Vaishali's e-Learning

Make your Learning Easier

Lecture Notes

Vaishali's e-Learning

Make Your Learning Easier

**Please visit our channel for detailed lectures entirely free of cost

Click Here

DIGITAL COMMUNICATION

Vaishali's e-Learning

Make your Learning Easier

Lecture Notes

Please visit our Youtube Channel to see all the videos recorded so far and learn from them.

The Team @ Vaishali's e-Learning puts a lot of effort to prepare thorough analysis of Subjects daily to help you with your learning and preparation with an aim to help you come out as a winner.

Working For Your Success Always,

Vaishali Kikan

Assistant Professor : KIET Group of Institutions

M.Tech : Delhi Technological University

GATE, NET

Probability & Statistics - Introduction

Origin:— game of chances/ gambling.

Random Signal:— $x(t) = A\cos(2\pi f t + \theta)$
deterministic → at t → A, f, θ are known.
$x(t)$ + noise → random signal.
$f \pm \Delta f = f'$
$x'(t) = A\cos(2\pi f' t + \theta)$
↳ random signal.

Output voltage of receiving antenna is random signal.
$T_x(V)$ + noise = $R_x(V)$ → Random signal.

Communication: essence is Randomness.
→ deterministic ⇒ P = (1)

Basics of Probability:—

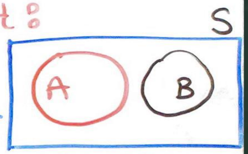
Outcome: End result of occurance of an event.
{H, T} → when toss a coin.

Random Experiment: outcome is random in nature.
→ All the initial conditions during all events are same.
→ All Possible outcomes are known in advance.
→ The outcome of a particular event is not known.

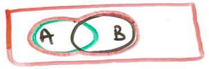
Random Event: Outcome of Random experiment with similar attributes.
eg: face value of dice
$f_1, f_2, f_3, f_4, f_5, f_6$.

Sample Space: Set/ Collection of all possible outcomes of a random experiment.
finite, discrete, uncountable.
eg: {H, T} → Coin, {1, 2, 3, 4, 5, 6} → dice.

Mutually Exclusive (disjoint) Event:
↳ dont have any common element.
A, B → mutually exclusive.

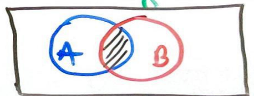


Union of event: A or B; A∪B



$n(A\cup B) = n(A) + n(B) - n(A\cap B)$

Intersection of events:— A and B; A∩B or AB



Shaded part → A∩B.

Occurance of event: Random experiment results in occurance of a particular event A.

Compliment of event: — elements which are not present in A. = $\bar{A}$

$\bar{A} = S - A$

Null of event:— {φ} ⇒ element which is not present in SS.

independent event: Occurance of one event does not depend upon the occurance of another event.

Please Visit Our Channel & Subscribe to get Instant Details About New Uploads

Vaishali's e-Learning
Make your Learning Easier

Probability & Statistics ⇒ Probability

① Relative frequency approach

→ experimental.

$$P = \lim_{n \to \infty} \left(\frac{n_A}{n} \right)$$

$n_A \to$ desired outcome

$n \to$ all outcomes

Toss a fair coin $\Rightarrow n \to \infty \Rightarrow P(H) \to \frac{1}{2}$

Probability :—

$0 \le P \le 1$

② classical approach :—

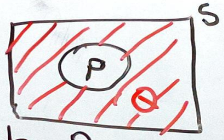
→ Theoratical.

$$P = \frac{\text{Desired outcome}}{\text{Total possible outcome or Sample space}}$$

Properties :—

$p \to$ probability (occurance)

$q \to$ compliment of p.



$p = \frac{P}{S}$ $\quad q = \frac{Q}{S}$

$p + q = \frac{P+Q}{S} = \frac{S}{S} = 1$

→ $p \ge 0$; $q \ge 0$; $p + q = 1$

→ $P(\phi) = 0$

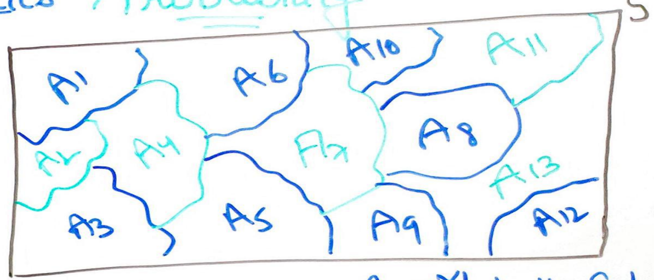
→ $P(A_1) + P(A_2) + \ldots\ldots + P(A_n) = 1$

Mutually exclusive events :—

$P(A \cap B) = 0$; $A \cap B = \phi$

$P(A \cup B) = P(A) + P(B) - \cancel{P(A \cap B)}^{0}$

$P(A \cup B) = P(A) + P(B)$



$A_1 = A_2 \ldots\ldots = A_n =$ Mutually Exclusive in nature.

Conditional probability :—

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad \text{— ①}$$

$$P(A/B) = \lim_{n \to \infty} \frac{n_{AB}}{n_B} = \lim_{n \to \infty} \frac{\frac{n_{AB}}{n}}{\frac{n_B}{n}}$$

$$= \frac{P(A \cap B)}{P(B)}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)} \quad \text{— ②}$$

$$P(A \cap B) = P(B/A)\,P(A) = P\left(\frac{A}{B}\right) P(B)$$

$$P(A \cap B \cap C) = P(A)\,P\left(\frac{B}{A}\right) P\left(\frac{C}{A \cap B}\right)$$

Lecture : https://www.youtube.com/playlist?list=PLXnsjPD8-xuuIC5GyfL8Iy6xysO2x4eMt

Please Visit Our Channel & Subscribe to get Instant Details About New Uploads

Bay's Theorem

Probability & Statistics ⇒ Probability

$P(B) = P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + \cdots\cdots + P(A_n)P\left(\frac{B}{A_n}\right)$

Total probability of the effect.

$$\boxed{P(A_1/B) = \frac{P(A_1)\,P(B/A_1)}{P(B)}}$$

$$= \frac{P(A_1)P(B/A_1)}{P(A_1)P(B/A_1) + P(A_2)P(B/A_2) + \cdots + P(A_n)P\left(\frac{B}{A_n}\right)}$$

$A_1 = A_2 \cdots\cdots = A_n$ = Mutually Exclusive

Bay's Theorem : Reverse probability of cause given that effect has happened.

$A = A \cap S$

$A = A \cap (A_1 \cup A_2 \cup A_3 \cdots \cup A_n)$

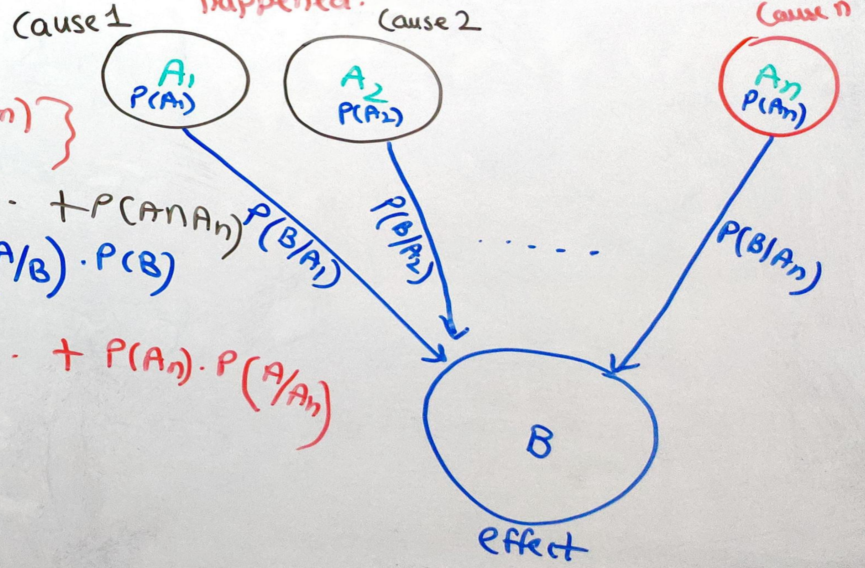
$A = (A \cap A_1) \cup (A \cap A_2) \cup \cdots (A \cap A_n)$

$P(A) = P\{(A \cap A_1) \cup (A \cap A_2) \cup \cdots (A \cap A_n)\}$

$P(A) = P(A \cap A_1) + P(A \cap A_2) + P(A \cap A_3) + \cdots + P(A \cap A_n)$

$P(A \cap B) = P(A) \cdot P(B/A) = P(A/B) \cdot P(B)$

$P(A) = P(A_1)P(A/A_1) + P(A_2)P(A/A_2) + \cdots\cdots + P(A_n) \cdot P(A/A_n)$



Lecture : https://www.youtube.com/playlist?list=PLXnsjPD8-xuuIC5GyfL8Iy6xysO2x4eMt

Probability & Statistics ⇒ Probability

Q From 21 tickets marked with 20 to 40 numerals, one is drawn at random. find the chance that it is multiple of 5.

$= \{20, 25, 30, 35, 40\}$ = favourable o/c

$$P = \frac{\text{favourable o/c}}{\text{Total Possible o/c}} = \frac{5}{21}$$

Q If You twice flip a coin, what is the probability of getting at least one head?

$SS = \{HH, HT, TH, TT\}$

$$P = \frac{3}{4}$$

Q Two dice are thrown together. find probability that

(i) The top faces are same.

(ii) The total of numbers on top is 9.

$$SS = \begin{Bmatrix} (1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\ (2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\ (3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\ (4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\ (5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\ (6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6) \end{Bmatrix}$$

(i) $P = 6/36 = 1/6$

(ii) $\{(4,5), (5,4), (6,3), (3,6)\} = 4/36 = \frac{1}{9}$

Q From a group of 15 chess players, 8 are selected to represent a group. What is the probability that the selected include 3 of the 4 best players?

Total → 15 ; selected → 8

$${}^{15}C_8 = \frac{15!}{8!\,7!} = 6435 \qquad {}^{n}C_i = \frac{n!}{(n-i)!\,i!}$$

15 players → 4 best player → 3 ; 11 other players → 5

$$P \Rightarrow \frac{{}^{4}C_3 + {}^{11}C_5}{6435} = \frac{1848}{6435}$$

Q A lot of ICs consist of 10 good, 4 with minor defect and 2 with major defects. Two chips are randomly chosen from the lot. What is the probability at least one chip is good.

at least → 2 good ; → 1 good

10 good, 6 defected

$$\frac{{}^{10}C_2}{{}^{16}C_2} = P_1$$

$$\frac{3 \cancel{9} \times \cancel{8}\,1}{8 \times \cancel{15}\,3} = \frac{3}{8}$$

$$\frac{{}^{10}C_1 + {}^{6}C_1}{{}^{16}C_2} = P_2$$

$$\frac{10+6}{8 \times 15} = P_2 = \frac{\cancel{16}\,2}{1\,\cancel{8} \times 15}$$

$$P_2 = \frac{2}{15}$$

$$P = P_1 + P_2 = \frac{3}{8} + \frac{2}{15}$$

Probability & Statistics

Q What is the probability that a leap year, selected at random, will contain 53 Sundays?

→ 366 days.

$7 \overline{)366}(52 \rightarrow$ full weeks
35
16
14
2

Remainder $= 2 \rightarrow$ S.S = {(Su, Mo), (Mo, Tu), (Tu, We), (We, Th), (Th, Fr), (Fr, Sa), (Sa, Su)}

$P = \frac{2}{7}$

$P = \frac{1}{7}$ {Su, Mo, Tu, We, Th, Fr, Sa}

Q An urn contains 4 white and 6 red balls. Two balls are drawn out together. What is the probability that both are red balls?

→ 4 → white, 6 → Red, 10 → Total balls.

$^{10}C_2$ = Total no. of outcomes.

$^{6}C_2$ = favourable outcome

$$P = \frac{\cancel{6} \times \cancel{5}/\cancel{2}}{\cancel{10} \times \cancel{9}/\cancel{2}} = \frac{2}{6} = \frac{1}{3}$$

Q 4 Persons are chosen at random from a group containing 3 men, 2 women and 4 children. Find Probability that exactly 2 of them are children.

3 → men, 2 → women, 4 → children

Total Person = 3 + 2 + 4 = 9

Total outcome = $^{9}C_4$

favourable outcome = $^{4}C_2 \times {}^{5}C_2$

$$\text{Total } P = \frac{^{4}C_2 \times {}^{5}C_2}{^{9}C_4} = \frac{10}{21}$$

Q A bag contains the numbers 1, 2, ..., 100 and two numbers are drawn at random. Find the probability that product is even.

Odd Product → odd when both numbers are odd.

100 numbers → 50 odd numbers.

favourable outcome = $^{50}C_2$

Total outcome = $^{100}C_2$

$$P_{odd} = \frac{^{50}C_2}{^{100}C_2} = \frac{\cancel{50} \times 49/\cancel{2}}{\cancel{100} \times 99/\cancel{2}} = 0.2475$$

$P_{even} = 1 - P_{odd} = 1 - 0.2475$
$= 0.7525$

Q A room has 3 electric lamps. From a collection of 10 electric bulbs of which 6 are good, 3 are selected at random and put in the lamps. Find the probability that room is lighted.

→ The probability that room is not lightened (P_N)

$$P_N = \frac{\text{favourable outcome}}{\text{Total outcome}}$$

Total → 10 bulbs → 6 → good, 4 → defected

T.O. = $^{10}C_3$; F.O. = $^{4}C_3$

$$P_N = \frac{^{4}C_3}{^{10}C_3} = \frac{1}{30}$$

$$P = 1 - P_N = 1 - \frac{1}{30} = \frac{29}{30}$$

Probability & Statistics

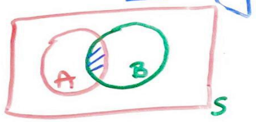
Q $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{3}$, $P(A\cap B)=\frac{1}{4}$

compute

(i) $P(\bar{A}\cap\bar{B})$

De morgan's law

$P(\overline{A\cup B}) = P(\bar{A}\cap\bar{B})$



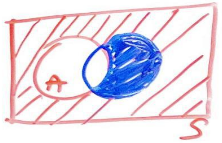
$P(A\cup B) = P(A)+P(B)-P(A\cap B) = \frac{1}{2}+\frac{1}{3}-\frac{1}{4}$

$= 7/12$

$P(\overline{A\cup B}) = 1-P(A\cup B)$

$1-7/12 = 5/12 = P(\bar{A}\cap\bar{B})$

(ii) $P(\bar{A}\cap B)$



$P(\bar{A}\cap B) = P(B)-P(A\cap B) = \frac{1}{3}-\frac{1}{4}$

$= 1/12$

Q $P(A+B)=\frac{3}{4}$; $P(A\cap B)=\frac{1}{4}$; $P(\bar{A})=\frac{2}{3}$

find $P(B)$; $P(\bar{A})=\frac{2}{3} \Rightarrow P(A)=1-P(\bar{A})$

$= 1-\frac{2}{3} = \frac{1}{3}$

$P(A+B) = P(A\cup B) = \frac{3}{4}$

$\frac{3}{4} = \frac{1}{3}+P(B)-\frac{1}{4} \Rightarrow P(B)=\frac{3}{4}-\frac{1}{3}+\frac{1}{4} = \frac{2}{3}$

Q 3 Students (A, B, C) are in a swimming race. A & B have same probability of winning, which is twice of C. Find the probability of A, B & C.

$A \rightarrow P(A)$; $B \rightarrow P(B)$; $C \rightarrow P(C)$

$P(A) = P(B) = 2P(C)$

$P(A)+P(B)+P(C) = 1$

$2P(C)+2P(C)+P(C) = 1$

$5P(C) = 1$

$P(C) = 1/5$

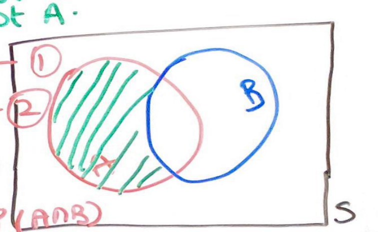
$P(A) = P(B) = 2P(C) = \frac{2}{5}$

Q find $P(A \text{ XOR } B)$; Occurance of A but not B or Occurance of B but not A.

$P(A-B)+P(B-A)$

$P(A-B) = P(A)-P(A\cap B)$ — ①

$P(B-A) = P(B)-P(A\cap B)$ — ②



$P(A \text{ XOR } B) = P(A-B)+P(B-A)$

$= P(A)-P(A\cap B)+P(B)-P(A\cap B)$

$= P(A)+P(B)-2P(A\cap B)$

Q The probability that a new airport will get an award for its design, an award for its efficient use of materials and both is 0.16, 0.24 & 0.11 respectively. What is the probability that it will get only one of two awards?

