Advanced Calculus Exam January 4, 1994

1. (20 pts.) Compute the minimum and maximum value of the function

$$f(x,y) = x^2 + y^2 + axy ,$$

in the region

$$x^2 + y^2 \leq 1$$

for each value of the parameter a. Make sure you consider all cases.

2. (20 pts.) Consider the plane

$$x + 2y + 2z = 1$$

and the line parameterised by t

$$x = t \qquad y = t - 1 \qquad z = 2t - 2$$

Find the mirror image of the line in the plane.

3. (20 pts.) Compute the volume of

$$\frac{1}{2}(x+y)^2 + (y+z)^2 + (x+z)^2 \leq 0$$

4. (20 pts.) For this problem you need to compute the work.

$$\oint \vec{F} \cdot d\vec{r}.$$

for the vector field

$$\vec{F} = -y\mathrm{i} + x\mathrm{j} + (x + y)\mathrm{k}$$

along the curve given by the intersection of the cylinder

$$x^2 + y^2 = 1$$

and the hyperbolic paraboloid

$$z = x^2 - y^2$$

traversed in an anti-clockwise sense relative to the positive z-axis.

a) Compute the line integral for the work directly.

b) Verify the results with Stoke's theorem for the portion of the hyperbolic paraboloid inside the cylinder.

5. (20 pts.) Determine if the following series converge. If you use a theorem state it clearly beforehand.

a)

$$\sum_{n=2}^{\infty} \frac{1}{n\ln(n)[\ln(\ln(n))]}$$

b)

$$\sum_{n=1}^{\infty} \left(1 - \frac{1}{n}\right)^{n^2}$$