Probability Distributions

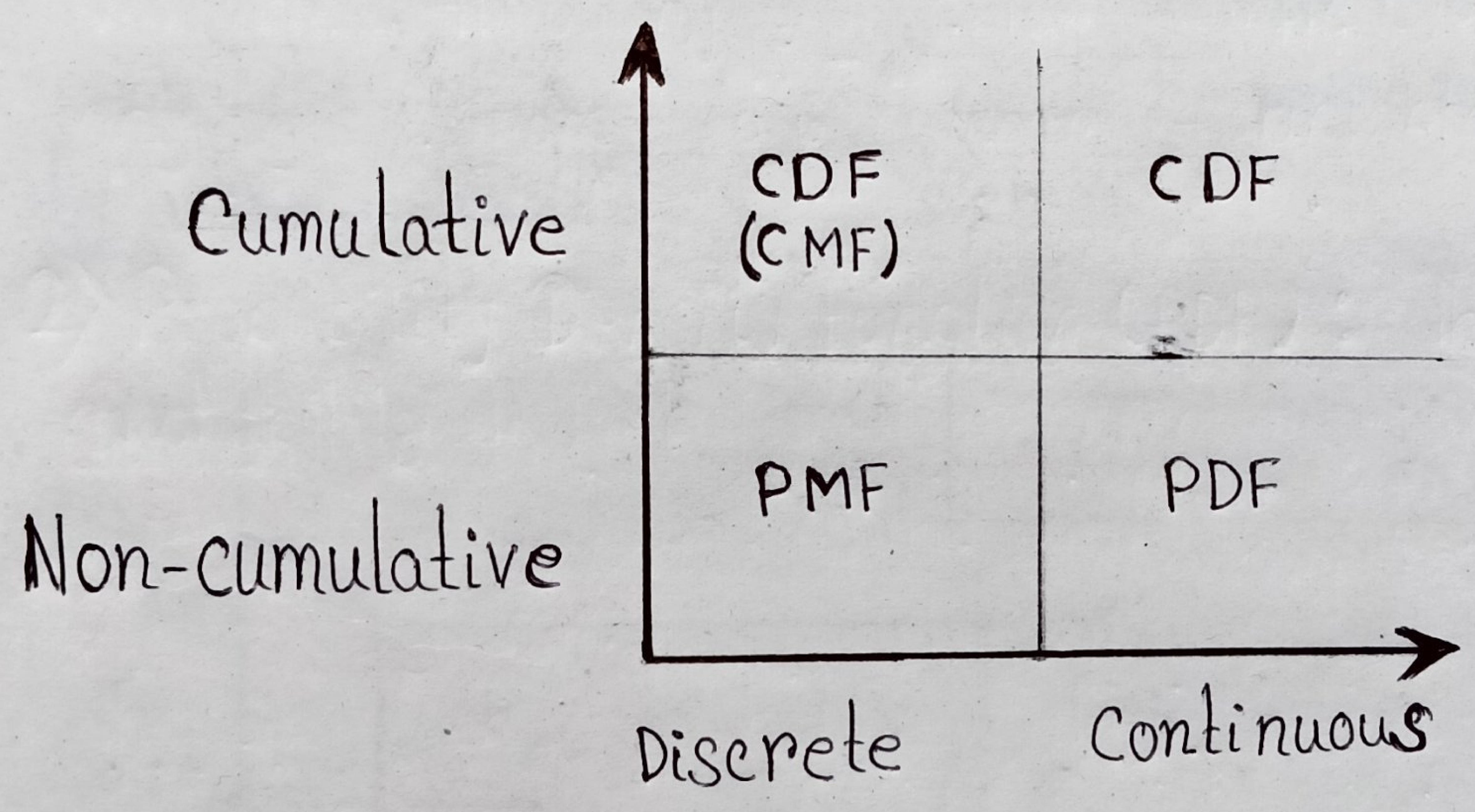
Probability Distribution Functions

1) Probability Mass Function (PMF) : The Probability distribution of a discrete random variable is described by the PMF, which is a statistical term.
The terms PDF and PMF are frequently misunderstood. The PDF is for continuous random variables, whereas the PMF is for discrete random variables. Throwing a dice, for example (you can only choose from 1 to 6 numbers (countable)).

