PRELIMINARY EXAM - ADVANCED CALCULUS 1/98

20 points per problem

Please show all work. To get full credit for a problem you need to **CLEARLY** describe your calculations.

1. Consider the plane curve defined by the equation (a is a positive constant)

$$y = a\cosh\left(\frac{x}{a}\right) \tag{1}$$

(a) Give a rough sketch of the curve.

(b) Determine the unit tangent $\hat{\mathrm{T}}$ and unit normal $\hat{\mathrm{N}}$ to the curve. Sketch these on the figure in part (a) for a particular point on the curve.

(c) Determine the arc length of the curve between $x = 0$ and $x = a$.

(d) Find the volume of the solid of revolution formed by rotating the region bounded by the lines $x = \pm a$, the x-axis, and equation (1) around the x-axis.

(e) What is the surface area of the volume of revolution described in part (d).

2. Find the area enclosed by the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ by first expressing the area as a line integral and then evaluating the integral.

3. (a) Compute the sum

$$\sum_{n=1}^{\infty} n\left(\frac{1}{2}\right)^{n-1}.$$

(b) For what values of x is this series convergent?

$$\sum_{n=1}^{\infty} \frac{3\cdot 5\cdots(2n+1)}{5\cdot 10\cdots(5n)}x^n$$

4. Find the shortest distance from the point $(1,1,1)$ to the surface defined by

$$2x^2 + 2y^2 = z^2.$$

5. Assume that $b > a > 0$ and $d > c > 0$. Consider the region R in the first quadrant that is bounded by the curves $y = ax^3$, $y = bx^3$, $x = cy^3$, $x = dy^3$. Make a change of coordinates $u = u(x,y)$, $v = v(x,y)$ that turns this region into a rectangular one in the (u,v)-plane. Use this coordinate transformation to determine the area of the region R.