How to normalize a signal for a given SNR

We will normalize a signal such that the Likelihood ratio (LR) test for it has a given signal-to-noise ratio (SNR) in noise with a given Power Spectral Density (PSD). [We often shorten this statement to say: "Normalize the signal to have a given SNR." It is understood in this statement that one is talking about the LR for Gaussian noise as the detection statistic.

Contents

We will reuse codes that have already been written. Path to folder containing signal and noise generation codes

```
addpath ../DSP/
addpath ../NOISE/
```

This is the target SNR

```
snr = 10;
```

Data generation parameters

```
nSamples = 2048;
sampFreq = 1024;
timeVec = (0:(nSamples-1))/sampFreq;
```

Generate the signal that is to be normalized

```
a1=10;
a2=3;
a3=3;
% Amplitude value does not matter as it will be changed in the normalization
A = 1;
sigVec = crcbgenqcsig(timeVec,1,[a1,a2,a3]);
```

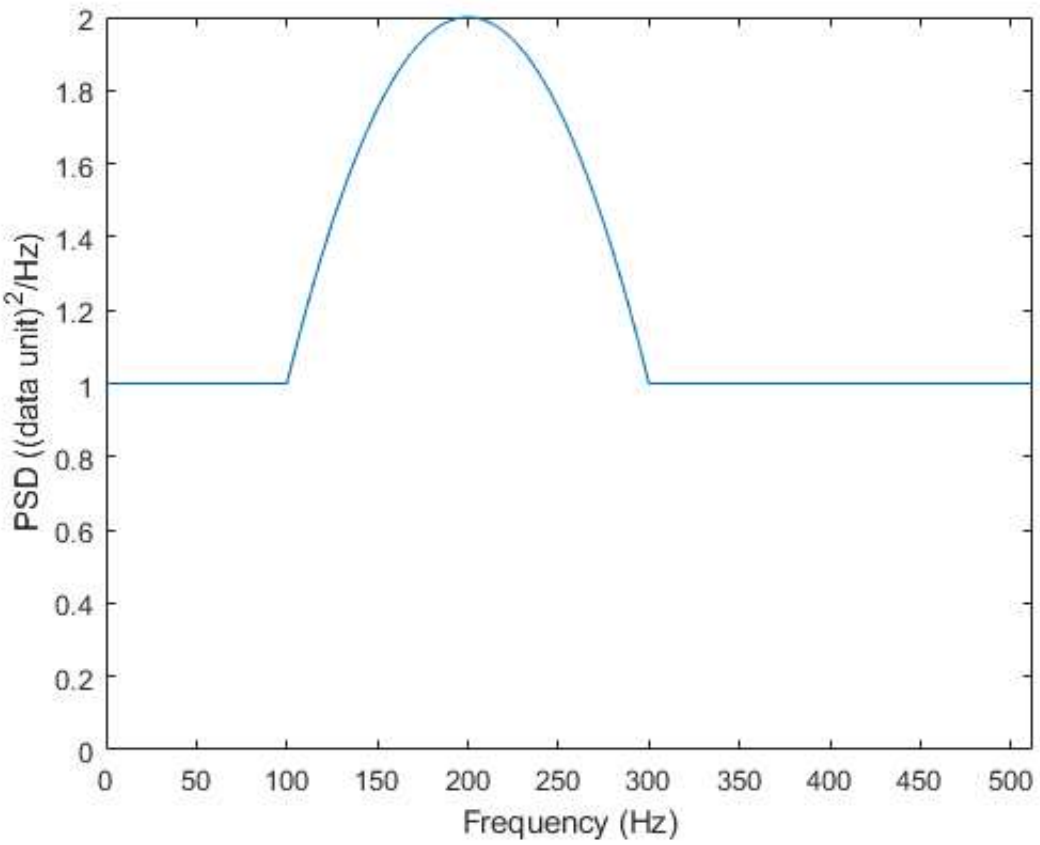
We will use the noise PSD used in colGaussNoiseDemo.m but add a constant to remove the parts that are zero. (Exercise: Prove that if the noise PSD is zero at some frequencies but the signal added to the noise is not, then one can create a detection statistic with infinite SNR.)

```
noisePSD = @(f) (f>=100 & f<=300).*(f-100).*(300-f)/10000 + 1;
```

