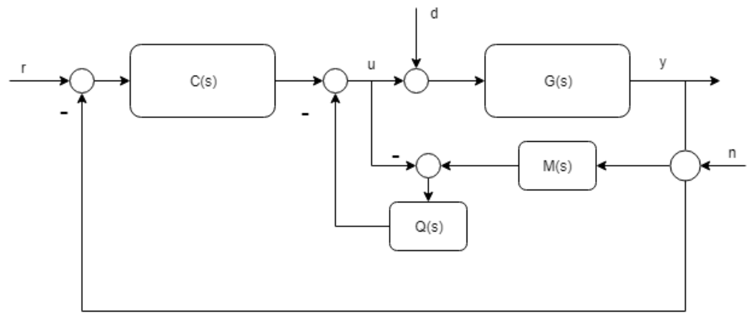
1: 20 Points

Consider the block diagram shown. This configuration is known as a disturbance observer and is used to cancel the effects of disturbances entering at the plant input node. You can assume that G is invertible throughout for simplicity.



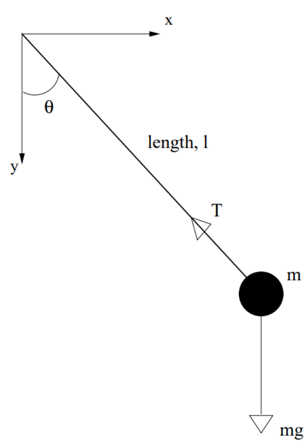
(a) Find the closed loop transfer function $T \triangleq \frac{Y}{R}$ and the sensitivity function $S \triangleq \frac{E}{R}$.

(b) Find the input sensitivity function $\frac{Y}{D}$. What happens when $Q = I$ and $M = G^{-1}$? What function does Q perform in the design?

2: 30 points

Consider the simple pendulum system shown below with a motor at the hinge that produces torque τ. The system dynamics are easy to derive from first principles.

$$\ddot{\theta} + \frac{g}{l}\sin\theta = \frac{\tau}{ml^2}$$



(a) Linearize the differential equation about equilibrium point $\theta = \frac{\pi}{2}$. Analyze the stability of the system in this configuration.

(b) Assume the mass is 1 kg and the length is 4 m. Design a PD controller (maps angle error to motor torque) that places the closed loop poles such that the closed loop system nominally has a 25% overshoot and a 2 second 2% settling time.

(c) Construct the nonlinear dynamics in Simulink and apply the controller designed in part (b). Plot the initial condition response starting at rest from $\theta_0 = 45^o$. For what range of initial conditions does the PD controller stabilize the nonlinear system? No need to derive this analytically - you can use the simulation to answer this. Be sure to include an image of your Simulink model in the solution.

3: 20 points

Consider the LTI system given by

$$\dot{x} = \begin{bmatrix} -\frac{3}{4} & -\frac{1}{4} \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 4 & 2 \end{bmatrix} x.$$

(a) Is the system controllable? Observable?

(b) Find a minimal realization. Design a controller for the minimal realization that achieves a time constant of 1 millisecond.