2) Probability Density Function (PDF) : The Probability distribution of a continuous random variable is described by the word PDF, which is a statistical term.
The Gaussian Distribution is the most common distribution used in PDF. If the features/random variables are Gaussian distributed, then the PDF will be as well. Because the single point represents a line that does not span the area under the curve, the probability of a single outcome is always 0 on a PDF graph.

3) Cumulative Density Function (CDF): The cumulative distribution function can be used to describe the continuous or discrete distribution of random variables.

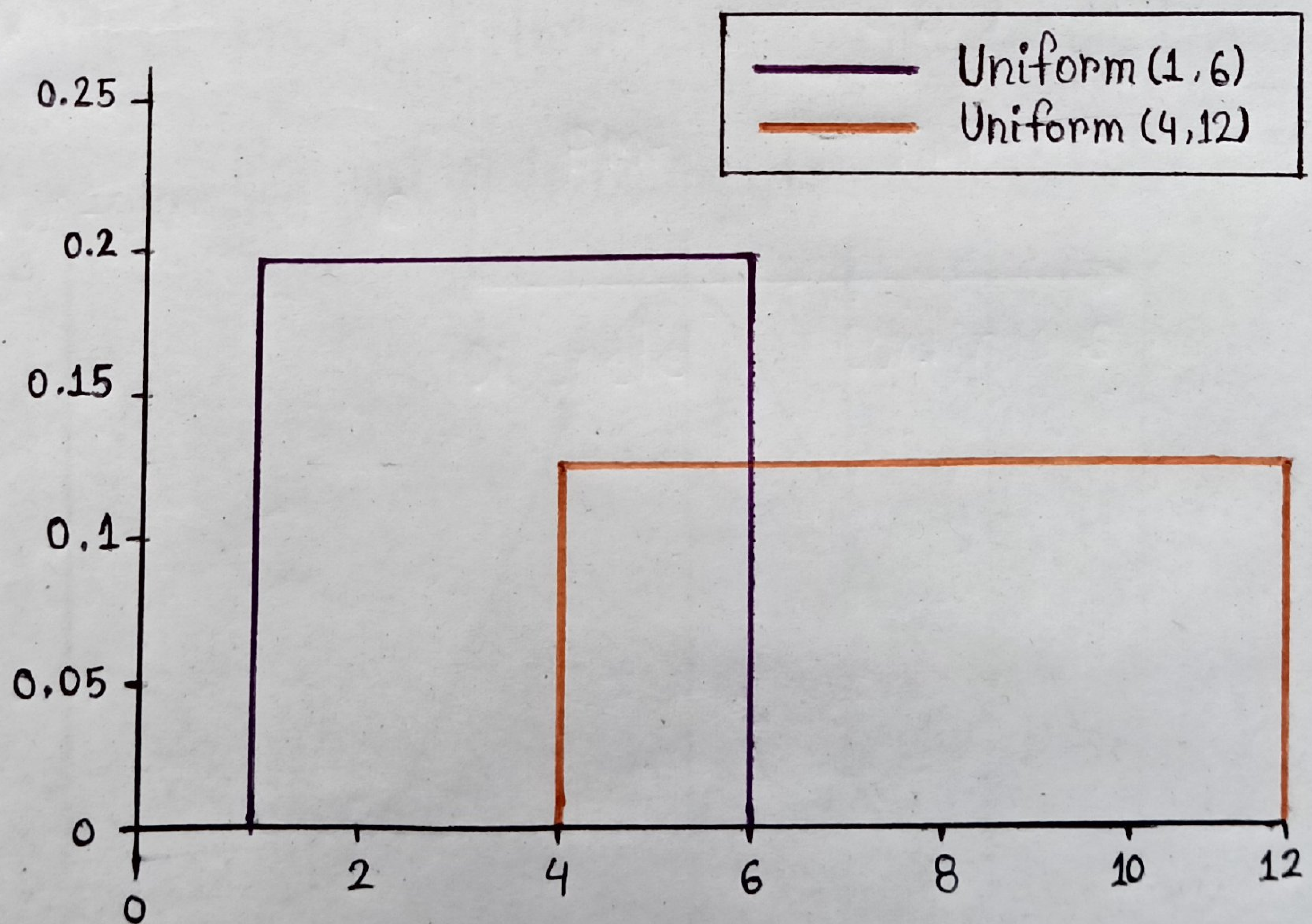
If X is the height of a person chosen at random, then F(x) is the probability of the individual being shorter than x. If F(180 cm) = 0.8, then an individual chosen at random has an 80% chance of being shorter than 180 cm (equivalently, a 20 percent chance that they will be taller than 180 cm).



