

MA 205, Discrete Mathematics

Problem Sheet 1, August 14, 2020

1. Express each of the following statements as a logical implication ($A \rightarrow B$). Also state its negation in English.
 - (a). If A holds, then B holds.
 - (b). A is true only if B is true.
 - (c). A is true whenever B is true.
 - (d). A is false only if B is false.
 - (e). A is a necessary condition for B.
 - (f). A is necessary and sufficient for B.
 - (g). A holds if and only if B holds.
2. Are these system specifications consistent? "Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded."
3. Four friends have been identified as suspects for an unauthorized access into a computer system. They have made statements to the investigating authorities. Alice said "Carlos did it." John said "I did not do it." Carlos said "Diana did it." Diana said "Carlos lied when he said that I did it."
 - a) If the authorities also know that exactly one of the four suspects is telling the truth, who did it? Explain your reasoning.
 - b) If the authorities also know that exactly one is lying, who did it? Explain your reasoning.
4. Suppose there are signs on the doors to two rooms. The sign on the first door reads "In this room there is a lady, and in the other one there is a tiger"; and the sign on the second door reads "In one of these rooms, there is a lady, and in one of them there is a tiger." Suppose that you know that one of these signs is true and the other is false. Behind which door is the lady?
5. Arvind announces to his class that he plans to surprise them with a quiz sometime next week. His students first wonder if the quiz could be on Friday of next week. They reason that it can't: if Arvind didn't give the quiz before Friday, then by midnight Thursday, they would know the quiz had to be on Friday, and so the quiz wouldn't be a surprise any more. Next the students wonder whether Arvind could give the surprise quiz Thursday. They observe that if the quiz wasn't given before Thursday, it would have to be given on the Thursday, since they already know it can't be given on Friday. But having figured that out, it wouldn't be a surprise if the quiz was on Thursday either. Similarly, the students reason that the quiz can't be on Wednesday, Tuesday, or Monday. Namely, it's impossible for Arvind to give a surprise quiz next week. All the students now relax, having concluded that Arvind must have been bluffing. And since no one expects the quiz, that's why, when Arvind gives it on Tuesday next week, it really is a surprise! What do you think is wrong with the students' reasoning?