

PRELIMINARY EXAM - Calculus March 27, 2006

Please show *all* your work and **CLEARLY** describe your calculations.

1. Find out whether or not the vector fields given below have a potential  $\varphi$ , and find  $\varphi$  if it exists.

(a)  $\mathbf{V} = (2xy^3)\mathbf{i} + (3x^2y^2 - 2yz)\mathbf{j} - y^2\mathbf{k}$

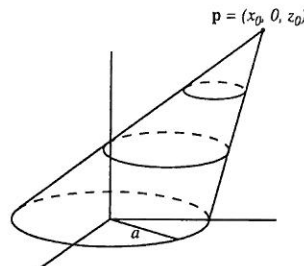
(b)  $\mathbf{V} = (2x^2y)\mathbf{i} + (3x^2y^2 - 2yz^2)\mathbf{j} - 2y^2z\mathbf{k}$

2. Show that if  $S$  is the surface bounding a volume  $V$  and  $f, g$  are scalar functions, then

$$\oint_S (f(\nabla g \cdot \mathbf{n}) - g(\nabla f \cdot \mathbf{n})) dS = \iiint_V (f\nabla^2 g - g\nabla^2 f) dV.$$

3. Evaluate  $\iint_S x^2 y^2 dS$  over the total surface (including the top and bottom) of the cylinder  $x^2 + y^2 = a^2$ ,  $z = 0$ ,  $z = h$ .

4. Compute the center of mass of a circular cone with base radius  $a$ , and apex at the point  $\mathbf{p} = x_0\mathbf{i} + z_0\mathbf{k}$  (see figure), where the mass density is  $\rho(x, y, z) = z$ .



5. Let  $\mathbf{F} = e^{x^2}(1 - y^2)\mathbf{i} - e^{y^2}(1 - x^2)\mathbf{j}$ , compute

$$\int_{-1}^1 \int_{-1}^1 \nabla \times \mathbf{F} dx dy.$$

6. Find all the critical points of the function  $f(x, y) = (2x^2 + 3y^2)e^{-(x^2+y^2)}$  in the domain  $-2 \leq x, y \leq 2$ , and determine if they are relative maxima or minima.