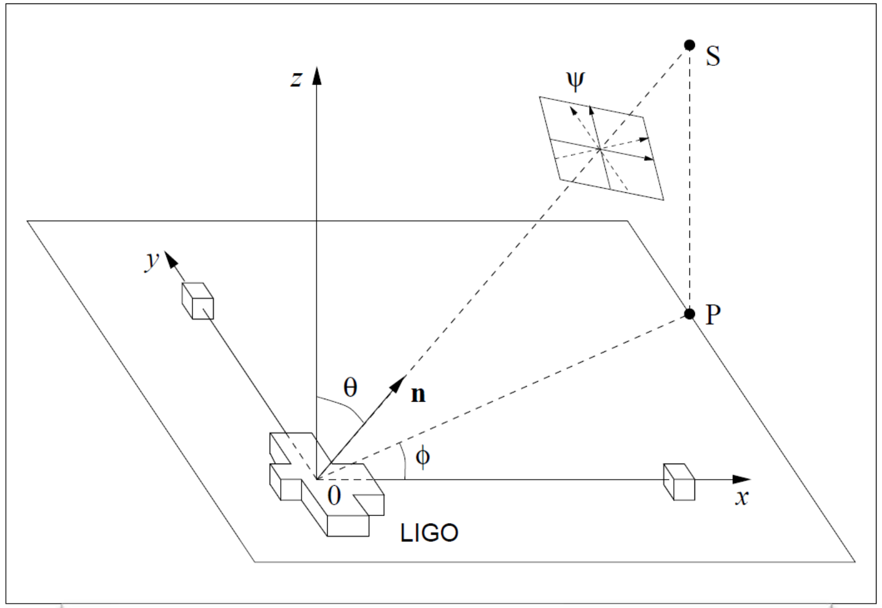
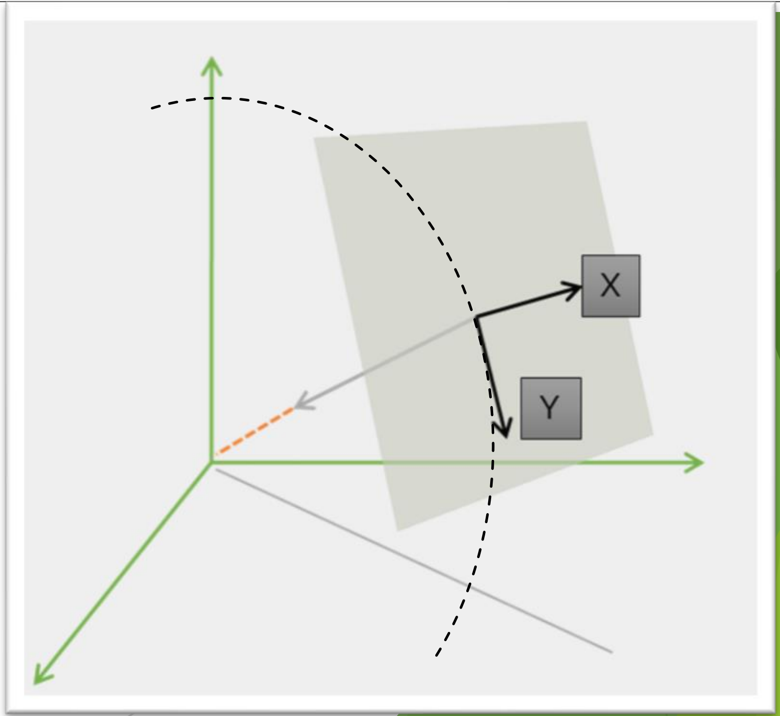
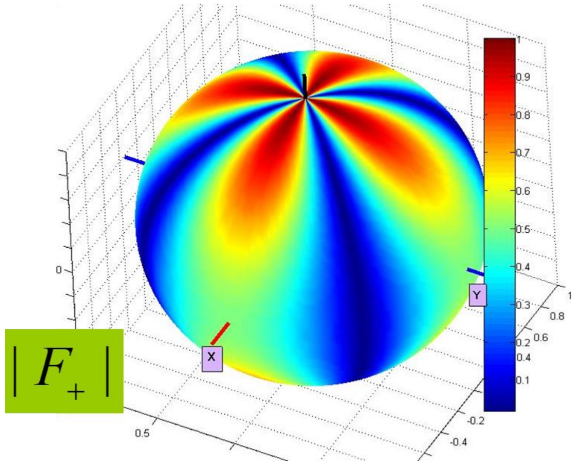
Lab Topic 2