$P(D \text{ XOR } E)$; $P(D)=0.16$; $P(E)=0.24$

$P(D\cap E)=0.11$

$P(D \text{ XOR } E) = P(D)+P(E)-2P(D\cap E)$

$= 0.16+0.24-2\times 0.11$

$= 0.40-0.22$

$P(D \text{ XOR } E) = 0.18$

Probability & Statistics - Conditional Probability

Q If two cards are drawn successively without replacement from a well shuffled pack of playing cards, find the probability that both are aces.

$P(A \cap B) = P(A) \cdot P(B/A)$ → first card → ace; second card → ace.

$$= \frac{^4C_1}{^{52}C_1} \times \frac{^3C_1}{^{51}C_1} = \frac{1}{221}$$

Q A manufacturer of airplane parts knows that the prob. is 0.7 that an order will be ready for shipment and will be delivered on time, and it is 0.8 that it will be ready for shipment. What is the probability that such an order will be delivered on time given that it was also ready for shipment?

$P(S \cap D) = 0.7$; $P(S) = 0.8$

$$P(D/S) = \frac{P(D \cap S)}{P(S)} = \frac{0.7}{0.8} = \frac{7}{8}$$

Q On tossing two coins simultaneously, find the probability that first coin shows Head given that second coin shows Tail.

$S = \{HH, HT, TH, TT\} \Rightarrow n = 4$

$A \rightarrow$ first coin shows head $= \{HH, HT\} \Rightarrow n = 2$

$B \rightarrow$ Second coin shows Tail $= \{HT, TT\} \Rightarrow n = 2$

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{2/4} = 1/2$$

$S = (A \cap B) = \{HT\} = n = 1$

Q Given a binary communication channel, where A is input and B is output. Let $P(A) = 0.4$; $P(B/A) = 0.9$ & $P(\bar{B}/\bar{A}) = 0.6$

find (i) $P(A/B)$
(ii) $P(A/\bar{B})$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = P\left(\frac{B}{A}\right) P(A) = 0.9 \times 0.4 = 0.36$$

$$P(A \cup B) \Rightarrow P(\bar{B}/\bar{A}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{A})} = \frac{P(\overline{A \cup B})}{1 - P(A)} = \frac{1 - P(A \cup B)}{1 - P(A)}$$

$$0.6 = \frac{1 - P(A \cup B)}{1 - 0.4} = 0.36 = 1 - P(A \cup B)$$

$\Rightarrow P(A \cup B) = 1 - 0.36 = 0.64$

$\boxed{P(A \cap B) = 0.36}$

$\boxed{P(A \cup B) = 0.64}$

$\boxed{P(B) = 0.6}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.64 = 0.4 + P(B) - 0.36$

$0.64 + 0.36 - 0.4 = P(B) \Rightarrow 0.6$

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)} = \frac{0.36}{0.6} = \frac{1}{6}$$

$$P\left(\frac{A}{\bar{B}}\right) = \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{P(A) - P(A \cap B)}{1 - P(B)} = \frac{0.4 - 0.36}{1 - 0.6} = \frac{0.04}{0.4} = 0.1$$

$= 0.1$

Q Let A & B be independent events with $P(A) = 0.5$ & $P(B) = 0.8$. Find the probability that neither of the events occur.

$P(B/A) = P(B)$; $P(A/B) = P(A)$

$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B}/\bar{A})$

$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B})$

$P(\bar{A}) = 1 - 0.5 = 0.5$; $P(\bar{B}) = 1 - 0.8 = 0.2$

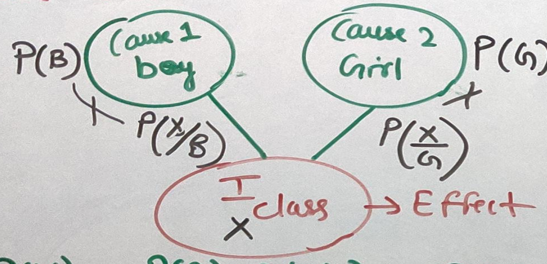
$P(\bar{A} \cap \bar{B}) = 0.5 \times 0.2$
$= 0.1$

Please Visit Our Channel & Subscribe to get Instant Details About New Uploads

Total Probability

Probability & Statistics

Q 2/3 of the students in a class are boys. It is known that the probability of a girl getting first class is 0·25 and that of a boy is 0·28. Find the probability that a student chosen at random will get first class.



$$P(X) = P(B)\cdot P\left(\frac{X}{B}\right) + P(G)\,P\left(\frac{X}{G}\right)$$

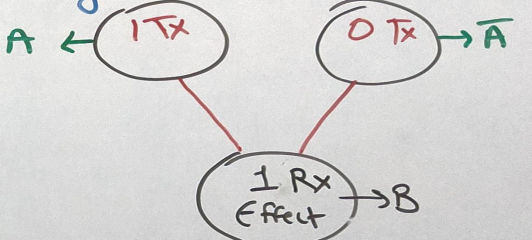
$P(B) = \frac{2}{3}$; $P(G) = 1 - \frac{2}{3} = \frac{1}{3}$

$P(X/B) = 0{\cdot}28$ $\quad$ $P(X/G) = 0{\cdot}25$

$$P(X) = \frac{2}{3}\times 0{\cdot}28 + \frac{1}{3}\times 0{\cdot}25$$

$$= 0{\cdot}27$$

Q For a certain binary communication channel, the probability that a transmitted '0' will be received as '0' is 0·95 & transmitted '1' will be received as '1' is 0·90. If prob. that '0' is transmitted is 0·4. Find the probability that '1' is received.



$P(\bar{A}) = 0{\cdot}4$ $\quad$ $P(A) = 1 - 0{\cdot}4 = 0{\cdot}6$

$P\left(\frac{B}{\bar{A}}\right) = 0{\cdot}95$ $\quad$ $P\left(\frac{B}{A}\right) = 0{\cdot}90$

$$P(B) = P(A)\,P\left(\frac{B}{A}\right) + P(\bar{A})\,P\left(\frac{B}{\bar{A}}\right)$$

$$P(B/\bar{A}) = 1 - P\left(\frac{\bar{B}}{\bar{A}}\right) = 1 - 0{\cdot}95 = 0{\cdot}05$$

$$P(B) = 0{\cdot}6 \times 0{\cdot}90 + 0{\cdot}4 \times 0{\cdot}05$$

$$= 0{\cdot}56$$

Please Visit Our Channel & Subscribe to get Instant Details About New Uploads

Probability & Statistics - Baye's Theorem

Q An urn contains 5 balls. Two balls are drawn at random and are found to be white. What is the probability that all balls being white?

A_1 : 2 balls are white
A_2 : 3 balls are white
A_3 : 4 balls are white
A_4 : 5 balls are white
(causes)

$P(A_1) = 1/4$
$P(A_2) = 1/4$
$P(A_3) = 1/4$
$P(A_4) = 1/4$

$X \rightarrow 2$ balls drawn are white

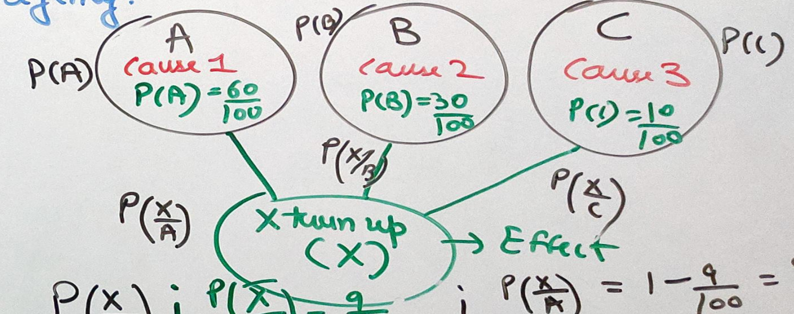
$$P\left(\frac{A_4}{X}\right) = \frac{P(A_4)\cdot P(X/A_4)}{P(X)}$$

$$P(X/A_1) = \frac{{}^2C_2}{{}^5C_2} = \frac{1}{10}$$
$$P(X/A_2) = \frac{{}^3C_2}{{}^5C_2} = 3/10$$
$$P(X/A_3) = \frac{{}^4C_2}{{}^5C_2} = 6/10$$
$$P(X/A_4) = \frac{{}^5C_2}{{}^5C_2} = 1$$

$$P(X) = \frac{1}{10}\times\frac{1}{4} + \frac{3}{10}\times\frac{1}{4} + \frac{6}{10}\times\frac{1}{4} + 1\times\frac{1}{4}$$

$$P(A_4/X) = \frac{1/4\times 1}{\frac{1}{40}+\frac{3}{40}+\frac{6}{40}+\frac{1}{4}} = \frac{1}{2}$$

Q The members of a consulting firm rent cars from rental agencies A, B & C as 60%, 30% & 10% respectively. If 9%, 20% & 6% of cars from A, B & C need turn up and if the rental car does not need turn up, what is the probability that it came from B agency?



$P\left(\frac{X}{A}\right)$; $P\left(\frac{X}{A}\right) = \frac{9}{100}$; $P\left(\frac{X}{A}\right) = 1 - \frac{9}{100} = \frac{91}{100}$

$P\left(\frac{X}{B}\right)$; $P\left(\frac{X}{B}\right) = \frac{20}{100}$; $P\left(\frac{X}{B}\right) = 1 - \frac{20}{100} = \frac{80}{100}$

$P\left(\frac{X}{C}\right)$; $P\left(\frac{X}{C}\right) = \frac{6}{100}$; $P\left(\frac{X}{C}\right) = 1 - \frac{6}{100} = \frac{94}{100}$

$$P\left(\frac{B}{X}\right) = \frac{P(B)\,P\left(\frac{X}{B}\right)}{P_r(X)}$$

$$P_r(X) = P(A)P(X/A) + P(B)P(X/B) + P(C)\cdot P(X/C)$$
$$= 0.6\times 0.91 + 0.3\times 0.8 + 0.1\times 0.94$$

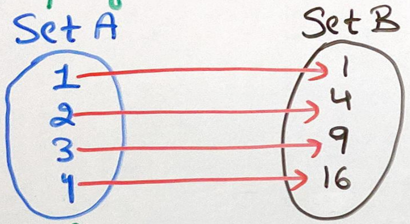
$$P(B/X) = \frac{0.3\times 0.8}{0.6\times 0.91 + 0.3\times 0.8 + 0.1\times 0.94} = \frac{3}{11}$$

Vaishali's e-Learning
Make your Learning Easier

Lecture : https://www.youtube.com/playlist?list=PLXnsjPD8-xuuIC5GyfL8Iy6xysO2x4eMt

Probability & Statistics - Random Variable

Function: Mapping of elements of set A to the elements of set B uniquely.



$f(x) = x^2$

Domain: Elements in Set A $\Rightarrow 1, 2, 3, 4$

Co domain: All elements in Set B $\rightarrow 1, 4, 9, 16, 25$

Range: Mapped elements in set B. $\Rightarrow 1, 4, 9, 16$

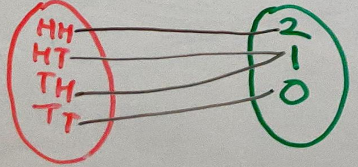
One One: Every element in set B is mapped.

Onto: Every element in set A is mapped to a unique element in set B.

Random Variable: Well defined, finite valued, Real function on sample space.

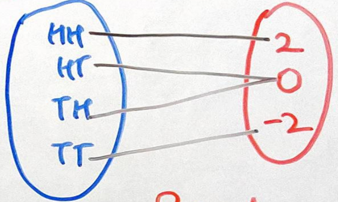
$SS = \{HH, HT, TH, TT\}$

Function: No. of Heads.



Random variable is denoted with the help of capital letter X, Y, Z, A, B, C .....

f_2: No. of heads − No. of tails



Random Variable

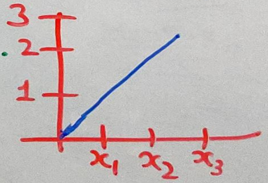
- Continuous R.V.
- Discrete R.V.
  - finite R.V.
  - Countable infinite R.V.

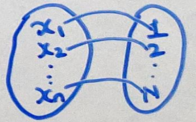
Continuous R.V.: The outcome varies continuously in an interval.

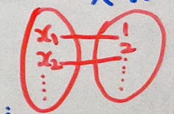
$x_1 = [0, 1]$

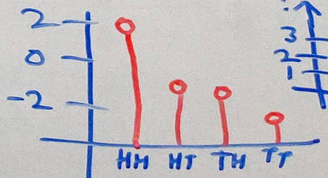
$x_2 = (1, 2]$

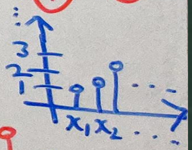
$x_3 = (2, 3]$











Countable infinite
↓
f^n should be one-one & onto

Please Visit Our Channel & Subscribe to get Instant Details About New Uploads

Probability & Statistics - PMF, CDF, PDF

1) Probability mass function :-

→ Discrete R.V.

$$P(x) = \begin{cases} P(x = x_i) = p_i & ; x = x_i \\ 0 & ; x \neq x_i \end{cases}$$

$i = 1, 2, 3 \ldots n$

1) Set form
2) Tabular form

} → Representation of PMF.

S.S. = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

X → no. of Heads.

3 {HHH} ⇒ $p = 1/8$

2 {HHT, HTH, THH} ⇒ $p = 3/8$

1 {HTT, THT, TTH} → $p = 3/8$

0 {TTT} ⇒ $p = 1/8$

$x_i = 0, 1, 2, 3$

Probability mass function :-

Set form:- $\{(x_1, p_1), (x_2, p_2), (x_3, p_3) \ldots\}$

$\{(0, 1/8), (1, 3/8), (2, 3/8), (3, 1/8)\}$

Tabular form:-

0	1	2	3
1/8	3/8	3/8	1/8

2) Cumulative distribution function :-

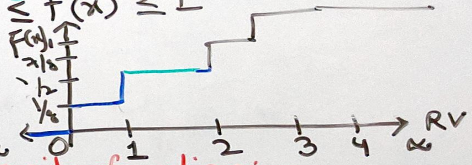
$F(x) = P(X \le x) = P(\omega : X(\omega) \le x)$

Properties:

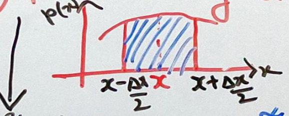
(i) monotonically increasing

(ii) Right continuous

(iii) $0 \le F(x) \le 1$



3) Probability density function :-



defined for Continuous R.V.

$$F(x) = \lim_{\Delta x \to 0} \frac{P\left(x + \frac{\Delta x}{2}\right) - P\left(x - \frac{\Delta x}{2}\right)}{\Delta x}$$

$$f_x(x) = \frac{dF(x)}{dx}$$

$$F(x) = \int_{-\infty}^{\infty} f(x)\,dx$$

Properties:-

1) $f_x(x) \ge 0$; $F(x)$ → monotonically ↑ ; $f_x(x) = \frac{d}{d(x)} F(x)$ → +ve

2) $\int_{-\infty}^{\infty} f_x(x)\,dx = 1 \Rightarrow \int_{-\infty}^{\infty} f_x(x)\,dx = \int_{-\infty}^{\infty} \frac{d}{dx} F(x)\,dx = F(x)\Big]_{-\infty}^{\infty} = F(\infty) - F(-\infty) = 1 - 0 = 1$

3) $\int_{-\infty}^{x} f_x(x)\,dx = F(x) = F(x)\Big]_{-\infty}^{x} = F(x) - F(-\infty) = F(x)$

4) $\int_{x_1}^{x_2} f_x(x)\,dx = F(x_2) - F(x_1)$

Lecture : https://www.youtube.com/playlist?list=PLXnsjPD8-xuuIC5GyfL8Iy6xysO2x4eMt

Probability & Statistics - PMF, CDF, PDF

Q Let X be discrete R.V. whose CDF is

$$F(x) = \begin{cases} 0 & \text{if } x < -3 \\ \frac{1}{6} & \text{if } -3 \le x < 6 \\ \frac{1}{2} & \text{if } 6 \le x < 10 \\ 1 & \text{if } x \ge 10 \end{cases}$$

find (i) $P(X \le 4)$, $P(-5 \le X \le 4)$, $P(X=6)$; $P(X=10)$

(iii) The PMF.

