Elementary Differential Equations Preliminary Exam, ES/AM

January 4, 1995

1. (34 points.) Solve the following differential equations:

$$y\,dx + (x + x^3 y^2)\,dy = 0,$$

$$y'' + y = 4x \sin x,$$

$$\left\{ \left[\left(D^2 - 1 \right)^2 - 5 \right]^2 - 16 \right\}^2 y = 0.$$

where $D \equiv \frac{d}{dx}$.

2. (14 points.) Find the general solution of the equation

$$u'''' + a(x)u''' + b(x)u'' + c(x)u' + d(x)u = 0$$

if $1, 1 + x, 1 + x^2$ are solutions of the equation.

3. (28 points.) Find the solution of the initial value problems
(a).

$$y' = \begin{pmatrix} 1 & 9 \\ -1 & -5 \end{pmatrix} y, \quad y(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix},$$

(b).

$$y'' - xy' + 2y = x, \quad y(0) = 0, \ y'(0) = 1$$

(Remark: the homogeneous equation has a polynomial solution).

4. (14 points.) Determine all real critical points of the system

$$\frac{dx}{dt} = x - x^2 - xy, \quad \frac{dy}{dt} = 3y - xy - 2y^2$$

and discuss their type and stability.

5. (10 points.) Can the function $f(x) = \cos x$ be expanded in a Fourier sine series over the interval $(0, \pi)$? Explain. Over the interval $(-\pi, \pi)$? Explain.