Indian Institute of Information Technology Guwahati

MA203: Probability and Random Processes **Problem Sheet-IV**

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(1) Find the expected number of throws of a fair die untill a 6 is obtained. [Ans:6]

(2) An urn contains 7 white and 3 res balls. Two balls are drawn together, at random from this urn. Compute the expected number and variance of white balls drawn.[Ans: 21/15, 84/225]

(3) For an RV X with p.d.f

$$f(x) = \begin{cases} \frac{x}{2}, & 0 \le x < 1; \\ \frac{1}{2}, & 1 \le x < 2; \\ \frac{3-x}{2}, & 2 \le x < 3, \end{cases}$$

show that moments of all order exist. Find the mean and the variance of X.

(4) (Random Walk Problem) Starting from the origin, unit steps are taken to the right with probability p and to the left with probability $1-p$. Assuming independent movements, find mean an variance of the distance moved from origin after n steps. [Ans: $n(2p-1), 4np(1-p)$]

(5) A man with n keys wants to open a door and tries the keys independently and at random. Find mean and variance of the number of trial required to open the door if (a) unsuccessful keys are not eliminated from further selection (b) unsuccessful keys are eliminated from further selection. [Ans: (a) n, n(n-1) (b) (n+1)/2, $(n^2-1)/12$]

(6) Obtain the mean, variance, the moment generating function, and the characteristic function of the following standard discrete distributions whose probability mass functions are given below:

(a) Discrete Uniform: $P(X=x) = \begin{cases} \frac{1}{n}, & x = 1, 2, \ldots, n; \\ 0, & \text{otherwise.} \end{cases}$

(b) Bernoulli : $P(X=x) = \begin{cases} p^x q^{1-x}, & x = 0, 1; \\ 0, & \text{otherwise.} \end{cases}$

(c) Binomial : $P(X=x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1, 2, \ldots, n; \\ 0, & \text{otherwise;} \end{cases}$ with $0 \le p \le 1$

(d) Poisson : $P(X=x) = \begin{cases} \frac{e^{-\lambda}\lambda^x}{x!}, & x = 0, 1, 2, \ldots; \\ 0, & \text{otherwise;} \end{cases}$ with $\lambda > 0$

(7) Obtain the mean, variance, the moment generating function, and the characteristic function of the following standard continuous distributions with probability density functions:

(a) Continuous Uniform: $f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b; \\ 0, & \text{otherwise.} \end{cases}$

(b) Exponential : $f(x) = \begin{cases} \lambda e^{-\lambda}, & x \ge 0; \\ 0, & \text{otherwise;} \end{cases}$ with $\lambda > 0$.

(c) Normal : $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\nu)^2}{2\sigma^2}}$ for all $x \in \mathbb{R}$ with $\nu \in \mathbb{R}$ and $\sigma > 0$.

(d) Standard normal: $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2} 2$ for all $x \in \mathbb{R}$

(8) A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of the time. It is agreed that his claim will be accepted if he correctly identifies at least 5 of the 6 cups. Find his chances of having the claim (i) accepted, (ii) rejected. [Ans: (a) 0.534 (b) 0.466]

(9) If $X \sim B(5,p)$ such that $9P(X=4) = P(X=2)$. [Ans: 1/4]

(10) Six coins are tossed 6,400 times. Using the Poisson distribution, find the approximate probability of getting six heads r times.

(11) Suppose that the number of calls coming into a telephone exchange between 10 and 11, say $X_1 \sim P(2)$. Similarly the number of called arriving between 11 and 12, say $X_2 \sim P(6)$. If x_1 and X_2 are independent, find the probability that more than 5 called come between 10 and 12. [Ans: 0.81]

(12) A boy and a girl decide to meet between 9 and 11 a.m. in a hotel. They decide not to wait for each other for more than 15 minutes. Assuming waiting time to be uniformly distributed, find the probability that they will meet.

(13) A sample of 100 item is taken at random from a batch known to contain 40% defective. Compare the probabilities that the sample contains atleast 44 defective obtained using binomial, poisson and normal.

(14) In a distribution exactly normal, 10.03% of items are under 25kg and 89.97% of items are under 70kg. Find mean and variance of the distribution. [Ans: 47.5, 309

(15) Let the MGF of RVs X and Y are given as $(\frac{4}{6} + \frac{2}{6}e^t)^9$ and $(\frac{6}{9} + \frac{3}{9}e^t)^{10}$ respectively. Let X and Y be two independent RVs then find the mean and variance of $X+Y$. Also find the $Pr(X+Y) = 4$.

(16) The local authorities in a city install 10,000 bulbs in the streets of the city. If these lamps have an average life 1,000 burning hours with a standard deviation of 200 hours, assuming normality, what number of bulbs might be expected to be fail (i) in the first 800 burning hours? (ii) between 800 and 1,200 burning hours? after what period of burning hours would you expect that (a) 10% of the lamps would fail? (b) 10% of the lamps would be still burning?

(17) Polystyrene spheres (balls) are produced with diameters that may be taken to be approximately normally distributed. A random sample of spheres has a mean diameter 151.3 mm and standard deviation 2.4 mm. What proportion of spheres has a diameter between 150 and 155 mm?

(18) If X, Y are independent normal variate with means 6, 7 and variances 9, 16 respectively, determine λ such that $P(2X + Y \leq \lambda) = P(4X - 3Y \geq 4\lambda)$. [Ans: $\frac{114\sqrt{2}+3\sqrt{13}}{6\sqrt{2}+4\sqrt{13}}$]