Analytical Decision Making

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Homework 1

Instructions:

1. The homework is a graded **individual** assignment.

2. Due date: **May 1st, 2018 at 12pm.**
    a. Late assignments won't be graded.
    b. No extension.

3. There are 3 exercises.

4. Please submit a Jupyter notebook with your answers and codes. I will use https://colab.research.google.com/ to check the notebooks, so make sure they work on this platform. You can also submit a word document saved in PDF and the codes independently.

5. In your optimization models, please make sure to be as clear as possible about your notations. For the Python codes, add comments throughout the code (e.g., "#This kind of comment") to help me follow what you are doing. No need to comment every line.

6. Please submit a copy of your assignment on Canvas AND email me a copy as a backup: orubel@ucdavis.edu

Exercise 1: Sensitivity Analyses

Content publishers such as The New York Times, The Washington Post and The Wall Street Journal generate revenue by using display advertisements. The Washington Post's website contains several different sections including Sports and National. The number of views each section gets per day can be estimated by analyzing historical data. Assume that the Sports section gets six million views per day and the National section get five million views per day. Assume four companies, GEICO, Delta, T-Mobile and Capital One, wish to advertise on the Sports and National sections of the Washington Post and they contract directly with the newspaper. For each company, the contract specifies the number of times its display ads are shown in these two sections. The contracts sometimes also specify a total number of page views that can originate from any section of the newspaper. The page views promised by The Washington Post to each advertiser are summarized in the left table below. Assume that the contract also specifies that The Washington Post receives $2.40 per click-through from each of the four companies. However, not every page view leads to a click. If every 1000 views leads to 5 clicks, the click-through rate is 0.5%. Newspapers use historical data and tracking technologies to determine click-through rates. Assume that the relevant click-through rates are given in the right table below.

Company	Sports	National	Total
GEICO	2 million	1 million	-
Delta	-	1 million	2 million
T-Mobile	1 million	1 million	3 million
Capital One	-	-	2 million

Contracts

Company	Sports	National
GEICO	2.5%	0.8%
Delta	2.0%	1.0%
T-Mobile	1.0%	3.0%
Capital One	1.5%	2.0%

Click-Through Rates

1. Write out the optimization problem. Make sure to detail the decision variables, the objective function and the constraints.
2. What is the optimal solution? Give the values of the decision variables and the optimal objective function value.
3. Conduct sensitivity analyses of the optimal solution to different click-through rates as given in the table below. Do one cell at time if you want, e.g., sensitivity of the solution to different CTRs for GEICO in the Sports section, then in the National section; then do the same for each advertiser.
4. Write no more than one paragraph (no more than 300 words) to describe the results of your analyses. Tables or graphs can be provided as supplemental material.

	Sports	*National*
Geico	From 2% to 3%	From 0.1% to 1.1%
Delta	From 1.5% to 2.5%	From 0.5% to 1.5%
T-Mobile	From 0.5% to 1.5%	From 2.5% to 3.5%
Capital One	From 1.5% to 2.0%	From 1.5% to 2.5%

Exercise 2: Operations Excellence

You operate two plants, i.e., A and B. Each plant makes two products, "standard" and "deluxe". A unit of standard gives a profit contribution of $10, while a unit of deluxe gives a profit contribution of $15.

Each plant uses two processes, grinding and polishing, for producing its products. Plant A has a grinding capacity of 80 hours per week and polishing capacity of 60 hours per week. For plant B, these capacities are 60 and 75 hours per week, respectively.

The grinding and polishing times in hours for a unit of each type of product in each factory are given in the tables below.

For Plant A

	Standard	Deluxe
Grinding	4	2
Polishing	2	5

For Plant B

	Standard	Deluxe
Grinding	5	3
Polishing	5	6

It is possible, for example, that plant B has older machines than plant A, resulting in higher unit processing times. In addition, each unit of each product uses 4 kg of a raw material, which we refer to as raw. The company has 120 kg of raw available per week. To start with, we will assume that plant A is allocated 75 kg of raw per week and plant B the remaining 45 kg per week. Each plant can build a very simple linear programming model to maximize its profit contribution.

1. Write the optimization models for each plant, i.e., make sure to detail the decision variables, the objective function and the constraints.
2. Provide the optimal solutions and profit levels for each factory. Give the values of the decision variables and the optimal objective function value.
3. Now, write the optimization model for the joint optimization of Plant A and B, and provide the optimal solution and profit levels.
4. Please comment on the results? How do they compare to the results obtained in Question 2.
5. In one paragraph (no more than 300 words), detail the recommendations you would provide to the CEO of the company to improve the firm's operations?

Exercise 3: Hotel La Quinta Motor Inns (LQM)

LQM is a middle-sized hotel chain that is considering expanding to more locations. LQM used data on 75 existing inn locations to build a linear regression model to predict "Profitability", computed at the operating margin, or earnings before interest and taxes divided by total revenue. They tried many independent variables and came up with the final model

$$\begin{aligned} Profitability = 39.05 &- (5.41 \times State\ Population\ per\ Inn) + (5.86 \times Price\ of\ the\ Inn) \\ &- (3.09 \times Square\ Root\ of\ the\ Median\ Income\ in\ the\ area) \\ &+ (1.75 \times College\ Students\ in\ the\ Area) \end{aligned}$$

All independent variables are significant and were normalized to have mean zero and standard deviation 1.

1. According to the regression equation given above, which variable positively affect Profitability? Which variable negatively affect Profitability? Does this intuitively make sense? Why?
2. Using this regression equation, LQM created a spreadsheet model to predict profitability. LQM collected data for several locations in California, which is provided in the excel spreadsheet on Canvas "LQM". Using this spreadsheet, compute the profitability for each hotel. Which one has the highest profitability? Which one has the lowest profitability?
3. LQM has a budget of $10 million to spend on hotels. Suppose we used a "greedy" approach where we selected the most profitable hotels until we ran out of budget. So we would start by the most profitable, and then if we had enough budget left, we would buy the hotel we predict to be the second most profitable, and so on.
    a. Describe what we would do with this approach, i.e., which hotels would we purchase?
    b. What would our total predicted profitability be? (This is the sum of the predicted profitability of all hotels we purchase.)
    c. If we are trying to maximize our total predicted profitability, is this a good approach? How about if we were trying to maximize the average predicted profitability of the hotels we select? How about if we had a budget of $20 million instead of $10 million?
4. Now, build an optimization model to select hotels given the $10 million budget.
    a. Write out the optimization problem. Make sure to detail the decision variables, the objective function and the constraints.
    b. What is the optimal solution? Give the values of the decision variables and the optimal objective function value.
    c. Does the optimal solution make sense intuitively? How does it compared to the greedy solution?
5. LQM thinks that buying too many hotels in one city is probably not a good idea and would prefer to diversify across as many cities as possible. Add constraint(s) to your model to limit the number of hotels purchased in any city to at most 2.
    a. What are the constraints that you need to add to the model? Intuitively, do you expect the new optimal objective function value to be larger, smaller or the same as before?
    b. Write the new optimization model.
    c. Solve the new model. Give the values of the decision variables and the optimal objective function value. How does this compare to the previous solution?
6. In one paragraph (no more than 300 words), describe how you would present your results to LQM. Do you have any recommendations for them to improve the regression model? How about to improve the optimization model?