

Differential Equations
ESAM Preliminary Examination
January 7, 2005, 1-3 p.m.

1. Solve the following differential equations for $y(x)$:

$$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = \sinh x,$$

$$\frac{dy}{dx} = xy(y-2),$$

$$\frac{dy}{dx} = \frac{x+3y-5}{x-y-1}.$$

2. Solve the system of equations

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} -3 & 0 & -4 \\ -1 & -1 & -1 \\ 1 & 0 & 1 \end{pmatrix} \mathbf{x}.$$

3. Consider the system

$$\begin{aligned} \frac{dx}{dt} &= y^2, \\ \frac{dy}{dt} &= x^2. \end{aligned}$$

(a) Sketch the phase portrait of the system.

(b) What is the qualitative difference in behavior of the solution with $x(0) = y(0) = -1000$ and the solution with $x(0) = -1000, y(0) = -1000.00001$?

4. Consider the system

$$\frac{d\vec{x}}{dt} = A\vec{x} + \vec{f}(t),$$

where \vec{x} and \vec{f} are vectors of size n and A is a constant $n \times n$ matrix. Characterize **all** matrices A so that for all periodic functions \vec{f} (irrespective of period) there will be at most one periodic solution.