Unit 4: Combinatorics

Topic 2: Permutation and Combination

Outline

1. Introduction
2. Permutation and Combination
   - Distinct objects without repetition
   - Distinct objects with repetition
   - Non-distinct objects
3. Circular Permutation

Introduction

The problem of arrangement of different objects is called permutation and problem of choosing different ways is called combination. In other words, order of the objects matters in permutation, whereas order of objects does not have any meaning in combination.

A **permutation** of a set of distinct objects is an ordered arrangement of these objects. An ordered arrangement of r objects of a set is called r-**permutation**.

A **combination** of a set of distinct objects is an unordered selection of these objects. An unordered selection of r objects of a set is called r-**combination**.

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Permutation and Combination

Permutation and Combination of distinct objects without repetition:

(i) The number of r-permutations of a set with n distinct elements without repetition is given by

$$P(n,r) := \frac{n!}{(n-r)!}.$$

(ii) The number of r-combinations of a set with n distinct elements without repetition is given by

$$C(n,r) := \binom{n}{r} = \frac{n!}{r!(n-r)!}.$$

Note the following:

1. $P(n, r) = r!C(n, r)$.
2. $C(n, r) = C(n, n - r)$.
3. $C(n, r) + C(n, r + 1) = C(n + 1, r + 1)$.
4. $C(n, 0) + C(n, 1) + \ldots + C(n, n) = 2^n$.

Permutation and Combination

Permutation and Combination of distinct objects allowing repetition:

(i) The number of r-permutations of a set with n distinct elements allowing repetition is given by n^r.
(ii) The number of r-combinations of a set with n distinct elements allowing repetition is given by

$$C(n+r-1, r) := \frac{(n+r-1)!}{r!(n-1)!}.$$

Permutation of non-distinct objects:

(i) The number of different permutations of n objects, where there are n_1 indistinguishable objects of type 1, n_2 indistinguishable objects of type 2,..., and n_k indistinguishable objects of type k, is

$$\frac{n!}{n_1!n_2!\cdots n_k!}.$$

Problems

Ex: In how many different orders can five runners finish a race if no ties are allowed?

Ex: A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes

1. are there in total?
2. contain exactly two heads?
3. contain at most three tails?
4. contain the same number of heads and tails?

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Ex: In how many different orders can five runners finish a race if no ties are allowed?
Ans: $P(5,5) = 5!$

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   Ans: $C(10,2)$
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4. contain the same number of heads and tails?
   Ans: $C(10,5)$

Problem

Ex: A club has 25 members.

1. How many ways are there to choose four members of the club to serve on an executive committee?
2. How many ways are there to choose a president, vice president, secretary, and treasurer of the club, where no person can hold more than one office?

Ex: How many ways are there to select four pieces of fruit from a bowl containing apples, oranges, and pears if the order in which the pieces are selected does not matter, only the type of fruit and not the individual piece matters, and there are at least four pieces of each type of fruit in the bowl?

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Ans: $C(3+4-1,2) = 15$

Problem

Ex: How many ways are there to assign three jobs to five employees if each employee can be given more than one job?

Ex: How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 17$, where x_1, x_2, x_3, and x_4 are nonnegative integers?

Ex: In how many ways 5 dots and 8 commas can be arranged?

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Ans: $\frac{13!}{5!8!}$

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Circular Permutation

The permutations discussed above so far can be termed as linear permutations as the objects were assumed to be arranged in a line. If we arrange objects in a circle or any closed curve, we call it as **circular permutation**.

- The number of circular permutations of n distinct objects taking all at a time is $(n-1)!$.

Ex: Find a formula for the number of circular r-permutations of n people.

Ex: How many different necklaces can be formed with 6 white and 5 red beads?

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Ans: $C(n,r)\cdot(r-1)!$ or $\frac{1}{r}P(n,r)$ if orientation matter. If orientation does not matter, then $\frac{1}{2}C(n,r)\cdot(r-1)!$ or $\frac{1}{2r}P(n,r)$

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Ex: How many different necklaces can be formed with 6 white and 5 red beads?
Ans: $\frac{1}{2\cdot 11}\frac{11!}{6!5!}$

Thank You

Any Question!!!