1: 20 points

MIMO Poles and Zeros

(a) Consider a system with plant model

$$G(s) = \frac{1}{s^2 + \frac{a}{50}s + a^2} \begin{bmatrix} s - a^2 & a(s+1) \\ -a(s+1) & s - a^2 \end{bmatrix}$$

These dynamics model the motion of a cylindrical satellite spinning at constant rate Ω about the z-axis with $a = (1 - I_{zz}/I_{xx})\Omega$. The controls affect the spin rates about x and y; as a becomes large, the system becomes increasingly sensitive to uncertainty. Assume $a = 100$. Find the poles and zeros of the system including their directions.

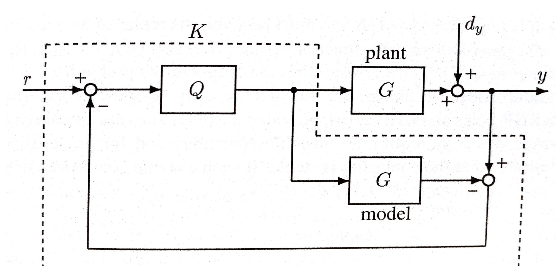
(b) Zeros found through the solution to the generalized eigenvalue problem are known as *invariant* zeros because they are unchanged by state feedback $u = Kx$. To see this, show that

$$\operatorname{rank}\left(\begin{bmatrix} A + BK - sI & B \\ C + DK & D \end{bmatrix}\right) = \operatorname{rank}\left(\begin{bmatrix} A - sI & B \\ C & D \end{bmatrix}\right).$$

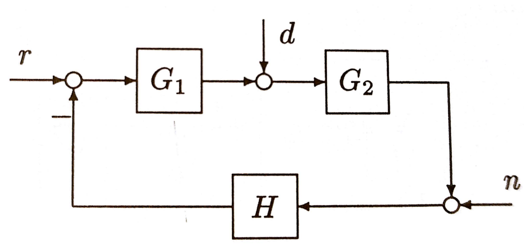
2: 20 Points

Internal Stability

(a) The control structure shown below is called Internal Model Control (IMC). Show that the IMC structure is internally unstable if Q is unstable.



(b) Recalling that a feedback system is internally stable $\iff$ all closed loop transfer functions are stable, find the conditions for internal stability of the feedback system shown. How do those simplify if $H(s)$ and $G_1(s)$ are both stable?



3: 10 points

Control Limitations

Consider the plant model

$$G(s) = \frac{s-1}{s(s-2)}.$$

Based on the RHP poles and zeros of the system, and assuming a conventional loop shape, what limitations are posed on the system bandwidth? Note that your answer might be somewhat nonsensical. What is the minimum peak of the sensitivity function?