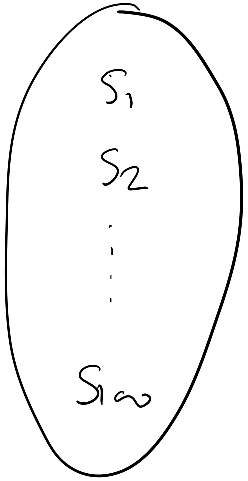
MA 203

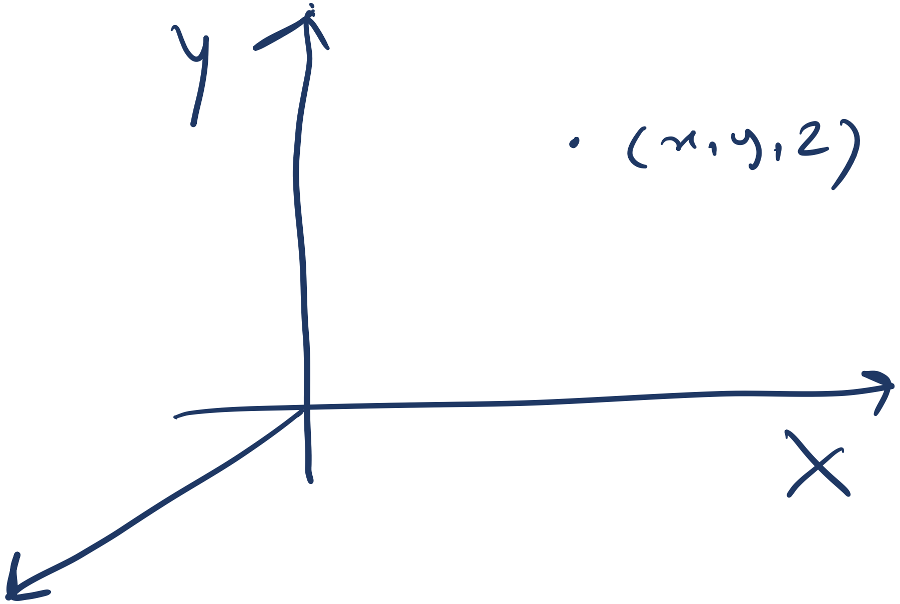
Joint Random Variable

X: Weight: 50 – 50 kg

Y: Height: 5 Feet 4 inch – 5 Feet 6 inch

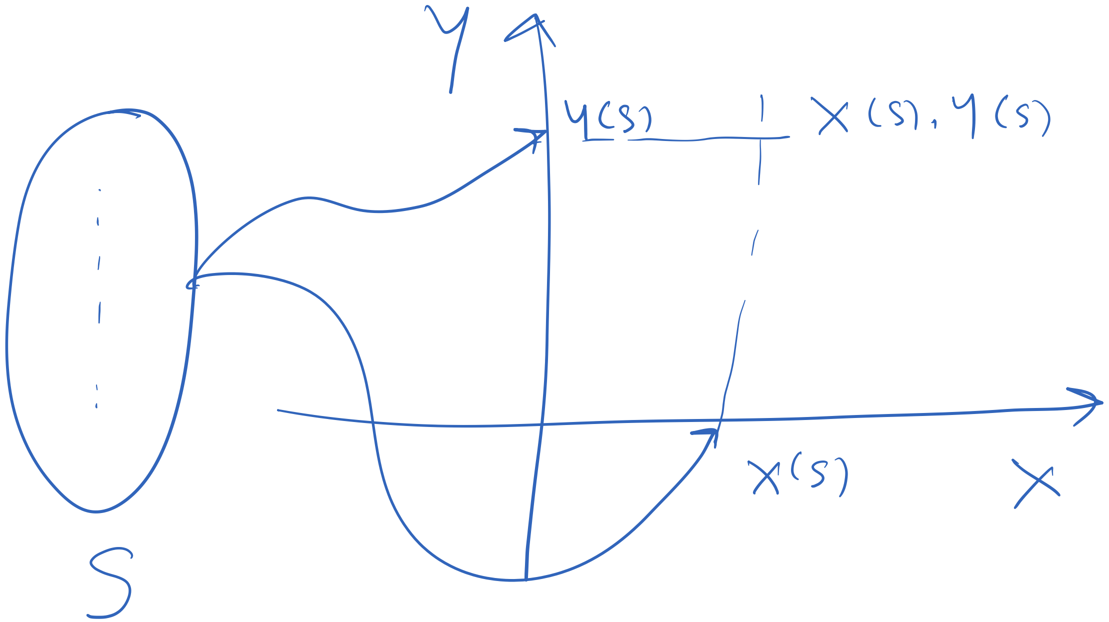


Require two RVs, One for height and one for Weight



Joint Random Variable

Let X and Y be two real random variables (RVs) defined on the same probability space (S, *F*, P) such that $X: S \to R_X$ and $Y: S \to R_Y$ where $R_X \subseteq \mathbb{R}$ and $R_Y \subseteq \mathbb{R}$. Then (X,Y) called a joint RV if for $s \in S$, $(X(s), Y(s)) \in R_X \times R_Y$.

