Indian Institute of Information Technology Guwahati

MA203: Probability and Random Processes **Problem Sheet-VII**

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(1) Define a random process. Identify the type of random process for the following stochastic process.
  (a) $\{W_k; k \in T\}$ where W_k be the time that the kth customer has to wait in the system before service.
  (b) The price of a share observed over days.
  (c) $\{Y(t); t \in T\}$ where $Y(t)$ denotes the number of phone calls recorded in the system at time t.

(2) Find out the mean, autocorrelation, power, auto-covariance, and variance functions for the following stochastic process $\{X(t); t \in T\}$: Check if the processes are WSS and White noise.
  (a) $X(t) = cos(2\pi ft + \theta)$; where $t \geq 0$ and f are constant and $\theta \sim U[-\pi, \pi]$.
  (b) $X(t) = A_0 + A_1 t + A_2 t^2$ where A_0, A_1, and A_2 are independent RVs with equal mean 0 and variance 1.
  (c) $X(t) = 1$ when there are even numbers of failures in the system during interval $[0, t]$ and $X(t) = -1$ when there are odd number of failures in the system during $[0, t]$.
  (d) $P(X(t) = n) = \frac{(at)^{n-1}}{(1+at)^{n+1}}; n = 1, 2, \ldots$ and $P(X(t) = 0) = \frac{at}{1+at}$.

(3) Let $Y_n = a_0 X_n + a_1 X_{n-1}; n = 1, 2, \ldots$, where X_i are iid RVs with equal mean 0 and variance 2. Is $\{Y_n; n \geq 1\}$ SSS? Is $\{Y_n; n \geq 1\}$ WSS?

(4) Let $X(t)$ and $Y(t)$ be independent Gaussian random processes with zero means and the same covariance function $C_X(\tau)$, where τ is the difference of two distinct time points, define the amplitude-modulated signal by $Z(t) = X(t)cos\omega t + Y(t)sin\omega t$. Find the pdf of $Z(t)$. Determine if $Z(t)$ is a SSS or/and WSS.

(5) Let $\{X_n : n \geq 1\}$ be a Gaussian rangom process with mean 0 and variance σ^2. If $Y_n = \frac{1}{2}(X_n + X_{n-1})$, check if Y_n is WSS or/and SSS. Is $\{Y_n : n \geq 1\}$ a Markov process?

(6) Let $\{X(t); t \geq 0\}$ follows the poisson process with average arrival rate of 5 people per 1/2 hour.
  (a) Find the probability of 10 arrivals in the interval of 10 minutes to 20 minutes.
  (b) Find the probability that any arrival has to wait for than 15 minutes.
  (c) $\text{P}(X(10) = 10 \mid X(20) = 15)$.
  (d) $\text{P}(X(20) = 15 \mid X(10) = 10)$.
  (e) $\text{P}(X(20) = 10 \mid X(19) = 8, X(18) = 6, X(17) = 4)$.

(7) Let $X_n, n = 0, 1, \ldots$ be a Markov chain (a discrete Markov process) with $\text{P}(X_0 = 0, X_1 = 1) = P(X_0 = 0, X_2 = 1) = P(X_1 = 1, X_2 = 1) = 1/3$. Compute $P(X_0 = 0, X_1 = 1, X_2 = 1)$.

(8) A certain calculating machine uses only the digits 0 and 1. It is supposed to transmit one of these digits through several stages. However, at every stage, there is a probability p that the digit that enters this stage will be changed when it leaves and a probability $q = 1 - p$ that it wont. Form a Markov chain to represent the process of transmission by taking as states the digits 0 and 1. What is the probability that the machine, after two stages, produces the digit 0 (i.e., the correct digit)? [Ans: $p^2 + q^2$]

(9) I have 4 umbrellas, some at home, some in the office. I keep moving between home and office. I take an umbrella with me only if it rains. If it does not rain I leave the umbrella behind (at home or in the office). It may happen that all umbrellas are in one place, I am at the other, it starts raining and must leave, so I get wet.
  (a) If the probability of rain is p, what is the probability that I get wet? [Ans: $\frac{pq}{q+4}$, where $q = 1 - p$.
  (b) Current estimates show that p = 0.6 in Guwahati. How many umbrellas should I have so that, if I follow the strategy above, the probability I get wet is less than 0.01? [Ans: 24]

(10) A critical part of a machine has an exponentially distributed lifetime with parameter α. Suppose that n spare parts are initially at stock, and let $N(t)$ be the number of spares left at time t.
  (a) Find $P(N(s + t) = j \mid N(s) = i)$.
  (b) Find the transition probability matrix.
  (c) Find $P_j(t)$.