Linear Algebra Exam

Friday, January 7, 2005 (9:00 – 11:00 AM)

Please show all work. To get full credit you must **clearly** describe your calculations.

1. (**20 points**) For what values of α and β does the set S span R^3?

$$S = \{(1,0,0),\ (1,1,0),\ (\alpha,1,\beta)\}.$$

2. (**20 points**) Find the eigenvalues (and their multiplicity) and corresponding eigenvectors of

$$A = \begin{pmatrix} -4 & 5 & 5 \\ -5 & 6 & 5 \\ -5 & 5 & 6 \end{pmatrix}$$

3. (**20 points**) Consider the system of equations

$$\begin{aligned} x - 3y + 2z &= 4 \\ 2x + y - z &= 1 \\ 3x - 2y + z &= \alpha \end{aligned}$$

(a) Determine a condition on the constant α that guarantees that the system has at least one solution.

(b) For this value of α determine the number of solutions that the system has and find this (these) solutions.

4. (**20 points**) Find a transformation $\mathbf{x} = \mathrm{P}\mathbf{y}$, $\mathbf{x} = (x_1, x_2, x_3)^\top$, $\mathbf{y} = (y_1, y_2, y_3)^\top$ which transforms the quadratic form

$$2x_1^2 + 4x_2^2 - 4x_3^2 + 6x_2x_3$$

to a sum (or difference) of squares

$$\alpha y_1^2 + \beta y_2^2 + \gamma y_3^2,$$

and determine α, β and γ.

5. (**20 points**) Prove that all eigenvalues of a Hermitian matrix are real and the eigenvectors corresponding to different eigenvalues are orthogonal.

6. (**20 points**) Compute A^n and $\exp(A)$ where A is the matrix

$$A = \begin{pmatrix} 1 & 2 \\ -1 & 4 \end{pmatrix}.$$