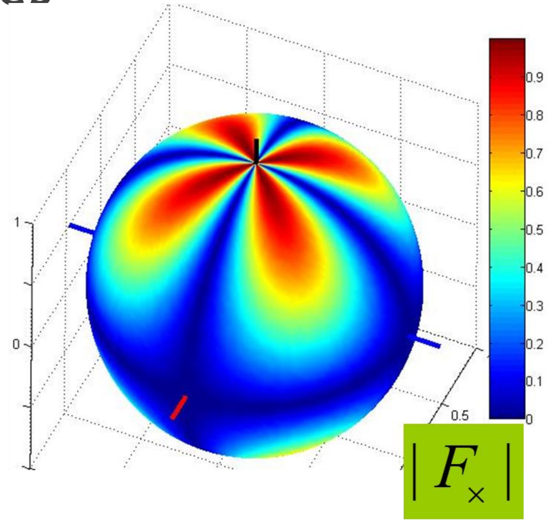
Antenna Patterns for LIGO

- Long wavelength and static detector approximation
- Write a code to calculate F_+ and $F_\times$ in an interferometer's local frame for input source direction
  - Source direction: (θ, ϕ) in detector frame
  - Plot them on a sphere using the **GWSIG/skyplot.m** function
- **Use burst convention**: Should agree with the pictures in the lecture slides



