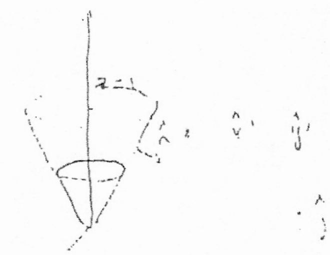
Advanced Calculus Exam — January 4, 1995

Please show all work. To get full credit for a problem you need to CLEARLY describe your calculations.



1. (20 pts.) Compute directly the flux.

$$\oint\!\!\oint \vec{F}\cdot\hat{n}\,dS$$

out of the cone

$$x^2+y^2=z^2 \qquad 0\le z\le 1$$

for the vector field

$$\vec{F}=xi+yj-zk.$$

Verify your answer with the divergence theorem.

$\hat{\imath}$

$\hat{k}$

$\frac{\sqrt{2}}{2}\hat{\imath}-\frac{\sqrt{2}}{2}\hat{k}$

2. (20 pts.) Evaluate

$$I=\int\!\!\int_R (x+y)^2\,dx\,dy$$

where R is the region

$$(x-y)^2+\frac{(x+y)^2}{4}\le 1\,.$$

3. (20 pts.) Consider the unit vector $\vec{A}=<2/3,2/3,1/3>$, and the two vectors $\vec{B}=<1,1,1>$ and $\vec{C}=<1,-1,0>$.

a) Find all vectors $\vec{U}$ such that

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$$\vec{U}\times\vec{A}=\vec{B}\,.$$

$\begin{vmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ x & y & z\\ 2/3 & 2/3 & 1/3\end{vmatrix} = \langle 1,1,1\rangle$

$\hat{\imath}\left(\frac{y}{3}-\frac{2z}{3}\right)-\hat{\jmath}\left(\frac{x}{3}-\frac{2z}{3}\right)+\hat{k}\left(\frac{2x}{3}-\frac{2y}{3}\right)$

b) ~~Find all vectors~~ $\vec{U}$ such that

$\vec{u}=\left(\begin{matrix}c\\ c\\ \frac{c-3}{2}\end{matrix}\right)$

$$\vec{U}\times\vec{A}=\vec{C}.$$

$y-2z=3$

$2z-x=3$

$2x-2y=3$

$\begin{pmatrix}0 & 1 & -2\\ -1 & 0 & 2\\ 2 & -2 & 0\end{pmatrix}$

$y-2z=3$

$2z-x=-3$

$2x-2y=0$

$y-x=0$

$x-y=0$

$x=y$

4. (20 pts.) Find the highest and lowest points of the intersection of the cylinder

$$x^2 + y^2 = 1$$

and the plane

$$2x + y - z = 4.$$

5. (10 pts.) Determine for what values of $p > 0$ the following series converges

$$\sum_{n=1}^{\infty} \left(\sin \frac{1}{n} \right)^p .$$

6. (10 pts) Determine

$$\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{1/x^2} .$$