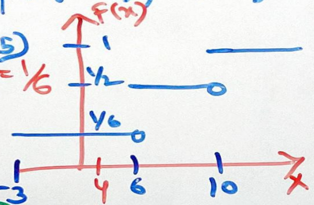
(i) $P(X \le 4) = 1/6$; $P(-5 \le X \le 4) = F(4) - F(-5) = 1/6 - 0 = 1/6$

$P(X=6) = \frac{1}{2} - \frac{1}{6} = 1/3$

$P(X=10) = 1 - 1/2 = 1/2$

(iii) $P(X=-3) = 1/6 - 0 = 1/6$

PMF: $\{(-3, 1/6), (6, 1/3), (10, 1/2)\}$



Q A R.V. X has PDF:

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

find (i) k

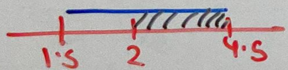
(ii) $P(1.5 < X < 4.5 \mid X > 2)$

$\sum_{i=0}^{7} p(x) = 1 \Rightarrow 0 + k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$= 10k^2 + 9k - 1 = 0$

$= (10k-1)(k+1) = 0$; $k = 1/10$; $\cancel{k = -1}$

(ii) $P(1.5 < X < 4.5 \mid X > 2) = \dfrac{P(1.5 < x < 4.5 \cap x > 2)}{P(X>2)}$



$= \dfrac{P(2 < x < 4.5)}{1 - P(X \le 2)}$

$= \dfrac{P(3) + P(4)}{1 - \{P(0) + P(1) + P(2)\}} = \dfrac{2/10 + 3/10}{1 - \{0 + 1/10 + 2/10}$

Q Given the PDF of R.V. X as

$$f(x) = \begin{cases} 12.5x - 1.25 & \text{for } 0.1 \le x \le 0.5 \\ 0 & \text{elsewhere} \end{cases}$$

find $P(0.2 \le X \le 0.3)$

$P(0.2 \le x \le 0.3) = \int_{0.2}^{0.3} f(x)\,dx$

$= \int_{0.2}^{0.3} 12.5x - 1.25\,dx = 1.25 \int_{0.2}^{0.3} (10x - 1)\,dx$

$= 1.25\{5x^2 - x\}_{0.2}^{0.3}$

$= 1.25\{5(0.3^2 - 0.2^2) - (0.3 - 0.2)\} = 0.1875$

Q A R.V. has PDF $f(x) = K(1+x)$; $2 \le x \le 5$

find $P(X \le 4)$.

$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{-\infty}^{\infty} K(1+x)\,dx = 1$

$\int_2^5 (k + kx)\,dx = 1 = \left[kx + \frac{kx^2}{2}\right]_2^5 = 1$

$= k\{5-2\} + k\left\{\frac{5^2}{2} - \frac{2^2}{2}\right\} = 1$

$\Rightarrow K = \frac{2}{27}$

$P(X \le 4) = \int_{-\infty}^{4} f_X(x)\,dx$

$\int_{-\infty}^{4} \frac{2}{27}(1+x)\,dx = \frac{2}{27}\int_2^4 (1+x)\,dx$

$= \frac{2}{27}\left\{x + \frac{x^2}{2}\right\}_2^4$

$= \frac{2}{27}\left\{4 - 2 + \frac{4^2}{2} - \frac{2^2}{2}\right\}$

$= \frac{16}{27}$

Probability & Statistics - PMF, CDF, PDF

Q If $p(x) = \begin{cases} x e^{-x^2/2} & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases}$

(i) Show $p(x)$ is a PDF of continuous RV X.
(ii) find its distribution function.

(i) $p(x) \rightarrow$ valid for only the $x \geq 0$

$$\int_{-\infty}^{\infty} p(x)dx = 1 \Rightarrow \int_0^{\infty} x e^{-x^2/2} dx \; ; \; x^2/2 = u \Rightarrow du = x\,dx$$

$$\Rightarrow \int_0^{\infty} e^{-u} du \Rightarrow [-e^{-u}]_0^{\infty} = 0 - (-1) = 1$$

$\Rightarrow$ It is a PDF of R.V. X.

(ii) $$F_X(x) = \int_{-\infty}^{x} f_X(x)dx$$

$$= \int_0^x x e^{-x^2/2} dx \Rightarrow x^2/2 = u \Rightarrow \int_0^{x^2/2} e^{-u} du$$

$$= [-e^{-u}]_0^{x^2/2} = -e^{-x^2/2} - (-e^{-0})$$

$$F(x) = 1 - e^{-x^2/2}$$

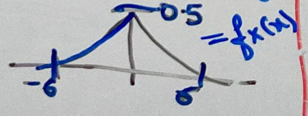
Q A continuous R.V. X has the PDF $f(x)$ given by $f(x) = Ce^{-|x|}$, $-\infty < x < \infty$
find the value of C and CDF of X.

$$\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_{-\infty}^{\infty} Ce^{-|x|} dx = 1$$

$$\Rightarrow 2\int_0^{\infty} Ce^{-x} dx = 1 \Rightarrow 2C\{-e^{-x}\}_0^{\infty} = 1$$

$$\Rightarrow C = 1/2$$

$$f_X(x) = \frac{1}{2} e^{-|x|}$$



CDF: when $x \leq 0 \rightarrow$ (i) ; when $x > 0 \rightarrow$ (ii)

(i) $$F(x \leq 0) = \int_{-\infty}^{x} \frac{1}{2} e^{-|x|} dx \; ; \; |x| = -x$$

$$= \int_{-\infty}^{x} \frac{1}{2} e^{-(-x)} dx = \frac{1}{2}\int_{-\infty}^{x} e^{x} dx$$

$$= \frac{1}{2}[e^x]_{-\infty}^{x} = \frac{1}{2}\{e^x - e^{-\infty}\}$$

$$= \frac{1}{2} e^x$$

(ii) $$F(x > 0) = P(X \leq x) = P(X \leq 0) + P(0 < X < x)$$

$$= F(0) + P(0 < X < x)$$

$$F(0) = \frac{1}{2} e^{-0} = \frac{1}{2}$$

$$\int_0^x f_X(x)dx = \int_0^x \frac{1}{2} e^{-|x|} dx \; ; \; |x| = x$$

$$= \frac{1}{2}\int_0^x e^{-x} dx = \frac{1}{2}[-e^{-x}]_0^x$$

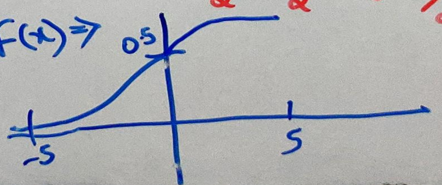
$$= \frac{1}{2}\{-e^{-x} - (-1)\}$$

$$= \frac{1}{2}\{1 - e^{-x}\}$$

$$= \frac{1}{2} - \frac{e^{-x}}{2}$$

$$F(x > 0) = \frac{1}{2} + \frac{1}{2} - e^{-x}/2 = 1 - \frac{e^{-x}}{2}$$

$F(x) \Rightarrow$



Probability & Statistics

Mean, Varience & Standard deviation

Expectation / Mean : Expected value of the outcome.

$E(x)$ = Depends upon Random Variable (RV)

Discrete RV

$$E(x) = \sum_i x_i f(x_i)$$

Continuous RV

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

Convergence condition required to find out $E(x)$.

$$\sum_i |x_i| f(x_i) < \infty \qquad \int_{-\infty}^{\infty} |x| f(x) < \infty$$

$$E\{g(x)\} = \sum_i g(x_i) f(x_i) \qquad E\{g(x)\} = \int_{-\infty}^{\infty} g(x) f(x)\,dx$$

Convergence condition : $\sum_i |g(x_i)| f(x_i) < \infty \qquad \int_{-\infty}^{\infty} |g(x)| f(x) < \infty$

Properties :-

① $E(c) = c \Rightarrow \int_{-\infty}^{\infty} c f(x)\,dx \Rightarrow c \underbrace{\int_{-\infty}^{\infty} f(x)\,dx}_{\text{PDF} \,=\, 1} = c \times 1 = c$

② $E(ax) = aE(x) \Rightarrow \int_{-\infty}^{\infty} ax f(x)\,dx \Rightarrow a\int_{-\infty}^{\infty} x f(x)\,dx \Rightarrow aE(x)$

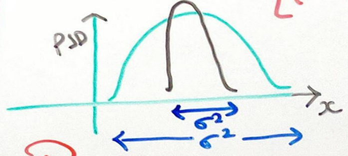
③ $E(ax+b) = aE(x)+b = \int_{-\infty}^{\infty}(ax+b) f(x)\,dx = \int_{-\infty}^{\infty} ax f(x)\,dx + \int_{-\infty}^{\infty} b f(x)\,dx$

$= a\underbrace{\int_{-\infty}^{\infty} x f(x)\,dx}_{E(x)} + b\underbrace{\int_{-\infty}^{\infty} f(x)\,dx}_{1}$

$= aE(x) + b$

Variance $= E[(x-\mu)^2] = \sum_i (x_i-\mu)^2 p_i$

↑ Expectation / mean.



σ^2 = Variance

σ = standard deviation

$\sigma = \sqrt{\text{Variance}}$

Properties :-

① $E[(x-\mu)^2] = E(x^2) - \mu^2$

↳ $E[(x^2+\mu^2-2x\mu)] \Rightarrow E(x^2)+\mu^2-2\underbrace{E(x)}_{\mu}\mu$

$E(x^2)+\mu^2-2\mu^2 = E(x^2)-\mu^2$

② $Var(bx+a) = b^2 Var(x)$; $Y = bx+a$; $\mu = b\mu+a$

$Var = E(Y-\mu)^2 \Rightarrow E(bx+\cancel{a}-b\mu-\cancel{a})^2$

$= E(bx-b\mu)^2$

$= b^2 E(x-\mu)^2$

$= b^2 Var(x)$

③ $Var(a) = 0$; put $b = 0$

$Var(a) = E(0) = 0$

④ $Var(x) = 0 \Rightarrow x = \mu$

$\int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx = 0$ ← non-ve (≥ 0)

$(x-\mu)^2 = 0$

$x = \mu$

Probability & Statistics - Mathematical Expectation

Q If Var(X) = 4, find Var(3X+8), where X is RV.

$Var(aX+b) = a^2 Var(X)$

$Var(3X+8) = (3)^2 Var(X)$

$= 9 \times 4 = 36$

Q Let X be a random variable with $E(X) = 1$ and $E[X(X-1)] = 4$. Find Var(X) and Var(2-3X).

$E(X) \to \mu$

$E[(X-1)X] = 4 \Rightarrow E[X^2 - X] = 4 \Rightarrow E(X^2) - E(X) = 4$

$E(X^2) - 1 = 4 \Rightarrow E(X^2) = 4+1 = 5$

$Var(X) = E(X^2) - \mu^2 = 5 - [E(X)]^2$

$= 5 - (1)^2 = 4$

$Var(2-3X) = (-3)^2 Var(X) = 9 \times 4 = 36$

Q If X is any R.V., show that standard R.V. has mean 0 and variance 1.

$Z = X - \mu/\sigma \to$ Standard Random Variable.

$E(Z) = 0$; $E\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma}E(X-\mu) = \frac{1}{\sigma}E(X) - \mu = \frac{1}{\sigma}(\mu-\mu)$

$E(Z) = 0$

$Var(Z) = E(Z-\mu)^2 = E\left(\frac{X-\mu}{\sigma} - \cancel{\mu}\right)^2 \Rightarrow E\left(\frac{X-\mu}{\sigma}\right)^2 = \frac{1}{\sigma^2}E(X-\mu)^2 = \frac{1}{\sigma^2} \times \sigma^2 = 1$

$Var(Z) = 1$

Q A coin is biased so that head is twice likely to appear as tail. When coin is tossed twice, find expected no. of head.

$H = 2T \Rightarrow H + T = 1 \Rightarrow 3T = 1 \Rightarrow T = \frac{1}{3} = q$

$H = 2/3 = p$

No. of head = R.V.

$\overset{0}{TT} \to q^2$ $\quad \overset{1}{HT, TH} = 2pq$ $\quad \overset{2}{HH} = p^2$

X	0	1	2
P(X)	1/9	4/9	4/9

$E(X) = \sum_i x_i p(x_i) = 0 \times \frac{1}{9} + 1 \times \frac{4}{9} + 2 \times \frac{4}{9}$

$= 4/3$

Q When a die is thrown, X denotes the number that turn up. Find E(X), E(X²) & Var(X).

X	1	2	3	4	5	6
P(X)	1/6	1/6	1/6	1/6	1/6	1/6

$E(X) = (1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}) = \frac{21}{6}$

$E(X^2) = (1 \times \frac{1}{6} + 4 \times \frac{1}{6} + 9 \times \frac{1}{6} + 16 \times \frac{1}{6} + 25 \times \frac{1}{6} + 36 \times \frac{1}{6}) = \frac{91}{6}$

$Var(X) = E(X^2) - E(X)^2 = \frac{91}{6} - \left(\frac{21}{6}\right)^2 = \frac{35}{12}$

Q A man draws 3 balls from an urn containing 5 white and 7 black balls. He gets Rs 10 for each white ball and ₹5 for black ball. Find his expectation.

Total o/c $= {}^{12}C_3$

3 BB (15) $\to {}^7C_3 / {}^{12}C_3$

2 BB + 1 WB (20) $\to \frac{{}^7C_2 \times {}^5C_1}{{}^{12}C_3}$

1 BB + 2 WB (25) $\to \frac{{}^7C_1 \times {}^5C_2}{{}^{12}C_3}$

3 WB (30) $\to {}^5C_3 / {}^{12}C_3$

X	P(X)
15	7/44
20	21/44
25	14/44
30	2/44

$E(X) = 15 \times \frac{7}{44} + \frac{21}{44} \times 20 + 25 \times \frac{14}{44} + 30 \times \frac{2}{44} = \frac{85}{4}$

Q A coin is tossed untill head appears. What is the expected number of toss?

$p \to$ head $= 1/2$; $q =$ Tail $= 1/2$

$= \underset{1}{p} + \underset{2}{qp} + \underset{3}{q^2p} + \underset{4}{q^3p} + \underset{5}{q^4p} + \dots \to RV$

$E(X) = 1 \times p + 2 \times q \times p + 3 \times q^2 p + 4q^3 p + \dots$

$= p[1 + 2q + 3q^2 + 4q^3 + 5q^4 + \dots]$

$= p(1-q)^{-2}$

$E(X) = p\,p^{-2} \Rightarrow p^{-1} = 2$

Probability & Statistics - Mathematical Expectation

Q In a continuous distribution, probability density is given by:

$f(x) = kx(2-x), \quad 0 < x < 2$

find mean & variance.

$\int_{-\infty}^{\infty} f(x)dx = 1 \Rightarrow \int_0^2 kx(2-x)dx = 1$

$k\int_0^2 (2x - x^2)dx = 1 \Rightarrow k\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$

$k\left[4 - \frac{8}{3}\right] = 1$

$k \times \frac{12-8}{3} = 1 \Rightarrow k = 3/4$

$E(x) = \int_{-\infty}^{\infty} x f(x)dx \quad ; f(x) = \frac{3}{4}x(2-x) \; ; 0 < x < 2$

$E(x) = \int_0^2 \frac{3}{4}x \cdot x(2-x)dx.$

$= \frac{3}{4}\int_0^2 x^2(2-x)\,dx = \frac{3}{4}\int_0^2 (2x^2 - x^3)dx$

$= \frac{3}{4}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2 = \frac{3}{4}\left\{\frac{2\times 8}{3} - \frac{16}{4}\right\}$

$E(x) = 1 = \mu$

$Var(x) = E(x-\mu)^2 = E(x^2) - \mu^2$

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)dx = \int_0^2 \frac{3}{4}(2x - x^2) \cdot x^2 dx$

$= \frac{3}{4}\int_0^2 (2x^3 - x^4)dx \Rightarrow \frac{3}{4}\left[\frac{x^4 \cdot 2}{4} - \frac{x^5}{5}\right]_0^2$

$= \frac{3}{4}\left\{\frac{2^5}{4} - \frac{2^5}{5}\right\} = \frac{6}{5}$

$Var = E(x^2) - E(x)^2 = \frac{6}{5} - (1)^2 = \frac{1}{5}$

Q The distribution function of a R.V. X is given by:

$F(x) = 1 - (1+x)e^{-x}, \quad x \geq 0$

find mean & variance.

$\frac{d}{dx}F(x) = f(x) \xrightarrow{} \text{Pdf} = \frac{d}{dx}\left[1 - (1+x)e^{-x}\right]$

$= 0 - 1 \times e^{-x} - (1+x)(-1)e^{-x}$

$= (1+x)e^{-x} - e^{-x} = \cancel{e^{-x}} - \cancel{e^{-x}} + xe^{-x}$

$f(x) = xe^{-x} \quad ; x \geq 0$

$E(x) = \int_{-\infty}^{\infty} f(x) \cdot x\,dx = \int_0^{\infty} x \cdot xe^{-x}dx$

$= \int_0^{\infty} x^2 e^{-x}dx = 2! = 2 = \mu$

$E(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f(x)\,dx. \Rightarrow \int_0^{\infty} x^2 \cdot xe^{-x}dx$

$= \int_0^{\infty} x^3 e^{-x}dx$

$= 3! = 6$

$Var(x) = E(x^2) - \mu^2$

$= 6 - 2^2$

$= 6 - 4$

$= 2$

Standard Probability Distributions

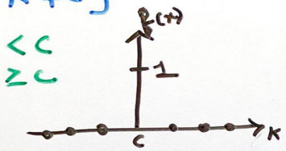
(1) Degenerate Distribution :–

$P_x = 1 \rightarrow$ at a single pt.

eg: dice having same number at all faces.

