Homework 7

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Question 1

```
clear;clc
```

The nominal plant is:

```
s = tf('s');
a = 100;
G = 1/(s^2+a/50*s+a^2)*[s-a^2,a*(s+1);-a*(s+1),s-a^2];
```

The uncertain model is:

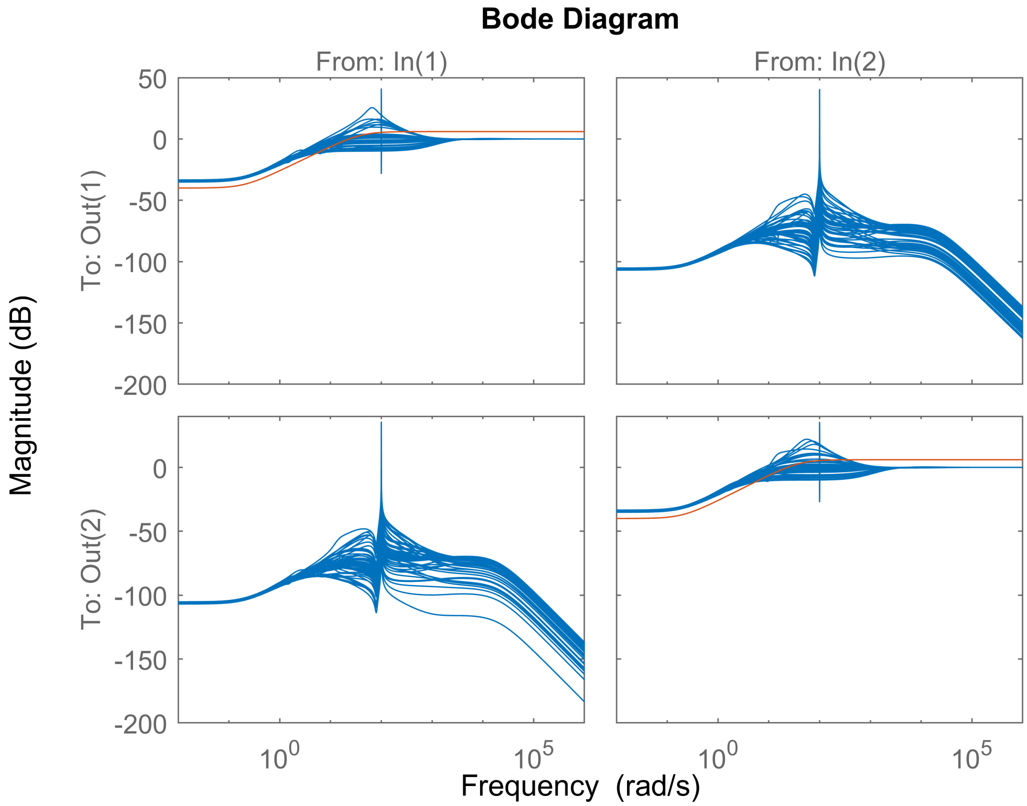
```
D1 = ultidyn('D1',[1,1]);
D2 = ultidyn('D2',[1,1]);
wI = (20*s+10)/(s+100);
WI = [wI,0;0,wI];
WI_unc = [D1*wI,0;0,D2*wI];
Gp = (eye(2)+WI_unc)*G;
```

Performance weight:

```
WP = (0.5*s+20)/(s+0.2)*eye(2,2);
```

(a) Design controller with mixsyn

```
[K_mix,CL_mix,GAM_mix] = mixsyn(Gp,WP,[],WI);  % use unc model
T_mix = feedback(Gp*K_mix,eye(2));
S_mix = eye(2)-T_mix;
S_mix_vec = usample(S_mix,50);
bodemag(S_mix_vec,inv(WP));
```



Check for RS:

```
isstable(T_mix.NominalValue)
```

```
ans = logical
   1
```

```
[stabmarg,~] = robstab(S_mix);
mu_RS = 1/stabmarg.LowerBound
```

```
mu_RS = 1.8530
```

Check for RP:

```
[perfmarg,~] = robustperf(WP*S_mix);
mu_RP = 1/perfmarg.LowerBound
```

```
mu_RP = 2.7291
```

The system didn't meet RS or RP goals since all mu values are greater than one.

