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(1) Let $X \sim Bin(n,p)$, where n is a positive integer and $p \in (0,1)$. Let $Y_1 = X^2$, and $Y_2 = \sqrt{X}$. Find the PMFs of Y_1 and Y_2.

(2) Let X be a random variable with PMF

$$f_X(x) = \begin{cases} \frac{1}{2}(\frac{2}{3})^x, & \text{if } x = 0,1,2,\ldots; \\ 0, & \text{otherwise.} \end{cases}$$

Find the CDF of $Y = \frac{X}{X+1}$ and hence determine the PMF of Y.

(3) Let X be a RV with PDF

$$f(x) = \begin{cases} \theta e^{-\theta x}, & \text{if } x \geq 0; \\ 0, & \text{otherwise.} \end{cases}$$

Find the PDF of $Y = (X - \frac{1}{\theta})^2$.

(4) If the MGF of a random variable X is $M_X(t) = \frac{1}{3t}(e^t - e^{-2t})$ for $t \neq 0$. Find the PDF of $Y = X^2$.

(5) Let X have the PDF

$$f(x) = \begin{cases} \frac{1}{2\sqrt{3}}, & \text{if } |x| < \sqrt{3}; \\ 0, & \text{otherwise.} \end{cases}$$

Find the actual probability $P(|X - \nu| \geq \frac{3\sigma}{2})$ and compare it with the upper bound obtained by Chebychev's inequality.

(6) Let X be a random variable with MGF $M(t)$, $|t| < h$, for some $h > 0$. Prove that $P(X \geq a) \leq e^{-at}M(t)$ for $0 < t < h$, and $P(X \leq a) \leq e^{-at}M(t)$ for $-h < t < 0$.

(7) Let X be a random variable such that $P(X \leq 0) = 0$ and $\nu = E(X)$ is finite. Show that $P(X \geq 2\nu) \leq 0.5$. 22. If X is a random variable such that $E(X) = 3$ and $E(X^2) = 13$, then determine a lower bound for $P(-2 < X < 8)$. (Ans: $\frac{21}{25}$.)

(8) Suppose we know that the number of items produced in a factory during a week is a RV with mean 500. Find an upper bound on the probability that this weeks production will be at least 1000? If the variance of a weeks production is known to equal to 100, then give a lower bound on the probability that this weeks production will be between 400 and 600? [Ans: 0.99.]

(9) Two numbers are independently chosen at random between 0 and 1. What is the probability that their product is less than a constant $k(0 < k < 1)$?

(10) Consider the following joint PMF of the random vector (X,Y)

X—Y	1	2	3	4
4	.08	.11	.09	.03
5	.04	.12	.21	.05
6	.09	.06	.08	.04

(a) Find marginal PMFs of X and Y.
(b) Find the probabilities $P(X + Y < 8), P(X + Y > 7), P(XY \leq 14)$.
(c) Find joint MGF of (X,Y), and the Correlation coefficient of X and Y.
(d) Find conditional expectation of X given $Y = 2$ and variance of Y given $X = 6$.

(11) Three balls are randomly placed in three empty boxes B_1, B_2, and B_3. Let N denote the total number of boxes which are occupied and let X_i denote the number of balls in the box $B_i, i = 1,2,3$.
(a) Find the joint PMF of (N, X_1).
(b) Find the joint PMF of (X_1, X_2).
(c) Find the marginal distributions of N and X_2.
(d) Find the marginal PMF of X_1 from the joint PMF of (X_1, X_2).

(12) The joint PDF of (X,Y) is given by

$$f_{XY}(x,y) = \begin{cases} 4xy, & \text{if } 0 < x < 1, 0 < y < 1; \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the marginal PDFs of X and Y.
(b) Find the joint CDF of (X,Y), and marginal CDFs of X and Y.

   (c) Find joint MGF of (X, Y).
   (d) Find the covariance matrix and verify whether X and Y are independent.
   (e) Find $P(\{0 < X < 0.5, 0.25 < Y < 1\})$ and $P(\{X + Y < 1\})$.
   (f) Find the marginal distributions of $U = \frac{X}{Y}$ and $V = XY$.

(13) Let the joint distribution of X and Y be $f(x, y) = 3e^{-x}e^{-3y}$; for $x, y \geq 0$ and 0 otherwise. Find the distribution of $\frac{X}{Y}$.

(14) Let X and Y be two independent poisson variates such that $P(X = 1) = P(X = 2)$ and $P(Y = 2) = P(Y = 3)$. Find the mean and variance for the random variable $Z = 2X - Y$. Find $P(X + Y \leq 10)$.

(15) Let the joint probability density function of X and Y be $f(x, y) = 5(x+y-3xy^2)$, if $0 < x < y < 1$ and 0, otherwise.
   (a) Calculate $E(X|Y = y)$ and $V(Y|X = x)$.
   (b) Is $E(XY) = E(X)E(Y)$.
   (c) Find $P(X + Y \leq 1)$.

(16) Let X and Y be jointly distributed random variables with $E(X) = E(Y) = 0, E(X^2) = E(Y^2) = 2$, and $\rho(X, Y) = 1/3$. Find $\rho(\frac{X}{3} + \frac{2Y}{3}, \frac{2X}{3} + \frac{Y}{3})$.

(17) Suppose that the random vector (X, Y) is uniformly distributed over the region $A = \{(x, y) : 0 < x < y < 1\}$. Find $\mathrm{Cov}(X, Y)$.