# Decision Tree Primer

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# Contents

# CHAPTER 1

# Decision Trees

The analysis of complex decisions with significant uncertainty can be confusing because 1) the consequence that will result from selecting any specified decision alternative cannot be predicted with certainty, 2) there are often a large number of different factors that must be taken into account when making the decision, 3) it may be useful to consider the possibility of reducing the uncertainty in the decision by collecting additional information, and 4) a decision maker's attitude toward risk taking can impact the relative desirability of different alternatives. This chapter reviews decision tree analysis procedures for addressing such complexities.

## 1.1 Decision Trees

To illustrate the analysis approach, a decision tree is used in the following example to help make a decision.

**Example 1.1**

**Product decision.** To absorb some short-term excess production capacity at its Arizona plant, Special Instrument Products is considering a short manufacturing run for either of two new products, a temperature sensor or a pressure sensor. The market for each product is known if the products can be successfully developed. However, there is some chance that it will not be possible to successfully develop them.

Revenue of $1,000,000 would be realized from selling the temperature sensor and revenue of $400,000 would be realized from selling the pressure sensor. Both of these amounts are net of production cost but do not include development cost. If development is unsuccessful for a product, then there will be no sales, and the development cost will be totally lost. Development cost would be $100,000 for the temperature sensor and $10,000 for the pressure sensor.

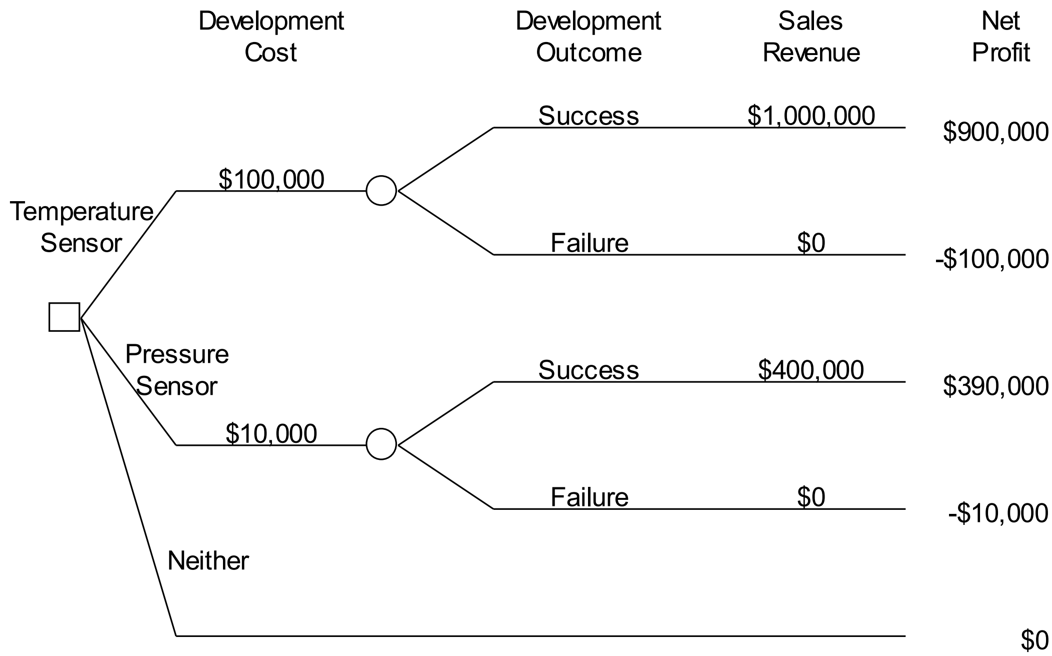


**Figure 1.1** *Special Instrument Products decision*

> **Question 1.1:** Which, if either, of these products should Special Instrument Products attempt to develop?

To answer Question 1.1 it is useful to represent the decision as shown in Figure 1.1. The tree-like diagram in this figure is read from left to right. At the left, indicated with a small square, is the decision to select among the three available alternatives, which are 1) the temperature sensor, 2) the pressure sensor, or 3) neither. The development costs for the "develop temperature sensor" and "develop pressure sensor" alternatives are shown on the branches for those alternatives. At the right of the development costs are small circles which represent the uncertainty about whether the development outcome will be a success or a failure. The branches to the right of each circle show the possible development outcomes. On the branch representing each possible development outcome, the sales revenue is shown for the alternative, assuming either success or failure for the development. Finally, the net profit is shown at the far right of the tree for each possible combination of development alternative and development outcome. For example, the topmost result of $900,000 is calculated as $\$1,000,000 - \$100,000 = \$900,000$. (Profits with negative signs indicate losses.)

The notation used in Figure 1.1 will be discussed in more detail shortly, but for now concentrate on determining the alternative Special Instrument Products should select. We can see from Figure 1.1 that developing the temperature sensor could yield the largest net profit ($900,000), but it could also yield the largest loss ($100,000). Developing the pressure sensor could only yield a net profit of $390,000, but the possible loss is limited to $10,000. On the other hand, not developing either of the sensors is risk free in the sense that there is no possibility of a loss. However, if Special Instrument Products decides not to attempt to develop one of the sensors, then the company is giving up the potential opportunity to make either $900,000 or $390,000. Question 1.1 will be answered in a following example after we discuss the criterion for making such a decision. ■

You may be thinking that the decision about which alternative is preferred depends on the probabilities that development will be successful for the temperature or pressure sensors. This is indeed the case, although knowing the probabilities will not by itself always make the best alternative in a decision immediately clear. However, if the outcomes are the same for the different alternatives, and only the probabilities differ, then probabilities alone are sufficient to determine the best alternative, as illustrated by Example 1.2.

**Example 1.2**

**Tossing a die.** Suppose you are offered two alternatives, each of which consists of a single toss of a fair die. With the first alternative, you will win $10 if any number greater than 4 is thrown, and lose $5 otherwise. With the second alternative, you will win $10 if any number greater than 3 is thrown, and lose $5 otherwise. In this case, since there are 6 faces on a die, the probability of winning is $2/6 = 1/3$ for the first alternative and $3/6 = 1/2$ for the second alternative. Since the consequences are the same for both alternatives and the probability of winning is greater for the second alternative, you should select the second alternative. ■

However, the possible outcomes are often not the same in realistic business decisions and this causes additional complexities, as illustrated by further consideration of the Special Instruments Product decision in Example 1.3.

**Example 1.3**

**Product decision.** Suppose that in Example 1.1 the probability of development success is 0.5 for the temperature sensor and 0.8 for the pressure sensor. Figure 1.2 is a diagram with these probabilities shown in parentheses on the branches representing the possible outcomes for each sensor development effort. (While only the probability of success is specified for each development effort, the probability of failure is determined by the rules of probability since the probabilities of development success and development failure must add up to one.)

A study of Figure 1.2 shows that having the probabilities does not resolve this decision for us. The figure shows that although the probability of development success is considerably lower for the temperature sensor than it is for the pressure sensor (0.5 versus 0.8), the net profit from successful development of the temperature sensor is considerably higher than the net profit from successful development

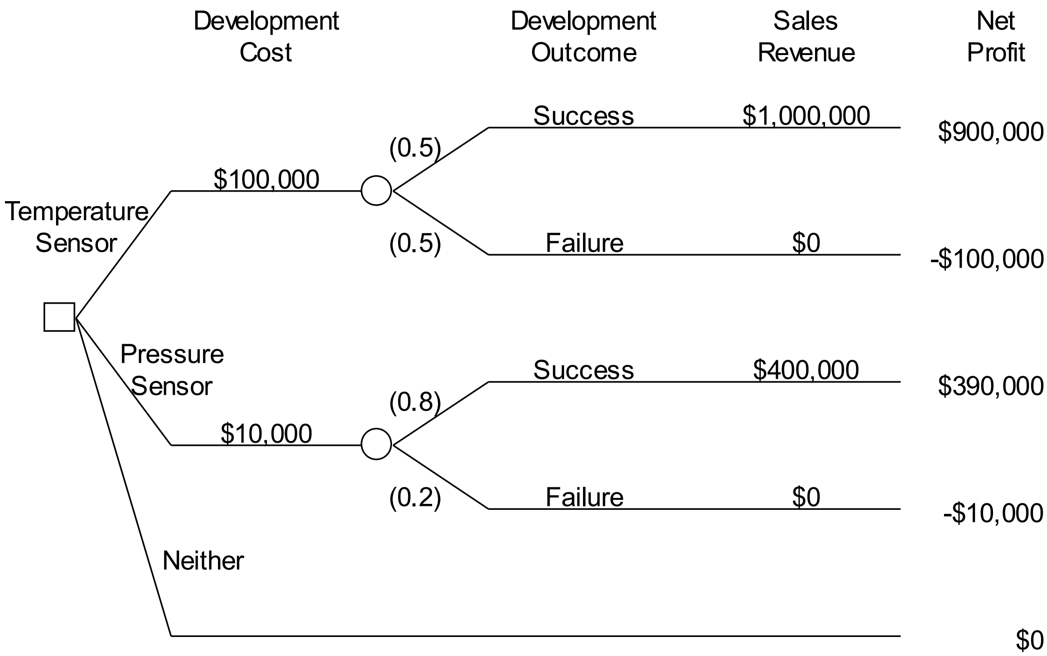


**Figure 1.2** *Special Instrument Products decision tree*

of the pressure sensor ($900,000 versus $390,000). That is, the alternative with the higher potential payoff has a lower probability that this payoff will actually be realized. ■

The resolution of this decision dilemma is addressed in the next section, but before doing this, Definition 1.1 clarifies the notation in Figures 1.1 and 1.2.

**Definition 1.1: Decision tree notation**

A diagram of a decision, as illustrated in Figure 1.2, is called a **decision tree**. This diagram is read from left to right. The leftmost node in a decision tree is called the **root node**. In Figure 1.2, this is a small square called a **decision node**. The branches emanating to the right from a decision node represent the set of decision alternatives that are available. One, and only one, of these alternatives can be selected. The small circles in the tree are called **chance nodes**. The number shown in parentheses on each branch of a chance node is the probability that the outcome shown on that branch will occur at the chance node. The right end of each path through the tree is called an **endpoint**, and each endpoint represents the final outcome of following a path from the root node of the decision tree to that endpoint.

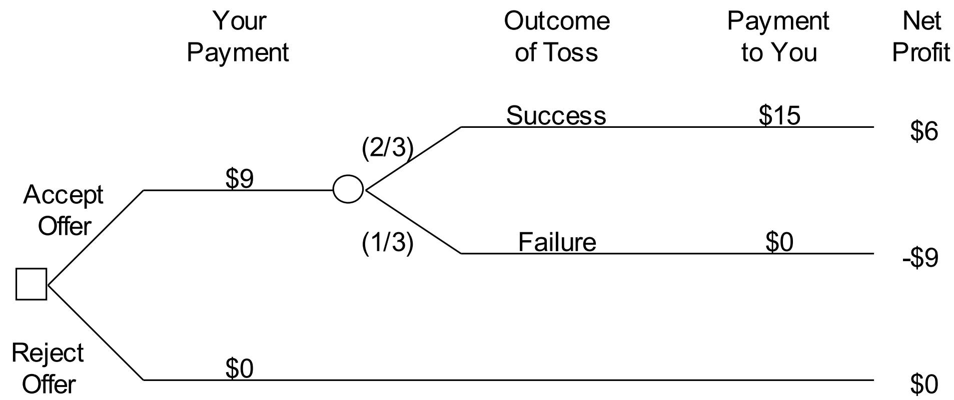


**Figure 1.3** *Die toss decision tree*

## 1.2 Expected Value

In order to decide which alternative to select in a decision problem, we need a decision criterion; that is, a rule for making a decision. Expected value is a criterion for making a decision that takes into account both the possible outcomes for each decision alternative and the probability that each outcome will occur. To illustrate the concept of expected value, we consider a simpler decision with lower stakes than the Special Instrument Products decision.