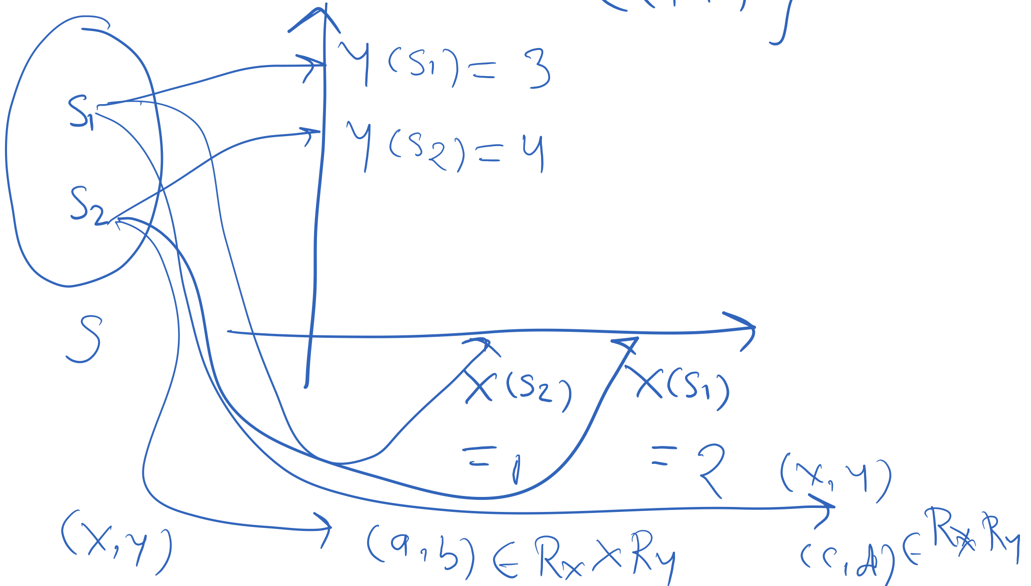


$(X, Y): S \longrightarrow R_X \times R_Y$

Suppose:

$R_X = \{1, 2\}$; $R_Y = \{3, 4\}$

$R_X \times R_Y = \{(1,2), (1,3), (2,3), (2,4)\}$



Notation:

1. (X, Y)

$X_1, X_2, \ldots, X_n$ — n RVs

$(X_1, X_2, \ldots, X_n)$

2. $T = \begin{bmatrix} X \\ Y \end{bmatrix}$ T: Random Vector
2- Dimensional

$T = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_n \end{bmatrix}$ T: Random Vector
n- dimensional

