Q.1

(a) We have $P(x) = \begin{cases} {}^{n}c_{x}\, p^{x} \cdot (1-p)^{n-x} & \text{if } x = 0,1,2,\dots n \\ 0 & \text{otherwise} \end{cases}$

$\therefore P(x \leq 2) = P(0) + P(1) + P(2)$ Here $n = 10$, $p = \frac{1}{6}$

$= {}^{10}c_{0} \cdot \left(\frac{1}{6}\right)^{0} \left(\frac{5}{6}\right)^{10} + {}^{10}c_{1} \cdot \left(\frac{1}{6}\right)^{1} \left(\frac{5}{6}\right)^{9} + {}^{10}c_{2} \left(\frac{1}{6}\right)^{2} \left(\frac{5}{6}\right)^{8}$

$= \left(\frac{5}{6}\right)^{8} \left\{ \frac{25}{36} + 10 \cdot \frac{5}{36} + 45 \cdot \frac{1}{36} \right\}$

$= \frac{5^8}{6^8} \cdot \frac{120}{36}$

$= \frac{24 \cdot 5^9}{6^{10}}$ [No step marking, Correct answer is 1

(b) $P(A \cap B) = \frac{1}{2}$, $P(A^c \cap B^c) = \frac{1}{3}$, $P(A) = P(B) = p$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow 1 - P(A^c \cap B^c) = 2p - \frac{1}{2}$

$\Rightarrow 1 - \frac{1}{3} = 2p - \frac{1}{2}$

$\Rightarrow 2p = \frac{3}{2} - \frac{1}{3} = \frac{7}{6}$

$\Rightarrow p = \frac{7}{12}$ [No step marking, correct answer is 1]

Q.2

L.H.S $= P(A \cap B) - P(A) \cdot P(B)$

$= P(A \cap B) - P(A) \cdot \{1 - P(B^c)\}$ [1 marks]

$\Rightarrow P(A) \cdot P(B^c) - \{P(A) - P(A \cap B)\}$

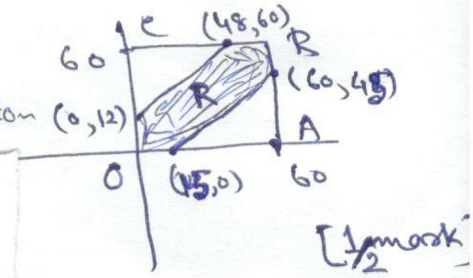
$= P(A) \cdot P(B^c) - P(A \cap B^c)$. [1 marks]

$=$ R.H.S

Q.3 Let the ~~train X~~ reaches the station at x past 8 AM and Y reches the station at y past Y AM. The variables x and y can take any value between 0 and 60. [1/2 marks]

Thus the sample space S is a square OABC of area $60 \times 60 = 3600$ unit^2.

[1/2 marks]



[1/2 mark]

The trains X & Y will meet at the station

if $x - y \leq 15$ or $y - x \leq 12$

$\Rightarrow x \leq y + 15$ $\quad \Rightarrow y - 12 \leq x$

i.e $y - 12 \leq x \leq y + 15$

Thus the favourable region for the trains to meet at the station is the region between $y - 12 = x$ & $y + 15 = x$, shaded region R in the graph. [1/2 marks]

The area of the region R is area of OABC $-$ 2(area of OAB)

$= 3600 - (\tfrac{1}{2} \cdot 48 \cdot 48) - \tfrac{1}{2} \times 45 \times 45$

$= 1435.5$ [1/2 marks]

$\therefore$ P(X & Y meet at station $= \dfrac{1435.5}{3600}$ [1/2 marks]

Q.4 We have

$$f_\theta(x) = \begin{cases} \theta^2 x e^{-\theta} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

~~Now~~ Clearly $\theta > 0$ & $x > 0 \Rightarrow \theta^2 x e^{-\theta} > 0$

$\Rightarrow f_\theta(x) \geq 0$ [1 marks]

Again $\displaystyle\int_{-\infty}^{\infty} f_\theta(x)\,dx = \int_0^{\infty} \theta^2 x e^{-\theta}\,dx$

$= \theta^2 e^{-\theta} \cdot \left[\dfrac{x^2}{2}\right]_0^{\infty}$,

which is undefined.

Therefore $f_\theta(x)$ is not a PDF for any RV. [2.5 marks]

Q.5 Let A be the event that the aircraft is present

B " " " " the radar generates an alarm

Given that

$P(A) = 0.05$; $P(B|A) = 0.99$;

$P(B|A^c) = 0.10$; $P(A^c) = 0.95$;

$P(B^c|A) = 0.01$; $P(B^c|A^c) = 0.90$ [1 marks].

Now

$P(A^c \cap B) = P(A^c).P(B|A^c)$ [½ mark]

$= 0.95 \times 0.10$

$= 0.095$ [½ mark]

$P(A \cap B^c) = P(A).P(B^c|A)$ [½ marks]

$= 0.05 \times 0.01$

$= 0.0005$ [½ marks]

and

$P(A|B) = \dfrac{P(A).P(B|A)}{P(A).P(B|A) + P(A^c).P(B|A^c)}$ [½ marks]

$= \dfrac{0.05 \times 0.99}{(0.05 \times 0.99) + (0.95 \times 0.10)}$ [½ marks]

$= \dfrac{0.0495}{0.1445}$

$= 0.3426$