COMPLEX VARIABLES PRELIMINARY EXAM

1. Calculate

   a) $\dfrac{1+i\,\tan\theta}{1-i\,\tan\theta}$

   b) $\dfrac{(1+i)^5-1}{(1+i)^5+1}$

   c) $\dfrac{(-1+\sqrt{3}i)^{15}}{(1-i)^{20}} + \dfrac{(-1-\sqrt{3}i)^{15}}{(1+i)^{20}}$

   d) $(-2+2i)^{1/3}$

2. a) Is the function $f(z) = \dfrac{z-i}{1-i\bar{z}}$ analytic?

   (Here $\bar{z}$ is the conjugate of z).

   b) Consider the harmonic function $u(x,y) = e^x\cos y$. Find $v(x,y)$ so that $f = u + iv$ is an entire function of $z = x + iy$ and $f(0) = 1$.

   c) Show that the curves $u = c_1$ and $v = c_2$ for a general analytic function $f = u + iv$ are orthogonal for any constants c_1 and c_2.

3. Classify the singularities of each function:

   a) $f(z) = e^{\frac{z}{z+1}}$

   b) $f = \dfrac{1}{\sinh z}$

   c) $f = \dfrac{z}{z^3+1}$

   d) $f = \dfrac{1}{1+nz}$

4. Express $f(z) = \dfrac{5+z}{4z^3-z^5}$ as two different infinite series in powers of z, one expanded about $z = 0$ and the other expanded about $z = \infty$. What is the radius of convergence of each?

5. Evaluate

   a) $\displaystyle\int_0^\infty \dfrac{t^\alpha}{t^2+1}\,dt,\ -1 < \alpha < 1$

   b) $\displaystyle\int_0^\infty \dfrac{dt}{t^8+1}$

6. Evaluate

   a) $\displaystyle\int_0^{2\pi} \dfrac{d\theta}{1-2p\cos\theta - p^2}$

   b) $\displaystyle\int_0^\infty \dfrac{\cos kx}{x^2+1}\,dx$