Introduction to Logic

Department of Science and Mathematics

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August 20, 2020

Translating from English into Logical Expressions

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What does $\forall x \ (S(x) \wedge C(x))$ represent?

Using two or more variables

When we are interested in the background of people in subjects besides calculus, we may prefer to use the two-variable quantifier $Q(x, y)$ for the statement "student x has studied subject y." Then we would replace $C(x)$ by $Q(x, \mathit{calculus})$ in both approaches to obtain $\forall x\ Q(x, \mathit{calculus})$ or $\forall x(S(x) \rightarrow Q(x, \mathit{calculus}))$.

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"There is a person x having the properties that x is a student in this class and x has visited Delhi."

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$\forall x\ (S(x) \rightarrow (G(x) \vee D(x)))$.
Note: We could use a two-place predicate $Q(x, y)$ to represent "x has visited y." Useful when more number of places are used.

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- Denote $S(m, y)$: Mail m is larger than y mega bytes and $C(m)$: Mail m will be compressed.
- Domain of m is the collection of all mails and y is a positive real number.
- $\forall m(S(m, 1) \to C(m))$

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- Let $S(n, x)$ denote "Network link n is in state x," where n has the domain of all network links and x has the domain of all possible states for a network link.
- $\exists u\ A(u) \rightarrow \exists n\ S(n, \text{available})$.

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Let $P(x)$: x is a lion, $Q(x)$: x is fierce and $R(x)$: x drinks coffee, respectively.

- $\forall x\ (P(x) \to Q(x))$.

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- $\forall x\ (P(x) \rightarrow Q(x))$.
- $\exists x (P(x) \land \neg R(x))$.

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- $\exists x(P(x) \wedge \neg R(x))$.
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- $\forall x\ (P(x) \to Q(x))$.
- $\exists x(P(x) \wedge \neg R(x))$.
- $\exists(Q(x) \wedge \neg R(x))$.

Compare the statements $\exists x(P(x) \wedge \neg R(x))$ and $\exists x(P(x) \to \neg R(x))$. (If there exists any creature (not necessarily lion) that does not drink cofee)

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Let $P(x)$: x is a hummingbird, $Q(x)$: x is large, $R(x)$: x lives on honey, and $S(x)$: x is richly colored, respectively.

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- $\forall x(P(x) \rightarrow R(x))$
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  For every real number x there is a real number y such that $x + y = 0$. (Existence of additive inverse of x)

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  For every real number x there is a real number y such that $x + y = 0$. (Existence of additive inverse of x)
- $\forall x \forall y \forall z (x + (y + z) = (x + y) + z)$ is the associative law for addition of real numbers

Translate into English the statement $\forall x \forall y((x > 0) \wedge (y < 0) \rightarrow (xy < 0))$, where the domain for both variables consists of all real numbers.

Translate into English the statement $\forall x \forall y((x > 0) \land (y < 0) \rightarrow (xy < 0))$, where the domain for both variables consists of all real numbers.
Solution: This statement says that for every real number x and for every real number y, if $x > 0$ and $y < 0$, then $xy < 0$.

Translate into English the statement $\forall x \forall y((x > 0) \wedge (y < 0) \rightarrow (xy < 0))$, where the domain for both variables consists of all real numbers.
Solution: This statement says that for every real number x and for every real number y, if $x > 0$ and $y < 0$, then $xy < 0$. That is, this statement says that for real numbers x and y, if x is positive and y is negative, then xy is negative.

Translate into English the statement $\forall x \forall y((x > 0) \land (y < 0) \to (xy < 0))$, where the domain for both variables consists of all real numbers.
Solution: This statement says that for every real number x and for every real number y, if $x > 0$ and $y < 0$, then $xy < 0$. That is, this statement says that for real numbers x and y, if x is positive and y is negative, then xy is negative. This can be stated more clearly as "The product of a positive real number and a negative real number is always a negative real number."

Quantification as loops

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To see whether $\forall x \forall y P(x, y)$ is true, we loop through the values for x, and for each x we loop through the values for y. If we find that $P(x, y)$ is true for all values for x and y, we have determined that $\forall x \forall y P(x, y)$ is true. If we ever hit a value x for which we hit a value y for which $P(x, y)$ is false, we have shown that $\forall x \forall y P(x, y)$ is false.

Similarly, to determine whether $\forall x \exists y P(x, y)$ is true, we loop through the values for x. For each x we loop through the values for y until we find a y for which $P(x, y)$ is true. If for every x we hit such a y, then $\forall x \exists y P(x, y)$ is true; if for some x we never hit such a y, then $\forall x \exists P(x, y)$ is false.

To see whether $\exists x \forall y P(x, y)$ is true, we loop through the values for x until we find an x for which $P(x, y)$ is always true when we loop through all values for y. Once we find such an x, we knowthat $\exists x \forall y P(x, y)$ is true. If we never hit such an x, then we know that $\exists x \forall y P(x, y)$ is false.