(b) Design with μ synthesis

Assume negative feedback, find the generalized plant:

```
systemnames = 'Gp WP';
inputvar = '[r{2};u{2}]';
outputvar = '[WP;r-Gp]';
```

```
input_to_Gp = '[u]';
input_to_WP = '[r-Gp]';
cleanupsysic = 'yes';
Pmu = sysic;
```

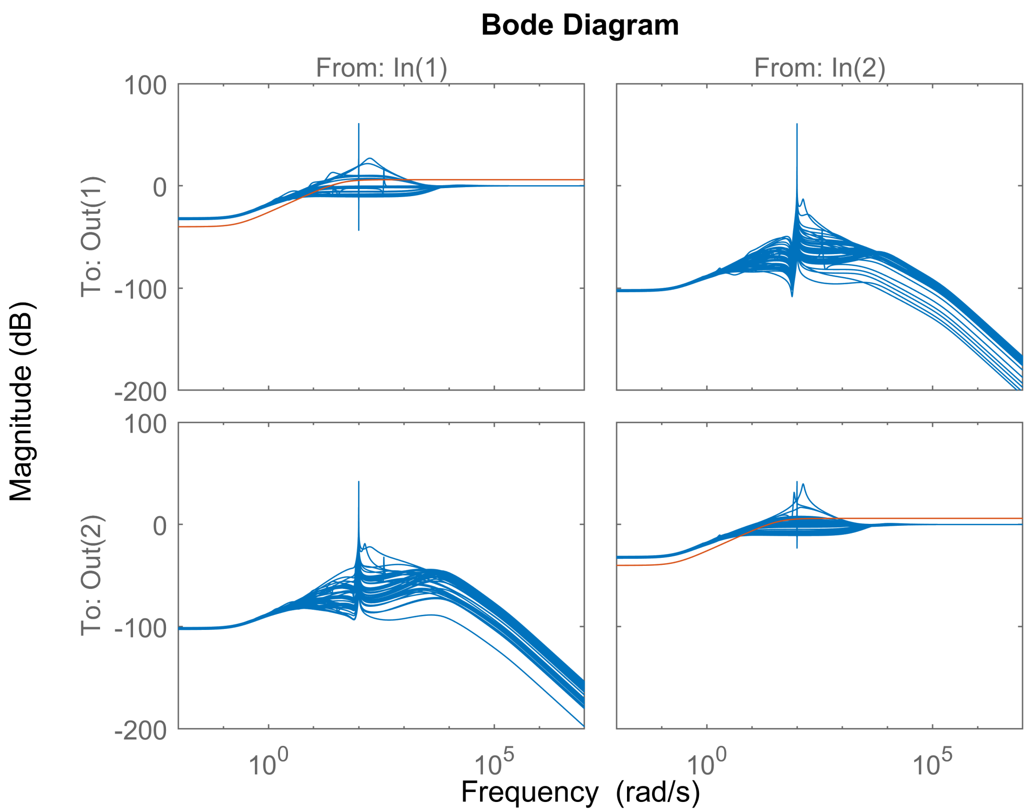
Minimize mu:

```
[K_mu,CLP_mu,mu] = musyn(Pmu,2,2);
```

```
D-K ITERATION SUMMARY:
-----------------------------------------------------------------
                       Robust performance               Fit order
-----------------------------------------------------------------
  Iter          K Step       Peak MU       D Fit             D
    1           2.717         2.711        2.734            12
    2           2.589         2.589        2.614            20
    3           2.589         2.588        2.588            20
    4            2.59         2.589        2.592            20

Best achieved robust performance: 2.59
```

```
T_mu = feedback(Gp*K_mu,eye(2));
S_mu = eye(2)-T_mu;
S_mu_vec = usample(S_mu,50);
bodemag(S_mu_vec,inv(WP));
```