Continuous Probability Distribution

1) **Uniform Distribution:** Uniform distribution is a sort of probability distribution in statistics in which all events are equally likely. Because the chances of drawing a heart, a club, a diamond, or a spade are equal, a deck of cards contains uniform distributions. Because the likelihood of receiving heads or tails in a coin toss is the Same, a coin has a uniform distribution.

A coin flip that returns a head or tail has a probability of $p = 0.50$ and would be represented by a line from the y-axis at 0.50.



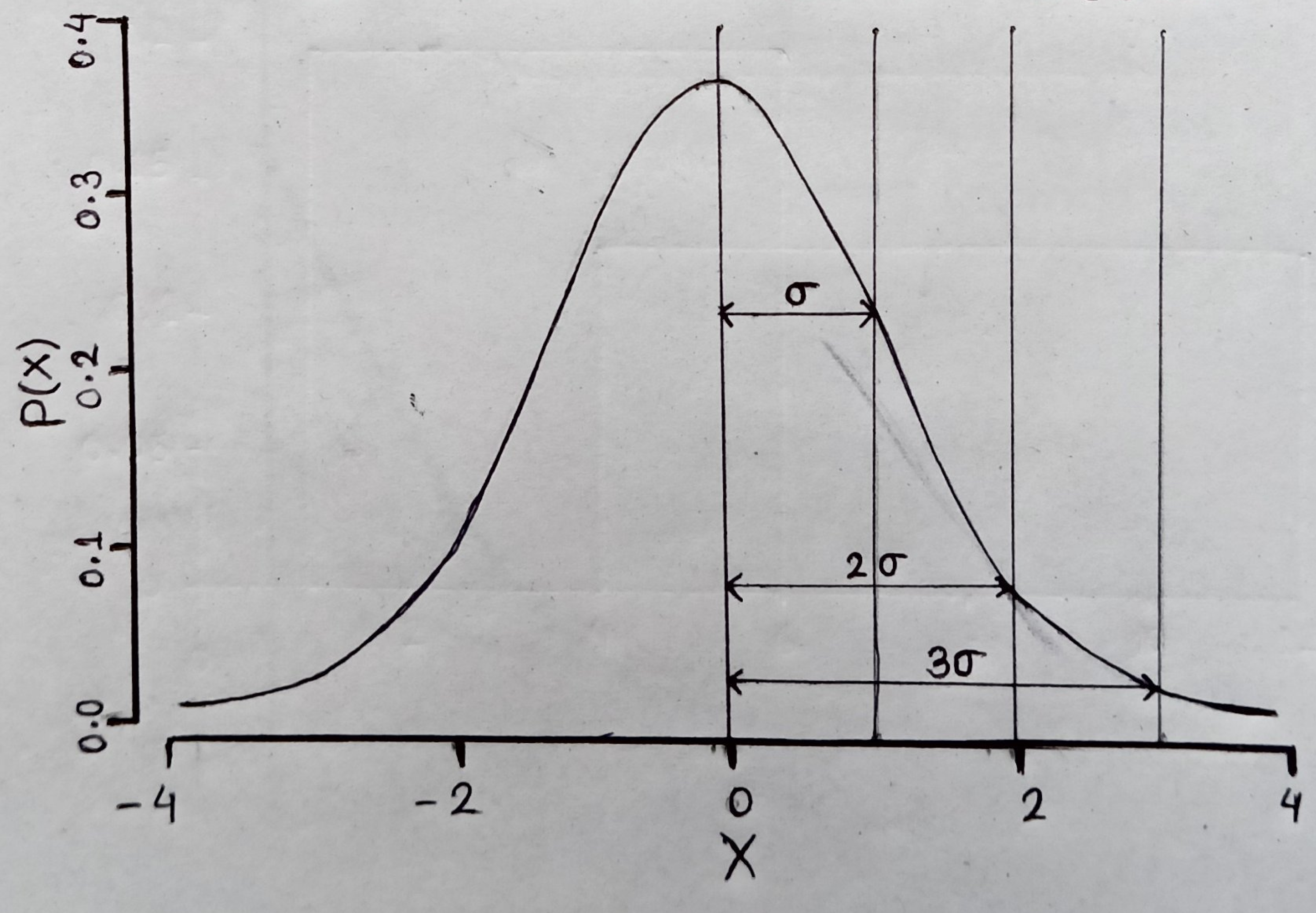
2) Normal/Gaussian Distribution :

