### **用AdaBoost算法学习一个强分类器**

****训练数据集****

| **序号** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| y | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 |

****解：****  
初始化数据权值分布  
D1=(w1,1,w1,2,…,w1,10)w1,i=0.1,i=1,2,…,10D\_1=(w\_{1,1},w\_{1,2},\dots,w\_{1,10})\\w\_{1,i}=0.1,i=1,2,\dots,10*D*1​=(*w*1,1​,*w*1,2​,…,*w*1,10​)*w*1,*i*​=0.1,*i*=1,2,…,10  
对于m=1m=1*m*=1,  
  (a)在权值分布为D1D\_1*D*1​的训练数据上，计算阈值ν\nu*ν*取[0.5,1.5,2.5,3.5,4.5,5.5,6.5,7.5,8.5]时分类误差率，

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ν\nu*ν* | 0.5 | 1.5 | 2.5 | 3.5 | 4.5 | 5.5 | 6.5 | 7.5 | 8.5 |
| 分类误差率 | 0.5 | 0.4 | 0.3 | 0.4 | 0.5 | 0.4 | 0.5 | 0.4 | 0.3 |

阈值取ν=8.5\nu=8.5*ν*=8.5时分类误差率最低，故基本分类器为  
G1(x)={1,−1,x&lt;8.5x≥8.5G\_1(x)=\begin{cases}1,&amp;x\lt8.5 \\-1,&amp;x\ge8.5\end{cases}*G*1​(*x*)={1,−1,​*x*<8.5*x*≥8.5​  
  (b)G1(x)G\_1(x)*G*1​(*x*)在训练数据集上的误差率e1=P(G1(xi)≠yi)=0.3e\_1=P(G\_1(x\_i)\neq y\_i) =0.3*e*1​=*P*(*G*1​(*xi*​)̸​=*yi*​)=0.3  
  ©计算G1(x)G\_1(x)*G*1​(*x*)的系数:α1=12log1−e1e1=0.4236\alpha\_1=\dfrac{1}{2}log\dfrac{1-e\_1}{e\_1}=0.4236*α*1​=21​*loge*1​1−*e*1​​=0.4236  
  (d)更新训练数据的权值分布：  
D2=(w2,1,w2,2,…,w2,10)D\_2=(w\_{2,1},w\_{2,2},\dots,w\_{2,10})*D*2​=(*w*2,1​,*w*2,2​,…,*w*2,10​)  
w2,i=w1,iZ1exp(−α1yiG1(xi)),i=1,2,…,10w\_{2,i} = \dfrac{w\_{1,i}}{Z\_1}exp(-\alpha\_1y\_iG\_1(x\_i)),i=1,2,\dots,10*w*2,*i*​=*Z*1​*w*1,*i*​​*exp*(−*α*1​*yi*​*G*1​(*xi*​)),*i*=1,2,…,10  
D2=(0.07142857,0.07142857,0.07142857,0.16666667,0.16666667,0.16666667,0.07142857,0.07142857,0.07142857,0.07142857)D\_2=(0.07142857,0.07142857,0.07142857,0.16666667,0.16666667,0.16666667,0.07142857,0.07142857,0.07142857,0.07142857)*D*2​=(0.07142857,0.07142857,0.07142857,0.16666667,0.16666667,0.16666667,0.07142857,0.07142857,0.07142857,0.07142857)  
f1(x)=α1G1(x)=0.4236G1(x)f\_1(x)=\alpha\_1G\_1(x)=0.4236G\_1(x)*f*1​(*x*)=*α*1​*G*1​(*x*)=0.4236*G*1​(*x*)  
  (e)分类器sign[f1(x)]sign[f\_1(x)]*sign*[*f*1​(*x*)]在训练数据集上有3个误分点

| **序号** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| G1(x)G\_1(x)*G*1​(*x*) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 |
| f1(x)f\_1(x)*f*1​(*x*) | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | -0.4236 |
| sign[f1(x)]sign[f\_1(x)]*sign*[*f*1​(*x*)] | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 |
| y | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 |