**Example 1.4**

**Rolling a die.** A friend proposes a wager: You will pay her \$9.00, and then a fair die will be rolled. If the die comes up a 3, 4, 5, or 6, then your friend will pay you \$15.00. If the die comes up 1 or 2, she will pay you nothing. Furthermore, your friend agrees to repeat this game as many times as you wish to play.

**Question 1.2:** Should you agree to play this game?

If a six-sided die is fair, there is a 1/6 probability that any specified side will come up on a roll. Therefore there is a 4/6 (= 2/3) probability that a 3, 4, 5, or 6 will come up and you will win. Figure 1.3 shows the decision tree for one play of this game.

At first glance, this may not look like a good bet since you can lose \$9.00, while you can only win \$6.00. However, the probability of winning the \$6.00 is 2/3, while the probability of losing the \$9.00 is only 1/3. Perhaps this isn't such a bad bet after all since the probability of winning is greater than the probability of losing.

The key to logically analyzing this decision is to realize that your friend will let you play this game as many times as you want. For example, how often would you expect to win if you play the game 1,500 times? Based on what you have learned about probability, you know that the proportion of games in which you will win over the long run is approximately equal to the probability of winning a single game. Thus, out of the 1,500 games, you would expect to win approximately $(2/3) \times 1,500 = 1,000$ times. Therefore, over the 1,500 games, you would expect to win a total of approximately $1,000 \times \$6 + 500 \times (-\$9) = \$1,500$. So this game looks like a good deal!

Based on this logic, what is each play of the game worth? Well, if 1,500 plays of the game are worth \$1,500, then one play of the game should be worth $\$1,500/1,500 = \$1.00$. Putting this another way, you will make an average of \$1.00 each time you play the game.

A little thought about the logic of these calculations shows that you can directly determine the average payoff from one play of the game by multiplying each possible payoff from the game by the probability of that payoff, and then adding up the results. For the die tossing game, this calculation is $(2/3) \times \$6 + (1/3) \times (-\$9) = \$1$. ∎

The quantity calculated in the manner illustrated in Example 1.4 is called the **expected value** for an alternative, as shown in Definition 1.2. Expected value is often a good measure of the value of an alternative since over the long run this is the average amount that you expect to make from selecting the alternative.

**Definition 1.2: Expected Value**

The **expected value** for an uncertain alternative is calculated by multiplying each possible outcome of the uncertain alternative by its probability, and summing the results. The **expected value decision criterion** selects the alternative that has the best expected value. In situations involving profits where "more is better," the alternative with the highest expected value is best, and in situations involving costs, where "less is better," the alternative with the lowest expected value is best.

**Example 1.5**

**Product decision.** The expected values for the Special Instrument Products decision are designated by "EV" in Figure 1.4. These are determined as follows: 1) For the temperature sensor alternative, $0.5 \times \$900,000 + 0.5 \times (-\$100,000) = \$400,000$, 2) for the pressure sensor alternative, $0.8 \times \$390,000 + 0.2 \times (-\$10,000) = \$310,000$, and 3) for doing neither of these \$0. Thus, the alternative with the highest expected value is developing the temperature sensor, and if the expected value criterion is applied, then the temperature sensor should be developed. ∎

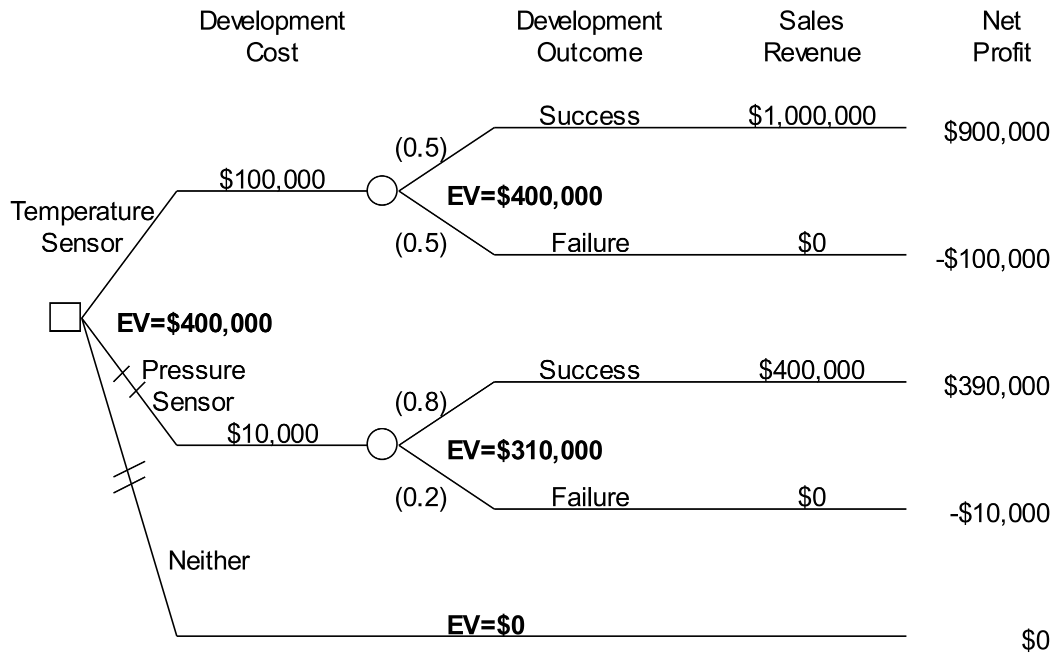


**Figure 1.4** *Special Instrument Products decision tree, with expected values*

Figure 1.4 illustrates some additional notation that is often used in decision trees. The branches representing the two alternatives that are less preferred are shown with crosshatching (//) on their branches. The expected value for each chance node is designated by "EV". Finally, the expected value at the decision node on the left is shown as equal to the expected value of the selected alternative.

## Xanadu Traders

We conclude this section by analyzing a decision involving international commerce. This example will be extended in the remainder of this chapter

**Example 1.6**

**Xanadu Traders.** Xanadu Traders, a privately held U.S. metals broker, has acquired an option to purchase one million kilograms of partially refined molyzirconium ore from the Zeldavian government for $5.00 per kilogram. Molyzirconium can be processed into several different products which are used in semiconductor manufacturing, and George Xanadu, the owner of Xanadu Traders, estimates that he would be able to sell the ore for $8.00 per kilogram after importing it. However, the U.S. government is currently negotiating with Zeldavia over alleged "dumping" of certain manufactured goods which that country exports to the United States. As part of these negotiations, the U.S. government has threatened to ban the import from Zeldavia of a class of materials that includes molyzirconium. If the U.S. government refuses to issue an import license for the molyzirconium after Xanadu has purchased it, then Xanadu will have to pay a penalty of \$1.00 per kilogram to the Zeldavian government to annul the purchase of the molyzirconium.

Xanadu has used the services of Daniel A. Analyst, a decision analyst, to help in making decisions of this type in the past, and George Xanadu calls on him to assist with this analysis. From prior analyses, George Xanadu is well-versed in decision analysis terminology, and he is able to use decision analysis terms in his discussion with Analyst.

Analyst: As I understand it, you can buy the one million kilograms of molyzirconium ore for \$5.00 a kilogram and sell it for \$8.00, which gives a profit of $(\$8.00 - \$5.00) \times 1,000,000 = \$3,000,000$. However, there is some chance that you cannot obtain an import license, in which case you will have to pay \$1.00 per kilogram to annul the purchase contract. In that case, you will not have to actually take the molyzirconium and pay Zeldavia for it, but you will lose $\$1.00 \times 1,000,000 = \$1,000,000$ due to the cost of annulling the contract.

Xanadu: Actually, "some chance" may be an understatement. The internal politics of Zeldavia make it hard for their government to agree to stop selling their manufactured goods at very low prices here in the United States. The chances are only fifty-fifty that I will be able to obtain the import license. As you know, Xanadu Traders is not a very large company. The \$1,000,000 loss would be serious, although certainly not fatal. On the other hand, making \$3,000,000 would help the balance sheet...

> **Question 1.3:** Which alternative should Xanadu select? Assume that Xanadu uses expected value as his decision criterion.

To answer this question, construct a decision tree. There are two possible alternatives, purchase the molyzirconium or don't purchase it. If the molyzirconium is purchased, then there is uncertainty about whether the import license will be issued or not. The decision tree is shown in Figure 1.5. Starting from the root node for this tree, it costs \$5 million to purchase the molyzirconium, and if the import license is issued, then the molyzirconium will be sold for \$8 million, yielding a net profit of \$3 million. On the other hand, if the import license is not issued then Xanadu will recover \$4 million of the \$5 million that it invested, but will lose the other \$1 million due to the cost of annulling the contract.

The endpoint net profits are shown in millions of dollars, and the expected value for the "purchase" alternative is $0.5 \times \$3 + 0.5 \times (-\$1) = \$1$, in millions of dollars. Therefore, if expected value is used as the decision criterion, then the preferred alternative is to purchase the molyzirconium. ∎

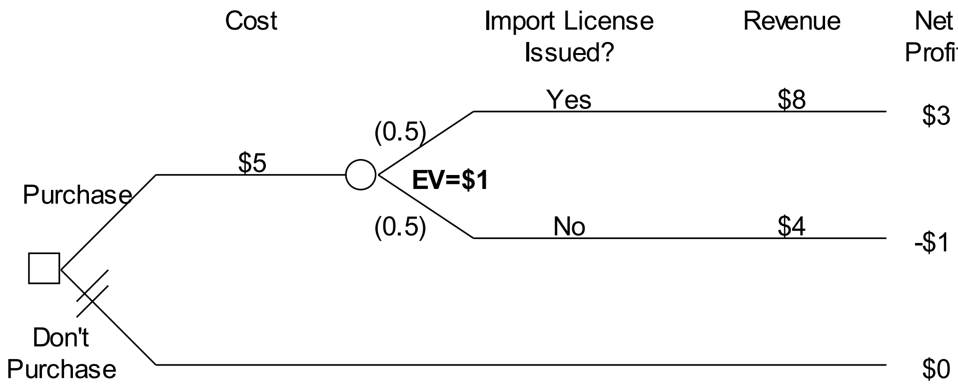


**Figure 1.5** *Xanadu Traders initial decision tree (dollar amounts in millions)*

## 1.3 Dependent Uncertainties

In this section, we consider an additional complexity that often occurs in business decisions: **dependent uncertainties**. Specifically, we will examine a case that illustrates this complexity of real world decisions, and review a procedure for analyzing decisions that include dependent uncertainties.

**Example 1.7**

**Xanadu Traders.** This is a continuation of Example 1.6. We now consider an expanded version of the decision that includes dependent uncertainties and extend the analysis procedure to handle this new issue. We continue to follow the discussion between Daniel Analyst and George Xanadu that started in Example 1.6.

