Lab Topic 4

SNR of LR test

- Joint pdf of noise:

$$p_{\bar{E}}(\bar{y}\,) = \frac{1}{(2\pi)^{N/2}|\mathbf{C}|^{1/2}} \exp\left(-\frac{1}{2}\|\bar{y}\|^2\right)$$

$$\|\bar{y}\|^2 = \bar{y}\boldsymbol{C}^{-1}\bar{y}^T$$

  - Let $\langle \bar{z}, \bar{y} \rangle = \bar{z}\boldsymbol{C}^{-1}\bar{y}^T$
  - LR statistic : $\Lambda = \frac{1}{2}\|\bar{y}\|^2 - \frac{1}{2}\|\bar{y} - \bar{s}\|^2 = \langle \bar{y}, \bar{s} \rangle - \frac{1}{2}\langle \bar{s}, \bar{s} \rangle \rightarrow \Lambda = \langle \bar{y}, \bar{s} \rangle$
  - SNR: $\|\bar{s}\|$

Inner product for stationary noise

- White Gaussian Noise

$$\langle \bar{x}, \bar{y} \rangle \rightarrow \sum_{k=0}^{N-1} x_k y_k$$

- Stationary Gaussian noise with Power Spectral Density (PSD) $S_n(f)$

$$\langle \bar{x}, \bar{y} \rangle \rightarrow \frac{\Delta}{N} \tilde{x} (\tilde{y}^{\dagger} ./ \bar{S}_n{}^{T}) = \frac{1}{T} \tilde{x} (\tilde{y}^{\dagger} ./ \bar{S}_n{}^{T})$$

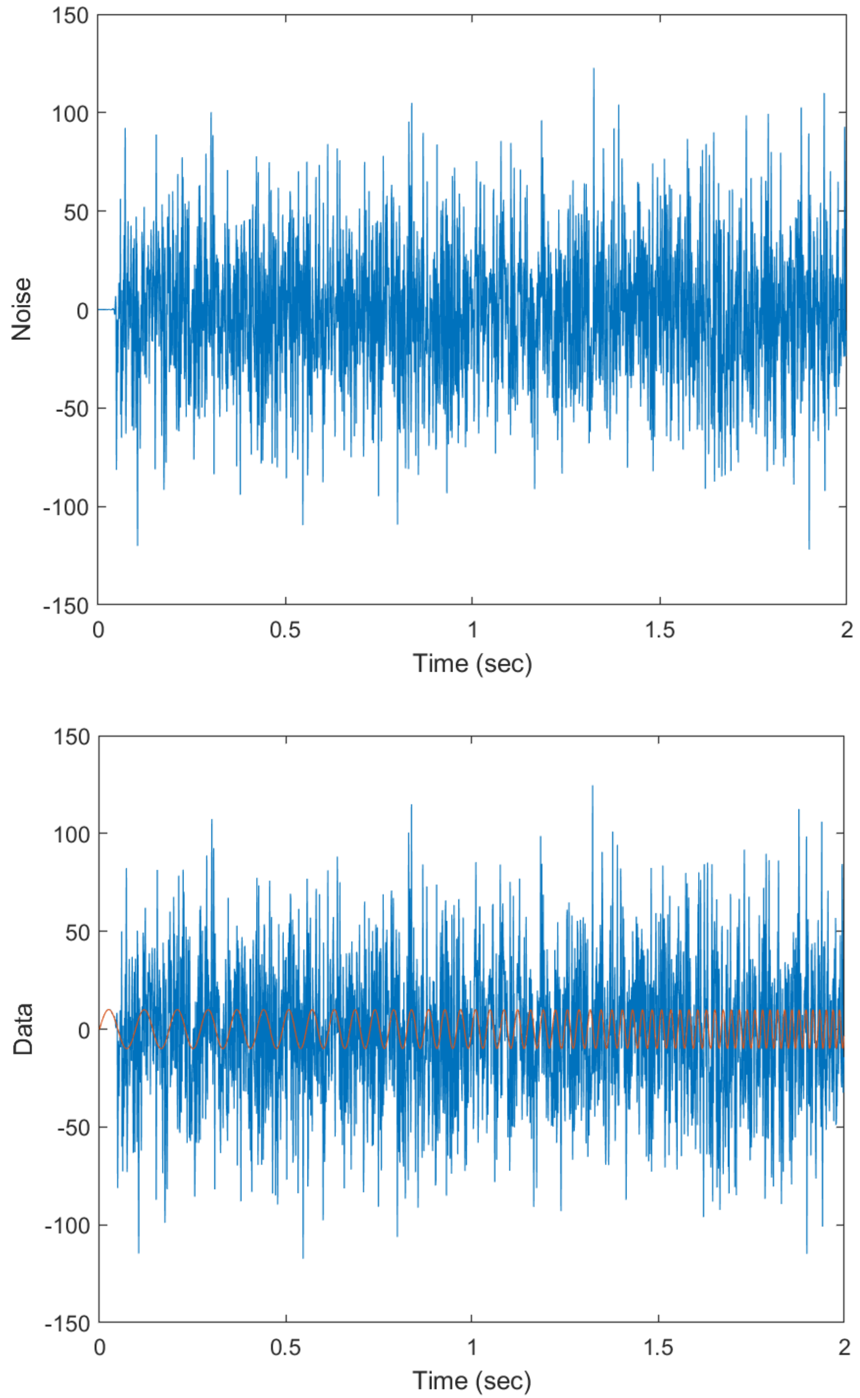
Where $\tilde{x} = \boldsymbol{F} \bar{x}$ is the DFT

