

PRELIMINARY EXAM - Calculus January 4, 2006

Please show *all* your work and **CLEARLY** describe your calculations.

1. Given the function

$$x \cos y + y \cos z + z \cos x = 1 \quad (1)$$

- (a) Find the derivatives $\partial z / \partial x$ and $\partial z / \partial y$
(b) Write down an equation for the plane tangent to the surface $z = f(x, y)$, implicitly described by equation (1) at $x = y = 0$.

2. Find out whether or not the vector fields given below have a potential φ , and find φ if it exists.

- (a) $V = (5x^2y - 4xy)\mathbf{i} + (3x^2 - 2y)\mathbf{j}$
(b) $V = (y + z)\mathbf{i} + (x + z)\mathbf{j} + (x + y)\mathbf{k}$

3. Compute the maximum and minimum values of the function $f(x, y) = 6 - 4x - 3y$, on the circle of unit radius $x^2 + y^2 = 1$.

4. Calculate the flux of the vector field $F = 1\mathbf{k}$ across the surface S in the direction away from the origin, where S is the upper unit hemisphere $x^2 + y^2 + z^2 = 1; z \geq 0$.

5. Show that if S is the surface bounding a volume V and f, g are scalar functions, then

$$\oint_S (f(\nabla g \cdot \mathbf{n}) - g(\nabla f \cdot \mathbf{n})) dS = \iiint_V (f \nabla^2 g - g \nabla^2 f) dV.$$

6. Evaluate $\iint_S x^2 y^2 dS$ over the total surface (including the top and bottom) of the cylinder $x^2 + y^2 = a^2, z = 0, z = h$.