Analyst: Maybe there is a way to reduce the risk. As I understand it, the reason you need to make a quick decision is that Zeldavia has also offered this deal to other brokers, and one of them may take it before you do. Is that really very likely? Perhaps you can apply for the import license and wait until you know whether it is approved before closing the deal with Zeldavia.

Xanadu: That's not very likely. Some of those brokers are pretty big operators, and dropping $1,000,000 wouldn't make them lose any sleep. I'd say there is a 0.70 probability that someone else will take Zeldavia's offer if I wait until the import license comes through. Of course, it doesn't cost anything to apply for an import license, so maybe it is worth waiting to see what happens...

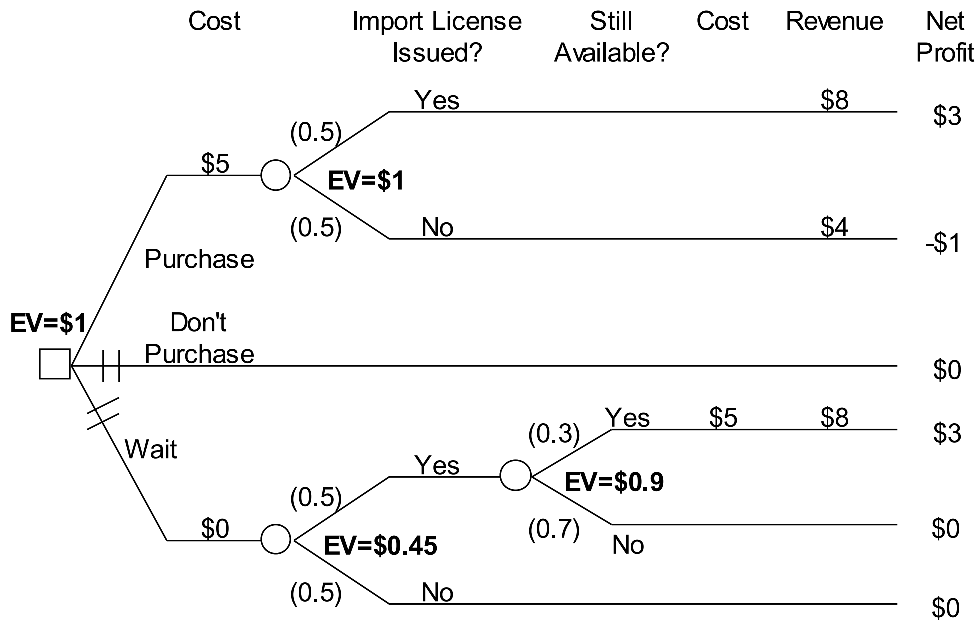


**Figure 1.6** *Xanadu Traders revised decision tree, with expected values*

> **Question 1.4:** Should Xanadu Traders wait to see if an import license is issued before purchasing the molyzirconium?

The decision tree for this revised problem is shown in Figure 1.6. The two alternatives at the top of this tree ("purchase" and "don't purchase") are the same as the alternatives shown in Figure 1.5. The third alternative ("wait") considers the situation where Xanadu waits to see whether it can obtain an import license before purchasing the molyzirconium. This alternative introduces a new analysis issue that must be addressed before the expected value for this alternative can be determined. This new issue concerns the fact that there are two stages of uncertainty for this alternative. First, the issue of an import license is resolved, and then there is a further uncertainty about whether the molyzirconium will still be available.

> **Question 1.5:** What is the expected value for the "wait" alternative?

The process of determining the expected value for this alternative involves two stages of calculation. In particular, it is necessary to start at the right side of the decision tree, and carry out successive calculations working toward the root node of the tree. Specifically, first determine the expected value for the alternative assuming that the import license is issued, and then use this result to calculate the expected value for the "wait" alternative prior to learning whether the import license is issued.

Examine Figure 1.6 to see how this calculation process works. As this figure shows, if the import license is issued, then there is a 0.3 probability that the molyzirconium will still be available. In this case, Xanadu will pay \$5 million for the molyzirconium, and sell it for \$8 million realizing \$3 million in net profit. If the molyzirconium is not still available, then Xanadu will not have to pay anything and will realize no net profit. Thus, the expected value for the situation after the uncertainty about the import license has been resolved is $0.3 \times \$3 + 0.7 \times \$0 = \$0.9$. This expected value is shown next to the lower right chance node on the decision tree in Figure 1.6.

From the discussion regarding expected value in Section 1.2, it follows that this \$0.9 million is the value of the alternative once the result of the import license application is known. Hence, this value should be used in the further expected value calculation needed to determine the overall value of the "wait" alternative. Thus, the expected value for the "wait" alternative is given by $0.5 \times \$0.9 + 0.5 \times \$0 = \$0.45$. This expected value is shown next to the lower left chance node on the decision tree in Figure 1.6. Since the expected value for the "wait" alternative is less than the \$1 million expected value for purchasing the molyzirconium right now, this alternative is less preferred than purchasing the molyzirconium right now. Xanadu should not wait, assuming that expected value is used as the decision criterion. ■

The process of sequentially determining expected values when there are dependent uncertainties in a decision tree, as demonstrated in Example 1.6, is called **decision tree rollback**. This term is defined in Definition 1.3.

**Definition 1.3: Decision Tree Rollback**

> The process of successively calculating expected values from the endpoints of the decision tree to the root node, as demonstrated in this section, is called a **decision tree rollback**.

## 1.4 Sequential Decisions

In addition to dependent uncertainties, real business decisions often include **sequential decisions**. This section considers an example that demonstrates how to address sequential decisions.

**Example 1.8**

**ABC Computer Company.** ABC Computer Company is considering submission of a bid for a government contract to provide 10,000 specialized computers for use in computer-aided design. There is only one other potential bidder for this contract, Complex Computers, Inc., and the low bidder will receive the contract. ABC's bidding decision is complicated by the fact that ABC is currently working on a new process to manufacture the computers. If this process works as hoped, then it may substantially lower the cost of making the computers. However, there is some chance that the new process will actually be more expensive than the current manufacturing process. Unfortunately, ABC will not be able to determine the cost of the new process without actually using it to manufacture the computers.

If ABC decides to bid, it will make one of three bids: $9,500 per computer, $8,500 per computer, or $7,500 per computer. Complex Computers is certain to bid, and it is equally likely that Complex will bid $10,000, $9,000, or $8,000 per computer. If ABC decides to bid, then it will cost $1,000,000 to prepare the bid due to the requirement that a prototype computer be included with the bid. This $1,000,000 will be totally lost regardless of whether ABC wins or loses the bidding competition.

With ABC's current manufacturing process, it is certain to cost $8,000 per computer to make each computer. With the proposed new manufacturing process, there is a 0.25 probability that the manufacturing cost will be $5,000 per computer and a 0.50 probability that the cost will be $7,500 per computer. Unfortunately, there is also a 0.25 probability that the cost will be $8,500 per computer.

> **Question 1.6:** Should ABC Computer Company submit a bid, and if so, what should they bid per computer?

A decision tree for this situation is shown in Figure 1.7. First, ABC must decide whether to bid and how much to bid. If ABC's bid is lower than Complex Computer's, then ABC must decide which manufacturing process to use. If ABC uses the proposed new manufacturing process, then the cost of manufacturing the computers is uncertain. The net profit figures (in millions of dollars) shown at the endpoints of the Figure 1.7 tree take into account the cost of preparing the bid, the cost of manufacturing the computers, and the revenue that ABC will receive for the computers. For example, examine the topmost endpoint value. It costs $1 million to prepare the bid, and ABC bids $9,500, which is lower than Complex Computers' bid of $10,000, and hence ABC wins the contract. Then the proposed new manufacturing process is used, and it costs $8,500 per computer to manufacture the 10,000 computers. Therefore, at this endpoint, ABC makes a net profit of $-1,000,000 - 10,000 \times \$8,500 + 10,000 \times \$9,500 = \$9,000,000 = \$9$

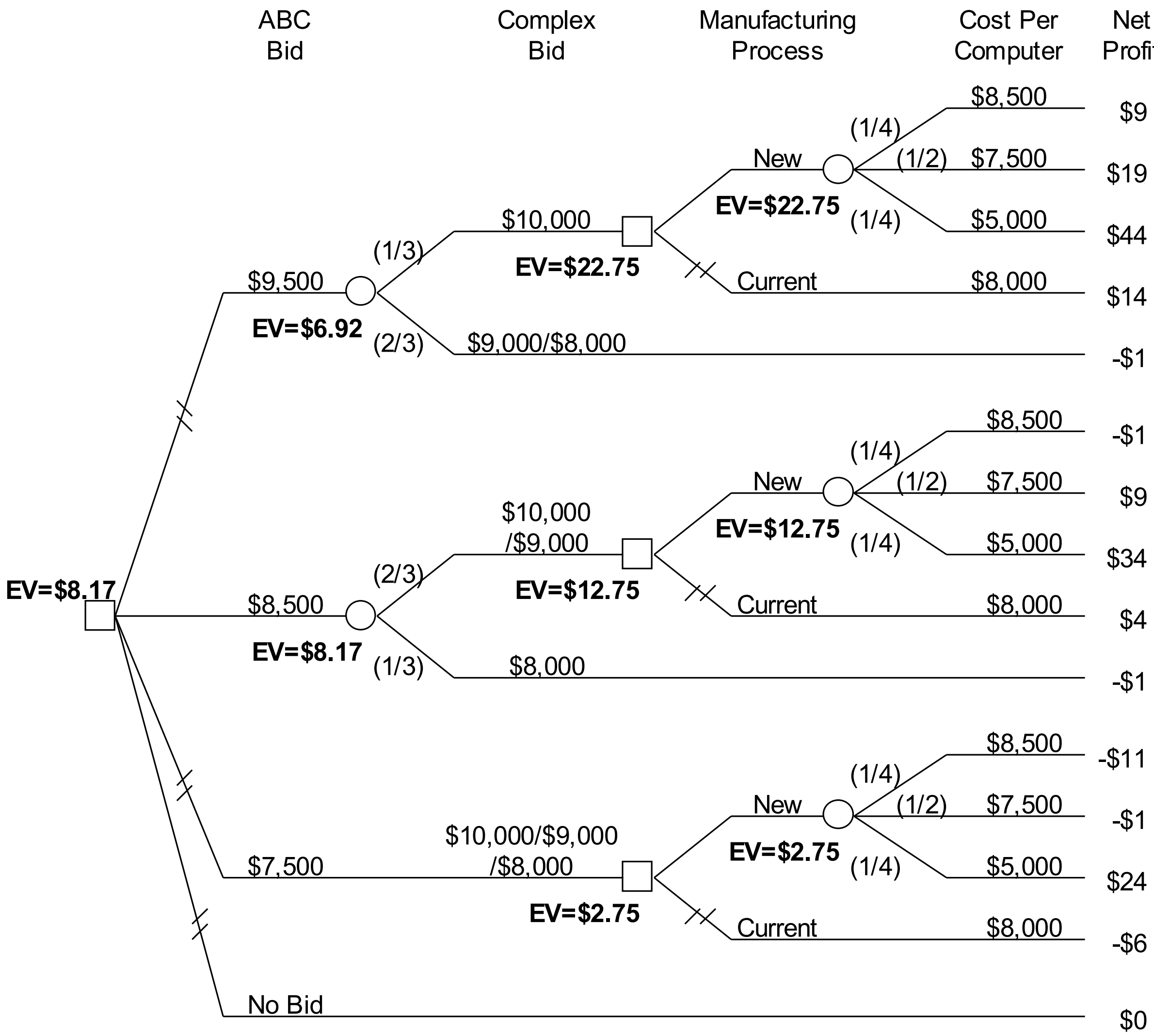


**Figure 1.7** *ABC Computer Company decision tree, with net profit in millions of dollars*

million. Verify the net profits shown at the other endpoints so that you better understand this process.

