Name:

1. Find the volume of the solid bounded by the two paraboloids $z = x^2 + 2y^2$ and $z = 12 - 2x^2 - y^2$.[10pts.]
2. Consider two spheres S_1 and S_2 of radius R and mass densities $\rho_1(x, y, z) = 1$ and $\rho_2(x, y, z) = C\sqrt{x^2 + y^2 + z^2}/R$, respectively. (a) Find C such that S_1 and S_2 have the same mass (b) For equal masses, which sphere has a higher moment of inertia I_z? [10pts]
3. Let $\mathbf{F}(\mathbf{r}) = \mathbf{r}/r^p$ with $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ and $r = \sqrt{x^2 + y^2}$ be a radially symmetric vector field and $p > 0$ a parameter. (a) Show that $\mathbf{F}$ is conservative, i.e. $\nabla \times \mathbf{F} = 0$. (b) Compute the potential ϕ with $F = \nabla\phi$. (c) For what value of p is $\mathbf{F}$ divergent free, i.e. $\nabla \cdot \mathbf{F} = 0$. (d) In this case, what differential equation (involving ∇) must ϕ fullfill? [10pts]
4. A particle moves from the point $(0, 0)$ to the point $(1, 0)$ along the curve $y = \alpha x(1 - x)$ in the force field $\mathbf{F} = (y^2 + 1)\mathbf{i} + (x + y)\mathbf{j}$. Find α so that the work done is a minimum. [10pts.]
5. Maxwell's curl equations are:

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \text{and} \quad \frac{1}{\mu_0}\nabla \times \vec{B} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

Assuming μ_0 and ϵ_0 are constant, show that

$$\frac{1}{\mu_0}\nabla \cdot (\mathbf{E} \times \mathbf{B}) + \frac{\partial}{\partial t}\left[\frac{1}{2\mu_0}|\mathbf{B}|^2 + \frac{\epsilon_0}{2}|\mathbf{E}|^2\right] = 0.$$

Note: The quantity $(\mathbf{E} \times \mathbf{B})/\mu_0$ is the electromagnetic energy flux, also called Poynting's vector, and

$$\frac{1}{2\mu_0}|\mathbf{B}|^2 + \frac{\epsilon_0}{2}|\mathbf{E}|^2$$

is the electromagnetic energy density.[10pts.]
6. Given a closed surface S bounding a three dimensional region T and the vector field $\mathbf{F}(\mathbf{r}) = \mathbf{r}$ with $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ show that the volume V of T is given by

$$V = \frac{1}{3}\oiint_S \mathbf{F} \cdot d\mathbf{S}. \quad [10\text{pts.}]$$

7. Use Stokes' Theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = 2x\mathbf{i} + x\mathbf{j} + 3y\mathbf{k}$ and where C is the ellipse in which the plane $z = x$ meets the cylinder $x^2 + y^2 = 4$, oriented counterclockwise as viewed from above.[10pts.]

Extra credit questions [5pts.]:

$$\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$ Stokes Thm

$$\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_T dV \, \nabla \cdot \mathbf{F}$$ Gauss's Thm

Name the above equalities and associate them with the correct faces (write the corresponding letter under the portrait).

Gauss

Stokes

	Score	Max
1:		10
2:		10
3:		10
4:		10
5:		10
6:		10
7:		10
EC:		
Σ		70