Joint Distribution Function

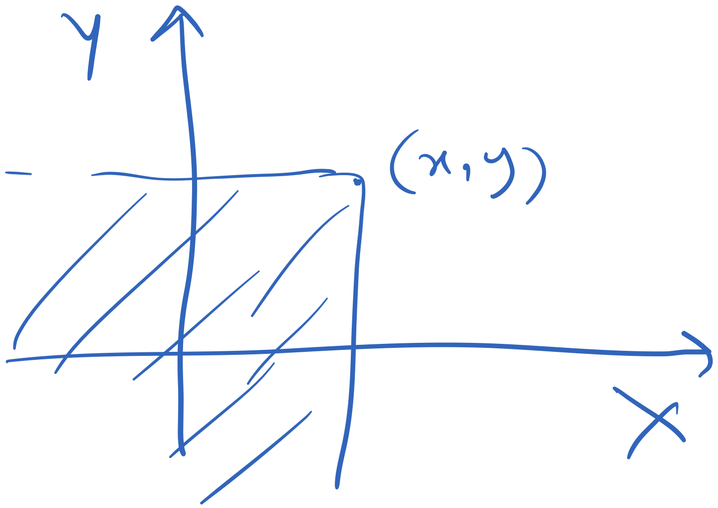
Case 1: When X and Y are continuous RVs

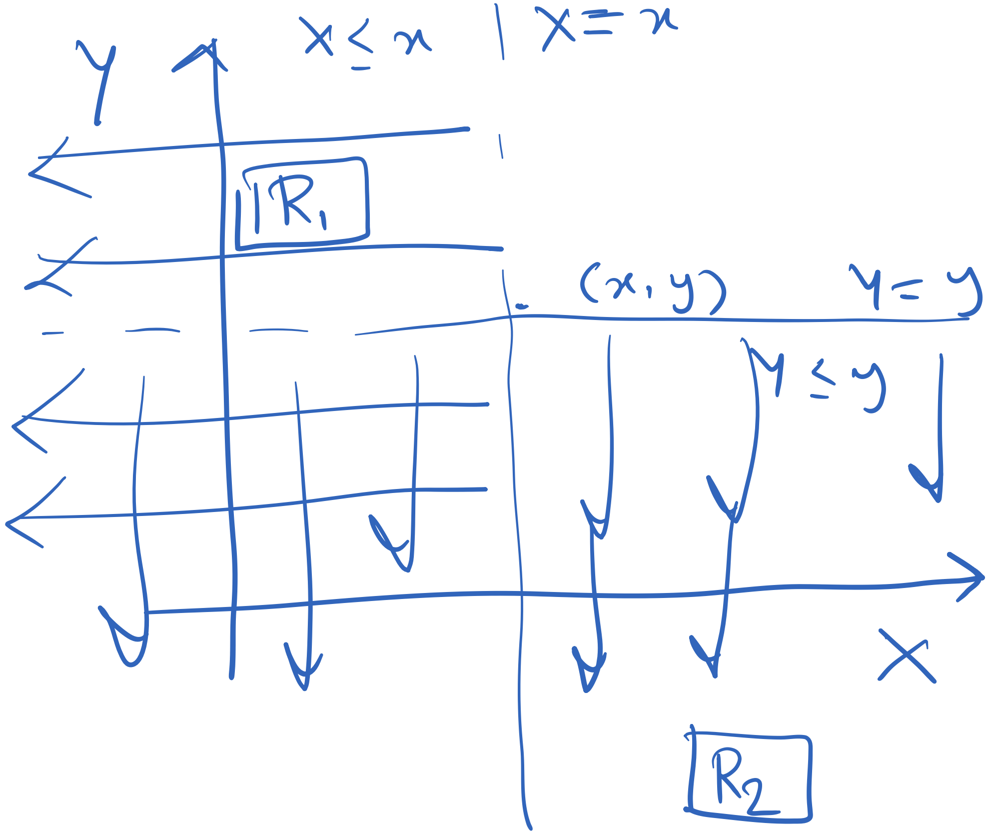
The joint cumulative distribution function (CDF) of X and Y is defined as