PDF: $f(k;c) = P(X=k) = \begin{cases} 1 & ; k=c \\ 0 & ; k \neq c \end{cases}$

CDF: $F(x) = P(X \le x) = \begin{cases} 0 & ; x < c \\ 1 & ; x \ge c \end{cases}$



$E(x) = \mu = c$; $Var(x) = 0$

$c \times 1 + (-1) \times 0 \ldots = c$; $E(x^2) - \mu^2$, $c^2 \times 1 - c^2 = 0$

(2) Bernoulli Distribution :–

→ Named after James Bernoulli.

→ experiment with 2 random outcomes (p, q) — p → success, q → failure

PDF: $f(x,p) = P(X=x) = \begin{cases} p & ; x=1 \\ 1-p & ; x=0 \end{cases}$

$\mu = p$; $0 \times (1-p) + 1 \times p = p$

$Var = pq$; $E(x^2) = 0 \times 1-p + 1 \times p = p$

$Var(x) = p - p^2 = p(1-p) = p \cdot q$

(3) Binomial Distribution :–

→ James Bernoulli discovered it.

→ distribution of no. of successes in a sequence of n independent Bernoulli trials.

$n \rightarrow$ no. of trials.

$p \rightarrow$ constant probability of success.

$q \rightarrow$ constant probability of failure $= 1-p$

PDF: $f(x,n,p) = P(X=x) = {}^nC_x \, p^x q^{n-x}$; $x = 0, 1, 2 \ldots n$

$E(x) = \mu = np$; $Var(x) = npq$

(4) Poisson Distribution :–

→ Discovered by Simon Denis Poisson

→ Discrete

→ limiting case of Binomial Distribution where:

(i) $n \rightarrow \infty$ (infinite trials)

(ii) $p \rightarrow 0$ *

(iii) $np = \mu$ {expected success} remain constant.

PDF: $f(x,\mu) = P(X=x) = \begin{cases} \dfrac{e^{-\mu}\mu^x}{x!} & ; x = 0,1,2 \ldots \\ 0 & , \text{otherwise} \end{cases}$

$E(x) = \mu$; $Var(x) = \mu$

(5) Geometric Distribution :–

→ x independent Bernoulli's trials untill a success occurs.

PDF: $f(x;p) = P(X=x) = q^{x-1} p$; $x = 0,1,2 \ldots$

$mean = 1/p$; $Var = q/p^2$

→ $x-1$ trials → failure ; x^{th} → success.

(6) Pascal Distribution / Negative binomial distribution :–

→ generalised geometric distribution {r success}

$f(x,p,r) = P(X=x) = \binom{x-1}{r-1} p^r q^{x-r}$; $x = 0,1,2 \ldots$

$mean = r/p$

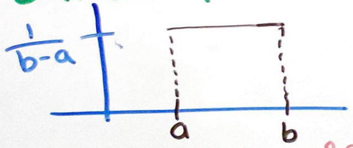
$varience = rq/p^2$

$x-1$ trial → $r-1$ success

x^{th} trial → r^{th} success.

Standard Probability Distributions

(1) Uniform Distribution function:-



$$f(x;a;b)=\begin{cases}\frac{1}{b-a} & \text{if } a<x<b\\ 0 & \text{o.w.}\end{cases}$$

$$F(x;a;b)=\begin{cases}0 & ;\ x<a\\ \frac{x-a}{b-a} & ;\ a<x<b\\ 1 & ;\ x\geq b\end{cases}$$

$\mu=\frac{a+b}{2}$; $\text{var}=\frac{(b-a)^2}{12}$

(2) Exponential Distribution:-

→ model time between independent events that occur at constant average rate.

$$f(x;\lambda)=\begin{cases}\lambda e^{-\lambda x} & ;\ x\geq 0\\ 0 & ;\ \text{o.w.}\end{cases}$$

mean $=1/\lambda$

Var $=1/\lambda^2$

(3) Gamma Distribution:-

$$f(x,\alpha,\lambda)=\begin{cases}\frac{\lambda e^{-\lambda x}(\lambda x)^{\alpha-1}}{\alpha!} & ;\ x\geq 0\\ 0 & ;\ \text{o.w.}\end{cases}$$

mean $=\alpha/\lambda$

Var $=\alpha/\lambda^2$

(4) Weibull Distribution:-

PDF: $$f(x;\alpha;\beta)=\begin{cases}\alpha\beta x^{\beta-1}e^{-\alpha x^{\beta}} & ;\ x\geq 0\\ 0 & \text{o.w.}\end{cases}$$

CDF: $$F(x;\alpha,\beta)=\begin{cases}0 & ;\ x\leq 0\\ 1-e^{-\alpha x^{\beta}} & ;\ x>0\end{cases}$$

$$\mu=\alpha^{-1/\beta}\,\Gamma(1+1/\beta)$$

$$\text{Var}=\alpha^{-2/\beta}\left[\Gamma(1+2/\beta)-\Gamma^2(1+1/\beta)\right]$$

Please Visit Our Channel & Subscribe to get Instant Details About New Uploads

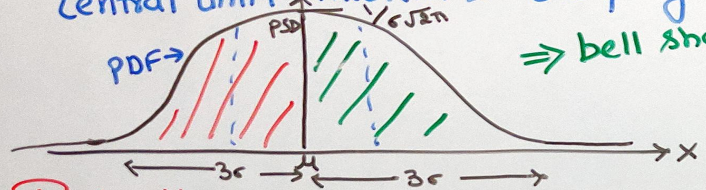
Vaishali's e-Learning
Make your Learning Easier

Probability and Statistics - Gaussian Distribution

Gaussian distribution / Normal distribution :-

→ used greatly in diverse fields.

→ used in digital communication in central limit theorem & Sampling theorem.



⇒ bell shaped structure

$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

PDF

mean → μ

Variance → σ^2

Properties :-

(i) Bell shaped function.

(ii) μ = mean , σ^2 = Variance,

(iii) $\int_{-\infty}^{\infty} f_x(x)\,dx = 1$

(iv) Symmetric about mean (μ)

(v) $P(x \le \mu) = P(x \ge \mu) = 1/2$

(vi) maximum value at $x = \frac{1}{\sigma\sqrt{2\pi}}$

(vii) mean = median = μ

(viii) $P(\mu-\sigma \le x \le \mu+\sigma) = 0.6825 = 68.25\%$

(ix) $P(\mu-2\sigma \le x \le \mu+2\sigma) = 0.9544 = 95.44\%$

(x) $P(\mu-3\sigma \le x \le \mu+3\sigma) = 0.995 = 99.5\%$

Gaussian Random Variable :-

$$F(x) = \int_{-\infty}^{x} f_x(x)\,dx$$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}\,dx$$

$$Y = \frac{\mu - x}{\sqrt{2}\sigma} \Rightarrow dy = -\frac{dx}{\sqrt{2}\sigma} \Rightarrow dx = -\sqrt{2}\sigma\,dy$$

$$x = -\infty \Rightarrow Y = \infty$$

$$x = x \Rightarrow Y = \frac{\mu - x}{\sqrt{2}\sigma}$$

$$F(x) = \int_{\infty}^{\frac{\mu-x}{\sqrt{2}\sigma}} \frac{1}{\cancel{\sigma}\cancel{\sqrt{2}}\sqrt{\pi}} \cdot e^{y^2} - \cancel{\sqrt{2}}\,\cancel{\sigma}\,dy$$

$$F(x) = -\int_{\infty}^{\frac{\mu-x}{\sqrt{2}\sigma}} e^{y^2}\,dy \cdot \frac{1}{\sqrt{\pi}}$$

$$F(x) = \frac{1}{\sqrt{\pi}} \int_{\frac{\mu-x}{\sqrt{2}\sigma}}^{\infty} e^{y^2}\,dy = \frac{1}{2}\left[\frac{2}{\sqrt{\pi}} \int_{\frac{\mu-x}{\sqrt{2}\sigma}}^{\infty} e^{y^2}\,dy\right]$$

$$F(x) = \frac{1}{2}\,\mathrm{erfc}\left\{\frac{\mu - x}{\sqrt{2}\sigma}\right\}$$

Lecture : https://www.youtube.com/playlist?list=PLXnsjPD8-xuuIC5GyfL8Iy6xysO2x4eMt

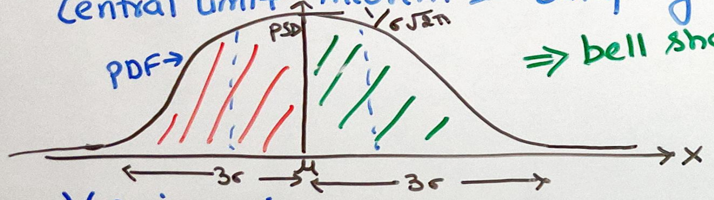
Please Visit Our Channel & Subscribe to get Instant Details About New Uploads

Probability and Statistics - Gaussian Distribution

Gaussian distribution / Normal distribution :-

→ used greatly in diverse fields.

→ used in digital communication in central limit theorem & Sampling theorem.



⇒ bell shaped structure

$$f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

PDF

mean → μ

variance → σ^2

Mean :- $\int_{-\infty}^{\infty} x\, f(x)\, dx$

$$E(x) = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$E(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x\, e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$

$$\frac{x-\mu}{\sqrt{2}\sigma} = Y \Rightarrow dx = \sqrt{2}\sigma\, dy$$

$$E(x) = \frac{1}{\cancel{\sigma\sqrt{2}}\sqrt{\pi}} \int_{-\infty}^{\infty} (\sqrt{2}\sigma Y + \mu) e^{-y^2} dy \cdot \cancel{\sqrt{2}\sigma}$$

$$E(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \sqrt{2}\sigma Y e^{-y^2} dy + \int_{-\infty}^{\infty} \mu e^{-y^2} dy \cdot \frac{1}{\sqrt{\pi}}$$

$$= \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \cancel{\int_{-\infty}^{\infty} Y e^{-y^2} dy}^{\,0} + \frac{\mu}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-y^2} dy$$

odd

$$= \frac{\mu}{\sqrt{\pi}} \int_{-\infty}^{\infty} e^{-y^2} dy = \frac{2\mu}{\sqrt{\pi}} \int_{0}^{\infty} e^{-y^2} dy$$

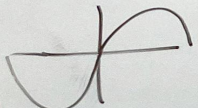
$$= \frac{\cancel{2}\mu}{\cancel{\sqrt{\pi}}} \frac{\cancel{\sqrt{\pi}}}{\cancel{2}}$$

$$E(x) = \mu$$

Variance :-

$$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f_x(x) dx$$

$$= \int_{-\infty}^{\infty} (x-\mu)^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$$



$$= \frac{1}{\cancel{\sigma\sqrt{2}}\sqrt{\pi}} \int_{-\infty}^{\infty} 2\sigma^2 y^2 e^{-y^2} \cdot \cancel{\sqrt{2}\sigma}\, dy$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} y^2 e^{-y^2} dy \quad ; \quad y^2 = t \Rightarrow 2y\,dy = dt$$

$$= \frac{2\sigma^2}{\sqrt{\pi}} \int_{-\infty}^{\infty} t e^{-t} \frac{dt}{2y} = \frac{\cancel{2}\sigma^2}{\cancel{2}\sqrt{\pi}} \int_{-\infty}^{\infty} t \cdot \frac{e^{-t}}{t^{-1/2}} dt$$

$$= \frac{\sigma^2}{\sqrt{\pi}} \int t^{-1/2} e^{-t} dt$$

$$= \frac{\sigma^2}{\cancel{\sqrt{\pi}}} \cdot \cancel{\sqrt{\pi}} = \sigma^2$$

Lecture : https://www.youtube.com/playlist?list=PLXnsjPD8-xuuIC5GyfL8Iy6xysO2x4eMt

Please Visit Our Channel & Subscribe to get Instant Details About New Uploads

Vaishali's e-Learning
Make your Learning Easier

2 Dimensional Random Variable

2-D Random Variable :-

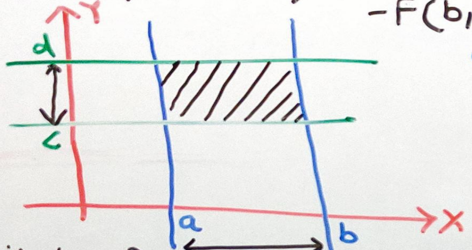
$$(X,Y)(\omega) = \{X(\omega), Y(\omega)\} \quad \omega \in S$$

Distribution function :-

$$F(x,y) = P(X \le x \,;\, Y \le y) \,;\, x,y \in R^2$$

Properties of Distribution function :-

① $P(a \le X \le b \,;\, c \le Y \le d) = F(b,d) - F(a,d) - F(b,c) + F(a,c)$



$a < b$

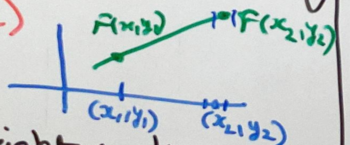
$c < d$

② Distribution f^n $[F(x,y)]$ is non decreasing

$F(x_1,y_1) < F(x_2,y_2)$

$x_1 < x_2$

$y_1 < y_2$



③ Distribution function is right continuous.

$F(x+0,y) = F(x,y+0) = F(x,y)$

④ $F(-\infty,y) = F(x,-\infty) = 0 = F(-\infty,-\infty)$

⑤ $F(\infty,\infty) = 1$

Probability mass function :-

$$F(x,y) = \sum_{x<x_i} \sum_{y<y_j} f(x,y) \rightarrow \text{PMF}$$

$X, Y \rightarrow$ Discrete R.V.

$$f(x,y) = \begin{cases} P(X = x_i \,;\, Y = y_j) = p_{ij} \,; & (x = x_i \,;\, y = y_j) \\ 0 \,; & \text{otherwise } X \ne x_i \,;\, Y \ne y_j \end{cases}$$

Properties of PMF :-

(i) $p_{ij} \ge 0$

(ii) $\sum_i \sum_j p_{ij} = 1$

Probability distribution function :-

$X, Y \rightarrow$ Continuous R.V.

$$F(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} \underbrace{f(x,y)}_{\rightarrow \text{PDF}} \, dv \, du$$

$$\underbrace{f(x,y)}_{\hookrightarrow \text{PDF}} = \frac{\partial^2 F(x,y)}{\partial x \, \partial y}$$

Properties of PDF :-

(i) $f(x,y) \ge 0$

(ii) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \, dy \, dx = 1$

Lecture : https://www.youtube.com/playlist?list=PLXnsjPD8-xuuIC5GyfL8Iy6xysO2x4eMt

2 Dimensional Random Variable - Questions

Q Suppose 2 fair coins are tossed simultaneously. We define two random variables X, Y as
X = No. of head in toss of first coin.
Y = No. of head in toss of second coin.

