PRELIMINARY EXAM – DIFFERENTIAL EQUATIONS 4/08

Note - all problems count equally

1. Consider the differential equation

$$x\frac{dy}{dx} = 3y + x^5 \cos x.$$

Determine the solution satisfying the initial condition $y(\pi/2) = 1$.

2. Consider the equation

$$y'' + \lambda y = 0,$$

with the boundary conditions

$$y'(0) = 0, \quad y'(1) = 0.$$

Determine all solutions for every value of λ $(-\infty < \lambda < \infty)$.
Next, consider the boundary conditions

$$y(0) = 0, \quad y(1) = 0.$$

and determine all solutions for every value of λ $(-\infty < \lambda < \infty)$.

3. Consider the equation

$$x'' + 9x = \cos(\omega t).$$

Determine the general solution for all values of ω.

4. Consider the equation

$$y' = yxe^{-x^2}.$$

Find the general solution.

5. Find the solution to the problem

$$x^2y'' - 3xy' + 3y = 0, \quad y(1) = 1, y'(2) = 0.$$

6. Consider the system of equations

$$\begin{aligned} x' &= -y - \alpha x\,, \\ y' &= x. \end{aligned}$$

(a) Determine the general solution for $\alpha = 0, 1, 2, 3$.

(b) For each case determine the type and stability of the critical point at the origin.

(c) For $\alpha = 1$ determine a particular solution to the forced system

$$\begin{aligned} x' &= -y - \alpha x + \cos t\,, \\ y' &= x, \end{aligned}$$

7. Consider the system

$$\begin{aligned} x' &= y - x^2\,, \\ y' &= x - y^2\,. \end{aligned}$$

Determine all critical points, their type and stability.