T Distribution

The T distribution (also called Student's T Distribution) is a family of distributions that look almost identical to the normal distribution curve, only a bit shorter and fatter. The t distribution is used instead of the normal distribution when you have small samples. (for more on this, see our example) The larger the sample size, the more the t distribution looks like the normal distribution. In fact, for sample sizes larger than 20, the distribution is almost exactly like the normal distribution.

How to Calculate the Score for a T Distribution

When you look at the t-distribution tables, you'll see that you need to know the "df". This means "degrees of freedom" and is just the sample size minus one.

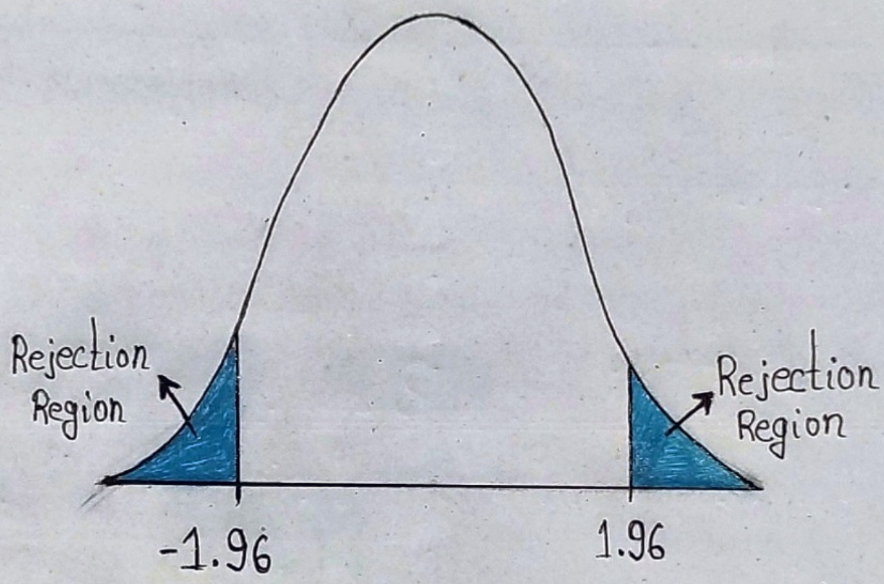
Step 1 : Subtract one from your sample size. This will be your degrees of freedom.

Step 2 : Look up the df in the left hand side of the t-distribution table. Locate the column under your alpha level (the alpha level is usually given to you in the note).

Uses

The T Distribution (and the associated t scores), are used in hypothesis testing when you want to figure out if you should accept or reject the null hypothesis.

The central region on this graph is the acceptance area and the tail is the rejection region, or regions. In this particular graph of a two tailed test, the rejection region is shaded blue. The area in the tail can be described with z-score or t-scores. For example, the image to the below shows an area in the tails of 5% (2.5% each side).



The z-score would be 1.96 (from the z-table), which represents 1.96 standard deviations from the mean. The null hypothesis will be rejected if z is less than -1.96 or greater than 1.96.

In general, this distribution is used when you have a small sample size (under 30) or you don't know the population standard deviation. For practical purposes (i.e. in real world), this is nearly always the case. So, unlike in your elementary statistics class, you'll likely be using it in real life situations more than the normal distribution. If the size of your sample is large enough, the two distributions are practically the same.

T Score Formula

A t-score is one form of a standardized test statistic. The t-score formula enables you to take an individual score and transform it into a standardized form > one which helps you to compare scores.
you'll want to use the t-score formula when you don't know the population standard deviation and you have a small sample (under 30).

The t score formula is :

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

Where
$\bar{x}$ = Sample mean
μ_0 = population mean
s = sample standard deviation
n = Sample size

If you have only one item in your sample, the square root in the denominator becomes $\sqrt{1}$. This means the formula becomes :

$$t = \frac{\bar{x} - \mu_0}{s}$$

In simple terms, the larger the t score, the larger the difference is between the groups you are testing. It's influenced by many factors including:

- How many items are in your sample.

- The means of your sample.
- The means of the population from which your sample is drawn.
- The standard deviation of your sample.

Uses

1. The upper and lower bounds of a confidence interval when the data are approximately normally distributed.

2. The p-value of the test statistic for t-tests and regression tests.

Example : Calculating the p-value from t-test

Problem : Bob wants to know if the mean height of a certain species of plant is equal to 15 inches. To test this, he collects a random sample of 20 plants and finds that the sample mean is 14 inches and the sample standard deviation is 3 inches. Conduct a t-test using a .05 alpha level of determine if the true mean height for the population is actually 15 inches.

Solution :

Step 1 : State the null and alternative hypotheses.

$H_0 : \mu = 15$

$H_a : \mu \neq 15$

Step 2 : Find the test statistic.

$$t = (\bar{x} - \mu) / (s/\sqrt{n}) = (14 - 15) / (3/\sqrt{20}) = -1.49$$

Step 3: Find the p-value for the test statistic.
To find the p-value by t-score, we need to use the t-distribution table with n-1 degrees of freedom. In our example, our sample size in n = 20, so n-1 = 19.

In the t-distribution table, we need to look at the row that corresponds to "19" on the left-hand side and attempt to look for the absolute value of our test statistic 1.49.

Notice that 1.49 does not show up in the table, but it does not fall between the two values 1.328 and 1.729.

We can look at the two alpha levels at the top of the table that correspond to these two number (1.328, 1.729). The alpha levels are 0.1 and 0.5.

This means that the p-value for a one-sided test is between 0.1 and 0.5. Let's call it .075. Since our t-test is two-sided, we need to multiply this value by 2. So, our estimated p.value is .075 * 2 = 0.15.

Step 4: Draw a conclusion.
Since this p-value is not less than our chosen alpha level of 0.5, we can't reject the null hypothesis. Thus, we don't have sufficient evidence to say that the true mean height of this species of plant is different than 15 inches.