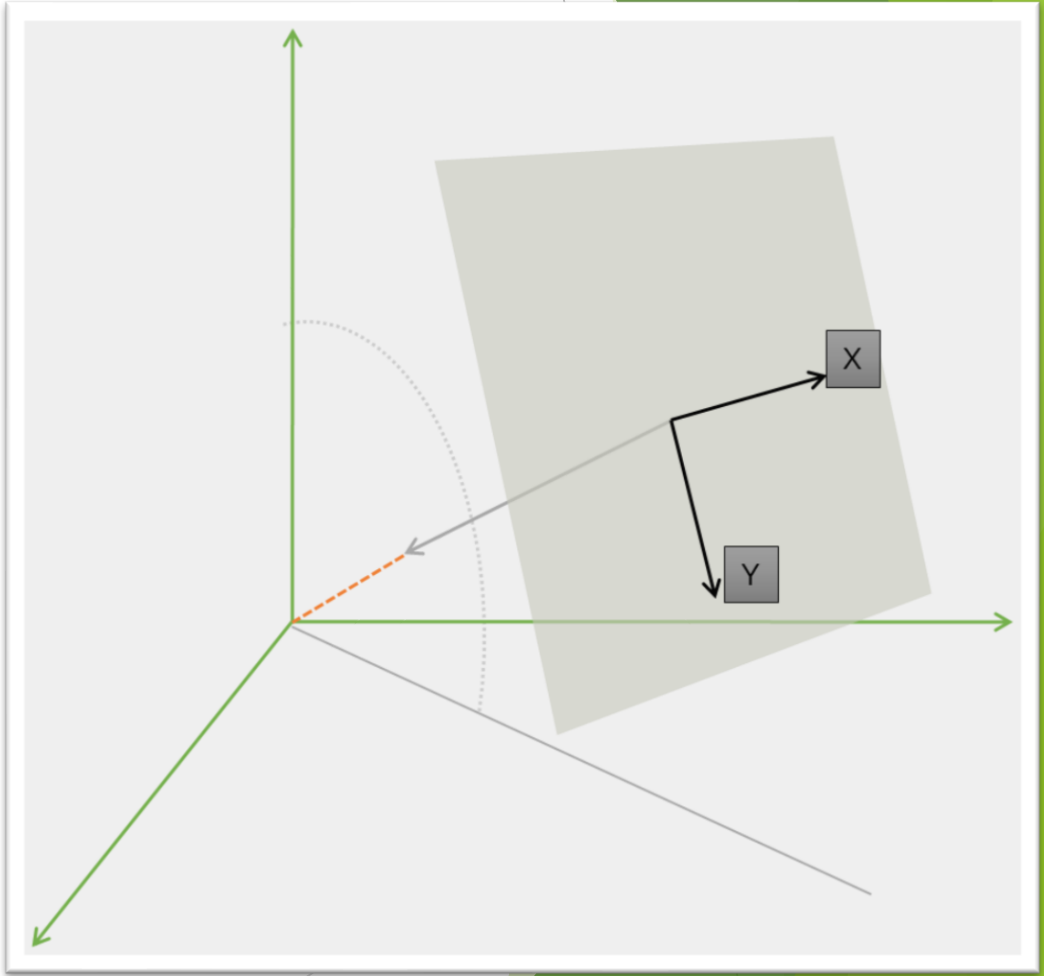






Obtaining wave tensor

- Use the expression for (a) polarization tensors, (b) Detector tensor, and (c) Contraction of polarization and detector tensors
- Use vector cross products (all vector components in detector frame):
  - Use **GWSIG/vcrossprod.m**
  - Detector frame $\hat{Z} = (0,0,1)$
  - Source direction in detector frame (for polar angles θ and ϕ): $\hat{n}$
  - Wave frame $\hat{x} \propto \hat{Z} \times \hat{n}$ (Note: must normalize)
  - Wave frame $\hat{y} = \hat{x} \times \hat{n}$
- If x and y are row matrices in Matlab:
  - x' is the transpose of x (Column matrix)
  - $x' * y$ is the same as $x \otimes y$



Strain signal

- Detector tensor:

$$\overleftrightarrow{D} = \frac{1}{2}(\hat{n}_X \otimes \hat{n}_X - \hat{n}_Y \otimes \hat{n}_Y)$$

- Wave tensor:

$$\overleftrightarrow{W} = h_+(t)\,\overleftrightarrow{e}_+ + h_\times(t)\overleftrightarrow{e}_\times$$

$$\overleftrightarrow{e}_+ = \hat{x}\otimes\hat{x} - \hat{y}\otimes\hat{y}; \quad \overleftrightarrow{e}_\times = \hat{x}\otimes\hat{y} + \hat{y}\otimes\hat{x}$$

- Matlab can calculate direct product:

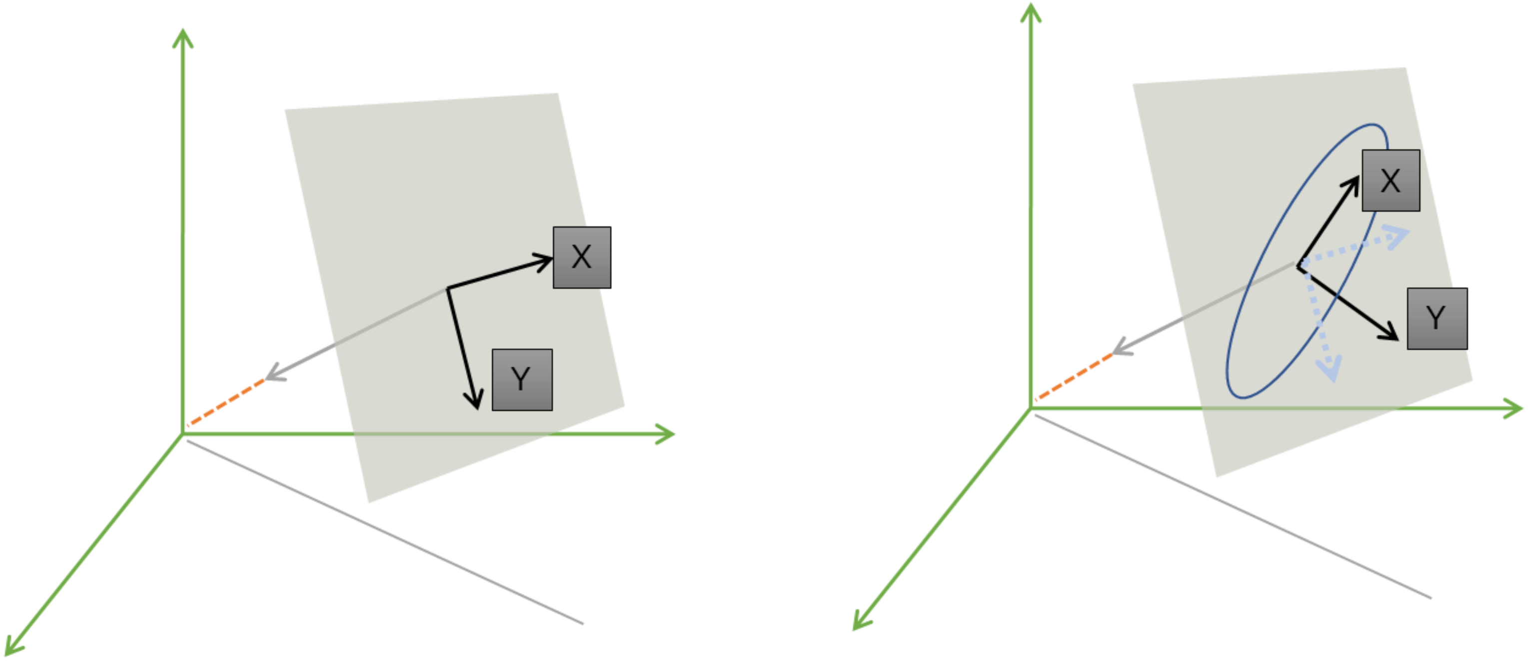
$$\underbrace{\left.\begin{matrix} a = [a_1, & a_2, & a_3] \\ b = [b_1, & b_2, & b_3] \end{matrix}\right\}}_{\text{2 row vectors in Matlab}} \rightarrow \underbrace{a' * b}_{Matlab} \rightarrow \hat{a}\otimes\hat{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}(b_1 \quad b_2 \quad b_3) = \begin{bmatrix} a_1b_1 & a_1b_2 & a_1b_3 \\ a_2b_1 & a_2b_2 & a_2b_3 \\ a_3b_1 & a_3b_2 & a_3b_3 \end{bmatrix}$$

- Strain signal: “Contraction of wave and detector tensors”

$$s(t) = \sum_{i,j=1}^{3} W_{ij}D_{ij} = W^{ij}D_{ij} = \overleftrightarrow{W}:\overleftrightarrow{D}$$

$$\rightarrow \mathrm{sum}(W(:)\ .*\ D(:))$$

- To use the above formula, all unit vector components must be written down in the same reference frame



