Introduction to Logic

Department of Science and Mathematics

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Discrete mathematics is the part of mathematics devoted to the study of discrete objects. (Here discrete means consisting of distinct or unconnected elements.) The kinds of problems solved using discrete mathematics include:

1. How many ways are there to choose a valid password on a computer system?
2. Is there a link between two computers in a network?
3. How can I identify spam e-mail messages?
4. How can I encrypt a message so that no unintended recipient can read it?
5. What is the shortest path between two cities using a transportation system?

A key reason for the growth in the importance of discrete mathematics is that information is stored and manipulated by computing machines in a discrete fashion.

Text/References:

1. Kenneth H. Rosen, Discrete Mathematics and its applications, 7th edition, McGraw - Hill, 2011.
2. C. L. Liu, Elements of Discrete Mathematics, 2/ed, Tata McGraw-Hill, 2000.

Evaluation methods:
Class activity, Quiz, group assignments and Viva voce.
Every Friday group assignments will be given and needs to be submitted on next Monday before 9:00 am.

Introduction to Logic

The rules of logic:

- give precise meaning to mathematical statements
- used to distinguish between valid and invalid mathematical arguments
- used in the design of computer circuits, the construction of computer programs, the verification of the correctness of programs

Proposition

A proposition is a declarative sentence that is either true or false, but not both.

- 1+2=3
- Guwahati is the capital of Assam
- 2+2=5
- Let x be an integer. Then $x + 3$ is an integer.

- Which day is today? (not a proposition)
- Let x and y be integers. Then $x + y = 3$. (not a proposition)
- Let x and y be integers such that $x + y = 3$. Then $y = 3 - x$. (proposition)

$*$: Poll

Is it a proposition or not (Yes/No)?

- It is 11:15 am.

$*$: Poll

Is it a proposition or not (Yes/No)?

- It is 11:15 am.
- The stock market will rise tomorrow.

We denote propositions using letters $p, q, r, s..$

The truth value proposition is true, denoted by T, if it is a true proposition, and the truth value of a proposition is false, denoted by F, if it is a false proposition.

- 1+2=3 - Truth value is T
- Guwahati is the capital of Assam - Truth value is F

New propositions, called compound propositions, are formed from existing propositions using logical operators.

Negation of a proposition

Definition

Let p be a proposition. The negation of p, denoted by $\neg p$ is the statement "It is not the case that p." The proposition $\neg p$ is read "not p." The truth value of the $\neg p$, is the opposite of the truth value of p.

- p: Discrete Mathematics course takes place in LG-II
- $\neg p$: Discrete Mathematics course does not take place in LG-II
- p: Guwahati is the capital of Assam
- $\neg p$: Guwahati is not the capital of Assam

Truth table for negation

p	$\neg p$
T	F
F	T

- p: Vandana's smartphone has at least 32 GB of memory
- $\neg p$: It is not the case that Vandana's smartphone has at least 32 GB of memory

- p: Vandana's smartphone has at least 32 GB of memory
- $\neg p$: It is not the case that Vandana's smartphone has at least 32 GB of memory

or

Vandana's smartphone has less than 32 GB of memory

$*$: Poll

Find $\neg p$ where

p : Vandana's smartphone has at most 32 GB of memory

1. Vandana's smartphone has at least 32 GB of memory
2. Vandana's smartphone has more than 32 GB of memory
3. Vandana's smartphone has less than 32 GB of memory

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p : Vandana's smartphone has at most 32 GB of memory

1. Vandana's smartphone has at least 32 GB of memory
2. Vandana's smartphone has more than 32 GB of memory
3. Vandana's smartphone has less than 32 GB of memory

$\neg p$: Vandana's smartphone has more than 32 GB of memory

Connectives

Connectives are logical operators that are used to form new propositions from two or more existing propositions.

Definition

Let p and q be propositions. The conjunction of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

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p: Raj's PC has more than 16 GB free hard disk space
q: The processor in Raj's PC runs faster than 1 GHz.
$p \wedge q$: Raj's PC has more than 16 GB free hard disk space, and the processor in Rajs PC runs faster than 1 GHz

or simply

$p \wedge q$: Raj's PC has more than 16 GB free hard disk space, and its processor runs faster than 1 GHz.

Truth table for $\wedge$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

In logic the word "but" sometimes is used instead of "and" in a conjunction.
"The sun is shining, but it is raining" is another way of saying "The sun is shining and it is raining."

