January 8 1992, 4-6 PM

DIFFERENTIAL EQUATIONS - PRELIMINARY EXAM

1. Find the general solution of

   a. $(x^{-1}y')' + x^{-3}y = 0$ (one solution has a simple form)

   b. $y'' - 2y' + 2y = \sin(x)$

2. Determine the two values of the constant α for which it is true that <u>all</u> solutions of the equation

   $$xy'' + (x - 1)y' - \alpha y = 0$$

   are <u>regular</u> at $x = 0$. Use Frobenius method. Do not determine the complete series.

3. Solve the following eigenvalue problem

   $$x^4 y'' + k^2 y = 0$$
   $$y(a) = y(b) = 0$$

   a. Show that the general solution of the differential equation can be expressed in terms of elementary functions after using the transformation: $y = t^{-1}z(t)$ where $t = x^{-1}$.

   b. Determine the eigenvalues $k = k_n$ and the corresponding eigenfunctions.

4a. Consider the following system of differential equations

$$\frac{dx}{dt} = ax - bxy = x(a - by)$$
$$\frac{dy}{dt} = - cy + dxy = y(dx - c)$$

where a, b, c and d are positive. Determine the steady states and their linear stability properties. Sketch a phase plane and discuss the trajectories in the phase plane near each singular point.

4b. Consider the modified system of equations

$$\frac{dx}{dt} = ax\frac{(K - x)}{K} - bxy$$
$$\frac{dy}{dt} = - cy + dxy$$

where $K > 0$. Same questions as in (4a).

5. Find the general solution of the nonlinear ordinary differential equation

   $$(1 + x^2)y'' + y'^2 + 1 = 0.$$

$y' = x$

$y(x) = [illegible]x + c$