Preliminary Examinations: Complex Variables **January 9, 2009**

Name: _____

Show all your work.

	Score
1	/12
2	/12
3	/12
4	/12
5	/13
6	/13
7	/13
8	/13
Total	/100

Problem 1 (12 points). Find the roots implied by $(-i)^{1/3}$.

Problem 2 (12 points). For

$$f(z) = e^{ix}e^{-|y|}$$

derive an expression for $f'(z)$ and indicate where $f'(z)$ exists.

Problem 3 (12 points). Evaluate the integral

$$\oint_C \frac{e^{-3z}}{\sin^2 z} dz$$

where the path C given by

$$x = \cos t, \qquad y = \sin t, \qquad 0 \leq t \leq 2\pi.$$

Problem 4 (12 points). Exploit the properties of analytic functions to evaluate the integral

$$\oint_C 2x(1-y) + i(x^2 - y^2 + 2y)\, dz$$

where the path C is given by $|z| = 1$.

Problem 5 (13 points). Use complex variables to determine

$$\int_0^\infty \frac{dx}{x^{1/2}(1+x)}.$$

Sketch the integration contour you use and show which, if any, of the contributions to the contour integral vanish.

Problem 6 (13 points). Use complex variables to determine

$$\int_0^\infty \frac{x \sin x}{1+x^2}\, dx.$$

Sketch the integration contour you use and show which, if any, of the contributions to the contour integral vanish.

Problem 7 (13 points). The Fourier transform of the solution $u(x)$ to the differential equation

$$\frac{du}{dx} + \alpha u = \delta(x), \qquad -\infty < x < \infty,$$

is given by

$$\tilde{u}(k) = \frac{1}{ik + \alpha}.$$

For $\alpha > 0$ determine $u(x)$ by the inverse transform

$$u(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{ikx}}{ik + \alpha}\, dk.$$

Sketch the integration contour you use and show which, if any, of the contributions to the contour integral vanish.

Problem 8 (13 points). Use the mapping $\zeta = \log z$ to solve the boundary-value problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \qquad -\infty < x < \infty, \quad y > 0,$$

with

$$u(x, 0) = H(x), \quad -\infty < x < \infty,$$

where $H(x)$ is the Heaviside function.