Find PMF. X: T → 0 → ½, H → 1 → ½ ; Y: T → 0 → ½, H → 1 → ½

$(X,Y) = (0,0), (0,1), (1,0), (1,1) \to p$

$p(0,0) = \frac{1}{2}\times\frac{1}{2}$; $p(0,1) = \frac{1}{2}\times\frac{1}{2}$; $p(1,0) = \frac{1}{2}\times\frac{1}{2}$

$p(1,1) = \frac{1}{2}\times\frac{1}{2}$

X \ Y	0	1
0	1/4	1/4
1	1/4	1/4

Q Let (X,Y) be continuous R.V. with joint PDF given by:—

$$f(x,y) = \begin{cases} e^{-(x+y)} & \text{if } 0 < x, y < \infty \\ 0 & ; \text{ otherwise} \end{cases}$$

find distribution function of (X,Y)

$$F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(x,y)\, dv\, du$$

$$= \int_0^x\int_0^y e^{-(x+y)}\, dv\, du = \int_0^x\int_0^y e^{-x}\cdot e^{-y}\, dv\, du$$

$$= \int_0^x e^{-x} du \int_0^y e^{-y} dv$$

$$= [-e^{-x}]_0^x\, [-e^{-y}]_0^y = (1-e^{-x})(1-e^{-y})$$

$$F(x,y) = \begin{cases} (1-e^{-x})(1-e^{-y}) & ; x, y \geq 0 \\ 0 & ; \text{o.w.} \end{cases}$$

Q Find K if $f(x,y) = K(1-x)(1-y)$ for $0 < x, y < 1$ to be the joint density function.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X}(x,y)\, dy\, dx = 1 \Rightarrow \int_0^1\int_0^1 K(1-x)(1-y)\, dx\, dy = 1$$

$$K\int_0^1 (1-x)\, dx \int_0^1 (1-y)\, dy = 1$$

$$K\left[x - \frac{x^2}{2}\right]_0^1 \left[y - \frac{y^2}{2}\right]_0^1 = 1$$

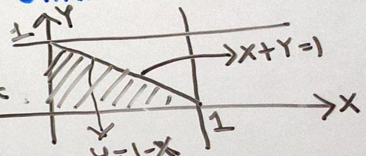
$$K\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 1$$

$$\boxed{K = 4}$$

Q Joint PDF of bivariate R.V. (X,Y) is given by:

$$f(x,y) = \begin{cases} 4xy & , 0 < x, y < 1 \\ 0 & , \text{otherwise.} \end{cases}$$

find $P(X+Y<1)$.



$$P(X+Y<1) = \int_0^1\int_0^{1-x} 4xy\, dy\, dx$$

$$= \int_0^1\int_0^{1-x} 4xy\, dy\, dx = \int_0^1 \left(4x\frac{y^2}{2}\right)_0^{1-x} dx$$

$$= \int_0^1 2x(1-x^2)\, dx$$

$$= \int_0^1 2x - 2x^3\, dx = \left[x^2 - \frac{2}{4}x^4\right]_0^1$$

$$= \frac{1}{6}$$

2 Dimensional Random Variable - Questions

Q If joint density function of R.V X, Y be

$$f(X,Y) = \begin{cases} e^{-(x+y)} & ; x \ge 0,\ y \ge 0 \\ 0 & ; \text{o.w.} \end{cases}$$

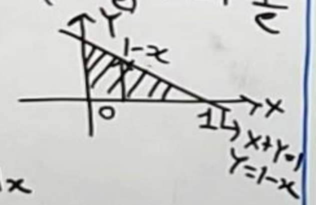
Find $P(X<1)$, $P(X+Y<1)$

$$P(X<1) = \int_0^1\int_0^\infty e^{-(x+y)}\,dy\,dx$$

$$= \int_0^1 e^{-x}dx \int_0^\infty e^{-y}dy$$

$$= [-e^{-x}]_0^1\,[-e^{-y}]_0^\infty = [1-e^{-1}][1-e^{-\infty}]$$

$$= (1-\tfrac{1}{e})\times 1 = 1-\tfrac{1}{e}$$



$$P(X+Y<1) = \int_0^1\int_0^{1-x} e^{-(x+y)}\,dy\,dx$$

$$= \int_0^1\int_0^{1-x} e^{-x}e^{-y}\,dy\,dx$$

$$= \int_0^1 [e^{-x}[-e^{-y}]_0^{1-x}\,dx$$

$$= \int_0^1 e^{-x}[-e^{-1+x}+e^0]\,dx$$

$$= \int_0^1 e^{-x}[e^{-1+x}+1]\,dx$$

$$= \int_0^1 (e^{-1}+e^{-x})\,dx = [e^{-1}x+[-e^{-x}]]_0^1$$

$$= e^{-1}+(-e^{-1})-(-e^0) = 1-\tfrac{2}{e}$$

Q If Joint PDF of X, Y is

$$f(x,y) = \begin{cases} 1/2 & ; x>0,\ y>0,\ x+y<2 \\ 0 & \text{elsewhere.} \end{cases}$$

find $P(X\le 1, Y\le 1)$, $P(X+Y<1)$, $P(X>2Y)$

$$P(X\le 1, Y\le 1) = \int_0^1\int_0^1 \tfrac{1}{2}\,dy\,dx = \tfrac{1}{2}\int_0^1 y\Big|_0^1 dx$$

$$= \tfrac{1}{2}\int_0^1 dx = \tfrac{1}{2}[x]_0^1 = \tfrac{1}{2}$$

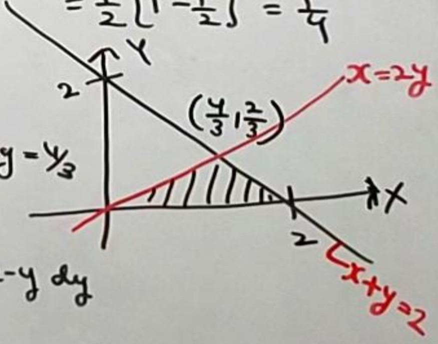
(ii) $$P(X+Y<1) = \int_0^1\int_0^{1-x} \tfrac{1}{2}\,dy\,dx = \tfrac{1}{2}\int_0^1 y\Big|_0^{1-x} dx$$

$$= \tfrac{1}{2}\int_0^1 1-x\,dx = \tfrac{1}{2}\left[x-\tfrac{x^2}{2}\right]_0^1$$

$$= \tfrac{1}{2}[1-\tfrac{1}{2}] = \tfrac{1}{4}$$

(iii) $P(X>2Y)$

$2y+y=2$
$3y=2$
$y=2/3$; $x=2y=4/3$



$$= \int_0^{2/3}\int_0^{2-y}\tfrac{1}{2}\,dx\,dy$$

$$= \tfrac{1}{2}\int_0^{2/3} x\Big|_0^{2-y}dy = \tfrac{1}{2}\int_0^{2/3} 2-y\,dy$$

$$= \tfrac{1}{2}\left[2y-\tfrac{y^2}{2}\right]_0^{2/3}$$

$$= \tfrac{1}{2}\left\{\tfrac{4}{3}-\tfrac{4}{9\times 2}\right\} = \tfrac{1}{3}$$

2 Dimensional Random Variable - Questions

Q If two random variables have joint density

$$f(x_1,x_2) = \begin{cases} x_1 x_2 & ; 0<x_1<1 ; 0<x_2<2 \\ 0 & \text{elsewhere.} \end{cases}$$

find the probability that both random variables take values less than 1.

$$\underset{\text{Joint CDF}}{P(x_1, x_2 < 1)} = \int_0^1\int_0^1 x_1 x_2 \, dx_2 \, dx_1$$

$$= \int_0^1 x_1 \, dx_1 \int_0^1 x_2 \, dx_2$$

$$= \left[\frac{x_1^2}{2}\right]_0^1 \left[\frac{x_2^2}{2}\right]_0^1$$

$$= \frac{1}{2}\cdot\frac{1}{2} = 1/4$$

Q If $f(x,y) = kye^{-x}, x>0; 0<y<2$ is the joint PDF of two R.V. X & Y. find K.

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dy\,dx = 1$$

$$1 = \int_0^{\infty}\int_0^2 kye^{-x}\,dy\,dx \Rightarrow K\int_0^2 y\,dy \int_0^{\infty} e^{-x}\,dx = 1$$

$$K\,\frac{y^2}{2}\Big|_0^2 (-e^{-x})\Big|_0^{\infty} = 1$$

$$K\cdot 2(-e^{-\infty} + e^0) = 1 \rightarrow K\cdot 2\times 1 = 1$$

$$K = 1/2$$

Q The joint PDF of R.V. X & Y is

$$f(x,y) = \begin{cases} cx(x-y) & ; 0<x<2 ; -x<y<x \\ 0 & \text{elsewhere} \end{cases}$$

Evaluate c.

$$\int_0^2\int_{-x}^{x} c\,x(x-y)\,dy\,dx = c\int_0^2\int_{-x}^{x} x^2 - xy\,dy\,dx$$

$$c\int_0^2 \left[x^2y - \frac{xy^2}{2}\right]_{-x}^{x} dx \Rightarrow c\int_0^2 x^3 + x^3 - \left[\cancel{\frac{x^3}{2}} - \cancel{\frac{x^3}{2}}\right] dx$$

$$c\int_0^2 2x^3\,dx \Rightarrow c\,\cancel{2}\frac{x^4}{\cancel{4}2}\Big|_0^2 = 1$$

$$8\times c = 1 \Rightarrow c = 1/8$$

Q find the Joint PDF of 2 R.V. X & Y whose joint distribution function is given by

$$F(x,y) = \begin{cases} 1-e^{-x} - e^{-y} + e^{-x-y} & ; x>0, y>0 \\ 0 & ; \text{elsewhere} \end{cases}$$

$$\frac{\partial^2 F(x,y)}{\partial x\,\partial y} = f(x,y)$$

$$\frac{\partial F(x,y)}{\partial y} = \frac{\partial(\cancel{1} - \cancel{e^{-x}} - e^{-y} + e^{-x}\cdot e^{-y})}{\partial y}$$

$$= e^{-y} - e^{-x}e^{-y}$$

$$\frac{\partial^2 F(x,y)}{\partial x\,\partial y} = 0 - \{-e^{-x}e^{-y}\}$$

$$f(x,y) = e^{-x}\cdot e^{-y} = e^{-x-y}$$

Marginal / Conditional Distribution

Marginal Distribution :—

$f_X(x) = \sum_y f_{X,Y}(x,y)$ {Discrete R.V.}

↳ Joint PDF

$f_Y(y) = \sum_x f_{X,Y}(x,y)$

Properties :—

(i) $f_X(x) \geq 0$; $f_Y(y) \geq 0$

(ii) $\sum_i f_X(x_i) = 1$; $\sum_i f_Y(y_i) = 1$

Continuous Random Variable :—

$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$

↳ Joint PDF

$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dx$

Properties :—

(i) $f_X(x) \geq 0$; $f_Y(y) \geq 0$

(ii) $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$; $\int_{-\infty}^{\infty} f_Y(y)\,dy = 1$

Independent Random Variables :—

$f_{X,Y}(x,y) = f_X(x) \cdot f_Y(y)$

Conditional Distribution :—

$P(A/B) = \frac{P(A \cap B)}{P(B)}$

$f_{X/Y}(x/y) = \frac{f_{X,Y}(x,y)}{f_Y(y)}$ | $f_{Y/X}(y/x) = \frac{f_{Y,X}(y,x)}{f_X(x)}$

Properties :—

(i) $f_{X/Y}(x/y) \geq 0$; $f_{Y/X}(y/x) \geq 0$

(ii) $\sum_x f_{X/Y}(x/y) = \frac{\sum_x f_{X,Y}(x,y)}{f_Y(y)} = \frac{\cancel{f_Y(y)}}{\cancel{f_Y(y)}} = 1$

Independence condition :—

(i) $f_{X/Y}(x/y) = \frac{f_X(x) \cdot \cancel{f_Y(y)}}{\cancel{f_Y(y)}} = f_X(x)$

(ii) $f_{Y/X}(y/x) = \frac{\cancel{f_X(x)} \cdot f_Y(y)}{\cancel{f_X(x)}} = f_Y(y)$

Q Check $\frac{1}{4}e^{-|x|} \cdot e^{-|y|}$ is independent or not

↳ Joint PDF

$f_{X,Y}(x,y) = \frac{1}{4}e^{-|x|} \cdot e^{-|y|} = \underbrace{\frac{1}{2}e^{-|x|}}_{f(x)} \cdot \underbrace{\frac{1}{2}e^{-|y|}}_{f(y)}$

⇒ ✓ independent

Marginal / Conditional Distribution

Q The joint distribution of X and Y is given by

$$f(x,y) = \frac{x+y}{21} \; ; \quad x = 1,2,3 \; ; \; y = 1,2.$$

Find the marginal distribution and mean of X & Y.

$$f_X(x) = \sum_y \underbrace{f(x,y)}_{\text{Joint PDF}} \quad \Rightarrow \frac{x+y}{21} = f(x,y)$$

$$f_X(1) = \sum_{y=1}^{2} f(1,y) = \frac{1+1}{21} + \frac{1+2}{21} = \frac{5}{21}$$

$$f_X(2) = \sum_{y=1}^{2} f(2,y) = \frac{2+1}{21} + \frac{2+2}{21} = \frac{7}{21} = \frac{1}{3}$$

$$f_X(3) = \sum_{y=1}^{2} f(3,y) = \frac{3+1}{21} + \frac{3+2}{21} = \frac{9}{21} = \frac{3}{7}$$

x	1	2	3
$f_X(x)$	$5/21$	$1/3$	$3/7$

$$E(X) = \sum_x x \, f_X(x)$$

$$= 1 \times \frac{5}{21} + 2 \times \frac{1}{3} + 3 \times \frac{3}{7}$$

$$f_Y(y) = \sum_{x=1}^{3} f(x,y)$$

$$f_Y(1) = \sum_{x=1}^{3} f(x,1) = \frac{1+1}{21} + \frac{2+1}{21} + \frac{3+1}{21} = \frac{9}{21}$$

$$f_Y(2) = \sum_{x=1}^{3} f(x,2) = \frac{1+2}{21} + \frac{2+2}{21} + \frac{3+2}{21} = \frac{12}{21}$$

Y	1	2
$f_Y(y)$	$3/7$	$4/7$

$$E(Y) = \sum_y y \, f_Y(y)$$

$$= 1 \times \frac{3}{7} + 2 \times \frac{4}{7}$$

$$= \frac{11}{7}$$

Statistical Average {2-D Random Variable}

$$E[g(x,y)] = \int_{x=-\infty}^{\infty}\int_{y=-\infty}^{\infty} g(x,y)\, \underbrace{f(x,y)}_{\text{Joint PDF}}\, dy\, dx \; ; \; g(x,y) = xy \Rightarrow E(xy) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy\, f(x,y)\, dy\, dx \;\rightarrow \text{1st order}$$

$$g(x,y) = x^2y^2 \Rightarrow E(x^2y^2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2y^2 f(x,y)\, dy\, dx \;\rightarrow \text{2nd order}$$

$$E_n[x^n y^n] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^n y^n f(x,y)\, dy\, dx \;\rightarrow \text{nth order expectation}$$

Q Let X and Y be random Variables with Joint density function

$$f(x,y) = \begin{cases} 4xy & ; 0 \le x, y \le 1 \\ 0 & ; \text{otherwise} \end{cases}$$

find E(XY).

$$g(x,y) = xy$$

$$E(xy) = \int_0^1\int_0^1 xy\, 4xy\, dy\, dx$$

$$= \int_0^1\int_0^1 4x^2y^2\, dy\, dx$$

$$= 4\int_0^1 x^2 dx \int_0^1 y^2 dy$$

$$= 4 \left.\frac{x^3}{3}\right|_0^1 \cdot \left.\frac{y^3}{3}\right|_0^1 = 4 \times \frac{1}{3} \times \frac{1}{3}$$

$$= 4/9$$

Q The joint PDF of random variable X and Y is given by:

$$f(x,y) = \begin{cases} 25 & ; 0.95 < x, y < 1.15 \\ 0 & ; \text{otherwise.} \end{cases}$$

find (i) P(XY<1) ; (ii) E(XY)

Conditional Probability

(i) $P(XY<1) \Rightarrow Y < 1/x$

CDF

$$\int_{-\infty}^{\infty}\int_{-\infty}^{1/x} f(x,y)\, dy\, dx$$

$$\int_{0.95}^{1.15}\int_{0.95}^{1/x} 25\, dy\, dx$$

$$25\int_{0.95}^{1.15} \left. y \right|_{0.95}^{1/x} dx$$

$$25\int_{0.95}^{1.15} \left(\frac{1}{x} - 0.95\right) dx$$

$$\left|\log(x) - 0.95x\right|_{0.95}^{1.15}$$

$$\log 1.15 - \log 0.95 + (0.95)^2 - 1.15 \times 0.95$$

$$= 0.025$$

(ii) $g(x,y) = xy$

$$\int_{0.95}^{1.15}\int_{0.95}^{1.15} xy\, 25\, dy\, dx$$

$$25\int_{0.95}^{1.15}\int_{0.95}^{1.15} xy\, dy\, dx$$

$$25\int_{0.95}^{1.15} x\, dx \int_{0.95}^{1.15} y\, dy$$

$$25 \left(\left.\frac{x^2}{2}\right|_{0.95}^{1.15}\right) \left.\frac{y^2}{2}\right|_{0.95}^{1.15}$$

$$= 1.102$$

Vaishali's e-Learning
Make your Learning Easier

Cauchy Schwarz Inequality

$E(XY)^2 \le E(X^2).E(Y^2)$; $X, Y \Rightarrow$ Random Variables.

Proof: Random Variable $\Rightarrow X - aY$

constant ← a

$E(X-aY)^2 \ge 0$ {mean square value is greater than, equal to zero)

$E[(X^2+a^2Y^2-2aXY)] \ge 0$

$\because E \rightarrow$ linear function

$E(X^2)+E(a^2Y^2)-E(2aXY] \ge 0$

$[E(X^2)+a^2E(Y^2)-2aE(XY)] \ge 0$

Let

$a = \dfrac{E(XY)}{E(Y)^2}$

$$E(X^2)+\frac{E(XY)^2}{E(Y^2)^{\cancel{2}}}\cancel{E(Y^2)} - \frac{2E(XY)^2}{E(Y^2)} \ge 0$$

$$\frac{E(X^2)E(Y^2)+E(XY)^2-2E(XY)^2}{E(Y^2)} \ge 0$$

$$E(X^2)E(Y^2) - E(XY)^2 \ge 0$$

$$E(X^2)E(Y^2) \ge E(XY)^2$$

$$E(XY)^2 \le E(X^2)E(Y^2)$$

Sum of Random Variables

$Z = X + Y$; $X, Y \rightarrow$ Random Variables.

