Numerical Methods

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What is a numerical method?

- Approximate problem solving techniques
- Example: Fast computing

$$\sum_{i=0}^{\infty} \frac{1}{n!} = e$$

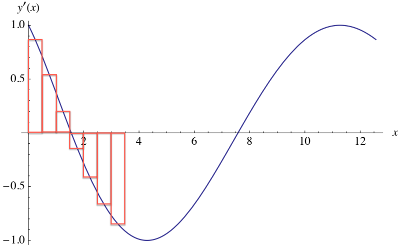
$$\frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} = 2.71666667 \approx e$$

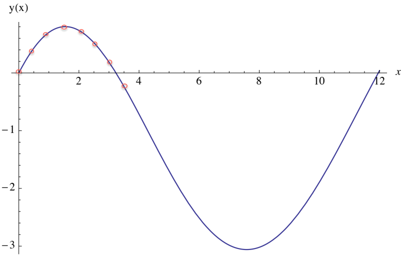
  - Computing the infinite series takes forever!
  - Summing 1^{st} 6 terms in the series results in approximation accurate to two decimal places

What is a numerical method?

- Example: Exact solution is unknown

$$y'(x) = \cos[(x^2 + x^3)^{\frac{1}{4}}]$$
$$y(0) = 0$$





- Sequence of approximations

Deriving a numerical method

- Example: Approximate $e \implies$ Taylor Series with $x = 1$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots$$

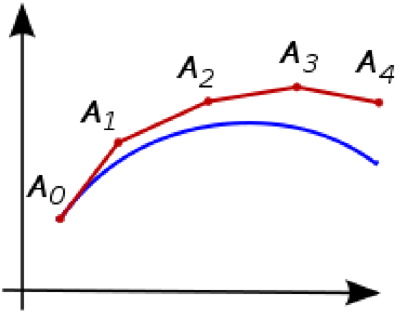
Deriving a numerical method

- Example: Difference equation for $y'(x) = f(x)$
  - Taylor Series for y about x

$$y(x+h) = y(x) + hy'(x) + \frac{1}{2}h^2 y''(x) + \cdots$$

$$\therefore y'(x) \approx \frac{y(x+h) - y(x)}{h} \approx f(x)$$

  - This is an approximation to the slope of $y(x)$



Deriving a numerical method

$$\frac{y(x+h)-y(x)}{h} \approx f(x)$$
$$\therefore y(x+h) \approx y(x) + hf(x)$$

$$y_{n+1} = y_n + hf(x_n) \qquad n = 1, 2, 3, \ldots$$
$$x_n = (n-1)h$$

- Let $f(x) = \cos[(x^2 + x^3)^{\frac{1}{4}}]$, $y(0) = 0$, and $h = 0.1$

$$\begin{aligned} & y_1 = 0 \\ x_1 = 0.0 \qquad & y_2 = 0 + 0.1 * \cos[(0.0^2 + 0.0^3)^{\frac{1}{4}}] = 0.1000 \\ x_2 = 0.1 \qquad & y_3 = 0.1000 + 0.1 * \cos[(0.1^2 + 0.1^3)^{\frac{1}{4}}] = 0.1900 \\ & \vdots \end{aligned}$$