Calculating the expected values shown on the Figure 1.7 decision tree requires addressing a new issue, namely what to do when there are multiple decision nodes in the tree. In this decision, the amount of the bid is the first decision, and if this is lower than the Complex Computers bid, then there is a second decision involving the type of manufacturing process to use. The calculation procedure for this situation is a straightforward extension of the calculation procedure that was demonstrated in the preceding section for dependent uncertainties.

This procedure will be illustrated by considering the topmost set of nodes in the Figure 1.7 tree. Start at the rightmost side of the tree, and calculate the expected value for the top rightmost chance node. This is determined as

$(1/4) \times \$9 + (1/2) \times \$19 + (1/4) \times \$44 = \$22.75$. At the top rightmost decision node, compare the expected values for the two branches. The expected value for the top branch of this decision node is \$22.75, and (since there is no uncertainty regarding the cost of the current manufacturing process) the expected value for the bottom branch is \$14. Since the top branch has the higher expected value, it is the preferred branch. That is, the proposed new manufacturing process should be used. Hence, the expected value for the "manufacturing process" decision node is equal to the expected value for the proposed new manufacturing process, which is \$22.75.

Now continue back toward the root of the decision tree by calculating the expected value for the top leftmost chance node in the tree. Since the expected value of the manufacturing process decision is \$22.75, and there is no uncertainty about the net profit if ABC loses the bid, the expected value for the top leftmost chance node is $(1/3) \times \$22.75 + (2/3) \times (-\$1) = \$6.92$.

A similar process is used to calculate the expected values for the other three branches of the root node, and the results are shown in Figure 1.7. These calculations show that an \$8,500 bid has the highest expected value, which is \$8.17 million. Hence, if ABC uses expected value as its decision criterion, then it should bid \$8,500. In addition, the calculations also show that ABC should use the proposed new manufacturing process if it wins the contract. The less preferred branches for each decision node have been indicated on the decision tree with cross hatching. ■

The complete specification of the alternatives that should be selected at all decision nodes in a decision tree is called a **decision strategy**.

**Definition 1.5: Decision Strategy**

> The complete specification of all the preferred decisions in a sequential decision problem is called the **decision strategy**. The decision strategy shown in Figure 1.7 can be summarized as follows: Bid \$8,500, and if you win the contract use the proposed new manufacturing process.

## 1.5 Exercises

**1.1** Arthrodax Company has been approached by Ranger Sound with a rush order offer to purchase 100 units of a customized version of Arthrodax's SoundScreamer audio mixer at \$5,000 per unit, and Arthrodax needs to decide how to respond. The electronic modifications of the standard SoundScreamer needed for this customized version are straightforward, but there will be a fixed cost of \$100,000 to design the modifications and set up for assembly of the customized SoundScreamers, regardless of the number of units produced. It will cost \$2,000 per unit to manufacture the circuit boards for the units. Since Arthrodax has some short term spare manufacturing capacity, the Ranger offer is potentially attractive. However, the circuit boards for the customized units will not fit into the standard SoundScreamer case, and Arthrodax must decide what to do about acquiring cases for the customized units as it decides whether to accept Ranger's purchase offer. An appropriate case can be purchased at $500 per case, but Arthrodax could instead purchase an injection molder to make the cases. It will cost $20,000 to purchase the molder, and there is a 0.6 probability that it will be possible to successfully make the cases using the molder. If the molder does not work, then the purchase price for the molder will be totally lost and Arthrodax must still purchase the cases at $500 per case. If the molder works, then it will cost $60 per case to make the cases using the molder. Regardless of which case is used, the cost of assembling the SoundScreamer circuit boards into the case is $20 per unit. Unfortunately, there is no way to test the molder without purchasing it. Assume that there is no other use for the molder except to make the cases for the Ranger order.

(i) Draw a decision tree for Arthrodax's decision about whether to accept the Ranger offer and how to acquire the cases for the customized SoundScreamers.

(ii) Using expected net profit as the decision criterion, determine the preferred course of action for Arthrodax.

**1.2** This is a continuation of Exercise 1.1. Assume that all information given in that exercise is still valid, except as discussed in this exercise. Ranger now tells Arthrodax that there is uncertainty about the number of customized SoundScreamers that will be needed. Specifically, there is a 0.35 probability that it will need 100 units, and a 0.65 probability that it will need 50 units. If Arthrodax will agree now to produce either number of units, then Ranger will pay $6,000 per unit if it ultimately orders 50 units, and will pay $5,000 per unit if it ultimately orders 100 units. The timing is such on this rush order that Arthrodax will have to make a decision about purchasing the injection molder before it knows how many units Ranger will take. However, Arthrodax will only need to purchase or manufacture the number of circuit boards and cases needed for the final order of either 50 or 100 units.

(i) Draw a decision tree for Arthrodax's decision about whether to accept the Ranger offer and how to acquire the cases for the customized SoundScreamers. Note that this is a situation with dependent uncertainties, as discussed in Section 1.3.

(ii) Using expected net profit as the decision criterion, determine the preferred course of action for Arthrodax.

**1.3** This is a continuation of Exercise 1.2. Assume that all information given in that exercise is still valid, except as discussed in this exercise. Assume now that Arthrodax could delay the decision about purchasing the injection molder until after it knows how many units Ranger will take.

(i) Draw a decision tree for Arthrodax's decision about whether to accept the Ranger offer and how to acquire the cases for the customized SoundScreamers. Note that this is a situation with sequential decisions, as discussed in Section 1.4.

(ii) Using expected net profit as the decision criterion, determine the preferred course of action for Arthrodax.

**1.4** Aba Manufacturing has contracted to provide Zyz Electronics with printed circuit ("PC") boards under the following terms: (1) 100,000 PC boards will be delivered to Zyz in one month, and (2) Zyz has an option to take delivery of an additional 100,000 boards in three months by giving Aba 30 days notice. Zyz will pay $5.00 for each board that it purchases. Aba manufactures the PC boards using a batch process, and manufacturing costs are as follows: (1) there is a fixed setup cost of $250,000 for any manufacturing batch run, regardless of the size of the run, and (2) there is a marginal manufacturing cost of $2.00 per board regardless of the size of the batch run. Aba must decide whether to manufacture all 200,000 PC boards now or whether to only manufacture 100,000 now and manufacture the other 100,000 boards only if Zyz exercises its option to buy those boards. If Aba manufactures 200,000 now and Zyz does not exercise its option, then the manufacturing cost of the extra 100,000 boards will be totally lost. Aba believes there is a 50% chance Zyz will exercise its option to buy the additional 100,000 PC boards.

(i) Explain why it might potentially be more profitable to manufacture all 200,000 boards now.

(ii) Draw a decision tree for the decision that Aba faces.

(iii) Determine the preferred course of action for Aba assuming it uses expected profit as its decision criterion.

**1.5** Kezo Systems has agreed to supply 500,000 PC FAX systems to Tarja Stores in 90 days at a fixed price. A key component in the FAX systems is a programmable array logic integrated circuit chip ("PAL chip"), one of which is required in each FAX system. Kezo has bought these chips in the past from an American chip manufacturer AM Chips. However, Kezo has been approached by a Korean manufacturer, KEC Electronics, which is offering a lower price on the chips. This offer is open for only 10 days, and Kezo must decide whether to buy some or all of the PAL chips from KEC. Any chips that Kezo does not buy from KEC will be bought from AM. AM Chips will sell PAL chips to Kezo for $3.00 per chip in any quantity. KEC will accept orders only in multiples of 250,000 PAL chips, and is offering to sell the chips for $2.00 per chip for 250,000 chips, and for $1.50 per chip in quantities of 500,000 or more chips. However, the situation is complicated by a dumping charge that has been filed by AM Chips against KEC. If this charge is upheld by the U. S. government, then the KEC chips will be subject to an antidumping tax. This case will not be resolved until after the point in time when Kezo must make the purchase decision. If Kezo buys the KEC chips, these will not be shipped until after the antidumping tax would go into effect and the chips would be subject to the tax. Under the terms offered by KEC, Kezo would have to pay any antidumping tax that is imposed. Kezo believes there is a 60%

chance the antidumping tax will be imposed. If it is imposed, then it is equally likely that the tax will be 50%, 100%, or 200% of the sale price for each PAL chip.

(i) Draw a decision tree for this decision.

(ii) Using expected value as the decision criterion, determine Kezo's preferred ordering alternative for the PAL chips.

**1.6** **Intermodular Semiconductor Systems case.** The Special Products Division of Intermodular Semiconductor Systems has received a Request for Quotation from Allied Intercontinental Corporation for 100 deep sea semiconductor electrotransponders, a specialized instrument used in testing undersea engineered structures. While Intermodular Semiconductor Systems has never produced deep sea electrotransponders, they have manufactured subsurface towed transponders, and it is clear that they could make an electrotransponder that meets Allied's specifications. However, the production cost is uncertain due to their lack of experience with this particular type of transponder. Furthermore, Allied has also requested a quotation from the Undersea Systems Division of General Electrodevices. Intermodular Semiconductor Systems and General Electrodevices are the only companies capable of producing the electrotransponders within the time frame required to meet the construction schedule for Allied's new undersea habitat project.

Mack Reynolds, the Manager of the Special Products Division, must decide whether to bid or not, and if Intermodular Semiconductor Systems does submit a bid, what the quoted price should be. He has assembled a project team consisting of Elizabeth Iron from manufacturing and John Traveler from marketing to assist with the analysis. Daniel A. Analyst, a consulting decision analyst, has also been called in to assist with the analysis.

Analyst: For this preliminary analysis, we have agreed to consider only a small number of different possible bids, production costs, and General Electrodevices bids.

Reynolds: That's correct. We will look at possible per-unit bids of \$3,000, \$5,000, and \$7,000. We will look at possible production costs of \$2,000, \$4,000, and \$6,000 per unit, and possible per-unit bids by General Electrodevices of \$4,000, \$6,000, and \$8,000.

Iron: There is quite a bit of uncertainty about the cost of producing the electrotransponders. I'd say there is a 50% chance we can produce them in a volume of 100 units at \$4,000 per unit. However, that still leaves a 50% chance that they will either be \$2,000 or \$6,000 per unit.

Analyst: Is one of these more likely than the other?

Iron: No. It's equally likely to be either \$2,000 or \$6,000. We don't have much experience with deep sea transponders. Our experience with subsurface towed transponders is relevant, but it may take some effort to make units that hold up to the pressure down deep. I'm sure we can do it, but it may be expensive.

Analyst: Could you do some type of cost-plus contract?

Reynolds: No way! This isn't the defense business. Once we commit, we have to produce at a fixed price. Allied would take us to court otherwise. They're tough cookies, but they pay their bills on time.