Generate the PSD vector to be used in the normalization. Should be generated for all positive DFT frequencies.

```
dataLen = nSamples/sampFreq;
```

```
kNyq = floor(nSamples/2)+1;
posFreq = (0:(kNyq-1))*(1/dataLen);
psdPosFreq = noisePSD(posFreq);
figure;
plot(posFreq,psdPosFreq);
axis([0,posFreq(end),0,max(psdPosFreq)]);
xlabel('Frequency (Hz)');
ylabel('PSD ((data unit)^2/Hz)');
```



Calculation of the norm

Norm of signal squared is inner product of signal with itself

```
normSigSqrd = innerprodpsd(sigVec,sigVec,sampFreq,psdPosFreq);
% Normalize signal to specified SNR
sigVec = snr*sigVec/sqrt(normSigSqrd);
```

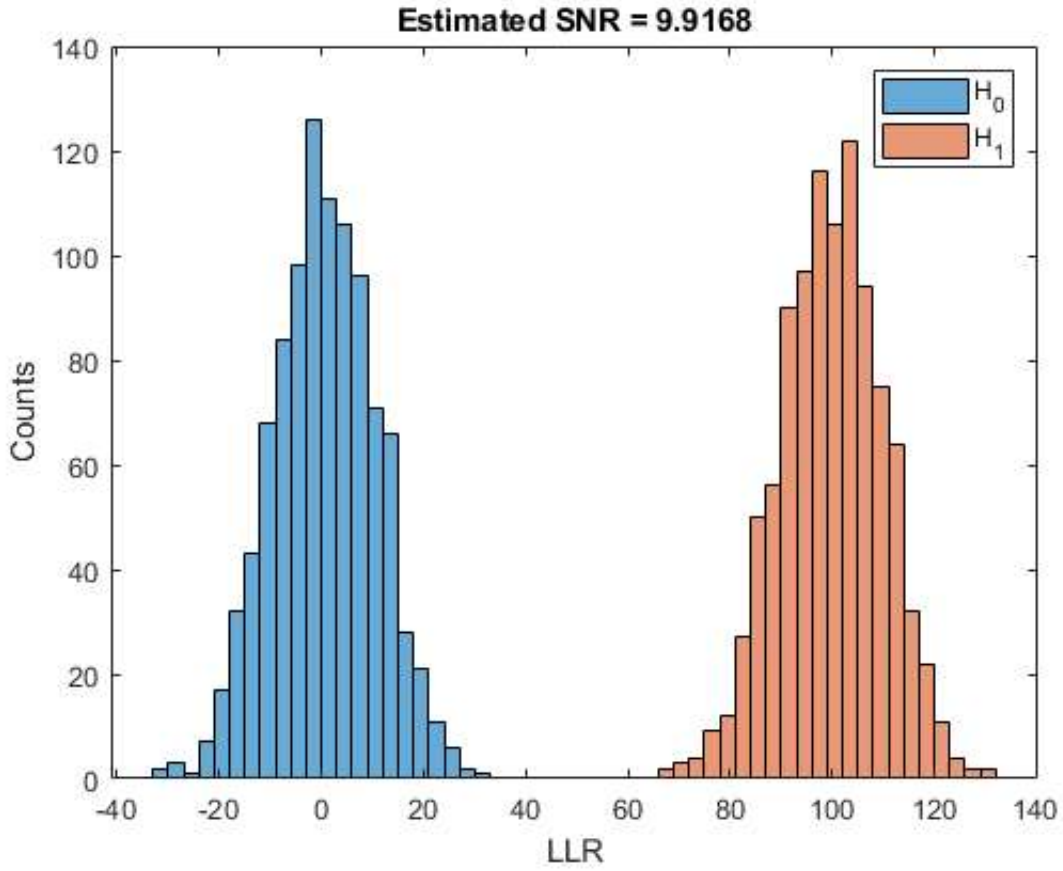
Test

```
%Obtain LLR values for multiple noise realizations
nH0Data = 1000;
llrH0 = zeros(1,nH0Data);
for lp = 1:nH0Data
    noiseVec = statgaussnoisegen(nSamples,[posFreq(:),psdPosFreq(:)],100,sampFreq);
    llrH0(lp) = innerprodpsd(noiseVec,sigVec,sampFreq,psdPosFreq);
end
```

```
%Obtain LLR for multiple data (=signal+noise) realizations
nH1Data = 1000;
llrH1 = zeros(1,nH1Data);
for lp = 1:nH0Data
    noiseVec = statgaussnoisegen(nSamples,[posFreq(:),psdPosFreq(:)],100,sampFreq);
    % Add normalized signal
    dataVec = noiseVec + sigVec;
    llrH1(lp) = innerprodpsd(dataVec,sigVec,sampFreq,psdPosFreq);
end
```

