Lab Topic 4

SNR of LR test

- Joint pdf of noise:

$$p_{\bar{E}}(\bar{y}) = \frac{1}{(2\pi)^{N/2}|\mathbf{C}|^{1/2}} \exp\left(-\frac{1}{2}\|\bar{y}\|^2\right)$$

$$\|\bar{y}\|^2 = \bar{y}\boldsymbol{C}^{-1}\bar{y}^T$$

  - Let $\langle \bar{z}, \bar{y} \rangle = \bar{z}\boldsymbol{C}^{-1}\bar{y}^T$
  - LR statistic : $\Lambda = \frac{1}{2}\|\bar{y}\|^2 - \frac{1}{2}\|\bar{y} - \bar{s}\|^2 = \langle \bar{y}, \bar{s} \rangle - \frac{1}{2}\langle \bar{s}, \bar{s} \rangle \rightarrow \Lambda = \langle \bar{y}, \bar{s} \rangle$
  - SNR: $\|\bar{s}\|$

Inner product for stationary noise

- White Gaussian Noise

$$\langle \bar{x}, \bar{y} \rangle \to \sum_{k=0}^{N-1} x_k y_k$$

- Stationary Gaussian noise with Power Spectral Density (PSD) $S_n(f)$

$$\langle \bar{x}, \bar{y} \rangle \to \frac{\Delta}{N} \tilde{x}(\tilde{y}^\dagger \,./\, \bar{S}_n{}^T) = \frac{1}{T} \tilde{x}(\tilde{y}^\dagger \,./\, \bar{S}_n{}^T)$$

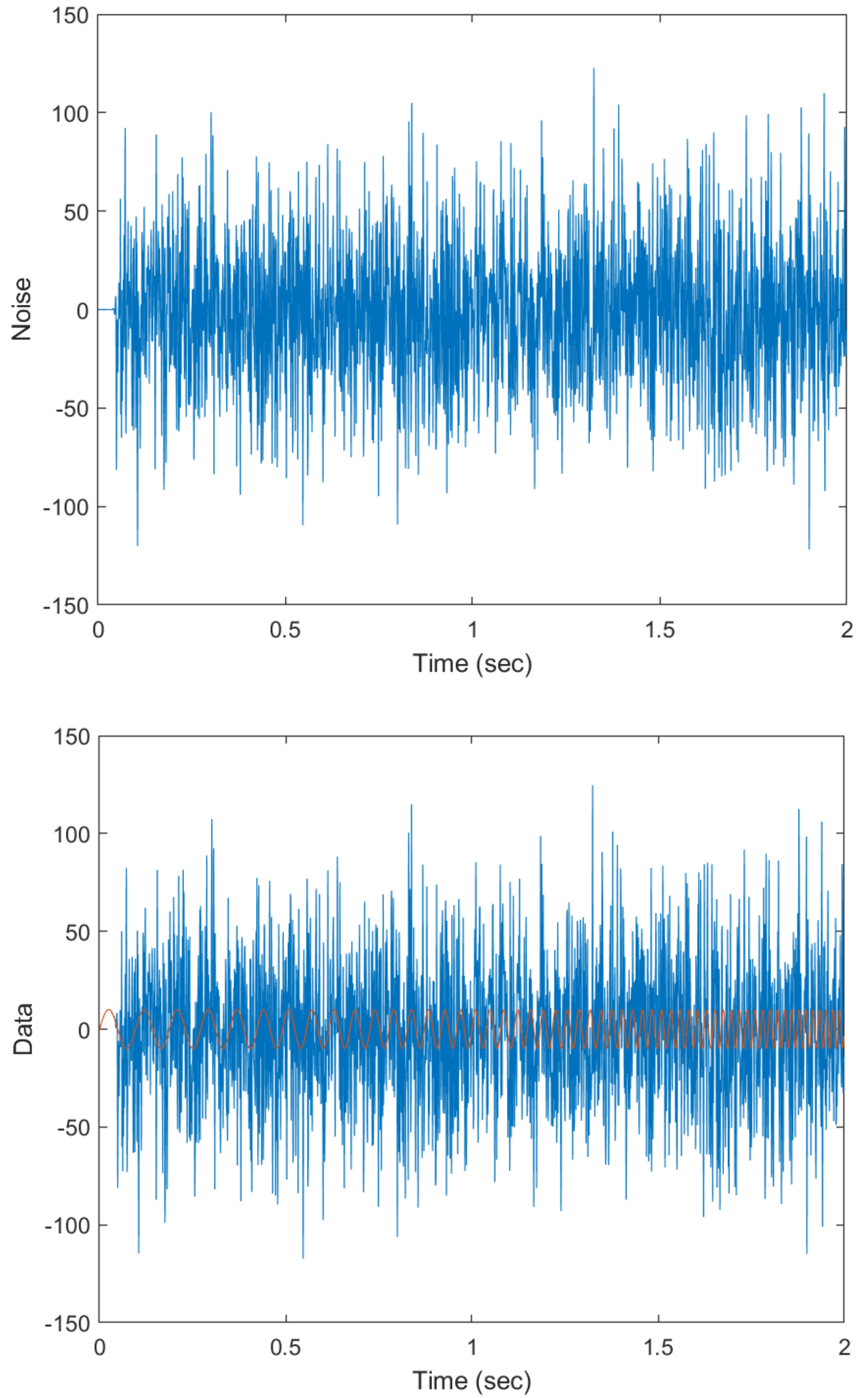
Where $\tilde{x} = \boldsymbol{F}\bar{x}$ is the DFT

