Jan. 92

ADVANCED CALCULUS

1. Show that if $f(x)$ is continuous for $0 \leq x \leq 1$, then

$$\lim_{n\to\infty} \frac{1}{n}\left[f(\frac{1}{n}) + f(\frac{2}{n}) + \ldots + f(\frac{n-1}{n}) + f(\frac{n}{n})\right] = \int_0^1 f(x)\,dx.$$

Hence, show that

$$\lim_{n\to\infty} \frac{1^2+2^2+\ldots+n^2}{n^3} = \frac{1}{3}$$

2. Find the length of the circumference of a circle

(a) using the parametric representation $x = a\cos\theta$, $y = a\sin\theta$.

(b) using the parametric representation $x = \frac{a(1-t^2)}{1+t^2}$, $y = \frac{2at}{1+t^2}$

3. Evaluate $\partial F/\partial n$ at a general point (x, y, z) on the surface S, where $\vec{n}$ is the outer normal to S, if:

(a) $F = x^2 - y^2$ and $S : x^2 + y^2 + z^2 = 4$

? $\rightarrow$ (b) $F = xyz$ and $S : x^2 + 2y^2 + 4z^2 = 8$

4. Evaluate the following contour integrals (where f and g are arbitrary smooth functions, and where closed contours are traversed in the usual counterclockwise sense):

? $\rightarrow$ a) $\oint_C 4xy^3\,dx + 6x^2y^2\,dy$; $C : x^2 + y^2 = 1$

b) $\oint_C f(x)\,dx + g(y)\,dy$; C : any smooth simple closed curve

c) $\oint_C \frac{-y\,dx + x\,dy}{x^2+y^2}$; C : any smooth simple closed curve not enclosing the origin

Check $\rightarrow$ d) $\oint_C \frac{-y\,dx + x\,dy}{x^2+y^2}$; C : any smooth simple closed curve enclosing the origin