

## Complex Variables Preliminary Examination

March 27, 2012

1. (10 points) Express  $f(x) = \frac{5+z}{4z^3 - z^5}$  as two different infinite series in powers of  $z$ , one expanded about  $z = 0$  and the other expanded about  $z = \infty$ . State and sketch the domain in which each series converges.

2. (10 points) Compute  $\int_0^\infty \frac{\cosh ax}{\cosh x} dx$ , where  $|a| < 1$ .

3. (10 points)

(a) Show that

$$F(z) = \int_0^\infty (1+t)e^{-zt} dt$$

converges only if  $\operatorname{Re}(z) > 0$ .

- (b) Find a function which is the analytic continuation of  $F(z)$  into the left half plane. Please clearly explain why your answer is correct.

4. (10 points) Verify that

$$u = \sin x \cosh y + 2 \cosh x \sin y + x^2 - y^2 + 4xy$$

satisfies Laplace's equation and find  $f(z)$ , where  $f(z)$  is a regular function whose real part is equal to  $u$ .

5. (10 points) Find **all** the values of  $(-8)^{1/6}$ , giving their real and imaginary parts, and plot them in the complex plane.
6. (10 points) Construct a fractional linear (Möbius) transformation that takes the curves  $|z| = 1$  and  $|z - 1| = 5/2$  onto concentric circles centered at the origin.