Indian Institute of Information Technology Guwahati

MA203: Probability and Random Processes **Problem Sheet-II on Probability**

---

(1) A box contains white and black balls. When two balls are drawn without replacement, suppose the probablity that both are white is $\frac{1}{3}$. (a) Find the smallest number of balls n the box. (b) How small cam the total number of balls be if black balls are even in number? [Ans: (a) 3 (b) 10]

(2) We have four boxes. Box 1 contains 2000 compnents of which 5% are defective. Box 2 contains 500 components of wich40% are defective. Box 3 and Box 4 contain 1000 each with 10% defective. We select at random one of the boxes and we remove at random a single component. Find the probability that the select component is defective. Find the probability that the defective component is drawn from Box 2.[Ans: 0.1625 and 1/1.625]

(3) Data on the readership of a certain magazine show that the proportion of male readers under 35 is 0.40 and over 35 is 0.20. If the proportion of readers under 35 is 0.70, find the proportion of subscribes that are 'females over 35 years'. Also calculate the probability that a randomly selected male subscribe is under 35 years of age. [Ans: 2/3]

(4) Urn 1 contains one white and two black marbles, urn 2 contains one black and two white marbles, and urn 3 contains three black and three white marbles. A die is rolled. If a 1, 2, or 3 shows up, urn 1 is selected; if a 4 shows up, urn 2 is selected; and if a 5 or 6 shows up, urn 3 is selected. Find the probability of drawing a white ball. [Ans: 4/9]

(5) There are two coins, a blue and a red one. We choose one of the two at random, each being chosen with probability 1/2, and proceed with two independent tosses. The coins are biased. With the blue coin, the probability of heads in any given toss is 0.99, whereas for the red coin it is 0.01. Let D be the event that the blue coin was selected. Let $H_i, i = 1, 2$, be the event that the ith toss resulted in head. Find $P(H_1), P(H_2), P(H_1 \cap H_2), P(H_1|D), P(H_2|D)$, and $P(H_1 \cap H_2|D)$. [Ans: 0.5, 0.5, 0.4901, 0.99, 0.99, and 0.9801.]

(6) The probability of X, Y, and Z becoming managers are 4/9, 2/9, and 1/3 respectively. The probability that the Bonus Scheme will be introduced if X, Y, and Z becomes managers are 3/10, 1/2, and 4/5 respectively. What is the probability that Bonus Scheme will be introduced? If the Bonus Scheme has been introduced, what is the probability that the manager appointed was X? [Ans: 23/45 and 6/23]

(7) If an aircraft is present in a certain area, a radar detects it and generates an alarm signal with probability 0.99. If an aircraft is not present, the radar generates a (false) alarm, with probability 0.10. We assume that an aircraft is present with probability 0.05. What is the probability of no aircraft presence and a false alarm? What is the probability of aircraft presence and no detection? If the radar generates a alarm, what is the probability of the presence of an aircraft?[Ans: 0.095, 0.0005, 0.343]

(8) A Slip of paper is given to person A who marks it either with a plus sign or a minus sign; the probability of his writing a plus sign is 1/3. A passes the slip to B, who may either leave it alone or change the sign before passing it to C. Next C passes the slip to D after perhaps changing the sign. Finally D passes it to a referee after perhaps changing the sign. The referee sees a plus sign on the slip. It is known that B, C, and D each change the sign with probability 2/3. Find the probability that A originally wrote a plus.[Ans: 13/41]

(9) Trains X and Y arrive at a station at random between 8 A.M. and 8.20 A.M. Train X stops for four minutes and train Y stops for five minutes. Assuming that the trains arrive independently of each other, find the probability that (a) train X arrives before train Y (b) the trains meet at the station (c) train X arrived before train Y given that the trains meet at station. [Ans: (a) 1/2 (b) 0.4 (c) 0.45]

(10) We have two coins; the first is fair and the second two-headed. We pick one of the coins at random, we toss it twice and heads shows both times. Find the probability that the coin picked is fair.

(11) Show that for any three events A, B and C,

(a) $P(A \cup B|C) = P(A|C) + P(B|C) - P(A \cap B|C)$

(b) $P(A \cap B|C) + P(A \cap \overline{B}|C) = P(A|C)$

(12) An urn contains four tickets marked with numbers 112,121,211,222. One ticket is drawn from the urn at random, and let A_i b3 the event that ith digit of the number of the ticket drawn is 1. Discuss independency of the events A_1, A_2, A_3.

(13) Let $S = (0,1)$ and $P(I) =$ length of I, where I is an interval in S. Let $A = (0, 1/2), B = (1/4, 1)$ and $C = (1/4, 11/12)$. Show that $P(A \cap B \cap C) = P(A)P(B)P(C)$, but $P(A \cap B) \neq P(A)P(B)$.

(14) Prove or disprove: Two events are independent if and only if they are mutually exclusive.

(15) If A and B are independent events, then (a) A and B^c are independent (b) A^c and B are independent (c) A^c and B^c are independent.

(16) If A, B, C are mutually independent evenets, the $A \cup B$ and C are also independent.

(17) A pair of dice is rolled n times. (a) Find the probability that "seven" will not show at all. (b) (Pascal) Fmd the probability of obtaining double six at least once.

(18) A coin with $P(h) = p = 1 - q$ is tossed n times. Show that the probability that the number of heads is even equals $\frac{1+(q-p)^n}{2}$.

(19) Consider the following three events: (a) At least I six is obtained when six dice are rolled, (b) at least 2 sixes are obtained when 12 dice are rolled, and (c) at least 3 sixes are obtained when 18 dice are rolled. Which of these events is more likely?