对m=2m=2*m*=2,  
  (a)在权值分布为D2D\_2*D*2​的训练数据上，计算阈值ν\nu*ν*取[0.5,1.5,2.5,3.5,4.5,5.5,6.5,7.5,8.5]时分类误差率，em=∑Gm(xi)≠yiwmie\_m=\sum\_{G\_m(x\_i)\neq y\_i} w\_{mi}*em*​=∑*Gm*​(*xi*​)̸​=*yi*​​*wmi*​

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ν\nu*ν* | 0.5 | 1.5 | 2.5 | 3.5 | 4.5 | 5.5 | 6.5 | 7.5 | 8.5 |
| 分类误差率 | 0.357 | 0.286 | 0.214 | 0.381 | 0.452 | 0.286 | 0.358 | 0.429 | 0.5 |

阈值取ν=2.5\nu=2.5*ν*=2.5时分类误差率最低，故基本分类器为  
G2(x)={1,−1,x&lt;2.5x≥2.5G\_2(x)=\begin{cases}1,&amp;x\lt2.5 \\-1,&amp;x\ge2.5\end{cases}*G*2​(*x*)={1,−1,​*x*<2.5*x*≥2.5​  
  (b)G2(x)G\_2(x)*G*2​(*x*)在训练数据集上的误差率e2=P(G2(xi)≠yi)=0.214e\_2=P(G\_2(x\_i)\neq y\_i) =0.214*e*2​=*P*(*G*2​(*xi*​)̸​=*yi*​)=0.214  
  ©计算G2(x)G\_2(x)*G*2​(*x*)的系数:α2=12log1−e2e2=0.6496\alpha\_2=\dfrac{1}{2}log\dfrac{1-e\_2}{e\_2}=0.6496*α*2​=21​*loge*2​1−*e*2​​=0.6496  
  (d)更新训练数据的权值分布：  
D3=(w3,1,w3,2,…,w3,10)D\_3=(w\_{3,1},w\_{3,2},\dots,w\_{3,10})*D*3​=(*w*3,1​,*w*3,2​,…,*w*3,10​)  
w3,i=w2,iZ1exp(−α2yiG2(xi)),i=1,2,…,10w\_{3,i} = \dfrac{w\_{2,i}}{Z\_1}exp(-\alpha\_2y\_iG\_2(x\_i)),i=1,2,\dots,10*w*3,*i*​=*Z*1​*w*2,*i*​​*exp*(−*α*2​*yi*​*G*2​(*xi*​)),*i*=1,2,…,10  
D3=(0.04545452,0.04545452,0.04545452,0.10606056,0.10606056,0.10606056,0.16666675,0.16666675,0.16666675,0.04545452)D\_3=(0.04545452,0.04545452,0.04545452,0.10606056,0.10606056,0.10606056,0.16666675,0.16666675,0.16666675,0.04545452)*D*3​=(0.04545452,0.04545452,0.04545452,0.10606056,0.10606056,0.10606056,0.16666675,0.16666675,0.16666675,0.04545452)  
f2(x)=0.4236G1(x)+0.6496G2(x)f\_2(x)=0.4236G\_1(x) + 0.6496G\_2(x)*f*2​(*x*)=0.4236*G*1​(*x*)+0.6496*G*2​(*x*)  
  (e)分类器sign[f2(x)]sign[f\_2(x)]*sign*[*f*2​(*x*)]在训练数据集上有3个误分点

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| G1(x)G\_1(x)*G*1​(*x*) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 |
| G2(x)G\_2(x)*G*2​(*x*) | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| α1G1(x)\alpha\_1G\_1(x)*α*1​*G*1​(*x*) | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | -0.4236 |
| α2G2(x)\alpha\_2G\_2(x)*α*2​*G*2​(*x*) | 0.6496 | 0.6496 | 0.6496 | -0.6496 | -0.6496 | -0.6496 | -0.6496 | -0.6496 | -0.6496 | -0.6496 |
| sign[f2(x)]sign[f\_2(x)]*sign*[*f*2​(*x*)] | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| y | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 |

对m=3m=3*m*=3  
  (a)在权值分布为D3D\_3*D*3​的训练数据上，计算阈值ν\nu*ν*取[0.5,1.5,2.5,3.5,4.5,5.5,6.5,7.5,8.5]时分类误差率，em=∑Gm(xi)≠yiwmie\_m=\sum\_{G\_m(x\_i)\neq y\_i} w\_{mi}*em*​=∑*Gm*​(*xi*​)̸​=*yi*​​*wmi*​

