PRELIMINARY EXAM - ADVANCED CALCULUS 1/99
20 points per problem

Please show all work. To get full credit for a problem you need to **CLEARLY** describe your calculations.

1. (a) Find the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{e^n(x-\pi)^n}{n^2}$$

(b) Determine if the following series converges. Provide justification for your answer.

$$\frac{1}{2n^2} \qquad \qquad \sum_{n=1}^{\infty} \frac{n+\sqrt{n}}{2n^3-1}$$

2. (a) Prove that $\vec{F} = (2xz^3 + 6y)\hat{i} + (6x - 2yz)\hat{j} + (3x^2z^2 - y^2)\hat{k}$ is a conservative force field.

(b) Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ where C is any path from $(1, -1, 1)$ to $(2, 1, -1)$.

(c) Give a physical interpretation of the results.

3. Find the maximum value of $f(x, y, z) = xyz$ on the portion of the plane $x + 2y + 3z = 6$ in the first octant ($x > 0$, $y > 0$, $z > 0$).

4. Let $f(x, y, x) = x^2 + y^2 + z^2$. Find the rate of change of the function f at the point $(3, 5, 4)$ along the curve of intersection of the surfaces $2x^2 - y^2 + 2z^2 = 25$ and $x^2 - y^2 + z^2 = 0$. page 513 # 12

5. Find the values of α for which the integral

$$I(\alpha) = \int\int_R (x^2 + y^2)^\alpha \ln(x^2 + y^2)\,dx\,dy$$

converges where R is the interior of the circle $x^2 + y^2 = 1$. Evaluate $I(\alpha)$ for these values of α.