Wave frame conventions

Include rotation due to polarization angle into the polarization tensors

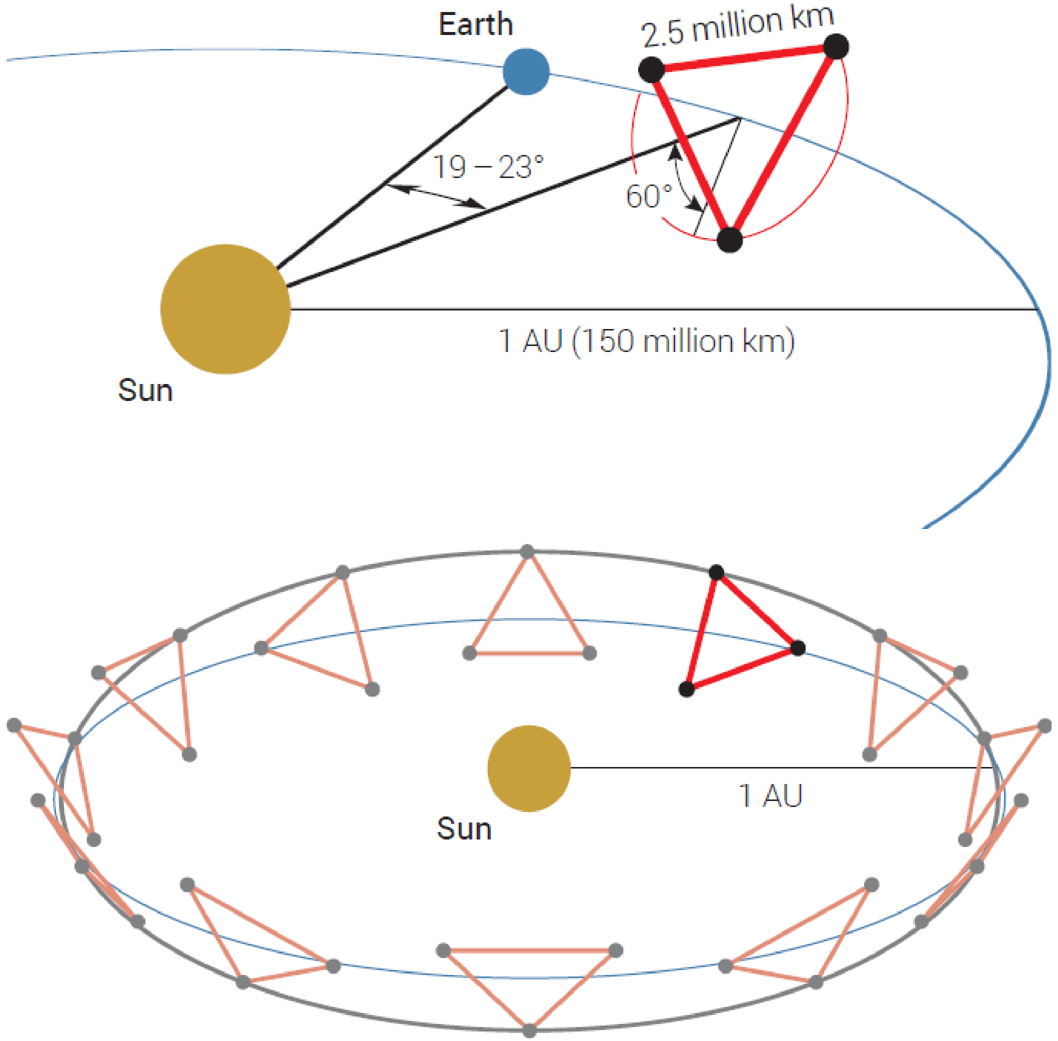
Antenna patterns for LISA

Toy LISA

- Rigid equilateral triangle formation of three satellites
- Actual LISA cannot be rigid because the satellites must follow Keplerian orbits
- Toy LISA is good for practicing data analysis because it allows fast generation of signals and templates
- However, Mock LISA Challenge Data requires more accurate LISA models

Reference frames and rotations needed

- We need to find the components of LISA arm vectors in the Solar System Barycentric (SSB) frame
- Get the detector tensor from the arm vectors
- The wave tensor will be written as usual (e.g., detector local frame) in the SSB
- Do contractions to get antenna pattern functions
- Introduce time delay: light travel travel time between the SSB and LISA centroid

