Topics: Normal Distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

   Ans:
   We have a normal distribution with $\mu = 45$ and $\sigma = 8$. Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour, you must have $X \leq 50$ so the question is to find $P(X > 50)$.

   $P(X > 50)$
   $= 1 - P(X \leq 50)$.
   $Z = (X - \mu)/\sigma$
   $= (X - 45)/8$

   Thus, the question can be answered by using the normal table to find
   $P(X \leq 50)$
   $= P(Z \leq (50 - 45)/8)$
   $= P(Z \leq 0.625)$
   $= 73.4\%$

   Probability that the service manager will not meet his demand will be
   $= 100 - 73.4$
   $= 26.6\%$ or 0.266.

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma = 6$. For each statement below, please specify True/False. If false, briefly explain why.
    A. More employees at the processing center are older than 44 than between 38 and 44.

Ans:

We have a normal distribution with $\mu = 38$ and $\sigma = 6$. Let X be the number of employees. So according to question

Probability of employees greater than age of 44= P (X>44)

P (X > 44)

= 1 – P (X ≤ 44).

Z = (X -μ)/σ

= (X - 38)/6

Thus, the question can be answered by using the normal table to find

P (X ≤ 44)

= P (Z ≤ (44 - 38)/6)

= P (Z ≤ 1)

= 84.1345%

Probability that the employee will be greater than age of 44

= 100-84.1345

= 15.86%

So, the probability of number of employees between 38-44 years of age

= P (X<44)-0.5

= 84.13450.5

= 34.1345%

Therefore, the statement that "More employees at the processing center are older than 44 than between 38 and 44" is TRUE.

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

Probability of employees less than age of 30 = P (X<30).

$Z = (X - \mu)/\sigma$

= (30 - 38)/6

Thus, the question can be answered by using the normal table to find

$P(X \leq 30)$

$= P(Z \leq (30 - 38)/6)$

$= P(Z \leq -1.333)$

= 9.12%

So, the number of employees with probability 0.912 of them being under age 30 = 0.0912*400

= 36.48 (or 36 employees).

Therefore, the statement B of the question is also 'TRUE'.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Ans: As we know that if X ~ N (μ1, σ1^2), and Y ~ N (μ2, σ2^2) are two independent random variables then,

X + Y ~ N (μ1 + μ2, σ1^2 + σ2^2), and

X − Y ~ N (μ1 − μ2, σ1^2 + σ2^2).

Similarly, if Z = aX + bY, where X and Y are as defined above, i.e., Z is linear combination of X and Y then,

Z ~ N (aμ1 + bμ2, a^2σ1^2 + b^2σ2^2).

Therefore, in the question

2X1~ N (2μ,4 σ^2) and

X1+X2 ~ N (μ + μ, σ^2 + σ^2) ~ N (2 u, 2σ^2)

2X1-(X1+X2) = N (4μ,6 σ^2)

4. Let X ~ N (100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

Ans: D.

The Probability of getting value between a and b should be 0.99.
So, the Probability of going wrong, or the Probability outside the a and b area is 0.01 (i.e., 1-0.99).

The Probability towards left from a = -0.005 (i.e., 0.01/2).
The Probability towards right from b = +0.005 (i.e., 0.01/2).

So, since we have the probabilities of a and b, we need to calculate X, the random variable at a and b which has got these probabilities.
By finding the Standard Normal Variable Z (Z Value), we can calculate the X values.
$Z = (X - \mu) / \sigma$

For Probability 0.005 the Z Value is -2.57 (from Z Table).
$Z * \sigma + \mu = X$
Z (-0.005) *20+100 = -(-2.57) *20+100 = 151.5
Z (+0.005) *20+100 = (-2.57) *20+100 = 48.5.
So, the option 'D' is correct.

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45

   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans:

Specify a Rupee range (centred on the mean) such that it contains 95 % probability for the annual profit of the company.
Mean Profit is Rs 540 million.
Standard Deviation is Rs 225.0 million
Range is Rs (99.00810347848784, 980.9918965215122) in Millions.

   B. Specify the 5th percentile of profit (in Rupees) for the company

Ans:

5th percentile of profit (in Million Rupees) is 170.0

   C. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

Making 1 loss: - Confidence Interval: 0.0477903522728147.
Making 2 loss: Confidence Interval: 0.040059156863817086.

Probability of Division 1 making a loss in a given year is more than Division 2.

THE END!!