The normal distribution, also known as the Gaussian distribution, is a symmetric probability distribution centred on the mean, indicating that data around the mean occur more frequently than data far from it. The normal distribution will show as a bell curve on a graph.

Points to remember :-

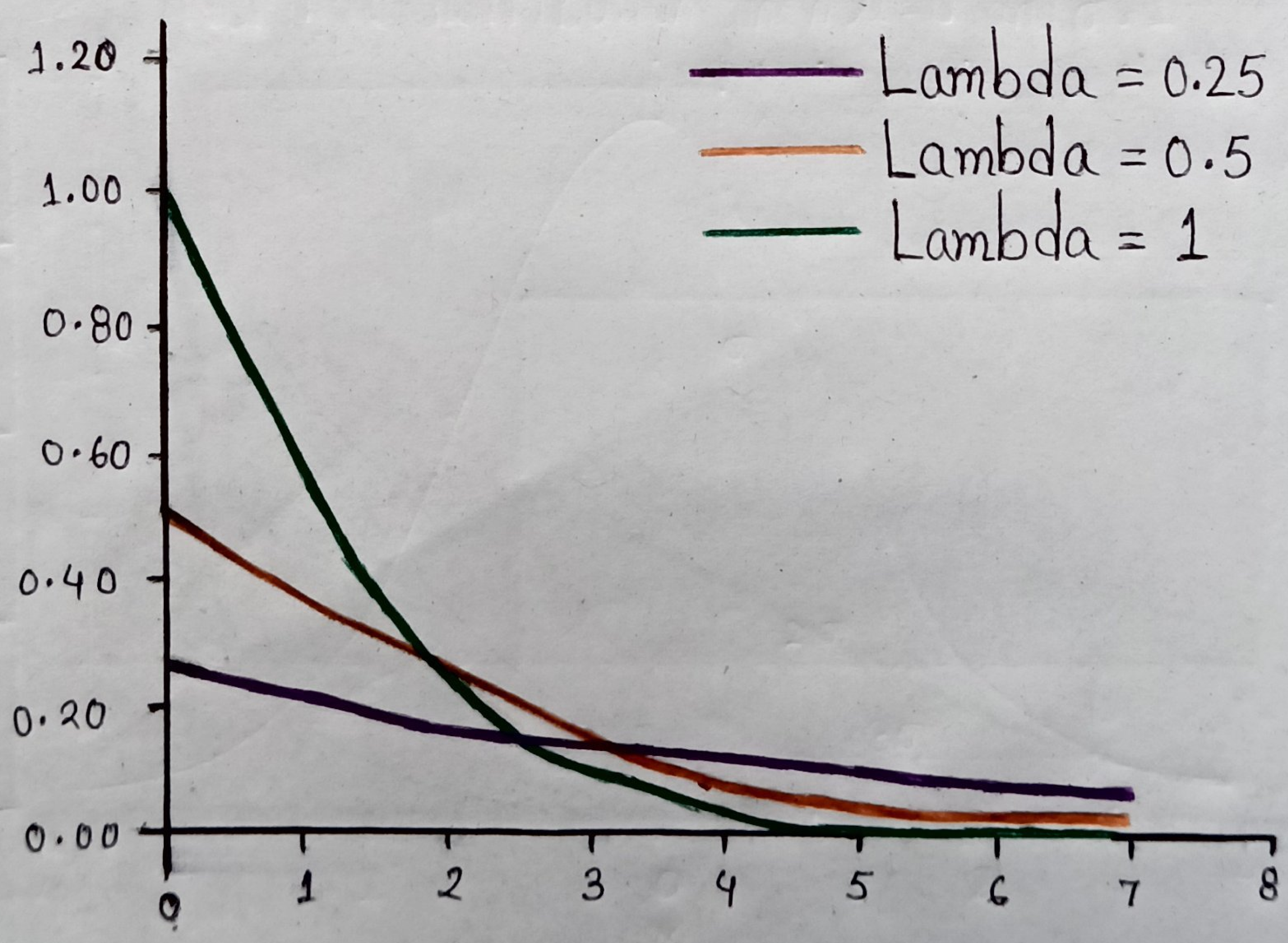
- A probability bell curve is referred to as a normal distribution.
- Although all symmetrical distributions are normal, not all normal distributions are symmetrical.
- Most pricing distributions aren't totally typical.