Signal to noise ratio estimate

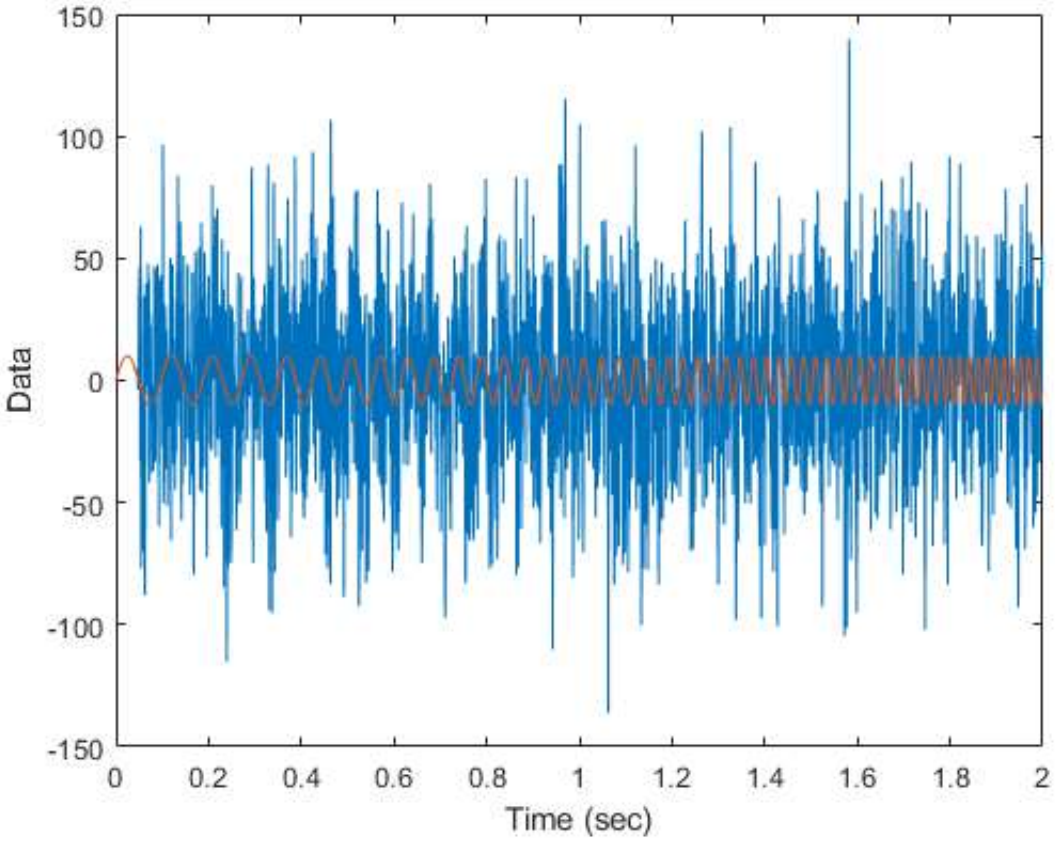
```
estSNR = (mean(llrH1)-mean(llrH0))/std(llrH0);

figure;
histogram(llrH0);
hold on;
histogram(llrH1);
xlabel('LLR');
ylabel('Counts');
legend('H_0','H_1');
title(['Estimated SNR = ',num2str(estSNR)]);
```



A data realization

```
figure;
```

```
plot(timeVec,dataVec);
hold on;
plot(timeVec,sigVec);
xlabel('Time (sec)');
ylabel('Data');
```



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