T is the length of the data $(\frac{\Delta}{N} = \frac{1}{N\,f_s} = \frac{1}{\mathrm{T}})$

$\bar{S}_n$ is $S_n(f)$ evaluated at the DFT frequencies

Normalization of signal amplitude: SNR

- Objective:
    1. Generate an N sample realization of colored gaussian noise with some specified Power Spectral Density (PSD)
    2. Normalize a given signal vector of N samples such that the Likelihood Ratio test for noise with the above PSD has a specified Signal to Noise Ratio (SNR)



Codes

- Generating stationary Gaussian noise: **NOISE/statgaussnoisegen.m**
  - Script **NOISE/colGaussNoiseDemo.m** has been modified to use this function
  - The function supplies the right scale factor for the generated noise (earlier we only wanted to get the shape of the PSD right, not its overall scale)
- Taking an inner product: **DETEST/innerprodpsd.m**
- Demo for normalization of signal: **DETEST/SNRcalc.m**

Exercise #1

- Follow the script DETEST/SNRcalc.m and write a function that calculates the normalization factor for a given signal and noise PSD
  - Inputs:
    - Signal vector
    - Sampling frequency
    - PSD at positive DFT frequencies
    - SNR
  - Outputs:
    - Normalized signal vector
    - Normalization factor
- Code should do a sanity check: Number of PSD values should match the number of positive DFT frequencies based on the length of the signal vector

Exercise #2

- Generate a realization of initial LIGO noise using **NOISE/statgaussnoisegen.m**
- Add the signal your team was assigned with an SNR=10 to the noise realization
- Time domain: Plot the data (signal+noise) realization and the signal
- Frequency domain: Plot the periodogram of the noise and data together
- Time-frequency domain: Plot the spectrogram of the data
- Can you see the signal in any of the three domains? Namely, the time, or the Fourier, or the time-frequency domains

Exercise #3

- Follow DETEST/SNRcalc.m to write a function to calculate the Likelihood Ratio for a given data vector
  - What are the inputs needed?
- Run this function over M realizations of data under H_0 and M realizations of data under H_1 $\rightarrow$ Estimate the SNR of the LR test as shown in SNRcalc.m
- Does the estimated SNR come close to the one you had normalized the signal with? (It should!)