Linear Algebra Exam

Friday, January 6, 2006 (9:00 – 11:00 AM)

Please show all work. To get full credit you must **clearly** describe your calculations.

1. **(20 points)** Let S

$$S = \{(1,1,1)^T,\ (-1,0,-1)^T,\ (-1,2,3)^T\}$$

be a basis for Euclidean space R^3 with the standard inner product

$$(a_1,a_2,a_3)^T \cdot (b_1,b_2,b_3)^T = a_1b_1 + a_2b_2 + a_3b_3.$$

Use the Gram-Schmidt procedure to transform S to an orthonormal basis.

2. **(20 points)** Find the eigenvalues (and their multiplicity) and corresponding standard eigenvectors (you need not find generalized eigenvectors) of

$$A = \begin{pmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -5 & -2 \end{pmatrix}$$

3. **(20 points)** Consider the system of equations

$$\begin{aligned} ax + by + cz &= r_1 \\ a^2x + b^2y + c^2z &= r_2 \\ a^3x + b^3y + c^3z &= r_3 \end{aligned}$$

(a) Determine condition(s) on the constants a, b, c that guarantee that the system has a unique solution for any r_1, r_2, r_3.

(b) If one of these conditions is not satisfied, state a condition (in words only) involving r_1, r_2, r_3 that will allow the system to have solutions. In this case how many solutions will the system have (you need not find such solutions, assuming they exist)?

4. **(20 points)** Transform the quadratic form

$$3x_1^2 + 2x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_2x_3$$

to a sum (or difference) of squares

$$\alpha y_1^2 + \beta y_2^2 + \gamma y_3^2,$$

and determine α, β and γ.

5. **(20 points)** Let Q_1 and Q_2 be square matrices. If Q_1 and Q_2 are orthogonal matrices, show that Q_1Q_2 is also an orthogonal matrix.