① mean :—

$m_Z = m_{X+Y} = m_X + m_Y$

mean of Sum of R.V = Sum of mean of R.V.

$$E(X+Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x+y) f_{X,Y}(x,y)\, dy\, dx$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x f(x,y)\, dy\, dx + \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} y f(x,y)\, dx\, dy$$

$$= \int_{-\infty}^{\infty} x \underbrace{\int_{-\infty}^{\infty} f(x,y)\, dy}_{\hookrightarrow f_X(x)} dx + \int_{-\infty}^{\infty} y \underbrace{\int_{-\infty}^{\infty} f(x,y)\, dx}_{\hookrightarrow f_Y(y)} dy$$

(marginal distribution of X) (marginal distribution of Y)

$$= \int_{-\infty}^{\infty} x f_X(x)\, dx + \int_{-\infty}^{\infty} y f_Y(y)\, dy$$

$$= E(x) + E(y)$$

$$m_{X+Y} = m_X + m_Y$$

$$m_Z = m_X + m_Y$$

② Variance :—

$$Var(Z) = E[X + Y - E(X+Y)]^2$$

$$\hookrightarrow E[Z - E(Z)]^2$$

$$= E(Z - m_Z)^2$$

$$\sigma_Z^2 = E(X + Y - E(X) - E(Y))^2$$

$$= E(\underbrace{X - E(X)}_{a} + \underbrace{Y - E(Y)}_{b})^2$$

$$= E[(X - E(X))^2 + (Y - E(Y))^2 + 2(X - E(X))(Y - E(Y))]$$

$$= E[(X - E(X)]^2 + E(Y - E(Y)]^2 + 2E(X - E(X)] E(Y - E(Y)]$$

$$\sigma_Z^2 = \sigma_X^2 + \sigma_Y^2 + Cov(X, Y)$$

Product of independent R.V :—

$$f(x,y) = f_X(x) \cdot f_Y(y)$$

$\hookrightarrow$ independence.

mean :— $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} xy f(x,y)\, dy\, dx$

$$\mu_Z = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\, y\, f_X(x) f_Y(y)\, dy\, dx$$

$$\mu_Z = \int_{-\infty}^{\infty} \underbrace{x f_X(x)\, dx}_{m_X} \cdot \int_{-\infty}^{\infty} \underbrace{y f_Y(y)\, dy}_{m_Y}$$

$$m_Z = m_X \cdot m_Y$$

Please Visit Our Channel & Subscribe to get Instant Details About New Uploads

Probability & Statistics

Effects of Gaussian Noise :-

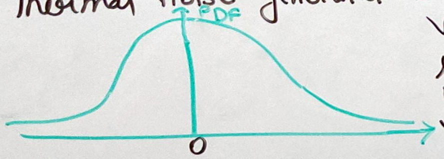
Noise → causes error

error rate → depend on signal & noise amplitude & characteristics of noise.

→ Noise of large amplitude spike will introduce error in the detected signal.

It is assumed to be gaussian.

eg: Thermal noise generated in a resistor.



$V \uparrow \Rightarrow PDF \downarrow$

$\mu = 0$

$max = V = 0$

Error Function:-

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

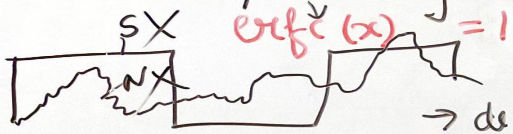
$$0 \le erf \le 1$$

$$x = 0 \Rightarrow erf(x) = 0$$

$$x = \infty \Rightarrow erf(x) = 1$$

Symmetry property: $erf(x) = -erf(-x)$

Complementary error function:-

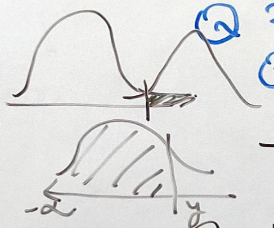


$$erfc(x) = 1 - erf(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2} du$$

$x \uparrow \Rightarrow erfc(x) \downarrow$

→ decreasing function of x

→ Power spectral density of signals



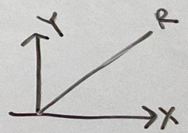
Q Factor :-

$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-x^2/2} dx \qquad \sigma^2 = 1 \ ; \ \mu = 0$$

→ Gaussian distribution with $\mu = 0$, $\sigma^2 = 1$

$$Q(-y) = 1 - Q(y)$$

Reyleigh's Distribution:-



→ continuous R.V.

→ produced from 2 Gaussian R.V.

$m_x = m_y = m$ & $\sigma_x = \sigma_y = \sigma$

$R = \sqrt{x^2 + y^2}$; $\phi = \tan^{-1}(Y/x)$

$R \Rightarrow \phi = 0$ to 2π even when $m = 0$

→ analyse narrow band Gaussian noise

$$f_R(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} \ ; \ r \ge 0$$

$$= 0 \ ; \ r < 0$$

$$mean = \sqrt{\frac{\pi}{2}}\sigma \ ; \ Var = \sigma_x^2 = \left[2 - \frac{\pi}{2}\right]\sigma^2$$

Vaishali's e-Learning

Make your Learning Easier

Lecture : https://www.youtube.com/playlist?list=PLXnsjPD8-xuuIC5GyfL8Iy6xysO2x4eMt

Please Visit Our Channel & Subscribe to get Instant Details About New Uploads

Chebyshev Inequality

$\sigma^2 \uparrow \Rightarrow$ spread $\uparrow$

$$P(\mu-\sigma \le x \le \mu+\sigma) \simeq 68\%$$
$$P(\mu-2\sigma \le x \le \mu+2\sigma) \simeq 95.4\%$$
$$P(\mu-3\sigma \le x \le \mu+3\sigma) \simeq 99.5\%$$
$\rightarrow$ Gaussian R.V.

Chebyshev inequality :—

$$P[|x-\mu| \le k\sigma] \ge 1-\frac{1}{k^2}$$

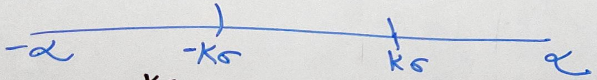
$\mu \rightarrow$ mean

$\sigma^2 \rightarrow$ varience $\Rightarrow \sigma =$ S.D. (standard Deviation)

Proof : Let $\mu = 0$

$$P[|x| \le k\sigma] \ge 1-\frac{1}{k^2}$$

$$\sigma^2 = \int_{-\infty}^{\infty} x^2 f_x(x)\,dx \quad \{\because \mu = 0\}$$
$\hookrightarrow$ 2nd order expectation



$$\sigma^2 = \int_{-\infty}^{-k\sigma} x^2 f_x(x)\,dx + \int_{-k\sigma}^{k\sigma} x^2 f_x(x)\,dx + \int_{k\sigma}^{\infty} x^2 f_x(x)\,dx$$

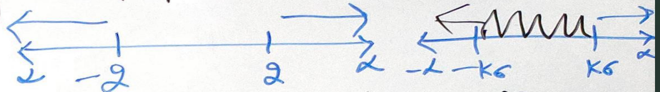
$X \rightarrow R.V \Rightarrow X^2 \ge 0$

$f_X(x) \ge 0$

$$\Rightarrow \int_{-k\sigma}^{k\sigma} x^2 f_X(x)\,dx \ge 0$$

$$\sigma^2 \ge \int_{-\infty}^{-k\sigma} x^2 f_X(x)\,dx + \int_{k\sigma}^{\infty} x^2 f_X(x)\,dx$$

$|x| \ge 2$ $\qquad$ $|x| \ge k\sigma$



$$\sigma^2 \Rightarrow \int_{|x| \ge k\sigma} x^2 f_X(x)\,dx \quad -①$$

Smallest value of $x = k\sigma$

if x is replaced by $k\sigma$ then also inequality in eq^n ① will hold.

$$\sigma^2 \ge \int_{|x| \ge k\sigma} k^2\sigma^2 f_X(x)\,dx$$

$$\cancel{\sigma^2} \ge k^2 \cancel{\sigma^2} \int_{|x| \ge k\sigma} f_X(x)\,dx$$
$\hookrightarrow$ XDF

$$\frac{1}{k^2} \ge P(|x| \ge k\sigma)$$

$$P(|x| \le k\sigma) \ge 1-\frac{1}{k^2}$$

$$K = 2 \Rightarrow 1-\frac{1}{4} = \frac{3}{4} = 75\%$$

$$K = 3 \Rightarrow 1-\frac{1}{9} = \frac{8}{9} = 89\%$$

$$P[|x-\mu| \le k\sigma) \ge 1-\frac{1}{k^2}$$

Lecture : https://www.youtube.com/playlist?list=PLXnsjPD8-xuuIC5GyfL8Iy6xysO2x4eMt

Correlation

Correlation → measure of dependence of one R.V. on another.

Let Random variables : X & Y

$Cov(XY) = E\{(x-m_x)(y-m_y)\}$

↳ expectation.

$= E[xy - m_y x - m_x y + m_y m_x\}$

E → linear operator

$= E(xy) - E(m_y x) - E(m_x y) + E(m_y m_x)$

$= E(xy) - m_y E(x) - m_x E(y) + m_x m_y$

$E(x) = m_x \quad ; \quad E(y) = m_y$

$= E(xy) - m_y m_x - m_y m_x + m_y m_x$

$= E(xy) - 2m_x m_y + m_x m_y$

$Cov(x,y) = E(xy) - m_x m_y\}$ → Relation b/w x & y

Correlation coefficient $(\rho_{XY}) = \dfrac{\sigma_{XY}}{\sigma_x \sigma_y}$

$Cov(xX) = E[(x-m_x)(x-m_x)\}$

$-1 \leq \rho_{XY} \leq 1$

Independent R.V:

$f(xy) = f_X(x) \cdot f_Y(y)$

$\mu_{XY} = \mu_X \mu_Y$

→ independent R.V. are uncorrelated.

→ uncorrelated R.V. are not always independent.

→ Joint Gaussian distribution are the exception.

CORRELATION

Q Let X and Y be jointly distributed with the correlation coefficient 1/2, $\sigma_X = 2$, $\sigma_Y = 3$, find Var(2X−4Y+3).

$\rho_{XY} = 1/2$; $\sigma_X = 2$; $\sigma_Y = 3$

$\rho_{XY} = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} \Rightarrow \frac{1}{2} = \frac{Cov(X,Y)}{2\times 3}$

$\Rightarrow Cov(X,Y) = 3$

$Var(2X-4Y+3) = 4\,Var(X) + 16\,Var(Y) - 8\,Cov(X,Y)$

$Var(X) = \sigma_X^2 = 4$; $Var(Y) \Rightarrow \sigma_Y^2 = 3^2 = 9$

$= 4\times 4 + 16\times 9 - 8\times 3$

Q The joint PMF of X & Y is

Y \ X	−1	+1
0	1/8	3/8
1	2/8	2/8

find correlation coefficient of (X,Y).

$f_X(x) =$

Y	0	1
f_X	1/2	1/2

$\sum_y f_{X,Y}(x,y)$

$E(X) = \mu = 0\times\frac{1}{2} + 1\times\frac{1}{2} = 1/2$

$E(X^2) = 0\times\frac{1}{2} + 1\times\frac{1}{2} = 1/2$

$\sigma_X^2 = E(X^2) - E(X)^2 = \frac{1}{2} - \frac{1}{4} = \frac{1}{2} \Rightarrow \sigma_X = \frac{1}{\sqrt{2}}$

$f_Y(y) =$

x	−1	1
$f_Y(y)$	3/8	5/8

$\Rightarrow E(X) = 2/8 = 1/4$

$E(X^2) = 1$

$\sigma^2 = 1 - \frac{1}{16} = \frac{15}{16} \Rightarrow \sigma_Y = \sqrt{15/16} = \frac{\sqrt{15}}{4}$

$Cov(X,Y) = E(XY) - E(X)\cdot E(Y)$

$= 0 - \frac{1}{2}\cdot\frac{1}{4} = -\frac{1}{8}$

$\Rightarrow E(XY) = -\frac{2}{8} + \frac{2}{8} = 0$

$\rho_{XY} = \frac{-1/8}{\frac{1}{\sqrt{2}}\cdot\sqrt{15}/4}$

Q If X and Y are 2 correlated random variables with same variance and if r is the correlation coefficient b/w X & Y find correlation between X & X+Y.

$\rho_{XY} = r = \frac{Cov(X,Y)}{\sigma\cdot\sigma} \Rightarrow Cov(X,Y) = r\sigma^2$

$Cov(X, X+Y) = E\{(X-\mu)\{(X-\mu)+(Y-\mu)\}\}$

$= E(X-\mu)^2 + E\{(X-\mu)(Y-\mu)\}$

$= \sigma^2 + Cov(X,Y) = \sigma^2 + r\sigma^2$

$= r(1+\sigma^2)$

$Var(X+Y) = Var(X) + Var(Y) + 2\,Cov(XY)$

$= \sigma^2 + \sigma^2 + 2r\sigma^2 \Rightarrow 2\sigma^2(1+r)$

$\rho_{X,X+Y} = \frac{r(1+\sigma^2)}{\sigma\cdot\sqrt{2(1+r)}\,\sigma} = \frac{r(1+\sigma^2)}{\sigma^2\sqrt{2(1+r)}}$

Q Let the RV X be uniformly distributed over (−1,1) and $Y = X^2$. Show X & Y are uncorrelated, but they are dependent.

$f_X(x,y) = \begin{cases} \frac{1}{2} & ; -1 \le x \le 1 \\ 0 & ; \text{o.w} \end{cases}$

$E(X) = \int_{-\infty}^{\infty} x\,\frac{1}{2}\,dx = \frac{1}{2}\int_{-1}^{1} x\,dx \rightarrow$ odd fn $= 0$

$E(Y) = E(X^2) = \int_{-1}^{1} x^2\,\frac{1}{2}\,dx = \frac{2}{2}\int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 = \frac{1}{3}$

$E(XY) = E(X\cdot X^2) = E(X^3) = \int_{-1}^{1}\frac{1}{2}x^3\,dx = 0$

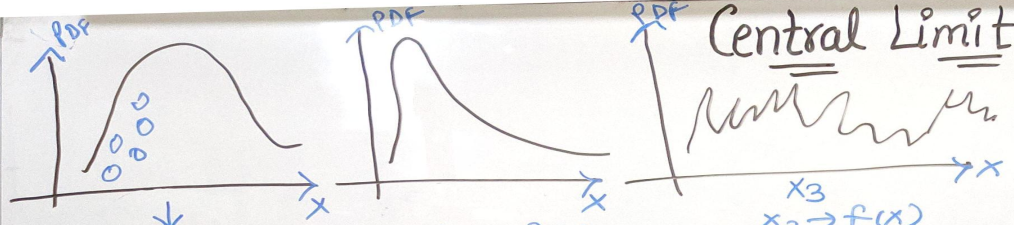
$Cov(X,Y) = E(XY) - E(X)E(Y)$

$= 0 - 0\times\frac{1}{3} = 0 \Rightarrow$ uncorrelated

$\therefore Y = X^2 \Rightarrow$ dependent

Please Visit Our Channel & Subscribe to get Instant Details About New Uploads

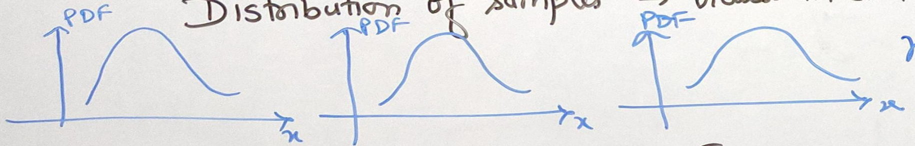
Central Limit Theorem {CLT}



$X_1 \to f(x)$ $\quad X_2 \to f(x)$ $\quad X_3 \to f(x)$

$n \to$ no. of samples $\to$ very large

Distribution of samples $\to$ Gaussian distribution



$n \to \alpha$ $\Downarrow$ Gaussian distribution

$S_n = X_1 + X_2 + X_3 + \cdots + X_n$ $\{X_1, X_2 \ldots X_n \to$ samples$\}$

$\Downarrow$

$X_1, X_2 \cdots X_n \to$ independent, identically distributed R.V.