Normal distribution when $\mu = 0$ and $\sigma = 1$

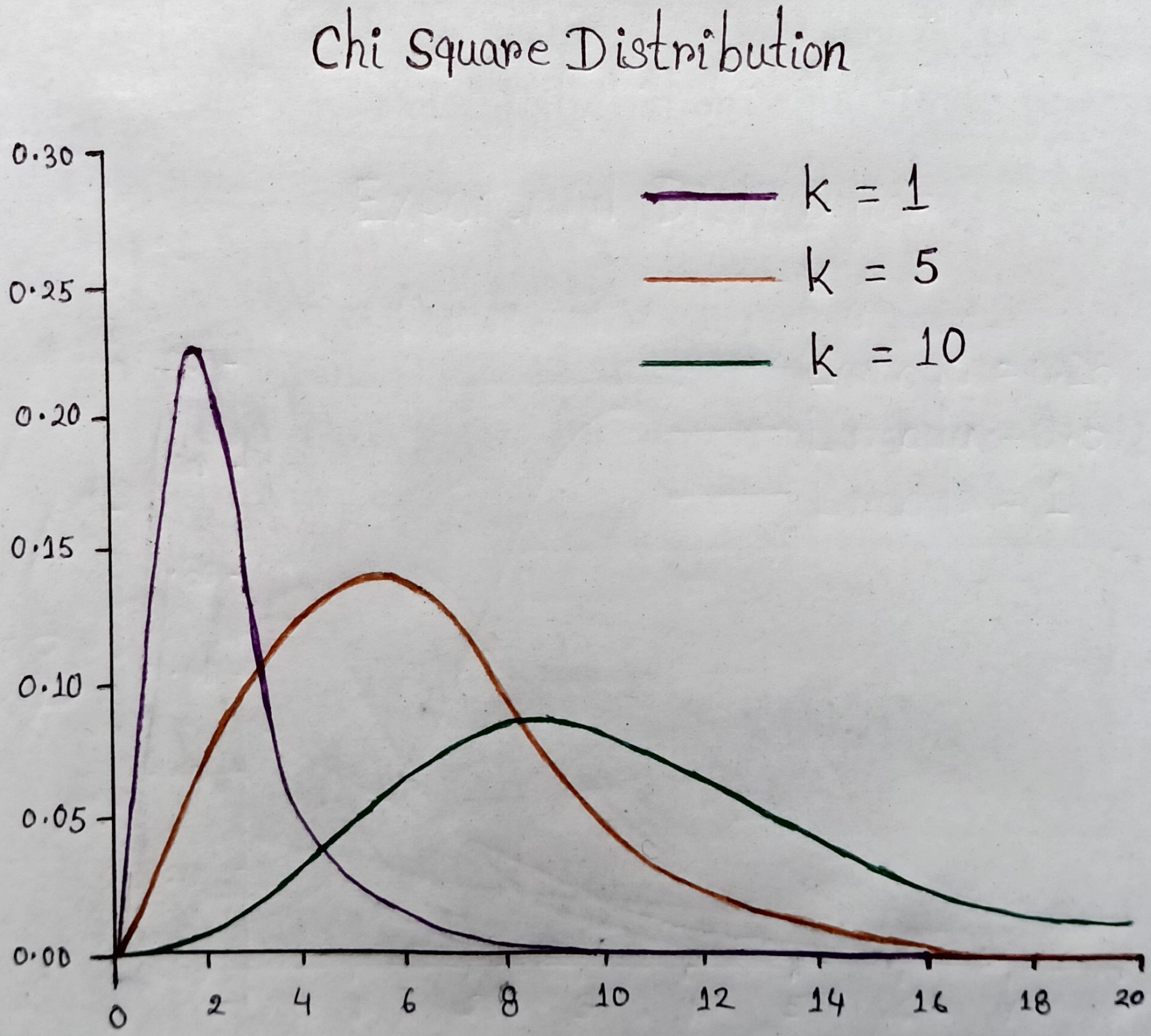


3) Exponential Distribution: The exponential distribution is a continuous used to estimate the time it will take for an event to occur. For example, in physics, it is frequently used to calculate radioactive decay, in engineering, it is frequently used to calculate the time required to receive a defective part on an assembly line, and in finance, it is frequently used to calculate the likelihood of a portfolio of financial assets defaulting. It can also be used to estimate the likelihood of a certain number of defaults occurring within a certain time frame.

Exponential Distribution



4) **Chi Square Distribution :** The Chi Square Distribution describes a sum of squared random variable's distribution. It's also used to determine whether a data distribution's goodness of fit is good, whether data series are independent, and to estimate confidence intervals around variance and standard deviation for a random variable from a normal distribution. Furthermore, the chi-square distribution is a subset of the gamma distribution.



Discrete Probability Distribution

1) **Bernoulli Distribution:** A Bernoulli distribution is a discrete probability distribution for a Bernoulli trial, which is a random experiment with just two outcomes (named "Success" or "Failure" in most cases). When flipping a coin, the likelihood of getting ahead (a "success") is 0.5. "Failure" has a chance of 1-P. (where p is the probability of success, which also equals 0.5 for a coin toss). For $n=1$, it is a particular case of the binomial distribution. In other words, it's a single-trial binomial distribution (e.g. a single coin toss).

BERNOULLI DISTRIBUTION

- A Bernoulli trial is an experiment with only two outcomes. An r.v. X has Bernoulli(P) distribution if

$$X = \begin{cases} 1 \text{ with probability } p \\ 0 \text{ with probability } 1-p \end{cases} ; 0 \le P \le 1$$