Iron: I want to emphasize that there is no problem making the electrotransponders and meeting Allied's schedule. The real issue is what type of material we have to use to take the pressure. We may be able to use molyaluminum like we do in the subsurface towed units in which case the cost will be lower. If we have to go to molyzirconium, then it will be more expensive. Most likely, we will end up using some of each, which will put the price in the middle.

Analyst: What is General Electrodevices likely to bid?

Traveler: They have more experience than we do with this sort of product. They have never made deep sea electrotransponders, but they have done a variety of other deep sea products. I spent some time with Elizabeth discussing their experience, and also reviewed what they did on a couple of recent bids. I'd say there is a 50% chance they will bid \$6,000 per unit. If not, they are more likely to bid low than high—there is about a 35% percent chance they will bid \$4,000 per unit.

Analyst: So that means there is 15% chance they will bid \$8,000.

Traveler: Yes.

Reynolds: Suppose we had a better handle on our production costs. Would that give us more of an idea what General Electrodevices would bid?

Iron: No. They use graphite-based materials to reinforce their transponders. The cost structure for that type of production doesn't have any relationship to our system using moly alloys.

(i) Draw a decision tree for the decision that Reynolds must make.

(ii) Determine the expected values for each of the alternatives, and specify which alternative Reynolds should select if he uses expected value as a decision criterion.

CHAPTER 2

# Risk Aversion

Expected value as a criterion for making decisions makes sense provided that the stakes at risk in the decision are small enough to “play the long run averages.” The range of decisions for which this is true covers many situations of practical business interest, but sometimes the stakes are high enough that this is not an appropriate assumption.

## 2.1 Risk Attitude

The value of a risky alternative to the decision maker may be different than the expected value of the alternative because of the risk that the alternative poses of serious losses. The concept of the **certainty equivalent** is useful for such situations, as shown in Definition 2.1.

**Definition 2.1: Certainty Equivalent**

The **certainty equivalent** for an alternative is the certain amount that is equally preferred to the alternative. An equivalent term for certainty equivalent is **selling price**.

**Example 2.1**

**Certainty equivalent.** Suppose that through a previous business deal you have come into possession of an uncertain alternative that has equal chances of yielding a profit of $10,000 or a loss of $5,000. The expected value for this alternative is $0.5 \times \$10,000 + 0.5 \times (-\$5,000) = \$2,500$. However, suppose that you decide that you would be willing to sell this alternative for $500 or more. Then, your certainty equivalent for the alternative is $500. ■

Using the concept of the certainty equivalent, it is possible to specify different attitudes toward risk taking, as shown in Definition 2.2.

**Definition 2.2: Risk Attitude**

> If your certainty equivalent for alternatives specified in terms of profits is less than the expected profit for an alternative, you are said to be **risk averse** with respect to this alternative. If your certainty equivalent is equal to the expected profit for the alternative, then you are said to be **risk neutral**. Finally, if your certainty equivalent is greater than the expected profit for the alternative, you are said to be **risk seeking**. These definitions are reversed for an uncertain alternative specified in terms of losses. That is, you are risk averse if your certainty equivalent is greater than the expected loss and risk seeking if your certainty equivalent is less than the expected loss.

Based on our earlier discussion in Chapter 1, if you are risk seeking with respect to the various decisions that you make, then over the long run you will probably go broke because on average you will not recover as much from the alternatives as you are willing to pay for them. This is not typical behavior in business, and therefore we will not consider risk seeking behavior further. (Note that there are situations where a risk seeking attitude may make sense in business. For example, suppose your business is in such serious trouble that it is going to go broke anyway unless you get lucky. You might as well "pray for rain" in such a situation and go against the odds. However, this is not a typical business situation.)

It is worth noting that the appropriate attitude toward risk taking can depend on the asset position of the organization taking the risk. A large Fortune 500 company may be able to play the odds and use expected value as its decision criterion in situations that would pose serious risks to a small "mom and pop" business.

## 2.2 Utility Functions

If certainty equivalents can be determined for the alternatives in a decision problem, then it is straightforward to determine the preferred alternative—simply select the alternative with the best certainty equivalent. This section discusses a procedure to determine certainty equivalents for the decision alternatives. The theory for how to determine certainty equivalents in a defensible manner has been developed, and we will present a practical procedure for using this theory that is appropriate for many realistic business decisions. Readers who are interested in the theory behind this approach should consult a decision analysis textbook.

Certainty equivalents can be determined using a modification of the procedure that we use to determine expected values. This modification involves introducing a new function, called the **utility function**. A typical utility function is shown in Figure 2.1. In this figure, the evaluation measure scale is shown on the horizontal axis, and the utility for each evaluation measure level is plotted on the vertical axis. The range of the evaluation measure in this example is from $-500$ to 2,000,

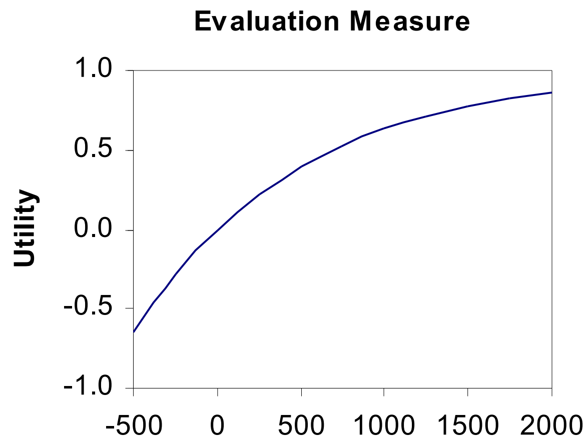


**Figure 2.1** *Illustrative utility function*

and this evaluation measure might, for example, represent the net profit from a business decision in thousands of dollars. Note that the exact numbers on the vertical scale do not have specific meanings, except that greater numbers represent more preferred levels of the evaluation measure. For example, if the evaluation measure is dollars of profit, then there is greater utility for an amount of $2,000 than an amount of $1,000.

The idea underlying the approach to calculating certainty equivalents is to first convert the possible outcomes in a decision problem to utilities using the utility function, and then calculate the expected value of these utilities for each alternative *using the same procedure that was used to calculate expected values.* After determined these *expected utilities* for each alternative, then it is straightforward to determine the certainty equivalent for each alternative using a procedure discussed later in this section.

**Definition 2.3: Utility Function**

> A **utility function** translates outcomes into numbers such that the expected value of the **utility** numbers can be used to calculate certainty equivalents for alternatives in a manner that is consistent with a decision maker's attitude toward risk taking.

Here is an intuitive explanation of why this calculation procedure using expected utilities makes sense as a way to take risk attitude into account. Examine the utility function in Figure 2.1. Note that this function drops off rapidly as the level of the evaluation measure becomes worse (more negative), while it grows less rapidly as the value of the evaluation measure becomes better (more positive). Intuitively, this is saying that the value that we lose from each unit of decrease of the evaluation measure becomes increasingly great as the level become more negative. Therefore, if we take an expected value of the utilities over the evaluation measure, alternatives that have a significant probability of yielding bad outcomes will be penalized more heavily in the calculation procedure than if expected value were used to evaluate the alternatives. Hence, an alternative with a significant chance of yielding bad outcomes will be downrated using a utility function from what would be true if expected value was used to evaluate alternatives.

## 2.3 The Exponential Utility Function

To implement the expected utility approach reviewed above, it is necessary to first determine a utility function. Both theory and practical experience have shown that it is often appropriate to use a particular form of utility function called the **exponential**. For risk averse decision makers, in decisions involving profits (more of the evaluation measure is better), this function has the form

$$u(x) = 1 - e^{-x/R}, \qquad R > 0$$

where $u(x)$ represents the utility function, $x$ is the evaluation measure, $R$ is a constant called the *risk tolerance*, and $e$ represents the exponential function. (The exponential function is often designated by "exp" on a financial calculator or in a spreadsheet program.)

In situations involving costs where less of the evaluation measure is preferred, the exponential utility function has the form

$$u(x) = 1 - e^{x/R}, \qquad R > 0$$

and in this case larger values of $x$ have lower utilities.

As noted above, the degree of risk aversion that is appropriate can depend on the asset position of the decision making entity, and $R$ represents the degree of risk aversion. As $R$ becomes larger, the utility function displays less risk aversion. (In fact, when $R$ approaches infinity, the decision maker becomes risk neutral.) The utility function plotted in Figure 2.1 is an exponential utility function with $R = 1,000$.

## 2.4 Assessing the Risk Tolerance

The following procedure can be used to determine the approximate value of $R$ for a particular decision maker: Ask the decision maker to consider a hypothetical alternative that has equal chances of yielding a profit of $r_o$ or a loss of $r_o/2$. Then ask the decision maker to specify the value of $r_o$ for which he or she would be indifferent between receiving or not receiving the alternative. (Or, put another way, ask the decision maker to adjust $r_o$ until the certainty equivalent for this hypothetical alternative is just equal to zero.) When the decision maker has adjusted $r_o$ in this way, then $R$ is approximately equal to $r_o$. Note that the expected value for this hypothetical alternative is $\text{EV} = 0.5 \times r_o - 0.5 \times (r_o/2) = 0.25 \times r_o$, and therefore as long as $r_o$ is greater than zero the decision maker is specifying a risk averse utility function.

We will now apply the expected utility approach to the Xanadu Traders decision.

**Example 2.2**

**Xanadu Traders.** This is a continuation of the Xanadu Traders decision in Example 1.8. We continue to follow the conversation between Daniel Analyst and George Xanadu.

Analyst: I understand from my previous work with you that financial risks of the size involved in this deal would be uncomfortable but would not sink Xanadu Traders. If you could, you would buy some insurance against the potential loss, but you are not going to avoid the deal just because of the possible loss.

Xanadu: That's correct.

Analyst: I recall that you told me in the past that you would be just willing to accept a deal with a fifty-fifty chance of making \$2,000,000 or losing \$1,000,000. However, if the upside were \$2,100,000 and the downside were \$1,050,000, you would not take the deal.

Xanadu: That's correct.

> **Question 2.1:** Taking into account Xanadu's attitude toward risk taking, what is the preferred alternative among those shown in Figure 1.6?

