Adv. Calc. Preliminary Examination

January 2012

Name: ______________________________

Instructions:

1. Show all work. Explain all steps. **Neatness counts.**
2. Use of notes, books, or calculators is not allowed.
3. If you need extra space, use the back of a page and make a note indicating it.
4. All problems count equally
5. Good luck!

1. Evaluate by Green's Theorem:

(a) $\oint_C ay\,\mathrm{d}x + bx\,\mathrm{d}y$ on any path (a and b are constants)

(b) $\oint_C \mathrm{e}^x \sin y\,\mathrm{d}x + \mathrm{e}^x \cos y\,\mathrm{d}y$ C : rectangle with vertices : $(0,0), (1,0), (1,\pi/2), (0,\pi/2)$

(c) $\oint_C (2x^3 - y^3)\,\mathrm{d}x + (x^3 + y^3)\,\mathrm{d}y$ $C:\ x^2 + y^2 = 1$

(d) $\oint_C \vec{F} \cdot \mathrm{d}\vec{r}$ $\vec{F} = \nabla(x^2 y)$ $C:\ x^2 + y^2 = 1$

(e) $\oint_C \vec{v} \cdot \vec{n}\,\mathrm{d}s$ $\vec{v} = (x^2 + y^2)\vec{\imath} - 2xy\vec{\jmath}$ $C:\ x^2 + y^2 = 1$

2. Evaluate

$$\oint_C \frac{y\,\mathrm{d}x - (x-1)\,\mathrm{d}y}{(x-1)^2 + y^2} \qquad C:\ x^2 + y^2 = 4$$

3. a) Let $\vec{\mathbf{F}} = x\vec{\mathbf{i}}+y\vec{\mathbf{j}}+z\vec{\mathbf{k}}$ and let S be the closed surface consisting of the paraboloid $z = 4 - x^2 - y^2$ and the plane $z = 3$.
(a) Without appealing to the divergence theorem, directly evaluate the surface integral

$$\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{n}}\, \mathrm{d}S,$$

where $\vec{\mathbf{n}}$ is the unit outer normal to S.

(b) Use the divergence theorem to evaluate the integral.

4. Evaluate

$$\iint_S \nabla \times \vec{F} \cdot \vec{n} \, \mathrm{d}S$$

both directly and by Stokes theorem, where $\vec{F} = y^2\vec{\imath} - xy\vec{\jmath} + z\vec{k}$ and S is the hemisphere $x^2 + y^2 + z^2 = 1$, $z \geq 0$.

5. Consider the equation

$$\sin(x + y^2) = \frac{1}{2}.$$

Determine all points in the $x - y$ plane where you can solve this equation locally for a function $y(x)$.

6. Find the maximum and minimum of $f(x, y) = (x-1)^2 + (y-1)^2$ subject to the constraint $x^2 + y^2 \leq 4$