# Performance: Digital Data, Analog Signals

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# Performance of various modulation schemes

- Measured by the bandwidth of the **modulated signal** (not the channel)
- -depends on a factor $r$ [1], which is determined by the modulation and filtering process
- Filters are used to filter out unwanted frequency components
- $0 < r < 1$

[1] Don't confuse this with r=no of data elements / no of signal elements

# Performance of various modulation schemes

- ASK and PSK, and under some assumptions, FSK: $B_T = (1 + r)R$ where $R$ is the bit rate
- MPSK: $B_T = \frac{1+r}{L}R = \frac{1+r}{\log_2 M}R$ where $L$ is the number of bits encoded per signal element, $M$ is the number of signal elements
- MFSK: $B_T = \frac{(1+r)M}{\log_2 M}R$
- Bandwidth efficiency: $\frac{R}{B_T}$

# Question

If a modulation scheme is highly bandwidth efficient, it means that

(A) it can send a higher number of bits per unit of bandwidth

(B) it can send a lower number of bits per unit of bandwidth

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| | $r=0$ | $r=0.5$ | $r=1$ |
|---|---|---|---|
| **ASK** | 1.0 | 0.67 | 0.5 |
| **Multilevel FSK** | | | |
| $M=4, L=2$ | 0.5 | 0.33 | 0.25 |
| $M=8, L=3$ | 0.375 | 0.25 | 0.1875 |
| $M=16, L=4$ | 0.25 | 0.167 | 0.125 |
| $M=32, L=5$ | 0.156 | 0.104 | 0.078 |
| **PSK** | 1.0 | 0.67 | 0.5 |
| **Multilevel PSK** | | | |
| $M=4, L=2$ | 2.00 | 1.33 | 1.00 |
| $M=8, L=3$ | 3.00 | 2.00 | 1.50 |
| $M=16, L=4$ | 4.00 | 2.67 | 2.00 |
| $M=32, L=5$ | 5.00 | 3.33 | 2.50 |

# $E_b/N_0$

- $E_b/N_0$ : the ratio of signal energy per bit to noise power density per hertz
- A parameter related to SNR that is more convenient for determining digital data rates and error rates
- Bit Error Rate is a function of $E_b/N_0$

Energy = Power * time
Energy per bit in a signal (analog or digital) that contains binary digital data, $E_b = ST_b$
$S$: signal power, $T_b$: time required to send 1 bit.
Since $R = \frac{1}{T_b}$, where $R$ is the data rate, $E_b = \frac{S}{R}$

$$\frac{E_b}{N_0} = \frac{S}{N_0 R} = \frac{S}{kTR}$$

# Why is $E_b/N_0$ important?

- $E_b/N_0$ : the ratio of signal energy per bit to noise power density per hertz
- Bit Error Rate is a **decreasing** function of $E_b/N_0$

$$\frac{E_b}{N_0} = \frac{S}{N_0 R} = \frac{S}{kTR}$$

- As the bit rate R increases, the transmitted signal power must increase relative to noise, to maintain the required $E_b/N_0$
- If the data rate were doubled, the bits would be more tightly packed together, and the same passage of noise might destroy more bits

# Question

For binary phase-shift keying $E_b/N_0 = 8.4dB$ is required for a bit error rate of $10^{-4}$ (1 bit error out of every 10,000). If the effective noise temperature is 290 K (room temperature) and the data rate is 2400 bps, what received signal level is required? Express the answer in dBW. $k = 1.38 * 10^{-23} J/K$

# Relating $E_b/N_0$ to SNR

$$\frac{E_b}{N_0} = \frac{S}{N_0 R}$$

$N_0$ is the noise power density in watts per hertz.
The noise $N$ in a signal with bandwidth B is $N = BN_0$.
Substituting for $N_0$,

$$\frac{E_b}{N_0} = \frac{S}{N_0 R} = \frac{SB}{NR}$$

# Relating $E_b/N_0$ to Spectral Efficiency

As per Shannon's formula, $C = B\log_2(1 + S/N)$. That is, $\frac{S}{N} = 2^{C/B} - 1$

$$\frac{E_b}{N_0} * \frac{R}{B} = \frac{S}{N}$$

Therefore $\frac{E_b}{N_0} = \frac{B}{R} * \frac{S}{N} = \frac{B}{R}(2^{C/B} - 1)$
Assuming $R = C$,

$$\frac{E_b}{N_0} = \frac{B}{C}(2^{C/B} - 1)$$

This relates the achievable spectral efficiency $C/B$ to $E_b/N_0$

# Question

For constant signal-to-noise ratio, an increase in data rate

(A) increases the error rate

(B) decreases the error rate

(C) has no effect on the error rate

# Bandwidth Efficiency for MFSK and MPSK

| | $r = 0$ | $r = 0.5$ | $r = 1$ |
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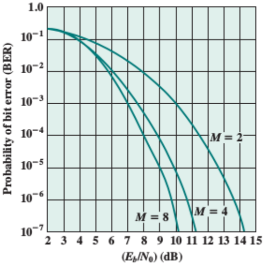
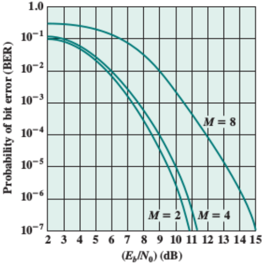
# Theoretical Bit Error Rate for MFSK and MPSK



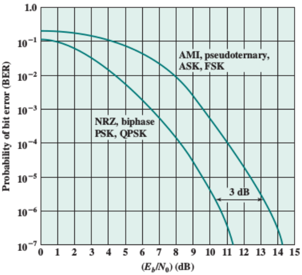
(a) Multilevel FSK (MFSK)



(b) Multilevel PSK (MPSK)

- For MFSK, BER decreases as M increases
- For MFSK, Bandwidth efficiency decreases as M increases
- For MPSK, BER increases as M increases
- For MPSK, Bandwidth efficiency increases as M increases
- For both, there is a tradeoff between bandwidth efficiency and BER

# BER for FSK, ASK, PSK, and QPSK




What is the bandwidth efficiency for FSK, ASK, PSK, and QPSK for a bit error rate of $10^{-7}$ on a channel with an SNR of 12 dB? For FSK and ASK, $(\frac{E_b}{N_0})dB$=14.2dB

$$\frac{E_b}{N_0} * \frac{R}{B} = \frac{S}{N}$$

$$\frac{R}{B_T} = \frac{S}{N}/(\frac{E_b}{N_0})$$

# Bandwidth efficiency for FSK, ASK, PSK, and QPSK

$$(\frac{R}{B_T})dB = (\frac{S}{N})dB - (\frac{E_b}{N_0})dB$$

For FSK and ASK, $(\frac{R}{B_T})dB = 12dB - 14.2dB = -2.2dB$

$$(\frac{R}{B_T}) = 0.6$$

Similarly, for PSK,

$$(\frac{R}{B_T}) = 12dB - 11.2dB = 0.8dB$$

$$(\frac{R}{B_T}) = 1.2$$

For QPSK, since the modulation rate determines the bandwidth and it is twice that of the data rate for PSK,

$$(\frac{R}{B_T}) = 2.4$$

# Bandwidth efficiency for FSK, ASK, PSK, and QPSK

- ASK and FSK exhibit the same bandwidth efficiency
- PSK is better, and even greater improvement can be achieved with multilevel signaling.