Introduction to Logic

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A direct proof of a conditional statement $p \to q$ is constructed when the first step is the assumption that p is true; subsequent steps are constructed using rules of inference, with the final step showing that q must also be true.

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1. Give a direct proof of the theorem "If n is an odd integer, then n^2 is odd."

2. Give a direct proof that if m and n are both perfect squares, then nm is also a perfect square.

Proof by contraposition

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3. Prove that if n is an integer and $3n + 2$ is odd, then n is odd.

4. Prove that if $n = ab$, where a and b are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$.

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Method of contradiction

10. Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction.

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11. Prove the theorem "If n is an integer, then n is odd if and only if n^2 is odd."

Proofs by equivalence

Sometimes a theorem states that several propositions are equivalent. Such a theorem states that propositions $p_1, p_2, p_3, \cdots, p_n$ are equivalent. This can be written as $p_1 \leftrightarrow p2 \leftrightarrow \cdots \leftrightarrow p_n$
$\equiv (p_1 \to p_2) \wedge (p_2 \to p_3) \wedge \cdots (p_n \to p_1)$.

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- Show that these statements about the integer n are equivalent:
  p_1: n is even.
  p_2: $n - 1$ is odd.
  p_3: n^2 is even.

Counter Examples

12. Show that the statement "Every positive integer is the sum of the squares of two integers" is false. (Give counter example)

Mistakes in Proofs

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The error is that $a - b$ equals zero; division of both sides of an equation by the same quantity is valid as long as this quantity is not zero.

Circular reasoning

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"Proof":
Suppose that n^2 is even. Then $n^2 = 2k$ for some integer k. Let $n = 2l$ for some integer l. This shows that n is even.

Exhaustive proof

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Proof by Cases

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15. Use a proof by cases to show that $|xy| = |x||y|$, where x and y are real numbers.

16. Formulate a conjecture about the final decimal digit of the square of an integer and prove your result.

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17. Show that there are no solutions in integers x and y of $x^2 + 3y^2 = 8$.

WITHOUT LOSS OF GENERALITY

- In the proof in Example 15, we can dismiss the case where $x < 0$ and $y \geq 0$, because it is the same as case where $x \geq 0$ and $y < 0$, with the roles of x and y reversed.

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- To shorten the proof, we could have proved cases together by assuming, without loss of generality, that $x \geq 0$ and $y < 0$.

Implicit in this statement is that we can complete the case with $x < 0$ and $y \geq 0$ using the same argument as we used for the case with $x \geq 0$ and $y < 0$, but with the obvious changes.

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We now illustrate a proof where without loss of generality is used effectively together with other proof techniques.

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- WLOG, we assume that x is odd, so that $x = 2m + 1$ for some integer k.

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- In (ii), $y = 2n + 1$ for some integer n, so that $xy = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1$ is odd. This completes the proof by contraposition.

19. What is wrong with this "proof?" If x is a real number, then x^2 is a positive real number.
Proof: " Let p_1 be "x is positive," let p_2 be "x is negative," and let q be "x^2 is positive." To show that $p_1 \rightarrow q$ is true, note that when x is positive, x^2 is positive because it is the product of two positive numbers, x and x. To show that $p_2 \rightarrow q$, note that when x is negative, x^2 is positive because it is the product of two negative numbers, x and x. This completes the proof.

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A Constructive Existence Proof

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We know that $\sqrt{2}$ is irrational. Consider the number $\sqrt{2}^{\sqrt{2}}$. If it is rational, we have two irrational numbers x and y with x^y rational, namely, $x = \sqrt{2}$ and $y = \sqrt{2}$. On the other hand if $\sqrt{2}^{\sqrt{2}}$ is irrational, then we can let $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$.

22. Show that if a and b are real numbers and $a \neq 0$, then there is a unique real number r such that $ar + b = 0$.

Tiling

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We color the squares of this checkerboard using alternating white and black squares. Observe that a domino in a tiling of such a board covers one white square and one black square. Next, note that this board has unequal numbers of white square and black squares.

Suppose we can use dominoes to tile a standard checkerboard with opposite corners removed.

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Suppose we can use dominoes to tile a standard checkerboard with opposite corners removed. Note that the standard checkerboard with opposite corners removed contains 64 - 2 $=$ 62 squares. The tiling would use $62/2 = 31$ dominoes. Note that each domino in this tiling covers one white and one black square. Consequently, the tiling covers 31 white squares and 31 black squares. However, when we remove two opposite corner squares, either 32 of the remaining squares are white and 30 are black or else 30 are white and 32 are black.

Suppose we can use dominoes to tile a standard checkerboard with opposite corners removed. Note that the standard checkerboard with opposite corners removed contains 64 - 2 = 62 squares. The tiling would use $62/2 = 31$ dominoes. Note that each domino in this tiling covers one white and one black square. Consequently, the tiling covers 31 white squares and 31 black squares. However, when we remove two opposite corner squares, either 32 of the remaining squares are white and 30 are black or else 30 are white and 32 are black. This contradicts the assumption that we can use dominoes to cover a standard checkerboard with opposite corners removed, completing the proof.