Advanced Calculus Exam January 8, 1997

Please show all work. To get full credit for a problem you need to CLEARLY describe your calculations.

1. (15 pts.) Given the sphere $(x-2)^2+(y-1)^2+(z-1)^2=4$ and the plane $x+2y+2z=15$, find the equation of that sphere which is the mirror image of the given sphere relative to the given plane.

2. (15 pts.) Does the series

$$\sum_{n=1}^{\infty}(\sqrt{n+2}-2\sqrt{n+1}+\sqrt{n})\sin n$$

converge?

3. (45 pts.)

(a) Compute the integral

$$\iint_{\Omega} xy^2\,dx\,dy,$$

where Ω is the region bounded by $y^2=4x$ and $x=1$.

(b) Find the area of the region bounded by the curve

$$(x^2+y^2)^3=x^4+y^4.$$

$(r^2)^2=(x^2+y^2)(x^2-y^2)$

(c) Compute the integral

$$\iiint_V (x^2+y^2)\,dx\,dy\,dz,$$

where V is the region bounded by $x^2+y^2=2z$ and $z=2$.

4. (10 pts.) Write the differential expression

$$w=x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}-y^2\frac{\partial^2 u}{\partial y^2}$$

in polar coordinates (r,θ) $(x=r\cos\theta,\ y=r\sin\theta)$.

5. (15 pts.) Find the work done by the inverse-square force field

$$\mathbf{F}(x,y,z)=\frac{\mathbf{r}}{r^3},\quad \mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$$

in moving a particle along the straight line segment from $(0,4,0)$ to $(0,4,3)$.