To see whether $\exists x \forall y P(x, y)$ is true, we loop through the values for x until we find an x for which $P(x, y)$ is always true when we loop through all values for y. Once we find such an x, we knowthat $\exists x \forall y P(x, y)$ is true. If we never hit such an x, then we know that $\exists x \forall y P(x, y)$ is false.
Finally, to see whether $\exists x \exists y P(x, y)$ is true, we loop through the values for x, where for each x we loop through the values for y until we hit an x for which we hit a y for which $P(x, y)$ is true. The statement $\exists x \exists y P(x, y)$ is false only if we never hit an x for which we hit a y such that $P(x, y)$ is true.

Order of Quantifiers

1. Let $P(x, y)$ be the statement $x + y = y + x$.
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Is it same as $\forall y \forall x P(x, y)$?
2. Let $Q(x, y)$ denote $x + y = 0$. What are the truth values of the quantifications $\exists x \forall y Q(x, y)$ and $\forall y \exists x Q(x, y)$ where the domain for all variables consists of all real numbers?

1. Let $Q(x, y, z)$ be the statement $x + y = z$. What are the truth values of the statements $\forall x \forall y \exists z \; Q(x, y, z)$ and $\exists z \forall x \forall y \; Q(x, y, z)$, where the domain of all variables consists of all real numbers?

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2. Translate the statement "The sum of two positive integers is always positive" into a logical expression.

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4. Use quantifiers to express the definition of the limit of a real-valued function $f(x)$ of a real variable x at a point a in its domain.

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3. Translate the statement "Every real number except zero has a multiplicative inverse."
4. Use quantifiers to express the definition of the limit of a real-valued function $f(x)$ of a real variable x at a point a in its domain.
For every real number $\epsilon > 0$ there exists a real number $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $0 < |x - a| < \delta$.

- Translate the statement $\forall x(C(x) \vee \exists y(C(y) \wedge F(x, y)))$ into English, where $C(x)$ is "x has a computer," $F(x, y)$ is "x and y are friends," and the domain for both x and y consists of all students in your school.

- Translate the statement $\forall x(C(x) \lor \exists y(C(y) \land F(x,y)))$ into English, where $C(x)$ is "x has a computer," $F(x,y)$ is "x and y are friends," and the domain for both x and y consists of all students in your school.

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The statement says that for every student x in your school, x has a computer or there is a student y such that y has a computer and x and y are friends. In other words, every student in your school has a computer or has a friend who has a computer.

Translate the statement $\exists x \forall y \forall z((F(x,y) \wedge F(x,z) \wedge (y \neq z)) \rightarrow \neg F(y,z)$ into English, where $F(a,b)$ means a and b are friends and the domain for x, y, and z consists of all students in your school.

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- In other words, there is a student none of whose friends are also friends with each other.

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$$\forall x((F(x) \wedge P(x)) \to \exists y M(x, y)) \equiv \forall x \exists y((F(x) \wedge P(x)) \to M(x, y))$$

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- $\exists \epsilon > 0 \forall \delta > 0 \exists x (0 < |x - a| < \delta \wedge |f(x) - L| \geq \epsilon)$.
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- $\forall L \exists \epsilon > 0 \forall \delta > 0 \exists x(0 < |x - a| < \delta \wedge |f(x) - L| \geq \epsilon)$.
  This last statement says that for every real number L there is a real number $\epsilon > 0$ such that for every real number $\delta > 0$, there exists a real number x such that $0 < |x - a| < \delta$ and $|f(x) - L| \geq \epsilon$.

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If $\sqrt{2} > \frac{3}{2}$, then $2 > (\frac{3}{2})^2$.

Table of rules of inference

p	$p \to q$	$\therefore q$
$\neg q$	$p \to q$	$\therefore \neg p$
$p \to q$	$q \to r$	$\therefore p \to r$
$p \vee q$	$\neg p$	$\therefore q$
p	$\therefore p \vee q$	
$p \wedge q$	$\therefore p$	
p	q	$\therefore p \wedge q$
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$p \vee q$	$\neg p \vee r$	$\therefore q \vee r$

1. Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a boat trip," and "If we take a boat trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset."

2. Show that the premises "If you send me an e-mail message, then I will finish writing the program," "If you do not send me an e-mail message, then I will go to sleep early," and "If I go to sleep early, then I will wake up feeling refreshed" lead to the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed."

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3. Use the rules of inference to show that the hypotheses "Jasmine is skiing or it is not snowing" and "It is snowing or Bart is playing hockey" imply that "Jasmine is skiing or Bart is playing hockey."

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4. Show that the premises $(p \wedge q) \vee r$ and $r \rightarrow s$ imply the conclusion $p \vee s$.

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4. Show that the premises $(p \wedge q) \vee r$ and $r \rightarrow s$ imply the conclusion $p \vee s$.

We can rewrite the premises $(p \wedge q) \vee r$ as two clauses (clause is a disjunction of variables or negations of these variables), $p \vee r$ and $q \vee r$. We can also replace $r \rightarrow s$ by the equivalent clause $\neg r \vee s$. Using the two clauses $p \vee r$ and $\neg r \vee s$, we can use resolution to conclude $p \vee s$.

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- $P(c)$ for some element c
  $\exists x P(x)$.
- If $\forall x(P(x) \rightarrow Q(x))$ is true, and if $P(a)$ is true for a particular element a in the domain of the universal quantifier, then $Q(a)$ must also be true