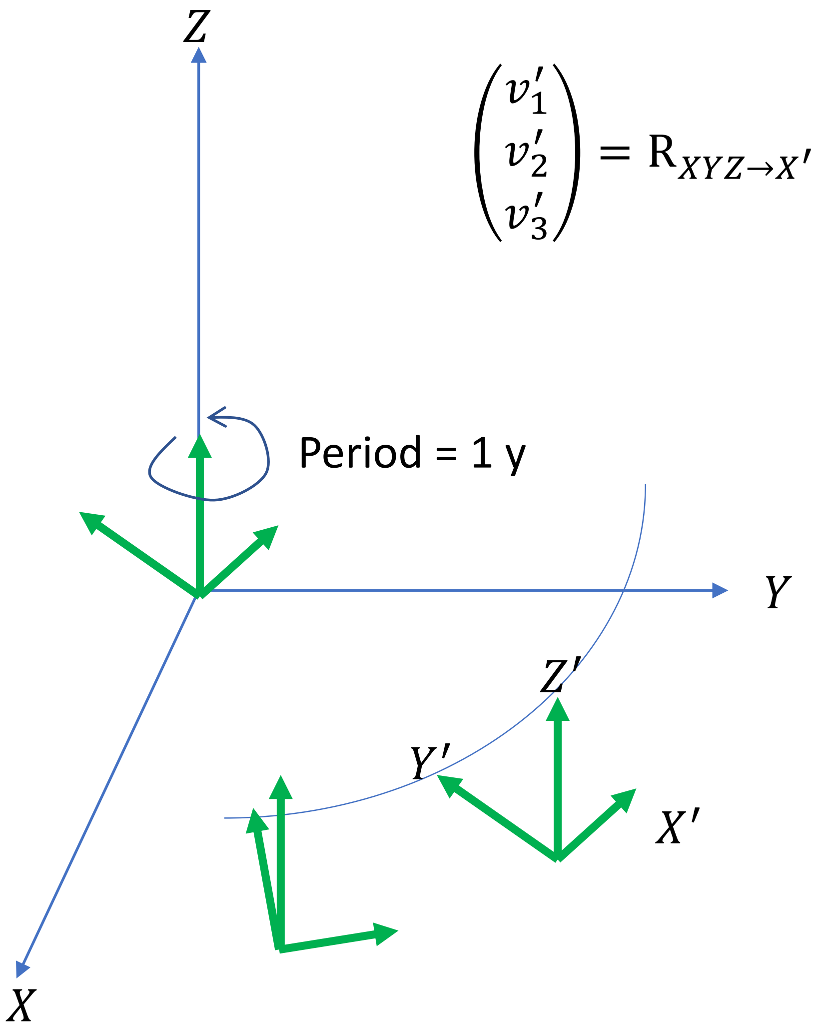


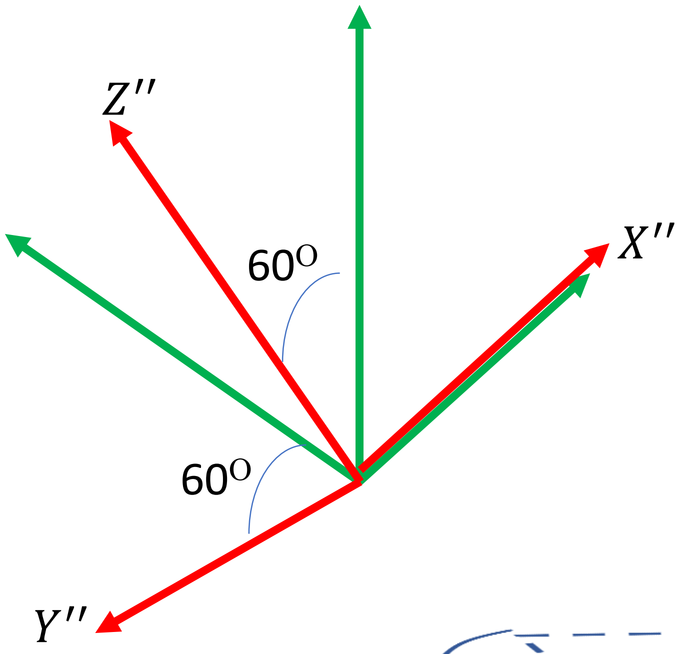
Obtain the arm components in the SSB frame