To answer this question, it is first necessary to determine Xanadu's utility function. This can be done using the information in the dialog. Using the concept of the risk tolerance, $r_o = \$2$ million when an uncertain alternative with equal chances of yielding a profit of $r_o$ or a loss of $r_o/2$ has a certainty equivalent of 0. Hence, $R$ is approximately equal to \$2 million. Therefore, Xanadu's utility function is

$$u(x) = 1 - e^{-x/2},$$

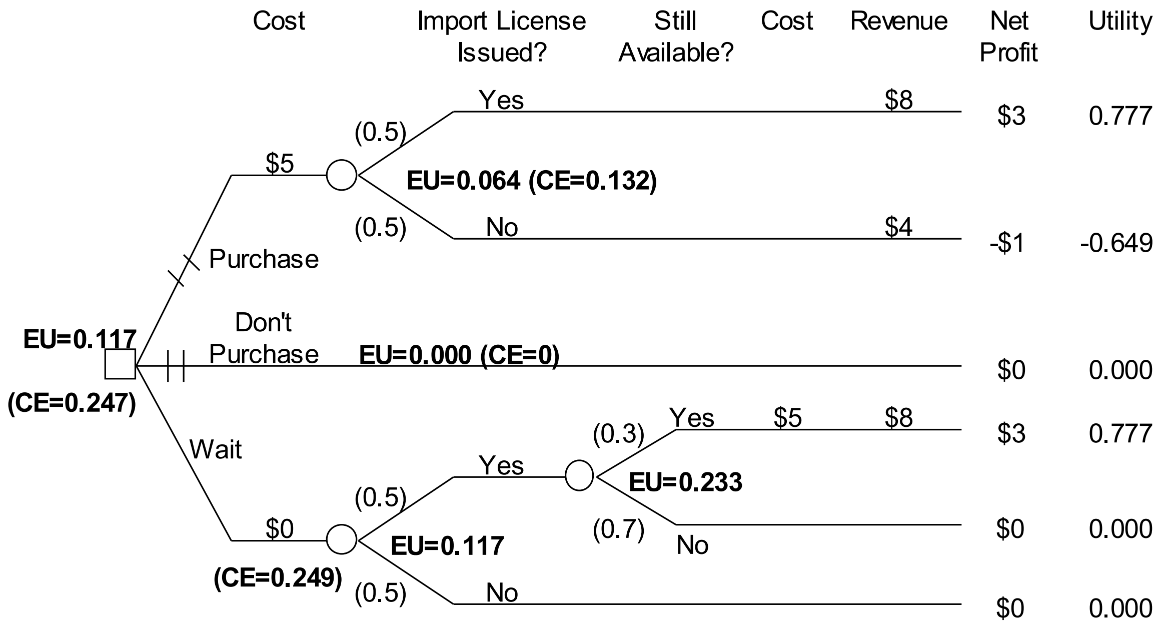


**Figure 2.2** *Xanadu Traders Expected Utility Analysis*

where $x$ is in millions of dollars. Using a spreadsheet or calculator, it is easy to find the utilities for each of the endpoint values in the Figure 1.6, and these are shown in Figure 2.2. In this figure, the utility numbers shown at the right side of the tree have been calculated using an exponential utility function with $R = \$2$ million. For example, the topmost utility number is given by $u(x) = 1 - e^{-3/2} = 0.777$.

Expected utility numbers are calculated in the same manner as expected values. For example, the expected utility for the topmost chance node is given by EU $= 0.5 \times (0.777) + 0.5 \times (-0.649) = 0.064$. This is the expected utility for the "purchase" alternative, and in a similar manner the expected utilities can be found for the "don't purchase" alternative (EU $= -1.000$) and the "wait" alternative (EU $= 0.117$). ∎

## 2.5 Certainty Equivalent for an Exponential Utility Function

Expected utility numbers do not have a simple intuitive interpretation, but there is a specific certainty equivalent corresponding to any specified expected utility. For an exponential utility function involving profits, it can be shown that the certainty equivalent is equal to

$$\text{CE} = -R \times \ln(1 - \text{EU}),$$

where CE is the certainty equivalent, EU is the expected utility, $R$ is the risk tolerance, and ln is the natural logarithm. Thus, the certainty equivalent for the "purchase" alternative in Figure 2.2 is given by CE $= -2 \times \ln[1 - 0.064] =$ \$0.132 million. The certainty equivalents are shown for all three alternatives in Figure 2.2, and larger certainty equivalents are more preferred.

In situations involving costs, where less of an evaluation measure is preferred to more, then the certainty equivalent is equal to

$$\text{CE} = R \times \ln(1 - \text{EU})$$

and alternatives with smaller certainty equivalents are more preferred in this case.

Since a certainty equivalent is the certain amount that is equally preferred to an alternative, the alternative with the greatest certainty equivalent is most preferred for situations where more of an evaluation measure is preferred to less. Therefore, taking Xanadu's risk attitude into account, the "purchase" alternative is no longer the preferred alternative, as it was with the expected value analysis. The "wait" alternative is now most preferred since it has a certainty equivalent of \$0.249 million, and the "purchase" alternative is now the second most preferred alternative with a certainty equivalent of \$0.132 million. The "don't purchase" alternative continues to be least preferred with a certainty equivalent of \$0.

Note that expected utilities can be directly used to rank alternatives in a decision problem. It can be shown that the alternative with the greatest expected utility will also have the most preferable certainty equivalent. (Note that this is true regardless of whether you are dealing with costs or profits, provided that you use the appropriate utility function formula given above.) Thus the three alternatives in Figure 2.2 could have been ranked directly using the expected utilities without calculating certainty equivalents. However, it is often preferable to calculate certainty equivalents since these are easier to intuitively interpret.

**Example 2.3**

**Xanadu Traders.** This example completes our study of Xanadu Traders. A comparison of the expected values and certainty equivalents for the three alternatives in Figure 2.2 is shown in Table 2.1. This demonstrates that the three alternatives have differing risks. There is no difference between the expected value and the certainty equivalent for the "don't purchase" alternative since there is no uncertainty with this alternative. The difference between the expected value and certainty equivalent is greatest for the "purchase" alternative indicating that it has the largest risk. This risk reduces the value of this alternative enough for Xanadu that it is no longer the most preferred alternative. The "wait" alternative also has a lower certainty equivalent than its expected value since this alternative has some risk. However, this risk is substantially lower than the risk for the "purchase" alternative, and hence this becomes the preferred alternative when Xanadu's risk attitude is taken into account. ■

| Alternative | Expected Value | Certainty Equivalent | Difference |
|---|---|---|---|
| Purchase | 1.000 | 0.132 | 0.868 |
| Don't Purchase | 0.000 | 0.000 | 0.000 |
| Wait | 0.450 | 0.249 | 0.201 |

**Table 2.1** *Comparison of expected values and certainty equivalents*

## 2.6 Exercises

**2.1** This is a continuation of Exercise 1.4. Assume that all the information in that exercise still holds, except assume now that Aba has an exponential utility function with a risk tolerance of \$100,000. Determine Aba's preferred course of action.

**2.2** This is a continuation of Exercise 1.5. Assume that all the information in that exercise still holds, except assume now that Kezo has an exponential utility function with a risk tolerance of \$750,000. Determine Kezo's preferred ordering alternative using this utility function.

**2.3** This is a continuation of the preceding exercise. (That is, assume that Kezo has an exponential utility function with a risk tolerance of \$750,000.) In an effort to attract Kezo's order, KEC Electronics has revised its offer as follows: At no increase in price, KEC will now provide Kezo with the right to cancel its entire order for a 10% fee after the outcome of the antidumping suit is known. However, KEC will not be able to accept any additional orders from Kezo once the outcome of the suit is known. Thus, for example, if Kezo has agreed to purchase 250,000 PAL chips from KEC at \$2.00 per chip, Kezo can cancel the order by paying \$50,000. This ability to cancel the order is potentially of interest to Kezo because it knows that AM Chips would be able to supply PAL chips after the outcome of the antidumping suit is known in time for Kezo to fill the Tarja order. However, Kezo knows that AM will increase the price of its chips if an antidumping tax is imposed. In particular, if a 50% tax is imposed, then AM will increase its chip price by 15%. If a 100% tax is imposed, then AM will increase its chip price by 20%. Finally, if a 200% tax is imposed, then AM will increase its chip price by 25%. Assuming that all other information given in the preceding exercise is still valid, determine Kezo's preferred alternative for the initial order of PAL chips as well as what Kezo should do if the antidumping tax is imposed.

CHAPTER 3

# The Value of Information

Often there is an option in a decision to collect additional information, and this chapter presents procedures for determining when it is worth collecting additional information.

## 3.1 Calculating the Value of Perfect Information

We begin by determining the value of **perfect** information. Perfect information removes all uncertainty about the outcomes for the decision alternatives. While there is rarely an option in real-world business decisions that would actually remove all uncertainty, the value of perfect information provides an easily calculated benchmark about the worth of collecting additional information. If all the available options for collecting information cost more than the value of perfect information, then these options do not need to be analyzed in further detail. This is because **imperfect** information cannot be worth more than **perfect** information.

**Box 3.1: The Value of Perfect Information**

> No source of information can be worth more than the **value of perfect information**.

The following example illustrates how to compute the value of perfect information.

**Example 3.1**

**Xanadu Traders.** This is a continuation of the Xanadu Traders decision that was discussed in Example 1.8. (Figure 1.6 shows this decision.) Suppose a source of perfect information existed that would let Xanadu know if the import license would be issued.

> **Question 3.1:** How much money would it be worth to obtain perfect information about issuance of the import license?

Figure 3.1 shows a decision tree with this (hypothetical) source of perfect information. The topmost three branches of the root node for this decision tree are the same as the corresponding branches in Figure 1.6. The lowest branch of the root node is the perfect information alternative. At a quick glance, the perfect information may appear to be similar to the "wait" alternative, since for both of these alternatives George Xanadu learns whether the license will be issued before he purchases the molyzirconium. However, with the perfect information alternative, information is available *immediately* about whether the license will be issued. Therefore, with the perfect information alternative, Xanadu does not run the risk that a competitor will purchase the molyzirconium before he learns whether the license will be issued.

Since the probability is 0.5 that the license will be issued, this is the probability that the perfect information source will report that the license will be issued. After learning this perfect information, Xanadu then can decide whether or not to purchase the molyzirconium. Of course, if Xanadu learns that the license will be issued, then he purchases the molyzirconium, and if Xanadu learns that the license will not be issued, then he does not purchase the molyzirconium.

By the standard calculation procedure, it is determined that the perfect information alternative has an expected value of \$1.5 million, and this is shown on the Figure 3.1 decision tree. Since the best alternative without perfect information ("purchase") has an expected value of \$1 million, the value of perfect information is $\$1.5 - \$1.0 = \$0.5$ million. Therefore, this places an upper limit on how much it is worth paying for *any* information about whether the license will be issued. It cannot be worth paying more than \$0.5 million for such information, since \$0.5 million if the value of perfect information. ∎

## 3.2 The Value of Imperfect Information

The calculation procedure is more complicated for determining the value of imperfect information. This procedure is illustrated by the following example.