$E(S_n) = E(X_1 + X_2 + X_3 \ldots + X_n)$

$= E(X_1) + E(X_2) + E(X_3) + \cdots + E(X_n)$

$= \mu + \mu + \mu + \cdots \mu$

$E(S_n) = n\mu$

$Var(S_n) = Var(X_1 + X_2 + \cdots + X_n)$

$= Var(X_1) + Var(X_2) + \cdots + Var(X_n)$

$Cov(X_i, X_j) \simeq 0$ {independent samples}

$= \sigma^2 + \sigma^2 + \cdots + \sigma^2$

$Var(S_n) = n\sigma^2$

$S_n = \sum x_i = N(\underbrace{n\mu}_{\text{mean}}, \underbrace{n\sigma^2}_{\text{variance}})$

$Z_n = \dfrac{X - n\mu}{\sqrt{n}\,\sigma}$

$\bar{X} = \dfrac{X_1 + X_2 + \cdots + X_n}{n}$

$E(\bar{X}) = \dfrac{E(X_1) + E(X_2) + \cdots + E(X_n)}{n}$

$= \dfrac{\mu + \mu + \cdots + \mu}{n} = \dfrac{n\mu}{n} = \mu$

$Var(\bar{X}) = Var\left\{\dfrac{X_1 + X_2 + \cdots + X_n}{n}\right\}$

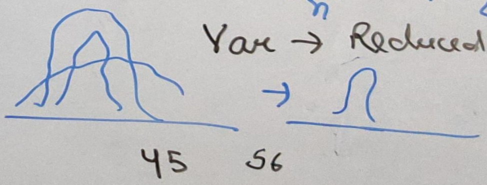
$= \dfrac{1}{n^2} Var(X_1) + Var(X_2) + \cdots + Var(X_n)$

$= \dfrac{1}{n^2}\{\sigma^2 + \sigma^2 + \cdots + \sigma^2\}$

$= \dfrac{1}{n^2} \cdot n \cdot \sigma^2 = \sigma^2/n$

$\bar{X} = \dfrac{\sum x_i}{n} = N\{\mu, \sigma^2/n\}$

Var $\to$ Reduced.



$n \to \alpha$

$Var(\bar{X}) \to 0$

H H/T

10000 — 5500 H, 4500 T

Lecture : https://www.youtube.com/playlist?list=PLXnsjPD8-xuuIC5GyfL8Iy6xysO2x4eMt

Central Limit Theorem

Q A random sample of size 100 is taken from a population whose mean is 60 and variance is 400. Using central limit theorem, with what probability can we assert that the mean of the sample will not differ from $\mu = 60$ by more than 4?

$n = 100$; $\mu = 60$; $\sigma^2 = 400$

$\mu' = \mu = 60$; $\sigma'^2 = \sigma^2/n = \frac{400}{100} = 4$

$\sigma' = 2$

$Z = \frac{X-60}{2} \to$ standard normal variate

$P\{|X-\mu| \le 4\} = P\left\{\frac{|X-60|}{2} \le 2\right\} = P\{|Z| \le 2\}$
$= P\{-2 \le Z \le 2\}$
$= 2P\{0 \le Z \le 2\} = 2 \times 0.4772$
$= 0.9544$

Q If $X_1, X_2 \ldots X_n$ are Poisson variables with parameter $\lambda = 2$, Use CLT to estimate $P(120 \le S_n \le 160)$, where $S_n = X_1 + X_2 + \cdots + X_n$, $n = 75$.

$\mu = n\lambda = \sigma^2 \Rightarrow \mu = \sigma^2 = 75 \times 2 = 150$

$Z = \frac{X-150}{\sqrt{150}}$; $P\{120 \le S_n \le 160\}$

$Z = \left\{\frac{120-150}{\sqrt{150}}\right\} = -2.4495$

$Z = \left\{\frac{160-150}{\sqrt{150}}\right\} = 0.8165$

$P\{-2.4495 \le Z \le 0.8165\}$
$P\{-2.4495 \le Z \le 0\} + P\{0 \le Z \le 0.8165\}$
$0.4929 + 0.2939$

Q A coin is tossed 200 times. find approximate probability that the number of heads obtained is between 80 to 120.

$n = 200$; $p = 1/2$; $q = 1/2$

$\mu = np = 200 \times \frac{1}{2} = 100$; $\sigma^2 = npq = 200 \times \frac{1}{4} = 50$

$Z = \left\{\frac{80-100}{\sqrt{50}}\right\} = -2.8284$

$Z = \left\{\frac{120-100}{\sqrt{50}}\right\} = 2.8284$

$P\{80 \le S_n \le 120\}$
$P\{-2.8284 \le Z \le 2.8284\}$
$= 2 \times 0.4977$
$= 0.9954$

Q 20 dice are thrown. find the approximate probability that the sum obtained is between 65 and 75 using CLT.

1	2	3	4	5	6	$\to X$
1/6	1/6	1/6	1/6	1/6	1/6	$P(x)$

$\mu = 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} \cdots + 6 \times \frac{1}{6} = \frac{7}{2}$

$\sigma^2 = 1 \times \frac{1}{6} + 4 \times \frac{1}{6} + 9 \times \frac{1}{6} + \cdots + 36 \times \frac{1}{6} = \frac{35}{12}$

$\mu' = 20 \times \frac{7}{2} = 70$; $\sigma'^2 = n\sigma^2 = 20 \times \frac{35}{12} = \frac{175}{3}$

$Z = \frac{X-70}{\sqrt{175/3}}$

$X = 65$
$Z = \frac{65-70}{\sqrt{175/3}} = -0.6547$

$X = 75$
$Z = \frac{75-70}{\sqrt{175/3}} = 0.6547$

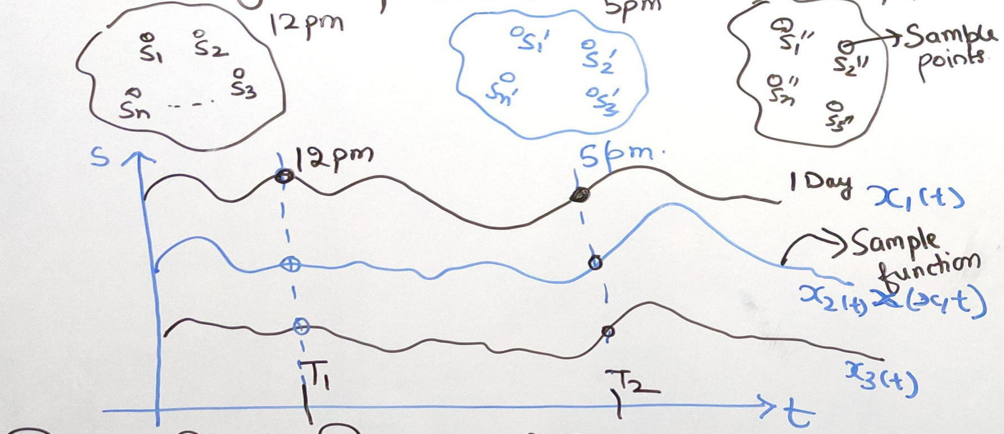
$P(-0.6547 \le Z \le 0.6547) = 2P\{0 \le Z \le 0.6547\}$
$= 2 \times 0.2422 = 0.4844.$

Random Process

$X(t)$; $X(t, x)$; $X(x, t)$

→ used to study statistical parameters of random signals like noise in the communication system.

Random Signal : future values cant be predicted.



Random Process : Projection of Random variable in the (Stochastic Process) form of sample function with respect to a continuous variable {generally time}.

Sample space :→ Collection of all sample points.

Ensemble :→ Collection of all sample functions

mean :- average value.

$$m_x = \int_{-\infty}^{+\infty} x f_X(x,t)\,dx \; ; \; t \to \text{constant}$$

mean wrt time :-

$$m(t) = \lim_{T\to\infty} \int_{-T/2}^{T/2} X(x,t)\,dt$$

CDF {Cumulative Distribution f^n}

$P\{X \le x\}$; $P\{X(t) \le x\}$

R.V

$F\{x_1, x_2 \dots x_n, t_1, t_2 \dots t_n\} =$

$P\{x(t) < x_1, x(t) < x_2, \dots, x(t) < x_n\}$

PDF {Probability Density function}

$$f(x,t) = \frac{\partial^n F\{x_1, x_2 \dots x_n, t_1, t_2 \dots t_n\}}{\partial t_1 \partial t_2 \partial t_3 \dots \partial t_n}$$

$R(t_1, t_2) = E\{x(t_1) \cdot x(t_2)\}$

↳ correlation at different times.

Autocorrelation :—

$R_X(\tau) = E(X(t) \cdot X(t+\tau))$

Properties :—

$R_X(\tau) = R_X(-\tau)$; max at $\tau = 0$; $= E(X^2(0))$

Classification of Random Process

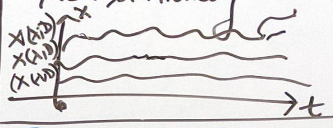
Random Process (State) ✓

$X(x,t)$

- Discrete (time)
  - Discrete ✓ — Discrete Random Sequence ✓
  - Continuous — Discrete Random Process
- Continuous (time)
  - Discrete — Continuous Random Sequence
  - Continuous — Continuous Random process

Random Process

- Stationary vs. Non stationary
- Wide sense stationary (weakly stationary)
- Ergodic Process



Stationary Random Process :-

Random Process whose statistical characteristics don't change with time.

→ Stationary shift of time origin dont have any effect on stationary random process.

Strictly Stationary :-

Joint CDF of original R.V. = new set of R.V. Obtained after shift τ

Original Joint CDF

$$F_{X(t_1),X(t_2)\dots X(t_n)}(x_1,x_2\dots x_n) = F_{X(t_1+\tau),X(t_2+\tau),\dots,X(t_n+\tau)}(x_1,x_2\dots x_n)$$

$X(t)$ → invariant wrt location of origin $t=0$.

Joint Stationary :-

$X(t_1)\dots X(t_n)$ & $Y(t_1)\dots Y(t_n)$ are invariant wrt location of origin $t=0$ $\forall\, i$ & j

$$f_{X(t_1)}(x_1) = f_{X(t_1+\tau)}(x_1) = f_X(x) \rightarrow \text{first order CDF}$$

$$f_{X(t_1)X(t_2)}(x_1,x_2) = f_{X(t_1-\tau)X(t_2-\tau)}(x_1,x_2) \rightarrow \text{Second order CDF}$$

Mean :- $m_X(t) = \int_{-\infty}^{\infty} x\, f_{X(t)}(x)\,dx = m_X$ → constant

Autocorrelation :- $R_X(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1 x_2\, f_{X(t_1)X(t_2)}(x_1,x_2)\,dx_1\,dx_2$ → Joint PDF

↳ does not depend.

Autocovariance $= C_X(t_2-t_1) = E\{[X(t_1)-m_X][X(t_2)-m_X]\} = R_X(t_2-t_1) - m_X^2$

Wide Sense Stationary :-

→ mean & autocorrelation are independent of shift of time origin.

$m_X(t) = \text{constant}$

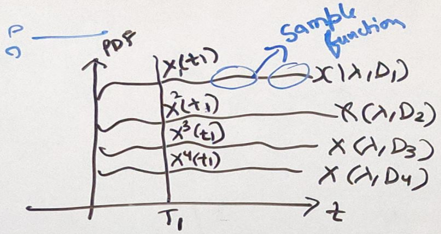
$R_X(t_1,t_2) = R_X(t_2-t_1)$ ✓

→ also called weakly stationary process

ERGODIC PROCESS

Conditions for a process to be ergodic process :—

(i) Condition on mean :—



Time average mean =

$$E(X(t)) = \frac{1}{2T}\int_{-T}^{T} x(t)\,dt$$

→ Time average mean = ensemble mean

$$\boxed{\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x(t)\,dt = m_x}$$

$$\frac{1}{2T}\int_{-T}^{T} x(t)\,dt = m(t)$$

$m(t)$ is Random Variable

$$E[m(t)] = E\,\frac{1}{2T}\int_{-T}^{T} x(t)\,dt$$

$$= \frac{1}{2T}\int_{-T}^{T} E(x(t))\,dt$$

$$= \frac{1}{2T}\int_{-T}^{T} \underbrace{m_x}_{\text{independent of time}}\,dt$$

$$= \frac{m_x}{2T}\int_{-T}^{T} dt = \frac{m_x}{2T}[t]_{-T}^{T}$$

$$= \frac{m_x}{2T} \times 2T = m_x$$

I $\boxed{\lim_{T\to\infty} \underbrace{m(t)}_{\text{Time mean}} = \underbrace{m_x}_{\text{Ensemble mean}}}$

II $\lim_{T\to\infty} Var[m(t)] = 0$

(ii) Condition on autocorrelation :—

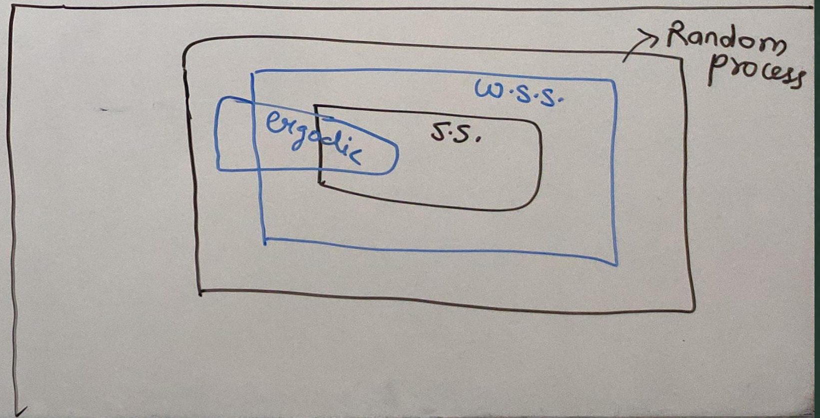
I $R_x(\tau, t) = \frac{1}{2T}\int_{-T}^{T} x(t)\,x(t+\tau)\,dt$

→ Time average autocorrelation f^n

I $\boxed{\lim_{T\to\infty} \underbrace{R_x(\tau, t)}_{\text{Random variable}} = R_x(\tau)}$

II Variance of autocorrelation f^n

$\boxed{Var\{R_x(\tau, t)\} = 0}$



Gaussian Process

Gaussian Process :—

$$\boxed{Y(t) = \int_0^T g(t)\,X(t)\,dt}$$

→ gaussian process

$g(t)$ → any function

mean square value of Random Variable $\boxed{Y}$ is a gaussian distributed Random Variable for every $g(t)$, then $x(t)$ is a Gaussian Process.

Gaussian Distribution ← $f_Y(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}$; μ = mean ; σ^2 = variance ; σ = standard deviation

Normalized Gaussian distribution: $\boxed{\mu = 0 \;;\; \sigma^2 = 1}$; $\boxed{f_Y(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2}}$ $= \boxed{N(\underbrace{0}_{\mu}, \underbrace{1}_{\sigma^2})}$

Properties :—

(i) when $x(t)$ is applied on a stable linear filter, $Y(t)$ obtained is gaussian process.

(ii) If Gaussian process is wide sense stationary, then it is strict sense stationary as well.

(iii) For $X(t_j)$

$\mu = m(t_j) = E(X(t_j)]$; $\boxed{j = 1, 2, \dots n}$; $k, j = 1, 2 \dots n$

$C_X(t_k, t_j) = E[(X(t_k) - m(t_k))(X(t_j) - m(t_j))]$ → autocorrelation fn

$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t)$

$x(t)$ → gaussian R.P.

$y(t)$ → Gaussian R.P.

(iv) If $E[(X(t_k) - m(t_k))(X(t_j) - m(t_j))] = 0$

→ uncorrelated when $X(t_i)$ are independent.

$X(t_1) \cdot X(t_2) \cdot X(t_3) \dots$

MULTIPLE RANDOM PROCESS

$X(t)$, $Y(t)$ → Two Random Process. X → Temperature of a city, Y → humidity

R_{XY} → Cross correlation function

$\boxed{R_{XY}(t_1, t_2)} = E\{X(t_1)Y(t_2)\}$ → correlation b/w $X(t_1)$ & $Y(t_2)$

Two Random process are jointly stationary if :-

(i) $X(t)$ & $Y(t)$ are wide sense stationary

(ii) $R_{XY}(t_1 t_2) = R_{XY}(t_2 - t_1) = R_{XY}(\tau)$; $t_2 - t_1 = \tau$ ← constant

Uncorrelated Process :-

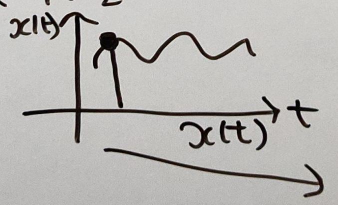
$\boxed{R_{XY}(\tau) = \bar{X}\,\bar{Y}}$ → $\bar{X}$ ⇒ mean of $X(t_1)$, $\bar{Y}$ ⇒ mean of $Y(t_2)$

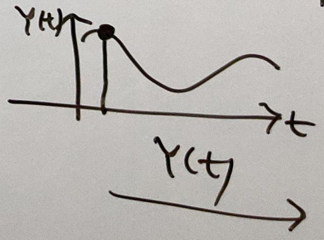
Incoherent / Orthogonal Process :-

$\boxed{R_{XY}(\tau) = 0}$

Independent process :-

$X(t_1)$ & $Y(t_2)$ are independent for all possible t_1 & t_2.





