Introduction to Logic

Department of Science and Mathematics

IIIT Guwahati.

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Predicates and Quantifiers

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Example

Let $P(x)$ denote the statement $x > 3$. What are the truth values of $P(4)$ and $P(2)$?

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Let $A(x)$ denote the statement : Computer x is under attack by an intruder. Suppose that of the computers on campus, only CS2 and MATH1 are currently under attack by intruders. What are truth values of A(CS1), A(CS2), and A(MATH1)?

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Let $Q(x, y)$ denote the statement : $x = y + 3$. What are the truth values of the propositions Q(1, 2) and Q(3, 0)?

Quantifiers

Quantification expresses the extent to which a predicate is true over a range of elements. We will focus on two types of quantification here: universal quantification, which tells us that a predicate is true for every element under consideration (domain), and existential quantification, which tells us that there is one or more element under consideration for which the predicate is true.

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THE UNIVERSAL QUANTIFIER : The meaning of the universal quantification of P(x) changes when we change the domain. The domain must always be specified when a universal quantifier is used; without it, the universal quantification of a statement is not defined.

Definition

The universal quantification of $P(x)$ is the statement $P(x)$ for all values of x in the domain. The notation $\forall x\ P(x)$ denotes the universal quantification of $P(x)$. Here $\forall x$ is called the universal quantifier. We read $\forall x\ P(x)$ as "for all x, P(x)" or "for every $x, P(x)$." An element for which $P(x)$ is false is called a counterexample of $\forall x\ P(x)$.

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- Let $P(x)$ be the statement "$x + 1 > x$." What is the truth value of the quantification $\forall x\ P(x)$, where the domain consists of all real numbers?
- Let $Q(x)$ be the statement "$x < 2$." What is the truth value of the quantification $\forall x\ Q(x)$, where the domain consists of all real numbers?

- What is the truth value of $\forall x\ P(x)$, where $P(x)$ is the statement $x^2 < 10$ and the domain consists of the positive integers not exceeding 4?

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A domain must always be specified when a statement $\exists x P(x)$ is used. We can also express existential quantification in many other ways, such as by using the words "for some, for at least one," or "there is."

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The quantifiers $\forall$ and $\exists$ have higher precedence than all logical operators from propositional calculus.