Definition

Let p and q be propositions. The disjunction of p and q, denoted by $p \vee q$, is the proposition "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

p: Raj's PC has more than 16 GB free hard disk space
q: The processor in Raj's PC runs faster than 1 GHz.
$p \vee q$: Raj's PC has more than 16 GB free hard disk space, or its processor runs faster than 1 GHz.

Truth table for $\vee$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The use of the connective or in a disjunction corresponds to inclusive or.

- inclusive or: Students who have taken calculus or computer science can take this class
- exclusive or : Students who have taken calculus or computer science, but not both, can enroll in this class.

Definition

Let p and q be propositions. The exclusive or of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

Truth table for $\oplus$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Determine whether an inclusive or, or an exclusive or, is intended.

a) Experience with $C++$ or Java is required.

b) Lunch includes soup or salad.

c) To enter the country you need a passport or a voter registration card.

d) Publish or perish.

Conditional Statements

Definition

Let p and q be propositions. The conditional statement $p \rightarrow q$ is the proposition "if p, then q." In the conditional statement $p \rightarrow q$, p is called the hypothesis and q is called the conclusion (or consequence).

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If India wins against England, then Pakistan will go to semi finals

Conditional Statements

Definition

Let p and q be propositions. The conditional statement $p \to q$ is the proposition "if p, then q." In the conditional statement $p \to q$, p is called the hypothesis and q is called the conclusion (or consequence).

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If India wins against England, then Pakistan will go to semi finals

Truth table for $\to$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Different expressions for if

- "if p, then q" , "if p, q" , "q if p"
- " p implies q", "q whenever p", "q when p","q follows from p"
- " p is sufficient for q" , "a sufficient condition for q is p"
- " q is necessary for p", "a necessary condition for p is q"

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- p only if q : p holds only if q holds
  i.e if q does not hold, then p does not hold
  i.e $\neg q \rightarrow \neg p$
  Therefore $p \rightarrow q$ and $\neg q \rightarrow \neg p$ represents the same statement.

Contd.

Another expression represents $p \to q$ is: q unless $\neg p$.

Example: She will not go unless you go with her.

q : She will not go

p : You don't go with her

If you don't go with her, she will not go

i.e If p, q.

Example: Let p be the statement " Anu learns discrete mathematics" and q the statement "Anu will find a good job." Express the statement $p \to q$ as a statement in English.

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Solution:

- If Anu learns discrete mathematics, then she will find a good job.
- Anu will find a good job when she learns discrete mathematics.
- For Anu to get a good job, it is sufficient for her to learn discrete mathematics.
- Anu will find a good job unless she does not learn discrete mathematics.

In mathematical reasoning, we consider conditional statements of a more general sort than we use in English. In English, conditional statements are used where there is a relationship between the hypothesis and the conclusion.

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  On the other hand, the statement
- If Juan has a smartphone, then $2 + 3 = 5$: True (irrespective of hypothesis since conclusion is true)
- If Juan has a smartphone, then $2 + 3 = 6$ is true if Juan does not have a smartphone, even though $2 + 3 = 6$ is false.

The if-then construction used in many programming languages is different from that used in logic. Most programming languages contain statements such as if p then S, where p is a proposition and S is a program segment (one or more statements to be executed). When execution of a program encounters such a statement, S is executed if p is true, but S is not executed if p is false.

1. Determine whether each of these conditional statements is true or false.
a) If $1 + 1 = 2$, then $2 + 2 = 5$.

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a) If $1 + 1 = 2$, then $2 + 2 = 5$.
b) If $1 + 1 = 3$, then $2 + 2 = 4$.
$*$ Poll
c) If $1 + 1 = 3$, then $2 + 2 = 5$.

1. Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 2$, then $2 + 2 = 5$.

b) If $1 + 1 = 3$, then $2 + 2 = 4$.

$*$ Poll

c) If $1 + 1 = 3$, then $2 + 2 = 5$.

d) If monkeys can fly, then $1 + 1 = 3$.

2.Determine whether each of these conditional statements is true or false.
a) If $1 + 1 = 3$, then unicorns exist.
b) If $1 + 1 = 3$, then dogs can fly.
c) If $1 + 1 = 2$, then dogs can fly.
d) If $2 + 2 = 4$, then $1 + 2 = 3$.