Math Questions Solutions

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$x_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ $y_1 = \begin{bmatrix} 0.5 \\ 1 \\ 2 \end{bmatrix}$

(a): $d(x,y) = 1 - \frac{x^T y}{\|y\| \cdot \|x\|}$

$\vec{x}_1^T \vec{y}_1 = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 0.5 \\ 1 \\ 2 \end{bmatrix} = 0.5 + 3 + 4 = 7.5$

$\|x_1\| = \sqrt{1^2+3^2+2^2} = \sqrt{14}$

$\|y_1\| = \sqrt{0.5^2+1^2+2^2} = \sqrt{5.25}$

$\therefore d(x_1, y_1) = 1 - \frac{7.5}{\sqrt{5.25} \cdot \sqrt{14}}$

(b): $\because d(\vec{x}, \vec{y}) = \|\vec{x} - \vec{y}\|$ $\vec{z} = \vec{x}_1 - \vec{y}_1 = \begin{bmatrix} 0.5 \\ 2 \\ 0 \end{bmatrix}$

$\therefore d(\vec{x}_1, \vec{y}_1) = \|\vec{z}\| = \sqrt{0.5^2+2^2+0^2} = \sqrt{4.25}$

(c): $\vec{x}_2 = 2\vec{x}_1 = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$

$d(\vec{x}_2, \vec{y}_1) = 1 - \frac{\vec{x}_2^T \vec{y}_1}{\|y\| \|x_2\|} = 1 - \frac{\begin{bmatrix} 2 & 6 & 4 \end{bmatrix} \begin{bmatrix} 0.5 \\ 1 \\ 2 \end{bmatrix}}{\sqrt{5.25} \cdot \sqrt{4+36+16}} = 1 - \frac{15}{\sqrt{5.25} \cdot \sqrt{56}} = 1 - \frac{15}{\sqrt{5.25}\sqrt{14} \cdot 2}$

$= 1 - \frac{7.5}{\sqrt{5.25}\sqrt{14}} =$

So for the cosin distance value, it will not change $d(\vec{x}_1, \vec{y}_1)$

(d): $d(\vec{x}_2, \vec{y}) = \|\vec{x}_2 - \vec{y}\|$

$\vec{x}_2 - \vec{y} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix} - \begin{bmatrix} 0.5 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 5 \\ 2 \end{bmatrix}$

$\therefore d(\vec{x}_2, \vec{y}) = \sqrt{1.5^2+5^2+4} = \sqrt{31.25} \neq \sqrt{4.25}$

So Euclidean distance changes