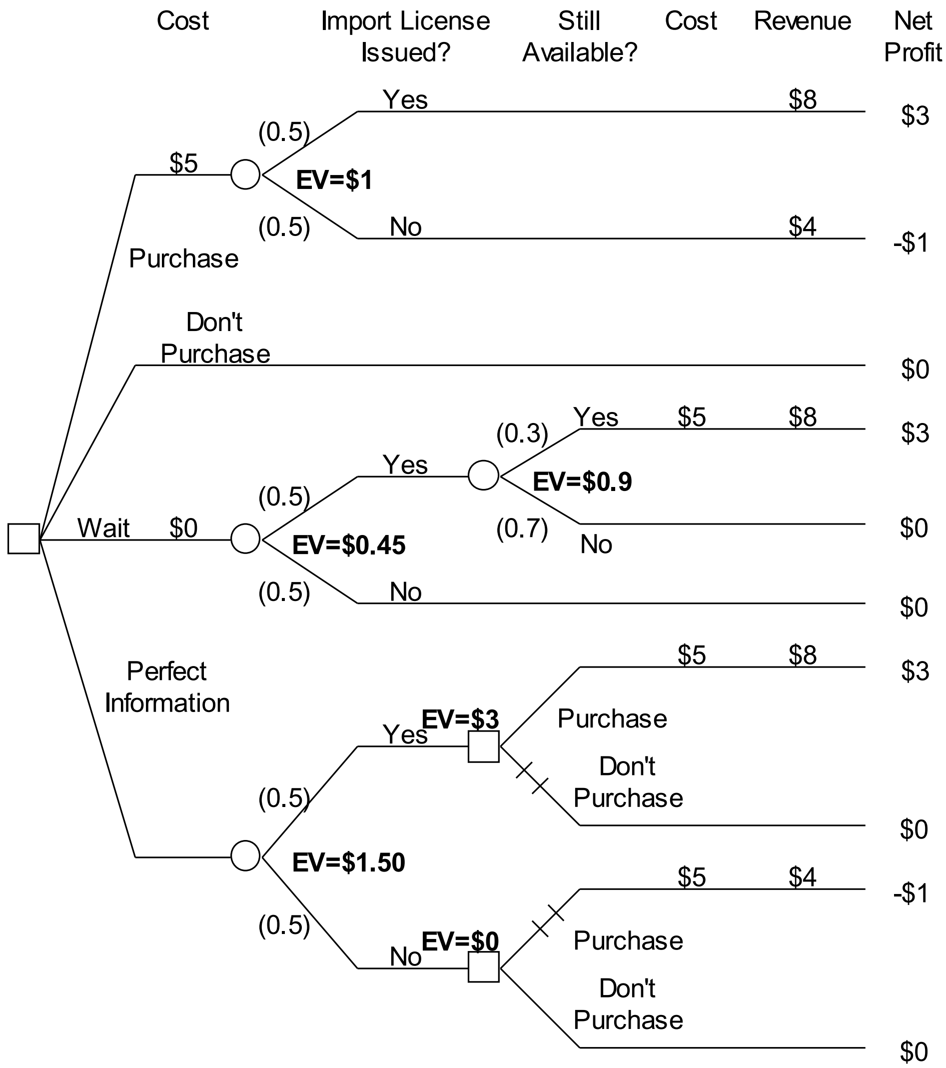


**Figure 3.1** *Xanadu Traders decision tree, with perfect information alternative*

**Example 3.2**

**Xanadu Traders.** Now consider a potential source of imperfect information in the Xanadu Traders case last discussed in Example 3.1. We continue with the discussion between Daniel Analyst and George Xanadu.

Analyst: Is there any way of obtaining additional information about the chances of obtaining a license other than waiting and seeing what happens? Perhaps there is something that doesn't take as long as waiting for the import approval.

Xanadu: Well, there's always John S. Lofton. He is a Washington-based business consultant with good connections in the import licensing bureaucracy. For a fee, he will consult his contacts and see if they think the license will be granted. Of course, his assessment that the license will come through is no guarantee. If somebody in Congress starts screaming, they might shut down imports from Zeldavia. They are really upset about this in the Industrial Belt, and Congress is starting to take some heat. On the other hand, even if Lofton thinks the license won't come through, he might be wrong. He has a pretty good record on calling these things, but not perfect. And he charges a lot for making a few telephone calls.

Analyst: How good has he been?

Xanadu: He's done some assessments for me, as well as other people I know. I'd say in cases where the import license was ultimately granted, he called it right 90% of the time. However, he hasn't been so good on the license requests that were turned down. In those cases, he only called it right 60% of the time.

Analyst: You commented earlier that he was expensive. How much would he charge?

Xanadu: This is a pretty standard job for him. His fee for this type of service is $10,000.

> **Question 3.2:** Should Xanadu hire Lofton, and if so, what is the maximum amount that he should pay Lofton for his services?

We know from our earlier analysis of the value of perfect information in Example 3.1 that the maximum amount that it could possibly be worth to purchases Lofton's services is $0.5 million. Since he would only charge $10,000 it is possible that it would be worth purchasing his services. However, it is clear from the discussion above that Lofton often makes mistakes, and perhaps Xanadu would not learn enough to warrant paying Lofton the $10,000.

A partial decision tree for the "Hire Lofton" alternative is shown in Figure 3.2. To simplify this tree, the possibility of hiring Lofton and then still waiting to see if the import license is issued has been eliminated from the tree. In this tree, each of the two subtrees starting from the decision nodes after the outcome of "predict import license issued?" has the same structure. Each of these subtrees also has the same structure as the top two branches of the decision tree in Figure 1.6. However, as we will see below, the probabilities on the "import license issued?" branches differ in Figure 3.2 from those in the Figure 1.6 tree. ■

## 3.3 Flipping a Probability Tree

In order to complete the analysis of the alternative in Example 3.2, we need the probabilities for the two branches labeled "predict import license issued?" in Figure 3.2. Additionally, we need the probabilities for the two sets of branches under the label "import license issued?" Unfortunately, as often happens in real problems, the information presented about Lofton's accuracy in his predictions is

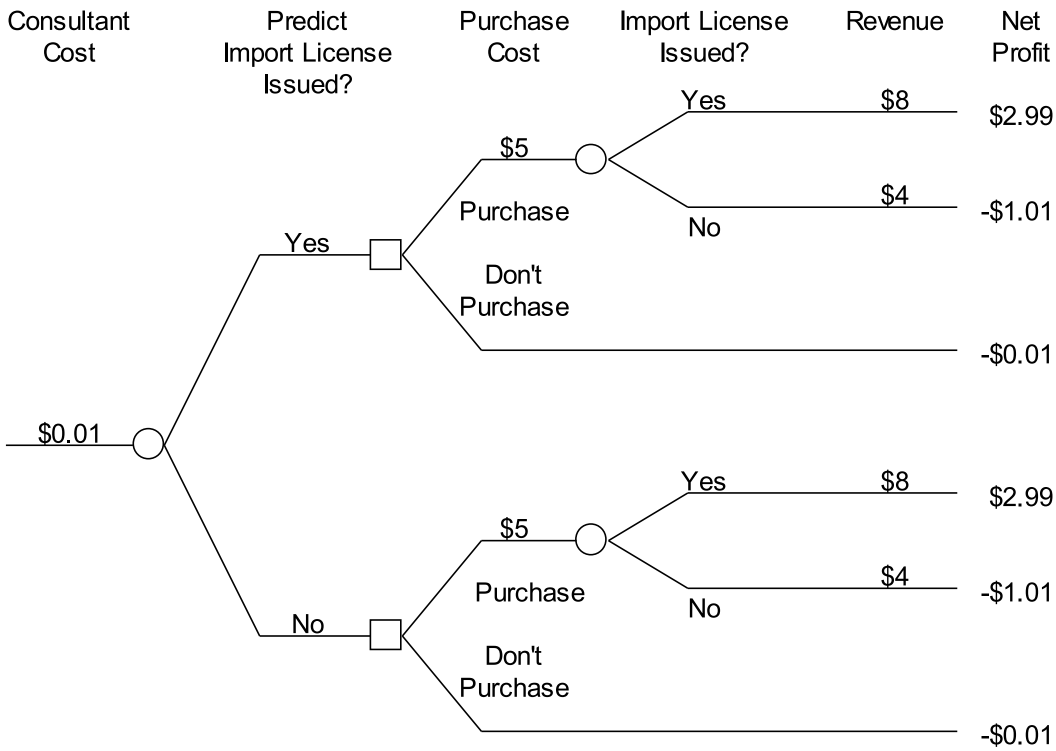


**Figure 3.2** *Hire consultant alternative*

not in a form that directly provides the required probabilities. Figure 3.3 shows in probability tree form the information that is given above about the accuracy of Lofton. The root node on the left side of the tree shows the probabilities for “import license issued?” specified in earlier discussions of this decision problem. The two chance nodes on the right side of the tree show the probabilities that Lofton will call the licensing decision right, based on the conversation presented in Example 3.2 between Daniel Analyst and George Xanadu.

Comparing Figure 3.2 with Figure 3.3, shows that the probabilities in Figure 3.3 are “backwards” from what is needed to assign probabilities to the branches of the chance nodes in Figure 3.2. That is, the probability of license approval is known, as well as the probability of Lofton’s different predictions, *given the actual situation* regarding license approval. However, the decision tree in Figure 3.2 requires the probability of Lofton’s different predictions and the probability of license approval *given Lofton’s predictions*. This is shown in Figure 3.4, where the probabilities marked A, B, C, D, E, and F are required. If these probabilities were known, then they could be inserted into the Figure 3.2 decision tree, and the expected value could be determined for the alternative of hiring Lofton.

This may seem like an odd way to present the information about Lofton’s accuracy, but information about the accuracy of an information source is often available in the form of Figure 3.3 when there is a historical record about the accuracy of the source. As an example, suppose that a new test instrument has

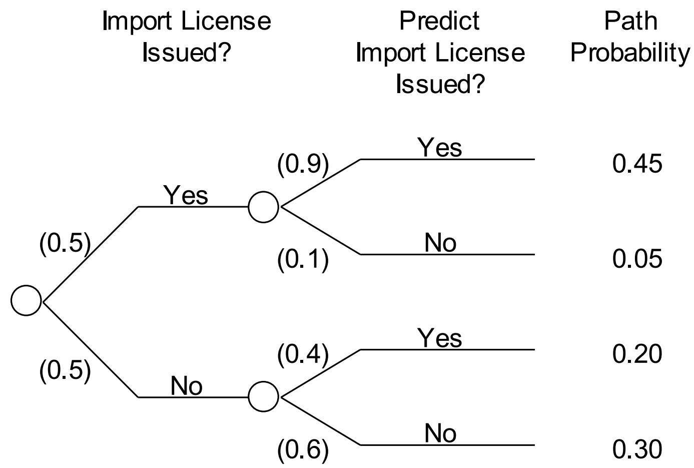


**Figure 3.3** *Accuracy of consultant*

been developed to use in testing for defects in the parts that are manufactured on a production line. How would the accuracy of the test instrument be determined? Probably by using the instrument on a series of parts that have previously been tested by other methods. Thus, it would be known whether the parts that are being tested are good or bad, and hence it would be possible to determine what fraction of good parts the test instrument correctly identifies as good, and what fraction of bad parts the test instrument correctly identifies as bad. This is analogous to the way that the information is presented for Lofton in Figure 3.3.

In a similar manner, the accuracy of a proposed medical diagnostic procedure for some medical condition is often determined by applying the diagnostic procedure to patients who are known to either have the condition or not have the condition. Information from such tests would be in the form of Figure 3.3. Thus, the form of the information shown in Figure 3.3 is common, and we need to know how to use such information when analyzing the value of a potential information source.

To proceed with the analysis of the alternative of hiring Lofton, we need to "flip" the probabilities from the tree in Figure 3.3 to determine the probabilities needed in Figure 3.4.

**Definition 3.1: Tree flipping**

**Tree flipping** is the process of calculating the probabilities for a probability tree with the order of the chance nodes reversed, as illustrated by Figures 3.3 and 3.4.

