MA203

Repeated trials

Repeated trials:
Given two experiments i.e., rolling a fair die and tossing a fair coin; find the probability that we get 2 on die and head on coin.

P(2 on die)=1/6
P(head on coin)=1/2
Assuming statistically independent, P(2 on die and head on coin)= P(2 on die)xP(head on coin)=1/12

Other way,
{1,2,3,4,5,6} {H,T}
Considering both as a single experiment, sample space is:
{1H,2H,3H,4H,5H,6H,1T,2T,3T,4T,5T,6T}
P(2 on die and head on coin)=1/12

Cartesian product: given 2 sets S1 and S2, product of the sets is a set S whose elements are all such pairs of elements of S1 and S2.

{H,T} {H,T} {HH,HT,TH,TT}

Bernoulli Trials:

Out of a set of n distinct objects, if k<n objects are taken out of n at a time, the total combinations are $\frac{n!}{(n-k)!k!} = \binom{n}{k}$ (orders are not considered)

Consider an experiment S and an event A with $P(A)=p$, $P(A^c)=q$ and $p+q=1$

Repeat the experiment n times and the resulting product space is $S_n=S \times S \times \ldots \times S$ (n times)

Define $p_n(k)$=P(A occurs k times in any order)

P(A occurs k times in a specific order) $=p^k q^{n-k}$

$p_n(k)=\binom{n}{k}p^k q^{n-k}$ Fundamental theorem

Considering event A as success and A^c as failure, $p_n(k)$ gives prob of k successes in n independent trials.

Eg. 1) A pair of dice is rolled n times. i) Find the probability that 7 will not show at all. ii) Find the probability of double six at least once.

Ans. Sample space of single roll of 2 dice consists of 36 elements.

i) A={seven}={(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)}

P(A)=6/36=1/6

P(A^c)=5/6

P(7 will not show at all)=$\binom{n}{0}(1/6)^0(5/6)^{n-0}$

ii) X: double six at least once

P(X)=1-P(X^c)=1-$(35/36)^n$

Suppose we are interested in no. of throws required to ensure a 50% success of obtaining double six at least once.

So, 1-$(35/36)^n$ >0.5

or, n=24.605

Historical importance: one of the first problems solved by Pascal and correctly interpret gambler's choice.

For a fixed n, consider $p_n(k)$ as a function of k

As k increases, $p_n(k)$ increases reaching a maximum for k=kmax

$$\frac{p_n(k-1)}{p_n(k)} = \frac{\binom{n}{k-1} p^{k-1} q^{n-k+1}}{\binom{n}{k} p^k q^{n-k}} = \frac{kq}{(n-k+1)p} < 1$$

if $k<(n+1)p$, $p_n(k-1)<p_n(k)$

If $k>(n+1)p$, then $p_n(k)$ is decreasing

Therefore, $p_n(k)$ is maximum for $kmax=[(n+1)p]$

If $k1=(n+1)p$ is an integer, then $\frac{p_n(k-1)}{p_n(k)}=1$

So, $p_n(k)$ has maximum values for k=k1 and k=k1-1

Most likely number of success out of n independent trials

Example 2. n=10, p=1/3, calculate most likely number of successes

Example 3. In New York state lottery, the player picks 6 numbers from a sequence of 1 through 51. At a lottery drawing, 6 balls are drawn at random from a box of 51 balls numbered 1 through 51. What is the prob that a player has k matches?

P(k matches) = $\frac{\binom{6}{k}\binom{51-6}{6-k}}{\binom{51}{6}}$

For perfect match, k=6 and prob is 1/18009460

Random variable:
Finite, single-valued function which maps the function into sample space.
For every outcome of sample space, we assign a number and this function is called RV.

Eg. S={HH,HT,TH,TT}
Function: number of heads

Outcome of S	HH	HT	TH	TT
RV	2	1	1	0