Introduction to Logic

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Propositional Equivalences

Definition

A compound proposition that is always true, no matter what the truth values of the propositional variables that occur in it, is called a tautology. A compound proposition that is always false is called a contradiction. A compound proposition that is neither a tautology nor a contradiction is called a contingency.

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- $p \wedge \neg p$.

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i.e $p \leftrightarrow q$ is a proposition with truth value T
i.e $p \leftrightarrow q$ is true

Logical Equivalences

Let T denotes the compound proposition that is always true and F denotes the compound proposition that is always false.

- $p \wedge T \equiv p$, $p \vee F \equiv p$ (Identity laws)
- $p \vee T \equiv T$, $p \wedge F \equiv F$ (Domination laws)
- $p \vee p \equiv p$, $p \wedge p \equiv p$ (Idempotent laws)
- $\neg(\neg p) \equiv p$ (Double negation law)

- $p \vee q \equiv q \vee p$
  $p \wedge q \equiv q \wedge p$ (Commutative laws)
- $(p \vee q) \vee r \equiv p \vee (q \vee r)$
  $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ (Associative laws)

Construct truth table and find an equivalent expression for $\neg(p \vee q)$

A. $\neg p \vee \neg q$

B. $\neg p \wedge \neg q$

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B. $\neg p \wedge \neg q$

- $\neg(p \vee q) \equiv \neg p \wedge \neg q$
  $\neg(p \wedge q) \equiv \neg p \vee \neg q$ (De Morgan's Laws)

- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.
  $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$(Distributive laws)
- $p \vee (p \wedge q) \equiv p$
  $p \wedge (p \vee q) \equiv p$ (Absorption laws)
- $p \vee \neg p \equiv T$
  $p \wedge \neg p \equiv F$ (Negation laws)

Logical equivalences involving conditional statements

Which among these are equivalent to $p \to q$?

1. $\neg p \vee q$
2. $\neg q \to \neg p$

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- $p \to q \equiv \neg p \lor q$.
- $p \to q \equiv \neg q \to \neg p$.

Logical equivalences involving conditional statements

Which among these are equivalent to $p \to q$?

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2. $\neg q \to \neg p$

- $p \to q \equiv \neg p \lor q$.
- $p \to q \equiv \neg q \to \neg p$.
- $p \lor q \equiv \neg p \to q$.
- $p \land q \equiv \neg(p \to \neg q)$.
- $\neg(p \to q) \equiv p \land \neg q$. (Use De Morgan's laws)

- $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$.
- $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$.
- $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$.
- $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$.

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- $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$. (Use De Morgan's laws)
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- Show that $(p \wedge q) \to (p \vee q)$ is a tautology.

Extension of De Morgan's laws

- $\neg(p_1 \vee p_2 \vee \cdots \vee p_n) \equiv (\neg p_1 \wedge \neg p_2 \wedge \cdots \wedge \neg p_n)$
- $\neg(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \equiv (\neg p_1 \vee \neg p_2 \vee \cdots \vee \neg p_n)$.

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Use De Morgan's laws to express the negations of

(a). Mike has a cellphone and he has a laptop computer.

- Mike does not have a cellphone or he does not have a laptop computer.

(b). Heather will go to the concert or Steve will go to the concert.

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- Heather will not go to the concert and Steve will not go to the concert.