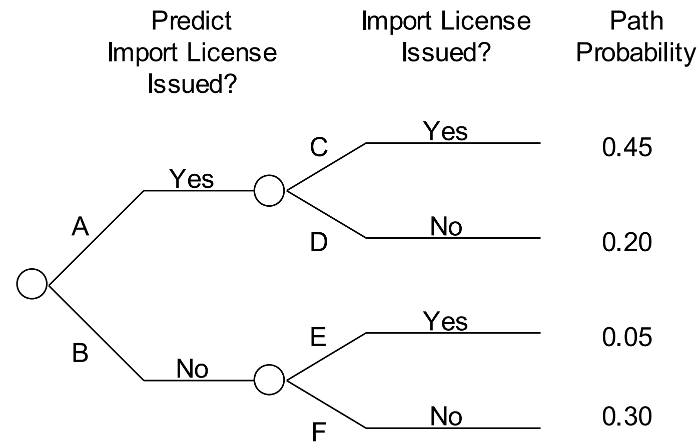


**Figure 3.4** *Probabilities needed for the decision tree*

## 3.4 Calculating "Flipped" Probabilities

It is straightforward to determine the probabilities in Figure 3.4. The key to doing this is to recognize that the *paths* from the root node to the endpoints are the same in the Figure 3.3 and Figure 3.4 trees, but they are arranged in a different order. The probabilities for these paths can be determined in Figure 3.3 by following the multiplication rule for probabilities. Namely, the probabilities on the branches along a path are multiplied to determine the probability of following that path. For example, the probability of following the topmost path in Figure 3.3 is determined as $0.5 \times 0.9 = 0.45$.

**Definition 3.2: Path probability**

A **path probability** is the probability of a particular sequence of branches from the root node to a specified endpoint in a probability tree. A path probability is determined by multiplying the probabilities on the branches included in the path.

Once the probabilities are determined for each path in Figure 3.3, they can be transferred to Figure 3.4, as shown at the right side of Figure 3.4. (The topmost and bottommost probabilities are transferred directly from the Figure 3.3 tree to the Figure 3.4 tree, and the other two path probabilities need to be reversed when they are transferred.)

Once the path probabilities are known, probabilities A and B can be determined. Probability A is the probability of a "yes" prediction regarding license approval, and this occurs only on the two topmost paths in the Figure 3.4 tree. Therefore, probability A is equal to the sum of the probabilities for the two topmost paths. That is, $A = 0.45 + 0.20 = 0.65$. Similarly, probability B is

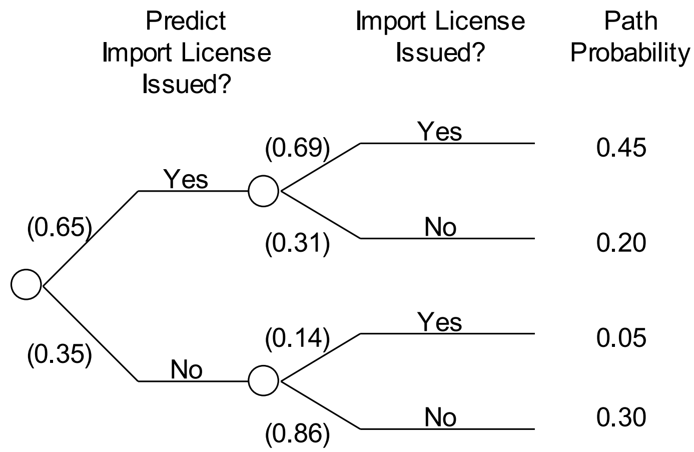


**Figure 3.5** *Decision tree probabilities*

equal to the sum of the probabilities for the two bottommost paths. That is, B = 0.05 + 0.30 = 0.35.

Once A and B are known, then C, D, E, and F can be determined using the multiplication rule. Thus, $A \times$ C = 0.45, or C = 0.45/A = 0.45/0.65 = 0.69 (rounded). Similarly, D = 0.20/A = 0.20/0.65 = 0.31 (rounded), E = 0.05/B = 0.05/0.35 = 0.14 (rounded), and F = 0.30/B = 0.30/0.35 = 0.86 (rounded). Figure 3.5 shows the probabilities filled in for the Figure 3.4 probability tree.

## 3.5 Finding the Expected Value of Imperfect Information

The probabilities in Figure 3.5 can now be transferred to the tree diagram in Figure 3.2, and the expected value can be calculated for the alternative of hiring Lofton by using the same process as in earlier decision trees. The result is shown in Figure 3.6, where the expected value for this alternative is \$1.13 million. Figure 1.6 shows that the best alternative without hiring Lofton only has an expected value of \$1 million, and so it is worth hiring Lofton. In fact, it is worth considerably more than \$10,000 to hire Lofton, since the alternative with hiring him for \$10,000 is worth \$1.13 million. In fact, it is worth it to hire Lofton as long as he costs less than $\$130,000 + \$10,000 = \$140,000$.

## 3.6 Exercises

**3.1** This is a continuation of Exercise 1.4. Assume that all the information presented in that exercise still holds. Determine the expected value of perfect information about whether Zyz will exercise its option.

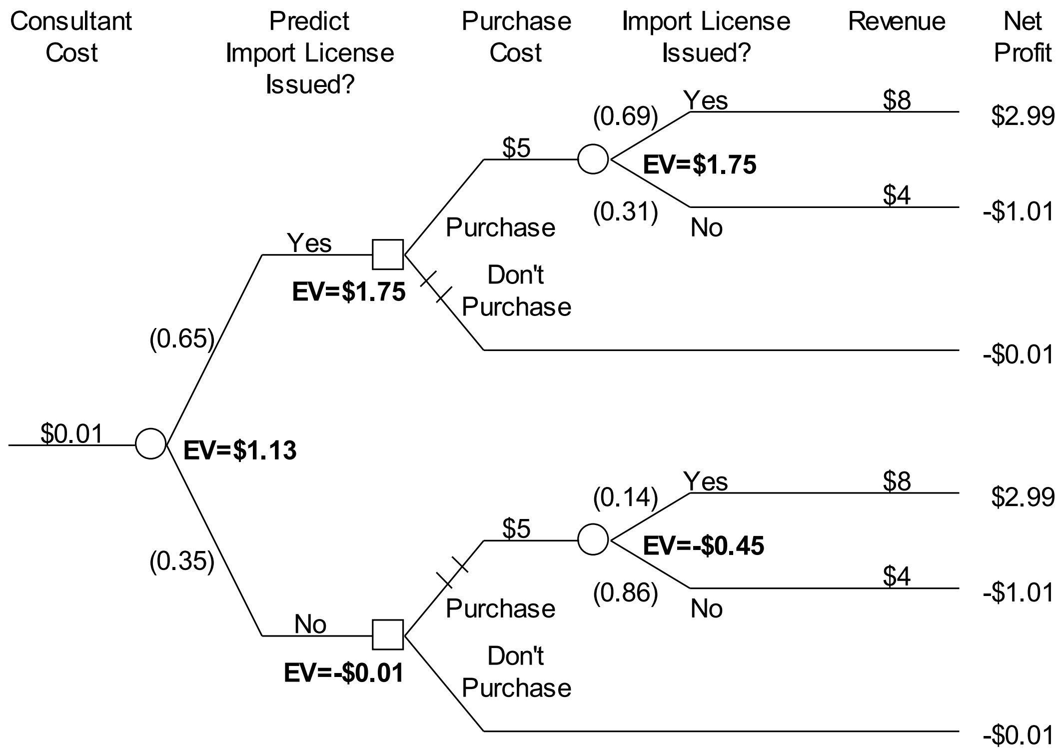


**Figure 3.6** *Hire consultant alternative, with expected value calculation*

**3.2** For the decision in the preceding exercise, Aba Manufacturing has created a new option: It can conduct some research and development in an attempt to lower the fixed setup cost associated with manufacturing a batch of the PC boards. This research and development would not be completed in time to influence the setup cost for the initial batch that Zyz has ordered, but would be completed before the second batch would have to be manufactured. The research and development will cost $25,000, and there is a 0.4 probability that it will be successful. If it is successful, then the fixed setup cost per batch will be reduced by $200,000 to $50,000. If the research and development is not successful, then there will be no reduction in the setup cost. There will be no other benefits from the research and development besides the potential reduction in setup cost for the Zyz reorder.

(i) Using expected profit as the decision criterion, determine whether Aba should undertake the research and development.

(ii) Using expected profit as the decision criteria, determine the value of learning for certain whether the research and development will be successful before a decision has to be made about whether to initially manufacture 100,000 or 200,000 PC boards.

**3.3** This is a continuation of Exercise 1.5. Assume that all the information presented in that exercise still holds. Using expected value as the decision criterion, determine the maximum amount that Kezo should pay for information about whether the antidumping tax will be imposed if this information can be obtained prior to making the ordering decision.

**3.4** A college athletic department is considering a mandatory drug testing policy for all its athletes. Suppose that the test to be used will give either a "positive" or a "negative" indication. From previous testing it is known that if the tested person is a drug user there is a 0.92 probability that the test will be "positive." In cases where the tested person is not a drug user, there is a 0.96 probability that the test will be "negative." Assume that 10% of the athletes to be tested are drug users.

(i) Determine the probability that a randomly selected athlete will test positive for drug use.
(ii) Assuming that a randomly selected athlete tests positive, determine the probability that he or she is actually a drug user.
(iii) Assuming that a randomly selected athlete tests negative, determine the probability that he or she is actually a drug user.
(iv) In light of the results above, discuss the potential advantages and disadvantages of introducing a mandatory drug testing program using this test.

**3.5** **Intermodular Semiconductor Systems, Part 2—The Value of Information.** This is a continuation of the case in Exercise 1.6. Assume that all information presented in that exercise still holds.

Analyst: Would it be possible to get a better handle on production costs before making the bid?

Iron: As I said earlier, the main issue is what it will cost to reinforce the electrotransponders to take the pressure. We could make up some material samples and borrow the high pressure chamber over in the Submersible Systems Division to do some tests. We'd get some information out of that, but there would still be a lot of uncertainty. Also, it would be expensive—I would have to put people on overtime to meet the bid schedule.

The main problem is that we don't have time to do very extensive testing before the bid is due. We could make up a rack of samples from materials we have in stock and take some measurements under pressure, but these materials aren't exactly the same as what we would use in the actual electrotransponders. Because of this, we would still not know for sure what we will have to do to make the electrotransponders work.

[This option was discussed at some length. Following this discussion Analyst summarizes as follows.]

Analyst: As I understand it, the result of doing material tests would be an indication that the production will either be "expensive" or "inexpensive." If molyaluminum is going to work, it is more likely that you will get an "inexpensive" result while if you have to use molyzirconium you are more likely to get an "expensive" result.

Iron: Yes. In previous cases when we have done tests like this and molyaluminum ultimately worked, then 80% of the time we had gotten an "inexpensive" indication. On the other hand, when it has worked out that we needed molyzirconium, then 90% of the time we had gotten an "expensive" indication.

Analyst: What about if a mixture worked?

Iron: We haven't gotten very much useful information in those cases. In cases where a mixture has worked, 60% of the time we had gotten an "inexpensive" indication and 40% of the time it came out "expensive."

Analyst: Based on our earlier discussion, I understand that if molyaluminum works the production costs will be $2,000 per unit, if molyzirconium is needed the costs will be $6,000 per unit, and if a mixture works the costs will be $4,000.

Iron: That's correct for the 100-unit quantity we are discussing here.

Reynolds: How much would the material tests cost?

Iron: There will be a lot of hand labor. I'll go talk with my people and get a figure back to you in a couple of hours.

[Iron leaves the meeting and later reports that it would cost $7,000 to conduct the material tests.]

(i) Determine the expected value of perfect information about what material must be used.

(ii) Determine whether it is worth doing the experiment that is outlined above.