Advanced Calculus Exam January 4, 1996

Please show all work. To get full credit for a problem you need to CLEARLY describe your calculations.

1. (20 pts.) Find the point (x, y, z) on the sphere

$$x^2 + y^2 + z^2 = 25$$

which is farthest from the point $(3, 4, 5\sqrt{3})$.

2. (20 pts.) Compute the flux

$$\Phi = \iint_S \vec{F} \cdot \mathbf{n}\, dS$$

? out of the bounding surface S of the hemisphere

$$x^2 + y^2 + z^2 \leq R^2, \quad z \geq 0$$

for the vector field

$$\vec{F} = x^3\mathbf{i} + y^3\mathbf{j} + z^3\mathbf{k}$$

($\mathbf{n}$ is the unit normal to S).

3. (15 pts.) Find the limits

check $$\lim_{n\to\infty} \left(\frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n} \right).$$

? ok $$\lim_{n\to\infty} \sum_{j=1}^{n} \frac{b^j}{(j+1)!}.$$

4. (20 pts.) Find the volume of the ellipsoid

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{a}\right)^2 + \left(\frac{z}{c}\right)^2 = 1.$$

5. (25 pts.) Let f be a scalar function defined on the surface S with perimeter C. Show that

check $$\oint_C f\, d\mathbf{r} = \iint_S \mathbf{n} \times (\nabla f)\, dS,$$

where $\mathbf{n}$ is the unit normal to S.

Hints:

1. Consider Stokes theorem with $\vec{F} = f\vec{a}$ for $\vec{a}$ constant.
2. $A \cdot (B \times C) = (A \times B) \cdot C$.