MA 205, Discrete Mathematics

Assignment 2, August 21, 2020

Attempt any 5 of the following.

1. Prove that if $m+n$ and $n+p$ are even integers, where m, n, and p are integers, then $m+p$ is even.
2. Use a direct proof to show that every odd integer is the difference of two squares.
3. Use a proof by contradiction to prove that the sum of an irrational number and a rational number is irrational.
4. Prove or disprove that the product of two irrational numbers is irrational.
5. Show that at least three of any 25 days chosen must fall in the same month of the year.
6. Use a proof by contradiction to showthat there is no rational number r for which $r^3+r+1=0$. [Hint: Assume that $r = a/b$ is a root, where a and b are integers and a/b is in lowest terms. Obtain an equation involving integers by multiplying by b^3. Then look at whether a and b are each odd or even.]
7. Prove that at least one of the real numbers $a_1, a_2, \cdots, a_n$ is greater than or equal to the average of these numbers.
8. Prove that these four statements about the integer n are equivalent: (i) n^2 is odd, (ii) $1-n$ is even, (iii) n^3 is odd, (iv) n^2+1 is even.
9. Prove the triangle inequality, which states that if x and y are real numbers, then $|x|+|y| \geq |x+y|$.
10. Prove that if x and y are real numbers, then $\max(x, y) + \min(x, y) = x + y$. [Hint: Use a proof by cases, with the two cases corresponding to $x \geq y$ and $x < y$, respectively.]
11. Formulate a conjecture about the decimal digits that appear as the final decimal digit of the fourth power of an integer. Prove your conjecture using a proof by cases.
12. Prove that there are infinitely many solutions in positive integers x, y, and z to the equation $x^2 + y^2 = z^2$. [Hint: Let $x = m^2 - n^2, y = 2mn$, and $z = m^2 + n^2$, where m and n are integers.]
13. Show that if r is an irrational number, there is a unique integer n such that the distance between r and n is less than $1/2$.
14. Prove that between every two rational numbers there is an irrational number.
15. Prove that between every rational number and every irrational number there is an irrational number.