T is the length of the data $\left(\frac{\Delta}{N} = \frac{1}{N\, f_s} = \frac{1}{\mathrm{T}}\right)$

$\bar{S}_n$ is $S_n(f)$ evaluated at the DFT frequencies

Normalization of signal amplitude: SNR

- Objective:
  1. Generate an N sample realization of colored gaussian noise with some specified Power Spectral Density (PSD)
  2. Normalize a given signal vector of N samples such that the Likelihood Ratio test for noise with the above PSD has a specified Signal to Noise Ratio (SNR)



Codes

- Generating stationary Gaussian noise: **NOISE/statgaussnoisegen.m**
  - Script **NOISE/colGaussNoiseDemo.m** has been modified to use this function
  - The function supplies the right scale factor for the generated noise (earlier we only wanted to get the shape of the PSD right, not its overall scale)
- Taking an inner product: **DETEST/innerprodpsd.m**
- Demo for normalization of signal: **DETEST/SNRcalc.m**

Exercise #1

- Follow the script **DETEST/SNRcalc.m** and write a function that calculates the normalization factor for a given signal and noise PSD
  - Inputs:
    - Signal vector
    - Sampling frequency
    - PSD at positive DFT frequencies
    - SNR
  - Outputs:
    - Normalized signal vector
    - Normalization factor
- Code should do a sanity check: Number of PSD values should match the number of positive DFT frequencies based on the length of the signal vector

Exercise #2

- Generate a realization of initial LIGO noise using **NOISE/statgaussnoisegen.m**
- Add the signal your team was assigned with an SNR=10 to the noise realization
- Time domain: Plot the data (signal+noise) realization and the signal
- Frequency domain: Plot the periodogram of the noise and data together
- Time-frequency domain: Plot the spectrogram of the data
- Can you see the signal in any of the three domains? Namely, the time, or the Fourier, or the time-frequency domains

Exercise #3

- Follow **DETEST/SNRcalc.m** to write a function to calculate the Likelihood Ratio for a given data vector
  - What are the inputs needed?
- Run this function over M realizations of data under H_0 and M realizations of data under H_1 $\rightarrow$ Estimate the SNR of the LR test as shown in SNRcalc.m
- Does the estimated SNR come close to the one you had normalized the signal with? (It should!)

