Assignment 5

MA 205 Discrete Mathematics

September 11, 2020

1. Give two reasons why the set of odd integers under addition is not a group.
2. Show that $\{1, 2, 3\}$ under multiplication modulo 4 is not a group but that $\{1, 2, 3, 4\}$ under multiplication modulo 5 is a group.
3. Show that the set $\{5, 15, 25, 35\}$ is a group under multiplication modulo 40. What is the identity element of this group?
4. Prove that the set of all 2×2 matrices with entries from $\mathbb{R}$ and determinant 1 is a group under matrix multiplication.
5. An abstract algebra teacher intended to give a typist a list of nine integers that form a group under multiplication modulo 91. Instead, one of the nine integers was inadvertently left out, so that the list appeared as 1, 9, 16, 22, 53, 74, 79, 81. Which integer was left out?
6. Show that in a group $(G, *)$, there is only one identity element. (i.e the identity element is unique).
7. Show that in a group $(G, *)$, the right and left cancellation laws hold; that is, for $a, b, c \in G$, $b * a = c * a$ implies $b = c$, and $a * b = a * c$ implies $b = c$.
8. For each element a in a group G, there is a unique element b in G such that $a * b = b * a = e$.(i.e inverse of an element in a group is unique.)
   We will use the symbol a^{-1} to denote the inverse of a in G.
9. Prove that in a group
   (a) $(a^{-1})^{-1} = a$ for all a.
   (b) For group elements a and b, $(a * b)^{-1} = b^{-1} * a^{-1}$.