POWER SPECTRAL DENSITY

$$S_x(f) = \int_{-\infty}^{\infty} R_x(t) e^{-j2\pi f t} dt$$ {fourier transform of auto correlation function}

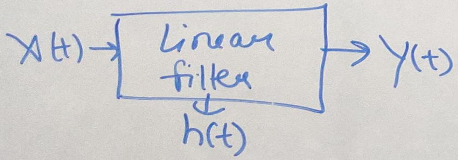
$$R_x(t) = \int_{-\infty}^{\infty} S_x(f) e^{j2\pi f t} df$$ {inverse fourier transform of P.S.D}

$$R_x(t) \xrightleftharpoons[\text{I.F.T}]{\text{F.T}} S_x(f)$$

Properties of Power Spectral Density (PSD):-

(i) $S_x(f) \rightarrow$ real & symmetric / even

$$S_x(f) = S_x(-f)$$



(ii) $S_x(f) \geq 0 \longrightarrow$ always +ve

(iii) $S_y(f) = S_x(f) \cdot |H(f)|^2$

$$R_y(f) = R_x(f) * R_h(f)$$

$$= R_x(f) * h(f) * h(f)$$

Random Process

Q Consider the Random process $X(t) = \cos(t+\phi)$, where ϕ is random variable with density function $f(\phi) = \frac{1}{\pi}$; where $-\frac{\pi}{2} < \phi < \frac{\pi}{2}$

Check whether or not the process is stationary.

$$E\{x(t)\} = \int_{-\pi/2}^{\pi/2} \frac{1}{\pi} \cos(t+\phi)\, d\phi$$

$$E\{\cos(t+\phi)\}$$

$$= \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \cos(t+\phi)\, d\phi$$

$$= \frac{1}{\pi} \left[\sin(t+\phi)\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{\pi} \left\{\sin(t+\pi/2) - \sin(t-\pi/2)\right\}$$

$$= \frac{1}{\pi} \left\{\sin(t+\pi/2) + \sin(\pi/2 - t)\right\}$$

$$= \frac{1}{\pi} \left\{\cos(t) + \cos(t)\right\}$$

$$= \frac{2}{\pi} \cos(t)$$

→ dependent upon 't'

$m_x = E\{x(t)\}$
$= E\{\cos(t+\phi)\}$

dependent upon 't'
→ not constant
⇒ it is not stationary

Please Visit Our Channel & Subscribe to get Instant Details About New Uploads

Vaishali's e-Learning

Make your Learning Easier

Random Process

Q A stationary random process $X=\{X(t)\}$ has autocorrelation function

$$R(\tau)=16+9e^{-|\tau|}$$

find the standard deviation of Process.

$$R(\tau)=E\left[X(t)\cdot X(t+\tau)\right]$$

$$R(0)=E\left[X(t)\cdot X(t)\right]$$

$$R(0)=E\left(X^2(t)\right]$$

$$E(X^2(t)]=R(0)=16+9e^0=16+9=25$$

$$\mu_X^2=\lim_{\tau\to\infty}R(\tau)$$

$$=16+9e^{-\infty}=16+9/e^{\infty}$$

$$\mu_X^2=16$$

$$Var(X)=E(X^2)-\mu_X^2$$

$$=25-16$$

$$=9$$

$$S.D=+\sqrt{Var}$$

$$=+\sqrt{9}$$

$$=+3$$

Q The autocorrelation function of a stationary process $X(t)$ is

$$R_{XX}(\tau)=9+2e^{-|\tau|}$$

find mean of $X(t)$ & mean of $Y=\int_0^2 X(t)\,dt$, $Var\{X(t)\}$

$$\mu_X^2=\lim_{\tau\to\infty}R_{XX}(\tau)=9+2^{-\infty}$$

$$\mu_X^2=9$$

$$\mu_X=3$$

$$R(0)=E(X^2(t)]=9+2=11$$

$$Var(X(t)]=11-9$$

$$E(X^2)-\mu_X^2$$

$$=2$$

$$E(Y)=E\{\int_0^2 x(t)\,dt$$

$$=\int_0^2 \underbrace{E(x(t)]}\,dt$$

$$=\int_0^2 3\,dt$$

$$E\left\{\int_0^2 x(t)\,dt\right\}=E\{Y\}$$

$$=3[t]_0^2$$

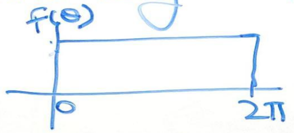
$$=3\times 2=6$$

Lecture : https://www.youtube.com/playlist?list=PLXnsjPD8-xuuIC5GyfL8Iy6xysO2x4eMt

Random Process – Numericals

Q Consider a random process, $x(t) = A\cos(\omega_0 t + \theta)$, where A, ω_0 are constant & θ is uniformly distributed in $[0, 2\pi]$. Find the stationarity of Random Process.

$$E = \int_{-\infty}^{\infty} x(t) f(\theta)\, d\theta$$



$$f(\theta) = \frac{1}{2\pi}$$

$$E(t) = \int_0^{2\pi} A\cos(\omega_0 t + \theta) \frac{1}{2\pi} d\theta$$

$$E(t) = \frac{A}{2\pi}\int_0^{2\pi} \cos(\omega_0 t + \theta)\, d\theta = \frac{A}{2\pi}\left[\sin(\omega_0 t + \theta)\right]_0^{2\pi}$$

$$= \frac{A}{2\pi}\left[\sin(\cancel{\omega_0 t} + 2\pi) - \sin(\cancel{\omega_0 t})\right] = \frac{A}{2\pi} \times 0 = 0$$

$$E(t) = 0$$

$$A\cdot C\cdot F = E\{x(t_1)x(t_2)\} = E\{A\cos(\omega_0 t_1 + \theta)\cdot A\cos(\omega_0 t_2 + \theta)\}$$

$$E[A^2 \cos(\omega_0 t_1 + \theta)\cdot\cos(\omega_0 t_2 + \theta)]$$

$$ACF = E\left[\frac{A^2}{2}\cdot 2\underbrace{\cos(\omega_0 t_1 + \theta)}_{A}\underbrace{\cos(\omega_0 t_2 + \theta)}_{B}\right] \quad ; \quad 2\cos A\cos B = \cos(A+B) + \cos(A-B)$$

$$ACF = E\left\{\frac{A^2}{2}\left[\cos(\omega_0(t_1+t_2) + 2\theta) + \cos(\omega_0(t_1 - t_2))\right]\right\}$$

$$ACF = \frac{A^2}{2}\left(\underbrace{E\cos(\omega_0(t_1+t_2) + 2\theta)}_{a'} + \underbrace{E(\cos(\omega_0 (t_1 - t_2)))}_{b'}\right)$$

$$a' = E\cos(\omega_0 t' + 2\theta) \Rightarrow a' = 0$$

$$b' = \int_0^{2\pi}\cos(\omega_0(t_1 - t_2))\cdot\frac{1}{2\pi}\, d\theta$$

$$b' = \cos[\omega_0(t_1 - t_2)]\cdot\int_0^{2\pi}\frac{1}{2\pi}\, d\theta$$

$$b' = \cos[\omega_0(t_1 - t_2)]\cdot\frac{1}{\cancel{2\pi}}[\cancel{2\pi} - 0]$$

$$b' = \cos[\omega_0(t_1 - t_2)]$$

$$ACF = \frac{A^2}{2}[0 + \cos(\omega_0(t_1 - t_2))]$$

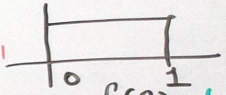
$$\boxed{ACF = \frac{A^2}{2}\cos[\omega_0(t_1 - t_2)]}$$

$ACF \Rightarrow f^n$ of $(t_1 - t_2 = \tau)$

$$ACF = \frac{A^2}{2}\cos(\omega_0 \tau)$$

$\Rightarrow$ W.S.S.

Q A random process is given by $x(t) = A\cos\omega_0 t$, ω_0 is constant. A is a random variable uniformly distributed in $[0, 1]$. Find mean & Autocorrelation function (ACF).



$$E = \int_{-\infty}^{\infty} x(t) f(A)\, dA = \int_0^1 A\cos\omega_0 t \cdot 1\, dA$$

$$= \cos\omega_0 t \int_0^1 A\, dA = \cos\omega_0 t \cdot \frac{A^2}{2}\Big|_0^1$$

$$E = \cos\omega_0 t\left[\frac{1}{2} - 1\right] = \frac{\cos\omega_0 t}{2}$$

$$ACF = E[x(t_1)\cdot x(t_2)] = E[A\cos\omega_0 t_1 \cdot A\cos\omega_0 t_2]$$

$$= E\left[\frac{A^2}{2}\cos(\omega_0(t_1 + t_2))\cdot\cos(\omega_0(t_1 - t_2))\right]$$

$$= \int_0^1 \frac{A^2}{2}\cos(\omega_0(t_1 + t_2))\cdot\cos\omega_0(t_1 - t_2)\, dA$$

$$ACF = \cos(\omega_0(t_1+t_2))\cdot\cos(\omega_0(t_1 - t_2))\frac{1}{2}\cdot\frac{1}{3}$$

$\rightarrow$ not WSS

Random Process - Numericals

Q A random process is given by $x(t) = A\cos\omega_0 t + B\sin\omega_0 t$, $\omega_0 \to$ constant. A & B are independent Random variables having 0 mean & variance σ^2. find mean & ACF of $x(t)$.

$x(t) = A\cos\omega_0 t + B\sin\omega_0 t$

$E(A\cdot B) = \underbrace{E(A)\cdot E(B)}_{=0}$ {independence condition}

$E(A) = E(B) = 0 \Rightarrow E(A^2) = E(B^2) = \sigma^2$

$\Rightarrow E(t) = E\{A\cos\omega_0 t + B\sin\omega_0 t\}$

$= E(A)\cos\omega_0 t + E(B)\sin\omega_0 t$

$= 0\cdot\cos\omega_0 t + 0\cdot\sin\omega_0 t = 0$

$ACF = E[x(t_1)\cdot x(t_2)]$

$= E\{(A\cos\omega_0 t_1 + B\sin\omega_0 t_1)(A\cos\omega_0 t_2 + B\sin\omega_0 t_2)\}$

$= E\{A^2\cos\omega_0 t_1\cos\omega_0 t_2 + AB\cos\omega_0 t_1\sin\omega_0 t_2 + AB\sin\omega_0 t_1\cos\omega_0 t_2 + B^2\sin\omega_0 t_1\sin\omega_0 t_2\}$

$= E(A^2)\cos\omega_0 t_1\cos\omega_0 t_2 + \cancel{E(AB)}\cos\omega_0 t_1\sin\omega_0 t_2 + \cancel{E(AB)}\sin\omega_0 t_1\cos\omega_0 t_2 + E(B^2)\sin\omega_0 t_1\sin\omega_0 t_2$

$= \sigma^2(\cos\omega_0 t_1\cos\omega_0 t_2 + \sin\omega_0 t_1\sin\omega_0 t_2)$

$= \sigma^2\cos(t_1 - t_2)\omega_0$; $t_1 - t_2 = \tau$

$ACF = \sigma^2\cos\tau\omega_0$

Q A random process is given by $x(t) = A\cos\omega_0 t + B\sin\omega_0 t$, $\omega_0 \to$ constant. A & B are independent R.V. having values -1 & 2 with probabilities $2/3$ & $1/3$ respectively. Find mean & ACF.

x_i	-1	2
$p(x_i)$	$2/3$	$1/3$

$\sum x_i p(x_i) = -1\times\frac{2}{3} + 2\times\frac{1}{3} = 0$

$E(A) = E(B) = 0$

$E(A\cdot B) = E(A)\cdot E(B) = 0$

$E(A^2) = E(B^2) = \sigma^2 = 1\times\frac{2}{3} + 4\times\frac{1}{3} = 2$

$E(t) = E\{A\cos\omega_0 t + B\sin\omega_0 t\}$

$= \cancel{E(A)}\cos\omega_0 t + \cancel{E(B)}\sin\omega_0 t \Rightarrow E(t) = 0$

$ACF \Rightarrow E(x(t_1)\cdot x(t_2))$

$\Rightarrow E\{(A\cos\omega_0 t_1 + B\sin\omega_0 t_1)(A\cos\omega_0 t_2 + B\sin\omega_0 t_2)\}$

$\Rightarrow E\{A^2\cos\omega_0 t_1\cos\omega_0 t_2 + AB\cos\omega_0 t_1\sin\omega_0 t_2 + AB\cos\omega_0 t_2\sin\omega_0 t_1 + B^2\sin\omega_0 t_1\sin\omega_0 t_2\}$

$\Rightarrow E(A^2)\cos\omega_0 t_1\cos\omega_0 t_2 + \underbrace{\cancel{E(AB)}}_{0}\cos\omega_0 t_1\sin\omega_0 t_2 + \underbrace{\cancel{E(AB)}}_{0}\cos\omega_0 t_2\sin\omega_0 t_1 + E(B^2)\sin\omega_0 t_1\sin\omega_0 t_2$

$= 2\{\cos\omega_0 t_1\cos\omega_0 t_2 + \sin\omega_0 t_1\sin\omega_0 t_2\}$

$= 2\cos(t_1 - t_2)\omega_0$; $t_1 - t_2 = \tau$

$ACF = 2\cos(\tau)\omega_0$

Random Process - Numerical

Q Let the random process $x(t)$ be defined by $x(t) = A+Bt$, where A and B are independent Random Variables, each uniformly distributed on $[-1,1]$. Find mean and auto correlation function.

$$f_X(x) = \frac{1}{b-a} \Rightarrow f_X(A) = \frac{1}{1-(-1)} = \frac{1}{2} \quad ; \quad f_X(B) = \frac{1}{1-(-1)} = \frac{1}{2}$$

$$E(x) = \frac{b+a}{2} \Rightarrow E(A) = \frac{1+(-1)}{2} = 0 \quad ; \quad E(B) = \frac{1+(-1)}{2} = 0$$

$$E(A^2) = \int_{-1}^{1} A^2 \cdot \frac{1}{2}\, dA = \frac{1}{2}\left|\frac{A^3}{3}\right|_{-1}^{1} = \frac{1}{2}\cdot\frac{1-(-1)}{3} = \frac{1}{3} = E(B^2)$$

$$E[x(t)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x(t)\, f_X(A)\, f_X(B)\, dA\, dB = \int_{-1}^{1}\int_{-1}^{1} (A+Bt)\cdot\frac{1}{2}\cdot\frac{1}{2}\, dA\, dB$$

$$\frac{A^2}{2} + BAt\Big|_{-1}^{1} \qquad \cancel{\frac{1}{2}} + Bt - \left(\cancel{\frac{1}{2}} - Bt\right) \qquad Bt + Bt = 2Bt$$

$$= \int_{-1}^{1}\left[\frac{A^2}{2} + BAt\right]_{-1}^{1}\cdot\frac{1}{4}\, dB$$

$$= \int_{-1}^{1} 2Bt\, dB = \frac{\cancel{2}B^2t}{\cancel{2}}\Big|_{-1}^{1} = t - t = 0$$

$$ACF = E\{x(t_1)\cdot x(t_2)\} = E\{(A+Bt_1)(A+Bt_2)\} = E\{A^2 + ABt_2 + ABt_1 + B^2t_1t_2\}$$

$$= E(A^2) + E(AB)t_2 + E(AB)t_1 + E(B^2)t_1t_2$$

$E(AB) = E(A)\cdot E(B)$ {Independence Condition}

$E(AB) = 0 \cdot 0 = 0$

$$= \frac{1}{3} + 0\cdot t_2 + 0\, t_1 + \frac{1}{3}t_1t_2$$

$$ACF = \frac{1}{3}(1 + t_1t_2)$$

Autocorrelation-Function-Numerical

Q If input-output is described by the differential equation $\frac{dy(t)}{dt} + y(t) = x(t)$, where $x(t)$ is input random process with 0 mean and covariance, $\sigma_{xx}(\tau) = \delta(\tau)$. Find the autocorrelation function of output.

$S_y(f) = |H(f)|^2 S_x(f)$ $\quad$ $S(f) = F.T.\{R(\tau)\}$

covariance property $\boxed{\sigma_{xx}(\tau) = R_x(\tau) - \mu_x^2}$ $\quad$ $\mu_x = 0$

$R_x(\tau) = \delta(\tau)$

$S_x(f) = F.T.\{\delta(\tau)\} = \int_{-\infty}^{\infty} \delta(\tau) e^{-j\omega t} df = 1$

$H(f)$

$\frac{dy(t)}{dt} + y(t) = x(t)$

$j\omega_0 y(f) + y(f) = x(t) \Rightarrow \frac{y(f)}{x(f)} = \frac{1}{1+j\omega} = H(f)$

$|H(f)| = \frac{1}{\sqrt{1+(j\omega)^2}}$ $\quad$ $|H(f)|^2 = \frac{1}{1+(j\omega)^2}$

$|H(f)|^2 = \frac{1}{1+(2\pi f j)^2}$

$S_y(f) = \frac{1}{1+(2\pi f j)^2}$

$S_y(f) = \frac{2}{2(1+(2\pi f j)^2)}$ $\xrightarrow{I.F.T}$ $\frac{1}{2} e^{-|\tau|} = R_y(\tau)$

THANK YOU

Click Here

Vaishali's e-Learning

Make your Learning Easier