Check for RS:

```
isstable(T_mu.NominalValue)  % NS
```

```
ans = logical
   1
```

```
[stabmarg,~] = robstab(S_mu);
mu_RS = 1/stabmarg.LowerBound
```

```
mu_RS = 2.0993
```

Check for RP:

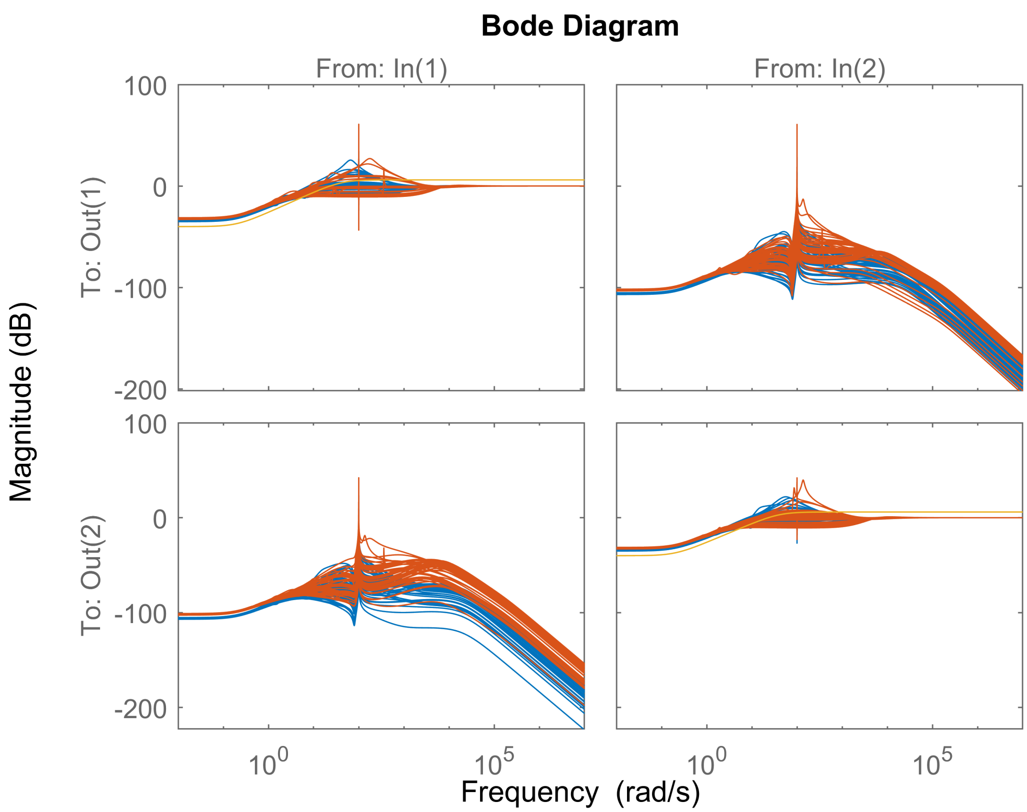
```
[perfmarg,~] = robustperf(WP*S_mu);
mu_RP = 1/perfmarg.LowerBound
```

```
mu_RP = 2.5879
```

The system didn't meet RS or RP goals since all mu values are greater than one.

Compare with hinf-mixsyn:

```
bodemag(S_mix_vec, S_mu_vec, inv(WP))
```



Question 2

(a) K stabilizes P means it's NS

1. RS:

The generalized plant is:

$$\begin{bmatrix} z \\ v \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -PW_1 & -W_2 & -P \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ u \end{bmatrix}$$

The LFT of this plant is:

$$N = \begin{bmatrix} 0 & 0 \end{bmatrix} + K(1+PK)^{-1}\begin{bmatrix} -PW_1 & -W2 \end{bmatrix}$$
$$N = -\begin{bmatrix} W_1T & W_2KS \end{bmatrix}$$

According to the small gain theory:

$$\|N\Delta\|_\infty = \|W_1T\Delta_1 + W_2KS\Delta_2\|_\infty < 1$$

Since $\|\Delta_1\|_\infty \le 1, \|\Delta_2\|_\infty \le 1$,

$$\| |W_1T| + |W_2KS| \|_\infty \le 1$$

2. RP:

In this case, Wp=1 according to the diagram, RP is equivalent to $\|W_PS_p\|_\infty = \|S_p\|_\infty \le 1$.

