Complex Variables Preliminary Examination
January 11, 2008

1. Evaluate
$$\left(-1+i\sqrt{3}\right)^{3/2}.$$

2. Expand the function
$$\frac{1}{z^2(z-1)}$$
in a Laurent series valid for $0<|z|<1$.

3. Use contour integration to evaluate the following integral. Explain carefully each step of the method you use for evaluation.
$$\int_0^\infty \frac{\log x}{1+x^2}\,dx$$

4. Let $f(z)=u(r,\theta)+iv(r,\theta)$ be an entire function. If $u(r,\theta)=r\sin\theta$ and $f(1)=i$, find $f(z)$.

5. Use conformal mapping to solve the boundary value problem,
$$u_{xx}+u_{yy}=0$$
in the domain $x^2+y^2>1$, with the boundary conditions
$$u=1+x \text{ on } x^2+y^2=1, \qquad u\to 1 \text{ as } x^2+y^2\to\infty$$
Hint: The mapping $\zeta=1/z$ conformally maps the exterior of the unit disk into its interior.