Complex Variables Preliminary Examination
January 11, 2007

1. Find two separate Laurent expansions of the function

$$\frac{1}{(iz+1)(z-2)}$$

about the point $z = 0$: one valid for $1 < |z| < 2$ and one valid for $|z| > 2$.

2. Suppose

$$f(z) = \frac{H(z)}{(z^2-1)(z-2)^2},$$

where $H(z)$ is an entire function. Determine the value of

$$\oint_C f(z)dz,$$

where C is taken counterclockwise around the circle

(a) $|z-5| = 2$

(b) $|z-5| = 5$

3. Use contour integration to evaluate the following integral. Explain carefully each step of the method you use for evaluation.

$$\int_0^{2\pi} \frac{\cos\theta}{2+\cos\theta}\, d\theta$$ (you need not simplify the answer)

4. Use contour integration to evaluate the following integral. Explain carefully each step of the method you use for evaluation.

$$\int_0^{\infty} \frac{x^{1/2}}{1+x^2}\, dx$$

5. Find the harmonic conjugate of

$$u(x,y) = xe^{-y}\cos x - ye^{-y}\sin x\,.$$

Find all complex analytic functions $f(z)$ of which u is the real part.

6. What is the image of the negative real line $\{z = x + i0 : x < 0\}$ under the map $f(z) = 1/(z+i)$?