Please choose five of the following six problems to solve. Clearly **indicate on the cover of your exam, which problems you have selected.** Please show all work. To get full credit for a problem, you need to CLEARLY show the details of your calculations.

Problem 1 (20 pts) Use Stokes' theorem to evaluate

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$$

where $\mathbf{F} = -yz\hat{\imath} + xz\hat{\jmath} + xy\hat{\mathbf{k}}$ and S is the area of the spherical cap $x^2 + y^2 + z^2 = 4$, $z \geq \sqrt{3}$. The vector $\mathbf{n}$ is the unit outward normal to the spherical cap.

Problem 2 (20 pts) For $\xi = x - y$, and $\eta = 2y$, convert the partial differential equation

$$x\frac{\partial^2 u}{\partial x^2} + 2(x+y)\frac{\partial^2 u}{\partial x \partial y} + y\frac{\partial^2 u}{\partial y^2} = 0$$

into one in terms of ξ and η.

Problem 3 (20 pts) Let $\mathbf{F}$ be a force field which is solenoidal ($\nabla \cdot \mathbf{F} = 0$) and consider a tube of force consisting of a surface with lateral side S and two end surfaces S_1 and S_2, with $\mathbf{F}$ directed along (parallel to) the lateral surface S. Show that the flux of $\mathbf{F}$ entering S_1 equals the flux of $\mathbf{F}$ leaving S_2.

Problem 4 (20 pts) Let P, Q be continuous functions with continuous partial derivatives and let $\dfrac{\partial P}{\partial y} = \dfrac{\partial Q}{\partial x}$ except at the points $(4,0)$, $(0,0)$, and $(-4,0)$. Let C_1 denote the circle $(x-2)^2+y^2 = 9$, let C_2 denote the circle $(x+2)^2+y^2 = 9$, let C_3 denote the circle $x^2+y^2 = 25$, and let C_4 denote the circle $x^2+y^2 = 1$. Given that $\oint_{C_1} P\,dx+Q\,dy = 11$, $\oint_{C_2} P\,dx + Q\,dy = 9$, and $\oint_{C_3} P\,dx + Q\,dy = 13$, find $\oint_{C_4} P\,dx + Q\,dy$.

Problem 5 (20 pts) Consider the curve $C = \{(x,y) : f(x,y) = 0\}$. Show that the sum of the squares of the directional derivatives of $f(x,y)$ at a point P on C along any two perpendicular directions, equals the square of the normal derivative of $f(x,y)$ at P.

Problem 6 (20 pts) Find the maximum and minimum values of the function

$$f(x,y) = 5x^2 + 6xy + 5y^2 - 6x - 6y$$

on the unit disk $\{(x,y) : x^2 + y^2 \leq 1\}$.