Indian Institute of Information Technology Guwahati

MA203: Probability and Random Processes **Problem Sheet-I on Probability**

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(1) Consider an experiment of throwing two unbiased dice. Construct the sample space and a σ-field corresponding to the events described as follows:
E_1: both the dice show same number
E_2: sum of points on two dice is even
E_3: sum of points on two dice is divisible by 3
E_4: sum is greater than or equal to 2 and is less than or equal to 12.

(2) A fair coin is tossed repeatedly until the first head appears. We note the number of tosses required to get the first head. Construct a probability space for the random experiment.

(3) Suppose that the lifetime of a computer memory chip is measured, and found that the proportion of chips whose lifetime exceeds t decreases exponentially at a rate α. Construct a probability space for the random experiment.

(4) Let $A, B,$ and C be three mutually exclusive and exhaustive events associated with a random experiment such that $P(B) = \frac{3}{2}P(A)$ and $P(C) = \frac{1}{2}P(B)$. Find $P(A)$. [Ans: $\frac{4}{13}$]

(5) Let $(S,\ \mathcal{F},\ P)$ be a probability space. Let $A, B \in \mathcal{F}$ be such that $P(A) = P(B) = \dfrac{2}{5}, P(A\cup B) = \dfrac{1}{2}$. Find $P(A \cap B)$ and $P(A \setminus B)$. [Ans: 3/10 and 1/10]

(6) A number x is selected at random in the interval $[-1, 1]$. For the events $A = \{x < 0\}, B = \{|x - 0.5| < 1\}$, and $C = \{x \geq 0.75\}$, find probabilities of B, $A \cap C$, and $A \cup B \cup C$. [Ans: 3/4, 0, 1]

(7) The sum of two non-negative quantities is equal to $2n$. Find the probability that the product of these two numbers is not less than $\frac{3}{4}$ times their greatest product. [Ans: 1/2]

(8) Given a class of $n \leq 365$ students, what is the probability of two students in the class having the same birthday? [Ans: $1 - \prod_{i=1}^{n-1}(1 - \frac{i}{365})$]

(9) Let a point is picked at random from the unit square $[0,\ 1] \times [0,\ 1]$. Find the probability that it is in the triangle bounded by $x = 1,\ y = 1$ and $x + y = 1$. [Ans: 1/2]

(10) A committee of 4 people is to be appointed from 3 officers from department A, 4 from deptt. B, 2 from deptt. C and 1 from D. Write down the sample space for this experiment along with the events and their probabilities in following cases:
  (a) There must be one from each deptt.
  (b) It should have at least one from the deptt. B.
  (c) A person from deptt. D must be in the committee.

(11) In a random arrangement of the letters of the word 'COMMERCE', find the probability that all vowels come together. [Ans: 3/28]

(12) An urn contains 6 red balls, 5 green balls and 4 blue balls. 9 balls were picked at random from the urn without replacement. What is the probability that out of the balls 4 are red, 3 are green and 2 are blue? [Ans: $\frac{{}^6C_4 \times {}^5C_3 \times {}^4C_2}{{}^{15}C_9}$]

(13) An urn contains 6 white, 4 red and 9 black balls. If three balled are drawn at random, find the probability that (a) two of the balls drawn are white (b) one is of each color (c) none is red (d) at least one is white.

(14) Let $S = \{0, 1, 2, \ldots\}$ be a sample space. Let $F = P(S)$. In each of the following cases, verify if $P(.)$ is a probability.
  (a) $P(A) = \sum_{x\in A} \frac{e^{-\lambda}\lambda^x}{x!}$ for $A \in F, \lambda > 0$.
  (b) $P(A) = \sum_{x\in A} p(1-p)^x$ for $A \in F, 0 < p < 1$.
  (c) $P(A) = 0$, if A has a finite number of elements, and $P(A) = 1$, if A has infinite number of elements, $A \in F$.

(15) Suppose (S, F, P) is a probability space and $A, B \in F$ be two mutually exclusive events. If $P(A) = 0.5$ and $P(B) = 0.3$, find the probabilities that (a) either A or B occurs (b) only A occurs (c) both A and B occurs (d) neither A nor B occurs. [Ans: (a) 0.8 (b) 0.5 (c) 0 (d) 1]

(16) In a communication system, data are transmitted randomly in groups of 8 bits called byte. If each bit is either '1' or '0' and there is no loss in the transmitting medium, what is the probability that a received byte contains at least two '1's. [Ans: 247/256]

(17) Three numbers are chosen at random and without replacement from the set $\{1, 2, ..., 50\}$. Find the probability that the chosen numbers are in (a) arithmetic progression, and (b) geometric progression. [Ans: (a) 3/100 (b) 23/19600]

(18) Let (S, F, P) be a probability space and let $A, B \in F$. Show that $P(A \cap B) - P(A)P(B) = P(A)P(B^c) - P(A \cap B^c) = P(A^c)P(B) - P(A^c \cap B) = P((A \cup B)^c) - P(A^c)P(B^c)$.

(19) Let $\{E_n\}_{n \geq 1}$ be an increasing sequence of events, then show that $P(\lim_{n\to\infty} E_n) = \lim_{n\to\infty} P(E_n)$, where $\lim_{n\to\infty} E_n = \cup_{n=1}^{\infty} E_n$.