 \hline
 \quad \quad \quad 1 \ 0 \ 0 \ 0
 \end{array}$$

No overflow occurred, because sign bit is same for $(R_1 + R_2)$.

(b) $R_1 = 0011$

$$R_2 = 0100$$

$$\begin{array}{r} R_1 + R_2 = \quad 0 \ 0 \ 1 \ 1 \\ \quad \quad 0 \ 1 \ 0 \ 0 \\ \hline \quad \quad 0 \ 1 \ 1 \ 1 \end{array}$$

No overflow occurred, because sign bit is same for $(R_1 + R_2)$.

(c) $R_1 = 1100$

$$R_2 = 1010$$

$$\begin{array}{r} R_1 + R_2 = \quad 1 \ 1 \ 0 \ 0 \\ \quad \quad 1 \ 0 \ 1 \ 0 \\ \hline \quad \quad 0 \ 1 \ 1 \ 0 \end{array}$$

Overflow occurred, because sign bit is different for $(R_1 + R_2)$.

(d) $R_1 = 1011$

$$R_2 = 1110$$

$$\begin{array}{r} R_1 + R_2 = \quad 1 \ 0 \ 1 \ 1 \\ \quad \quad 1 \ 1 \ 1 \ 0 \\ \hline \quad \quad 1 \ 0 \ 0 \ 1 \end{array}$$

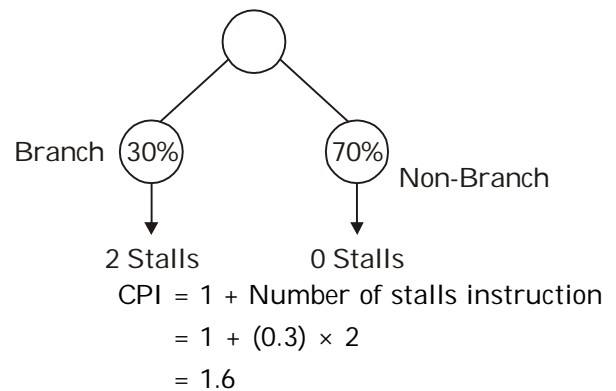
No overflow occurred, because sign bit is same for $(R_1 + R_2)$.

62. A processor X1 operation at 2 GHz has a standard 5-stage RISC instruction pipeline having a base CPI (cycles per instruction) of one without any pipeline hazards. For a given program P that has 30% branch instructions, control hazards incur 2 cycles stall for every branch. A new version of the processor X₂ operating at same clock frequency has an additional branch predictor unit (BPU) that completely eliminate stalls for correctly predicted branches. There is neither any savings nor any additional stalls for wrong predictions. There are no structural hazards and data hazards for X₁ and X₂. If the BPU has a prediction accuracy of 80%, the speed upto (rounded off to two decimal places) obtained by X₂ over X₁ in executing P is ____.

Sol: (1.42)

$$\text{Cycle time } (t_p) = 2 \text{ GHz} = 0.5 \text{ nanoseconds}$$

$$K = 5, X_1 \Rightarrow \text{without branch prediction.}$$



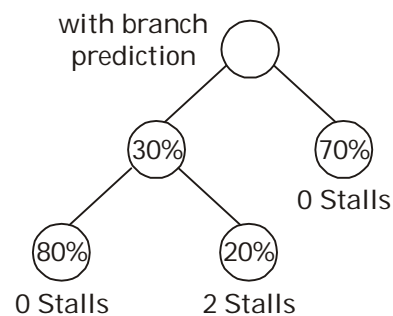
Average instruction execution time

$$= \text{CPI} \times \text{Cycle time}$$

$$= 1.6 \times 0.5 \text{ nsec}$$

$$= 0.8 \text{ ns.}$$

$X_2 \Rightarrow$ "If BPU predicted correct branch then it eliminate stalls but if BPU predicted wrong branch then BPU not add any additional stalls **but remaining stalls we present.**"



$$\text{CPI} = 1 + (0.3 \times 0.2 \times 2)$$

$$= 1.12$$

Average instruction execution time

$$= \text{CPI} \times \text{Cycle time}$$

$$= 1.12 \times 0.5 \text{ ns} = 0.56 \text{ ns}$$

$$\text{Speed up} = \frac{0.8}{0.56} = 1.42$$

63. Consider a demand paging system with four page frames (initially empty) and LRU page replacement policy. For the following page reference string.

7, 2, 7, 3, 2, 5, 3, 4, 6, 7, 7, 1, 5, 6, 1

the page fault rate, defined as the ratio of number of page faults to the number of memory accesses (rounded off to one decimal place_ is ____.

