Introduction to Logic

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August 10, 2020

Converse, Contrapositive and Inverse

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p	q	$p \to q$	$\neg q$	$\neg p$	$\neg q \to \neg p$
T	T	T	F	F	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

When two compound propositions always have the same truth value we call them equivalent, so that a conditional statement and its contrapositive are equivalent.

- Check whether the converse and the inverse of a conditional statement are equivalent.
- Is any of it equivalent to the original conditional statement?

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- Contrapositive: If the home team does not win, then it is not raining.
- Converse : If the home team wins, then it is raining.
- Inverse : If it is not raining, then the home team does not win.
  Only the contrapositive is equivalent to the original statement.

Biconditional

Definition

Let p and q be propositions. The biconditional statement $p \leftrightarrow q$ is the proposition p if and only if (iff) q. The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called bi-implications.

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$p \rightarrow q$	$q \rightarrow p$	$p \rightarrow q \wedge q \rightarrow p$
T	T	T
F	T	F
T	F	F
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Note that $p \leftrightarrow q$ has exactly the same truth value as $(p \rightarrow q) \wedge (q \rightarrow p)$ and hence equivalent.

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- p is necessary and sufficient for q.
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- if p then q, and conversely.
- q iff p.

Let
p :You can take the flight
q : You buy a ticket.
Then $p \leftrightarrow q$ is the statement:
You can take the flight if and only if you buy a ticket.

* Poll

Determine whether these biconditionals are true or false.

a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.

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b)$1 + 1 = 2$ if and only if $2 + 3 = 4$.
c) $1 + 1 = 3$ if and only if monkeys can fly.
d) $0 > 1$ if and only if $2 > 1$.

Truth tables of Compound Propositions

Exercise:

1. Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$.

Precedence of Logical Operators : We will generally use parentheses to specify the order in which logical operators in a compound proposition are to be applied. For instance, $(p \vee q) \wedge (\neg r)$ is the conjunction of $p \vee q$ and $\neg r$.

Order of precedence of Logical Operators is $\neg, \wedge, \vee, \rightarrow, \leftrightarrow$.

$\neg p \vee q$ is the conjunction of $\neg p$ and q, namely, $(\neg p) \vee q$, not the negation of the conjunction of p and q, namely $\neg(p \vee q)$.

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e) If it is below freezing, it is also snowing.

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f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
g) That it is below freezing is necessary and sufficient for it to be snowing.

2. Let p, q, and r be the propositions.
p :You have the flu.
q :You miss the final examination.
r :You pass the course.
Express each of these propositions as an English sentence.
(a). $p \to q$
(b). $\neg q \leftrightarrow r$
(c). $q \to \neg r$
(d). $(p \to \neg r) \lor (q \to \neg r)$
(e). $(p \land q) \lor (\neg q \land r)$.

3. Write each of these propositions in the form p if and only if q in English.
a) For you to get an A in this course, it is necessary and sufficient that you learn how to solve discrete mathematics problems.
b) If you read the newspaper every day, you will be informed, and conversely.
c) It rains if it is a weekend day, and it is a weekend day if it rains.
d) You can see the wizard only if the wizard is not in, and the wizard is not in only if you can see him.

4. State the converse, contrapositive, and inverse of each of these conditional statements.
a) If it snows today, I will ski tomorrow.
b) I come to class whenever there is going to be a quiz.
c) A positive integer is a prime only if it has no divisors other than 1 and itself.