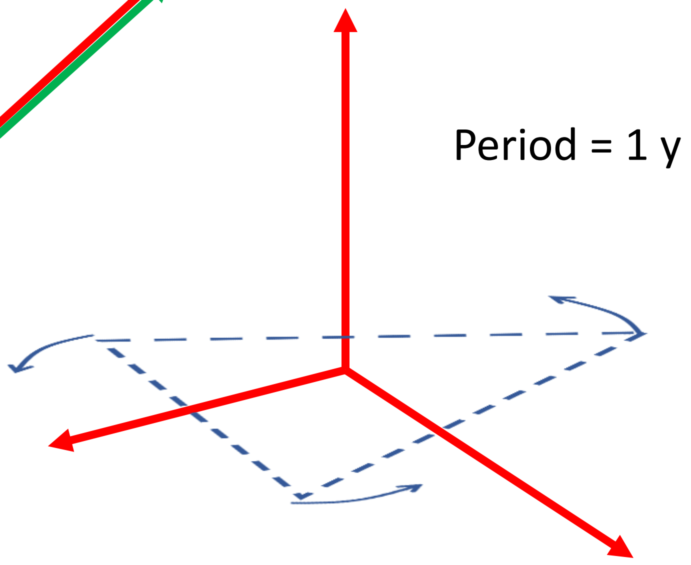
$$\begin{pmatrix} v'_1 \\ v'_2 \\ v'_3 \end{pmatrix} = \mathrm{R}_{XYZ\to X'Y'Z'} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$$

$$\mathrm{R}_{XYZ\to X'Y'Z'} = \begin{pmatrix} \hat{x}'.\hat{x} & \hat{x}'.\hat{y} & \hat{x}'.\hat{z} \\ \hat{y}'.\hat{x} & \hat{y}'.\hat{y} & \hat{y}'.\hat{z} \\ \hat{z}'.\hat{x} & \hat{z}'.\hat{y} & \hat{z}'.\hat{z} \end{pmatrix}$$

$$RR^T = I$$







Antenna patterns

- Use the expressions in Sec IIIB of the paper arXiv:1207.4956v1 to obtain the detector tensors for the two Michelson TDI combinations
- Obtain the antenna patterns for each TDI combination by contracting each polarization tensor with the respective detector tensor
- Write a code:
  - Inputs: Source direction, time instant
  - Outputs: F_+, F_x for each TDI combination

To do

- Generate h_+, h_x that are sinusoidal
- Assume some sky location and polarization angle for the GW source
- Generate the detector response of LISA (no doppler shift included)
- Take FFT of the detector response and compare to the FFT of h_+

LISA response

- LISA antenna pattern generation
  - Use detector tensors from arXiv:1207, Sec III.B (one TDI combination only)
- LISA detector response including doppler shift
  - $h_{+,\times}(t) \to h_{+,\times}(t - \frac{\hat{n}.\bar{x}_d}{c})$
  - $\hat{n}$: Wave propagation direction
  - $\bar{x}_d(t)$: LISA centroid
- Plot LISA response for a monochromatic source
  - $h_+(t) = A\sin(\omega_0 t)\,; h_\times(t) = \left(\frac{A}{2}\right)\cos\omega_0 t$
  - One parameter missing: polarization angle (but we will ignore it)
  - Compare FFT of the response to that of $h_+(t)$

Doppler shifts

Detector response: Moving detector

- Detector located at $\bar{x}_d$ in the plane GW wave field
- Response: $s(t) = W_{ij}D^{ij} = F_+(\hat{n})h_+\left(\omega t - \bar{k}.\bar{x}_d\right) + F_\times(\hat{n})h_\times(\omega t - \bar{k}.\bar{x}_d)$
- $\hat{n}$: Direction to GW source
- $\bar{k} = -2\pi\hat{n}/\lambda; \qquad \omega = 2\pi f; \qquad f\lambda = c$
- For a moving detector, $\bar{x}_d$ is a function of time

To do

- Write a code to calculate the position vector of the LISA centroid
- Take the same GW sinusoidal signals as before and calculate the two TDI responses but including the doppler shifts this time.

Direction determination

Effect of sky location

- Take the same h_+, h_x but at different sky locations and see that the waveforms differ.
- This allows LISA to acquire directionality for long-lived sources.