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ν\nu*ν* | 0.5 | 1.5 | 2.5 | 3.5 | 4.5 | 5.5 | 6.5 | 7.5 | 8.5 |
| 分类误差率 | 0.409 | 0.455 | 0.5 | 0.394 | 0.288 | 0.182 | 0.348 | 0.485 | 0.318 |

阈值取ν=5.5\nu=5.5*ν*=5.5时分类误差率最低，故基本分类器为  
G2(x)={−1,1,x&lt;5.5x≥5.5G\_2(x)=\begin{cases}-1,&amp;x\lt5.5 \\1,&amp;x\ge5.5\end{cases}*G*2​(*x*)={−1,1,​*x*<5.5*x*≥5.5​  
  (b)G3(x)G\_3(x)*G*3​(*x*)在训练数据集上的误差率e3=P(G3(xi)≠yi)=0.7520e\_3=P(G\_3(x\_i)\neq y\_i) =0.7520*e*3​=*P*(*G*3​(*xi*​)̸​=*yi*​)=0.7520  
  (d)更新训练数据的权值分布：  
D4=(w4,1,w4,2,…,w4,10)D\_4=(w\_{4,1},w\_{4,2},\dots,w\_{4,10})*D*4​=(*w*4,1​,*w*4,2​,…,*w*4,10​)  
w4,i=w3,iZ1exp(−α3yiG3(xi)),i=1,2,…,10w\_{4,i} = \dfrac{w\_{3,i}}{Z\_1}exp(-\alpha\_3y\_iG\_3(x\_i)),i=1,2,\dots,10*w*4,*i*​=*Z*1​*w*3,*i*​​*exp*(−*α*3​*yi*​*G*3​(*xi*​)),*i*=1,2,…,10  
D4=(0.125,0.125,0.125,0.06481478,0.06481478,0.06481478,0.10185189,0.10185189,0.10185189,0.125)D\_4=(0.125,0.125,0.125,0.06481478,0.06481478,0.06481478, 0.10185189,0.10185189,0.10185189,0.125)*D*4​=(0.125,0.125,0.125,0.06481478,0.06481478,0.06481478,0.10185189,0.10185189,0.10185189,0.125)  
f3(x)=0.4236G1(x)+0.6496G2(x)+0.7520G3(x)f\_3(x)=0.4236G\_1(x) + 0.6496G\_2(x)+0.7520G\_3(x)*f*3​(*x*)=0.4236*G*1​(*x*)+0.6496*G*2​(*x*)+0.7520*G*3​(*x*)  
  (e)分类器sign[f3(x)]sign[f\_3(x)]*sign*[*f*3​(*x*)]在训练数据集上有0个误分点

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| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| G1(x)G\_1(x)*G*1​(*x*) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 |
| G2(x)G\_2(x)*G*2​(*x*) | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| G3(x)G\_3(x)*G*3​(*x*) | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 |
| α1G1(x)\alpha\_1G\_1(x)*α*1​*G*1​(*x*) | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | 0.4236 | -0.4236 |
| α2G2(x)\alpha\_2G\_2(x)*α*2​*G*2​(*x*) | 0.6496 | 0.6496 | 0.6496 | -0.6496 | -0.6496 | -0.6496 | -0.6496 | -0.6496 | -0.6496 | -0.6496 |
| α3G3(x)\alpha\_3G\_3(x)*α*3​*G*3​(*x*) | -0.7520 | -0.7520 | -0.7520 | -0.7520 | -0.7520 | -0.7520 | 0.7520 | 0.7520 | 0.7520 | 0.7520 |
| sign[f3(x)]sign[f\_3(x)]*sign*[*f*3​(*x*)] | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 |
| y | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 |

于是最终的分类器为  
G(x)=sign[f3(x)]=0.4236G1(x)+0.6496G2(x)+0.7520G3(x)G(x)=sign[f\_3(x)]=0.4236G\_1(x) + 0.6496G\_2(x)+0.7520G\_3(x)*G*(*x*)=*sign*[*f*3​(*x*)]=0.4236*G*1​(*x*)+0.6496*G*2​(*x*)+0.7520*G*3​(*x*)