Test significance

- For any detection statistic $\Gamma(\bar{y})$, a function of the data $\bar{y}$, the **significance** of detection made with the statistic is defined as :
  - "The probability of observing a value of $\Gamma \geq$ its observed value for given data if the null hypothesis is true"

$$\Pr\left\{\Gamma \geq \Gamma^{(observed)} \middle| H_0\right\} = \int_{\Gamma^{(observed)}}^{\infty} \underbrace{p_\Gamma(x|H_0)}_{Estimated}\, dx$$

$\Gamma^{(observed)} = \Gamma(\bar{y})$: The value of Γ obtained for the given data $\bar{y}$

Estimating probability

$$\Pr\left\{\Gamma \geq \Gamma^{(observed)} \middle| H_0\right\} = \int_{\Gamma^{(observed)}}^{\infty} \underbrace{p_\Gamma(x|H_0)}_{Estimated}\, dx$$

- Frequentist probability: Obtain a large number of trial values of Γ
  - Trial value $\Rightarrow$ generate a data realization and compute the Γ value for it
  - Then,

$$\Pr\{\Gamma \geq x\} \approx \frac{Number\ of\ times\ \Gamma \geq x}{Total\ number\ of\ data\ \ realizations}$$

- The approximation above becomes better as the total number of trials increases

Exercise #4: Part 1

- Write a function to calculate the GLRT for the unknown amplitude case

$$L_G = \langle \bar{y}, \bar{q} \rangle^2 \text{ where } \bar{q} = \frac{\bar{s}}{||\bar{s}||}$$

  - Inputs:
    - Data vector $\bar{y}$
    - Noise PSD at positive DFT frequencies
  - Output:
    - GLRT value
- This function is similar to the one you wrote earlier in Exercise #3 except that the data inner product is computed with the template vector $\bar{q}$, which is the signal vector normalized to have SNR=1
  - Note: A new function **DETEST/normsig4psd.m** is available for normalizing a signal (it is the solution to **Lab Topic 4→Exercise #1**)

Exercise #4: Part 2

- You have been provided 3 data realizations named **DETEST/data<n>.mat** where $n = 0, 1, 3$
- You are told that the signal in each data realization is a quadratic chirp but the amplitude is unknown (maybe zero also!)
- Signal parameters: a1=10; a2=3; a3=3;
- You will calculate the GLRT for this signal, using the function coded in **Exercise #4: Part1,** for each of the 3 data realizations
  - The quadratic chirp signal is the one coded in **DSP/crcbgenqcsig.m**
  - and the noise PSD used is the one in **DETEST/SNRCalc.m**

    ```
    noisePSD = @(f) (f>=100 & f<=300).*(f-100).*(300-f)/10000 + 1;
    ```
- Use simulations to estimate the significance of the GLRT values for the 3 data realizations
- Finally, **git push** a text files into **DETEST** called **Team<n>_testSig.txt** containing the significances you found for the 3 data realizations (each value on a new line starting from $n = 1$). In the last line of the file, write the number of data realizations your simulated to estimate the significance

Exercise #5

- Extend the function in Exercise #4 Part 1 to create a new function:
  - Inputs:
    - Data vector
    - Parameter values $a_1,\ a_2,\ a_3$ for a quadratic chirp
    - Noise PSD vector at positive DFT frequencies
  - Output:
    - value of Likelihood ratio maximized over amplitude: $\langle \bar{y}, \bar{q}(\Theta) \rangle^2$

      where Θ is the set of input parameters $a_1,\ a_2,\ a_3$
- See the script **DETEST/LRcalc.m** for an example of the steps involved for a specific set of values for Θ
- Test: Repeat **Exercise #4 Part 2** using this function: You should get identical values for the significance

Conclusion: Lab Topic 4

- The function created in Exercise #5 will serve as the fitness function that will be optimized by PSO to obtain the GLRT
$$L_G = \max_{\Theta} \langle \bar{y}, \bar{q}(\Theta) \rangle^2$$
- This optimization problem and the results associated with it are fully discussed in the textbook
- The main difference is that we have generalized the textbook problem to the case of colored Gaussian noise