Sol: (0.6)

Given page reference string,

7, 2, 7, 3, 2, 5, 3, 4, 6, 7, 7, 1, 5, 6, 1

$$\text{Page fault rate} = \frac{\text{Number of page faults}}{\text{Number of memory access}}$$

There are four page frames are given and LRU page replacement policy is used.

7	2	7	3	2	5	3	4	6	7	7	1	5	6	1
					5		5	5	7		7	7		
			3		3		3	3	3		1	1		
	2		2		2		2	6	6		6	6		
7	7		7		7		4	4	4		4	5		

Number of Page fault = 9

Number of memory access = 15

$$\text{Page fault rate} = \frac{9}{15} = 0.6$$

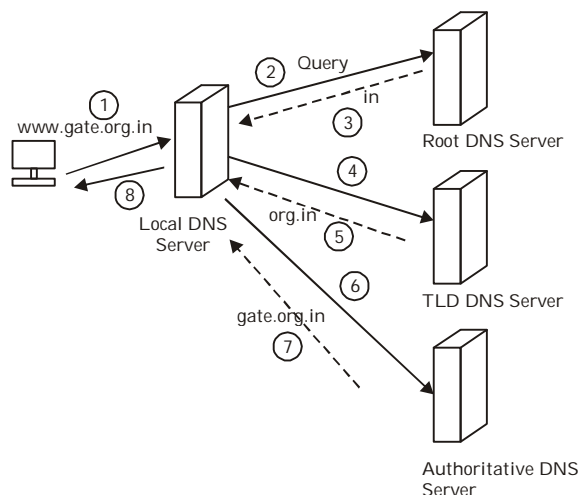
64. Consider the resolution of the domain name www.gate.org.in by a DNS resolver. Assume that no resource records are cached anywhere across the DNS servers and that iterative query mechanism is used in the resolution. The number of DNS query-response pairs involved in completely resolving the domain name is ____.

Sol: (4)

Given domain name,

www.gate.org.in

Iterative query mechanism is used in the resolution,



There are 4-pairs involved in completely resolving the domain name.

65. Consider the following resource :

$$f_1 = 1;$$

$$f(2n) = 2f(n) - 1, \text{ for } n \geq 1$$

$$f(2n + 1) = 2f(n) + 1, \text{ for } n \geq 1$$

Then, which of the following statements is/are TRUE ?

- (a) $f(2^n + 1) = 2^n + 1$ (b) $f(5 \cdot 2^n) = 2^{n+1} + 1$
(c) $f(2^n - 1) = 2^n - 1$ (d) $f(2^n) = 1$

Sol: (b, c, d)

Eliminating option is the best way for solving this question $\Rightarrow f(2n)$ is even function and $f(2n + 1)$ is odd function.

$$f(1) = 1$$

$$f(2) = f(2 \cdot 1) = 2f(1) - 1 = 2 - 1 = 1 \quad [f(2n) = 2f(n) - 1]$$

$$f(3) = f(2 \cdot 2 + 1) = 2f(2) + 1 = 3 \quad [f(2n + 1) = 2f(n) + 1]$$

$$f(4) = f(2 \cdot 2) = 2f(2) - 1 = 2 \cdot 1 - 1 = 1$$

$$f(5) = f(2 \cdot 2 + 1) = 2f(2) + 1 = 2 \cdot 1 + 1 = 3$$

$$f(6) = f(2 \cdot 3) = 2f(3) - 1 = 2 \cdot 3 - 1 = 5$$

$$f(7) = f(2 \cdot 3 + 1) = 2f(3) + 1 = 7$$

$$f(8) = f(2^3) = f(2 \cdot 4) = 2f(4) - 1 = 2 \cdot 1 - 1 = 1$$

$$f(9) = f(2 \cdot 4 + 1) = 2f(4) + 1 = 2 \cdot 1 + 1 = 3$$

$$f(10) = f(5 \cdot 2) = 2f(5) - 1 = 2 \cdot 3 - 1 = 5$$

:

$$f(20) = f(5 \cdot 4) = f(10 \cdot 2) = 2f(10) - 1 = 2 \cdot 5 - 1 = 9$$

- Computation of $f(2)$, $f(4)$ and $f(8)$ show that $f(2^n) = 1$ is correct.
- Computation of $f(3)$ and $f(7)$ show that $f(2^n - 1) = 2^n - 1$ is correct.
- Computation of $f(10)$ and $f(20)$ show that $f(5 \cdot 2^n) = 2^{n+1} + 1$
- Computation of $f(5)$ and $f(9)$ show that $f(2^n + 1) = 2^n + 1$ is not correct.