

Problem 1 (20 points) Solve the initial value problem

$$3y^2 \frac{dy}{dx} = xy^3 + x + 3y^3 + 3, \quad y(0) = 0.$$

Problem 2 (20 points) Solve the initial value problem

$$(x+3) \frac{dy}{dx} - (x+1)y = \frac{1}{x+3}, \quad y(0) = 1.$$

Problem 3 (25 points) Solve the initial value problem

$$x'' + 2x' + x = 2 \sin t, \quad x(0) = 0, \quad x'(0) = 1$$

Problem 4 (40 points) Using the method of undetermined coefficients write the form of a particular solution of the following equations. You do not have to compute the coefficients.

(a)  $x'' + 5x' + 4x = e^t + t + \sin t$

(b)  $x'' + 5x' + 4x = e^{-4t}$

(c)  $x'' + 5x' + 4x = te^{-t}$

(d)  $x'' + 4x = 2 \sin 2t$

(e)  $x'' + 4x = t \sin 2t$

Problem 5 (25 points) Solve the initial value problem

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} 5 & 2 \\ -2 & 1 \end{pmatrix} \vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

Problem 6 (25 points) Solve the initial value problem

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} -1 & 3 \\ -3 & -1 \end{pmatrix} \vec{x}, \quad \vec{x}(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Problem 7 (15 points) Find a particular solution of the system

$$\frac{d\vec{x}}{dt} = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix} \vec{x} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^t$$

Problem 8 (30 points) Find all real critical points of the system of equations

$$\frac{dx}{dt} = (2+x)(y-x+2), \quad \frac{dy}{dt} = y+x.$$

Determine the type and stability of the critical points.