$$P(X=x) = p^x (1-p)^{1-x} \text{ for } x = 0, 1; \text{and } 0 < P < 1$$

2) Binomial Distribution: A binomial distribution can be thought of as simply the probability of a SUCCESS or FAILURE outcome in an experiment or survey that is repeated multiple times. The binomial is a type of distribution that has two possible outcomes (the prefix "bi" means two, or twice). For example, a coin toss has only two possible outcomes: heads or tails and taking a test could have two possible outcomes: pass or fail.

Binomial distribution has three criteria:

- The number of observations or trials is fixed.
- Each observation or trial is independent.
- The probability of success (tails, heads, fail or pass) is exactly the same from one trial to another.

Binomial Distribution Formula

$$P(X) = {}_nC_x\, p^x (1-p)^{n-x}$$

- n = Total number of events.
- r (or) x = Total number of successful events.
- p = Probability of success on a single trial.
- nCr = [n!/r! (n-r)]!
- 1-p = Probability of failure.

3) **Poisson Distribution :** A Poisson distribution is a probability distribution used in statistics to show how many times an event is expected to happen over a certain amount of time. To put it another way, it's a count distribution. Poisson distributions are frequently accustomed comperhend independent events that occur at a gradual rate during a selected timeframe. The Poisson distribution is a discrete function, which means the variable can only take values from a (possibly endless) list of possibilities. To put it another way, the variable can't take all of the possible values in any continuous range. The Variable can only take the values 0, 1, 2, 3, etc., with no fractions or decimals, in the Poisson distribution (a discrete distribution).

Poisson Distribution Formula

$$P(X=x) = \frac{\lambda^x e^{-\lambda}}{x!}$$

Where

- $x = 0, 1, 2, 3, \ldots\ldots$
- λ = mean number of occurrences in the interval.
- e = Euler's constant ≈ 2.71828