$$F_{X,Y}(x, y) = P(X \le x, Y \le y).$$

$X \le x$; Event X lies in R_1

$Y \le y$; Event Y lies in R_2





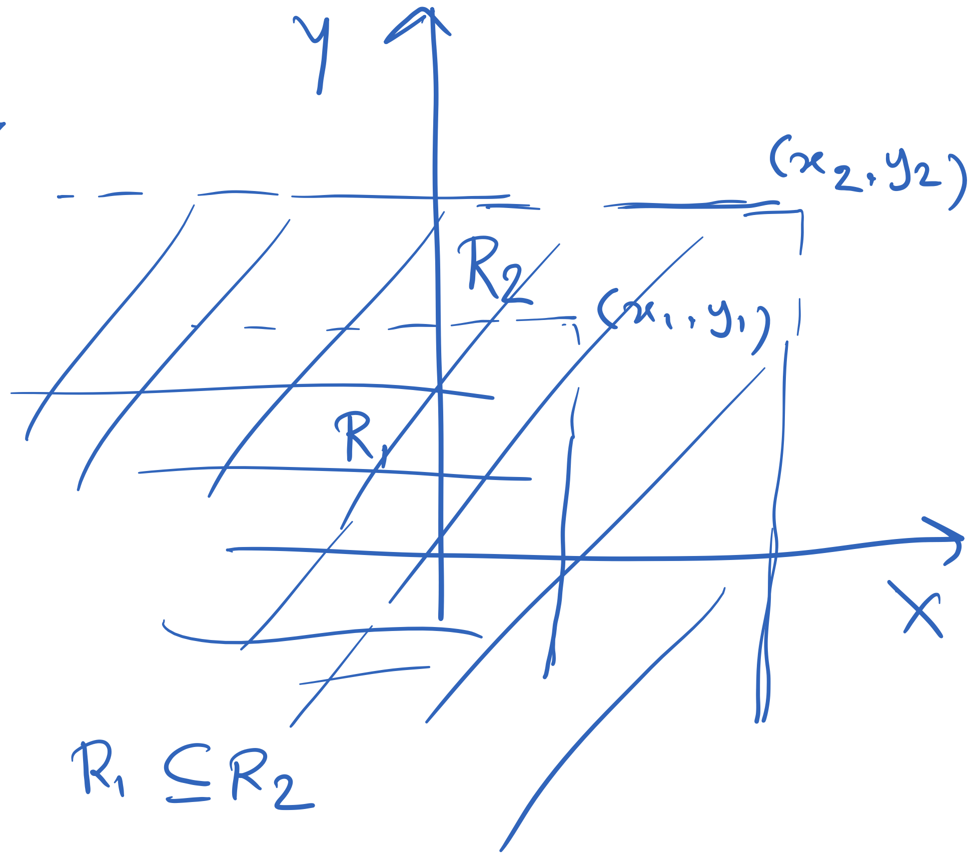
Properties of Joint CDF

1. $F_{X,Y}(x_1, y_1) \leq F_{X,Y}(x_2, y_2)$ if $x_1 \leq x_2$ and $y_1 \leq y_2$.

