MA 205, Discrete Mathematics
Problem Sheet 3, August 28, 2020

1. Suppose that g is a function from A to B and f is a function from B to C.
   - Show that if both f and g are one-to-one functions, then $f \circ g$ is also one-to-one.
   - Show that if both f and g are onto functions, then $f \circ g$ is also onto.
2. Suppose that f is a function from A to B, where A and B are finite sets with $|A| = |B|$. Show that f is one-to-one if and only if it is onto.
3. Determine whether each of these sets is finite, countably infinite (an infinite set which is countable). For those that are countably infinite, exhibit a one-to-one correspondence between $\mathbb{N}$ and that set. (Explain why your mapping is one -one and onto.)
   a) the odd positive integers
   b) the integers
   c) the integers that are multiples of 7
   d) the integers greater than 10
   e) all bit strings not containing the bit 0
4. Show that subset of any countable set is countable.
5. Show that if A and B are sets and $A \subset B$ then $|A| \leq |B|$. (Hint: Apply definition in Slide 24, Lec 8).