

Numerical Methods

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Numerics courses offered by ESAM

- ▶ 446-1,2 Numerical Solution of Partial Differential Equations (Core classes)
- ▶ 445 Iterative Methods for Elliptic Equations
- ▶ 447 Boundary Integral Methods
- ▶ 448 Numerical Methods for Random Processes
- ▶ 449 Numerical Methods for Moving Interfaces
- ▶ Other numerics courses offered outside dept. e.g. Finite Element Methods or Computation Fluid Dynamics
- ▶ Instructor's notes suffice for ESAM numerics courses

What is a numerical method?

- ▶ Approximate problem solving technique
- ▶ Example: Fast computing

$$\sum_{n=0}^{\infty} \frac{1}{n!} = e$$

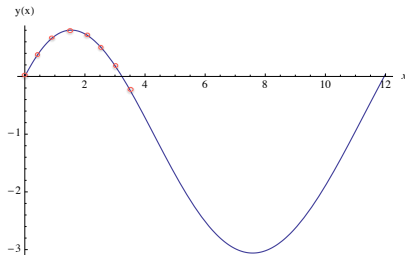
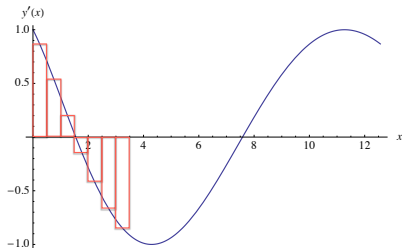
$$\frac{1}{1} + \frac{1}{1} + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} + \frac{1}{120} = 2.71666667 \approx e$$

- ▶ Computing the infinite series takes forever!
- ▶ Summing 1st 6 terms in the series results in approximation accurate to two decimal places

What is a numerical method?

- Example: Exact solution is unknown

$$y'(x) = \cos[(x^2 + x^3)^{\frac{1}{4}}]$$
$$y(0) = 0$$



- Sequence of approximations

Deriving a numerical method

- ▶ Example: Approximate $e \implies$ Taylor Series with $x = 1$

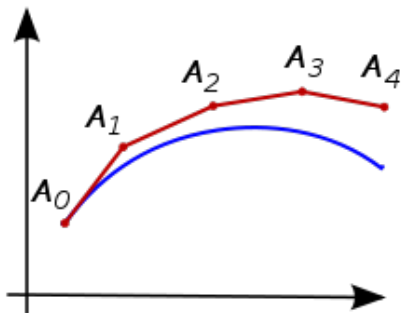
$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots$$

Deriving a numerical method

- ▶ Example: Difference equation for $y'(x) = f(x)$
 - ▶ Taylor Series for y about x

$$y(x+h) = y(x) + hy'(x) + \frac{1}{2}h^2y''(x) + \dots$$
$$\therefore y'(x) \approx \frac{y(x+h) - y(x)}{h} \approx f(x)$$

- ▶ This is an approximation to the slope of $f(x)$



Deriving a numerical method

$$\frac{y(x+h) - y(x)}{h} \approx f(x)$$
$$\therefore y(x+h) \approx y(x) + hf(x)$$

$$y_{n+1} = y_n + hf(x_n) \quad n = 1, 2, 3, \dots$$
$$x_n = (n-1)h$$

- Let $f(x) = \cos[(x^2 + x^3)^{\frac{1}{4}}]$, $y(0) = 0$, and $h = 0.1$

$$y_1 = 0$$

$$x_1 = 0.0 \quad y_2 = 0 + 0.1 * \cos[(0.0^2 + 0.0^3)^{\frac{1}{4}}] = 0.1000$$

$$x_2 = 0.1 \quad y_3 = 0.1000 + 0.1 * \cos[(0.1^2 + 0.1^3)^{\frac{1}{4}}] = 0.1900$$

⋮