Proof: $\{X \leq x_1, Y \leq y_1\} \subseteq \{X \leq x_2, Y \leq y_2\}$

$P\{X \leq x_1, Y \leq y_1\} \leq P\{X \leq x_2, Y \leq y_2\}$

$F_{X,Y}(x_1, y_1) \leq F_{X,Y}(x_2, y_2)$



$R_1 \subseteq R_2$

2. $F_{X,Y}(x, -\infty) = 0$ $= P\{X \leq x, \underbrace{Y \leq -\infty}\}$

Impossible Event

3. $F_{X,Y}(\infty, \infty) = 1$ $= P(X \leq \infty, Y \leq \infty)$

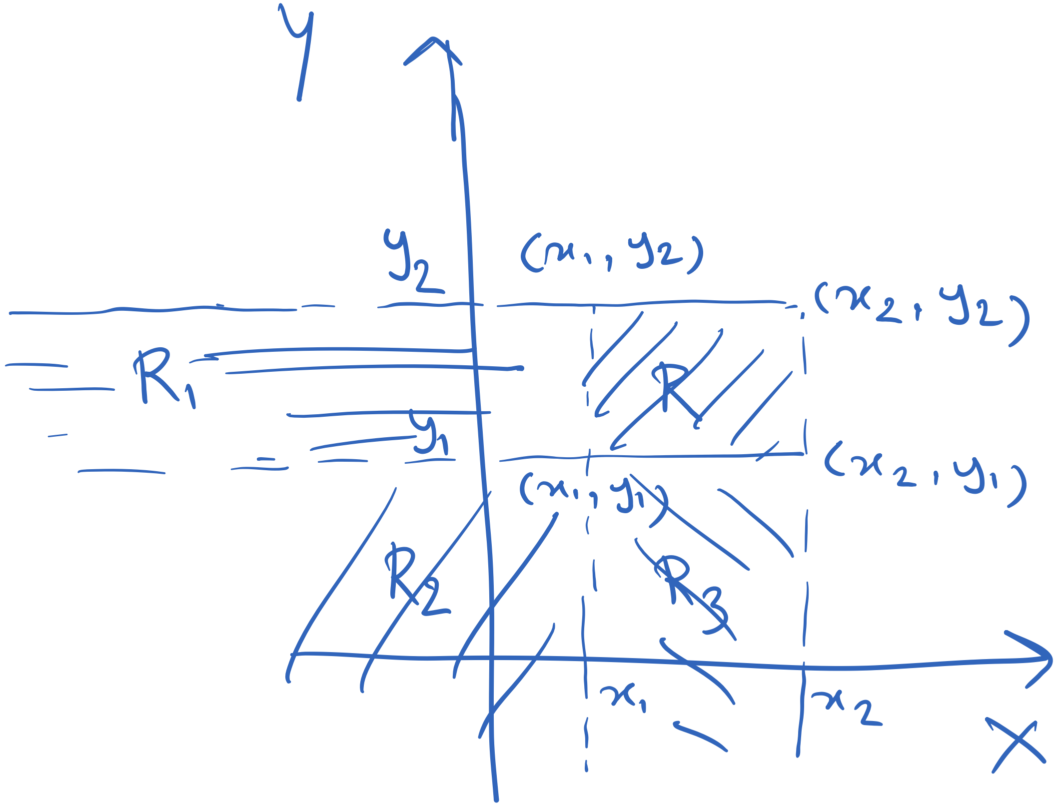
4. If $x_1 < x_2$ and $y_1 < y_2$, then

$$P(x_1 < X \leq x_2, y_1 < Y \leq y_2) = F_{X,Y}(x_2, y_2) - F_{X,Y}(x_1, y_2) - F_{X,Y}(x_2, y_1) + F_{X,Y}(x_1, y_1).$$

Proof: $$F_{X,Y}(x_2,y_2) - \underbrace{\{F_{X,Y}(x_1,y_2) - F_{X,Y}(x_1,y_1)\}}_{R_1}$$

$$- \underbrace{\{F_{X,Y}(x_2,y_1) - F_{X,Y}(x_1,y_1)\}}_{R_3}$$

$$- \underbrace{F_{X,Y}(x_1,y_1)}_{R_2}$$



$$= F_{X,Y}(x_2,y_2) - F_{X,Y}(x_1,y_2)$$
$$- F_{X,Y}(x_2,y_1)$$
$$+ F_{X,Y}(x_1,y_1)$$

5. $F_X(x) = F_{X,Y}(x, +\infty)$

<u>Proof:</u> $F_X(x) = P(X \le x)$

$= P(X \le x, Y \le \infty)$

$= F_{X,Y}(x, \infty)$