

PRELIMINARY EXAM – DIFFERENTIAL EQUATIONS 1/08

Note - all problems count equally

1. Consider the differential equation

$$x^2 y' + y = 1.$$

Determine the solution satisfying the initial condition $y(1/2) = 0$.

2. Consider the equation

$$y'' + \lambda y = 0,$$

with the boundary conditions

$$y(0) + y'(0) = 0, \quad y\left(\frac{\pi}{2}\right) + y'\left(\frac{\pi}{2}\right) = 0.$$

Determine all solutions for every value of λ ($-\infty < \lambda < \infty$).

3. Consider the equation

$$x'' + 2x' + x = \exp(\alpha t).$$

Determine the general solution for all values of α .

4. Find the general solution to the equation

$$y' = \frac{1}{x + y}.$$

5. Find the solution to the problem

$$x^2 y'' + xy' + y = 0, \quad y(1) = 2, y'(1) = 3.$$

6. Consider the system of equations

$$\begin{aligned}x' &= y, \\y' &= -x - \delta y.\end{aligned}$$

- (a) Determine the general solution for $\delta = 0, 1, 2, 3$.
- (b) For each case determine the type and stability of the critical point at the origin.
- (c) For $\delta = 1$ determine the general solution to the forced system

$$\begin{aligned}x' &= y + t, \\y' &= -x - \delta y + 1.\end{aligned}$$

7. Consider the system

$$\begin{aligned}x' &= x^2 - 2x - xy, \\y' &= y^2 - 4y + xy.\end{aligned}$$

Determine all critical points, their type and stability.