The disturbance can be seen as another uncertanty.

Use the Nyquist plot as uncertainty disk:

$$|W_3| + |W_1L| + |W_2K| < |1+L| \ \ \forall\omega$$

Therefore,

$$\|W_3S\|_\infty + \|W_1T\|_\infty + \|W_2KS\|_\infty \le 1$$

(b) The generalized plant is:

$$\begin{bmatrix} z_I \\ z_O \\ v \end{bmatrix} = \begin{bmatrix} 0 & 0 & W_{1I} \\ W_{1O}GW_{2I} & 0 & W_{1O}G \\ -GW_{2I} & -W_{2O} & -G \end{bmatrix} \begin{bmatrix} w_I \\ w_O \\ u \end{bmatrix}$$

The LFT of this plant is:

$$\begin{aligned} M &= \begin{bmatrix} 0 & 0 \\ W_{1O}GW_{2I} & 0 \end{bmatrix} + \begin{bmatrix} W_{1I} \\ W_{1O}G \end{bmatrix} K(I+GK)^{-1}\begin{bmatrix} -GW_{2I} & -W_{2O} \end{bmatrix} \\ &= \begin{bmatrix} 0 & 0 \\ W_{1O}GW_{2I} & 0 \end{bmatrix} + \begin{bmatrix} -W_{1I}K(I+GK)^{-1}GW_{2I} & -W_{1I}K(I+GK)^{-1}W_{2O} \\ -W_{1O}GK(I+GK)^{-1}GW_{2I} & -W_{1O}GK(I+GK)^{-1}W_{2O} \end{bmatrix} \\ &= \begin{bmatrix} -W_{1I}K(I+GK)^{-1}GW_{2I} & -W_{1I}K(I+GK)^{-1}W_{2O} \\ W_{1O}GW_{2I} - W_{1O}GK(I+GK)^{-1}GW_{2I} & -W_{1O}GK(I+GK)^{-1}W_{2O} \end{bmatrix} \end{aligned}$$

Assume:

$$T_I = K(I+GK)^{-1}G, T = GK(I+GK)^{-1}, S = (I+GK)^{-1}$$

The LFT can be written as:

$$
\begin{aligned}
M &= \begin{bmatrix} -W_{1I}T_IW_{2I} & -W_{1I}KSW_{2O} \\ W_{1O}(I-T)GW_{2I} & -W_{1O}TW_{2O} \end{bmatrix} \\
&= \begin{bmatrix} -W_{1I}T_IW_{2I} & -W_{1I}KSW_{2O} \\ W_{1O}SGW_{2I} & -W_{1O}TW_{2O} \end{bmatrix} \\
&= \begin{bmatrix} W_{1I} & 0 \\ 0 & -W_{1O}0 \end{bmatrix} \begin{bmatrix} -T_I & -KS \\ SG & -T \end{bmatrix} \begin{bmatrix} W_{2I} & 0 \\ 0 & W_{2O} \end{bmatrix}
\end{aligned}
$$

(c)

```
clear;clc
```

The plant model:

```
s = tf('s');
G = 1/(75*s+1)*[-87.8,1.4;-108.2,-1.4];
K = 0.7/s/G;
wI = (s+0.2)/(0.5*s+1);
WI = [wI,0;0,wI];
eps = 1e-3;
WP = (s/2+0.05)/(s+eps)*eye(2);
```

1. diagonal uncertainty:

```
D1 = ultidyn('D1',[1,1]);
D2 = ultidyn('D2',[1,1]);
D_diag = [D1,0;0,D2];
Gp_diag = G*(eye(2)+D_diag*WI);
S_diag = eye(2)-feedback(Gp_diag*K,eye(2));
[perfmarg_diag,~] = robustperf(WP*S_diag);
mu_diag = 1/perfmarg_diag.LowerBound
```

```
mu_diag = 0.9666
```

2. full-block uncertainty:

```
D1 = ultidyn('D1',[1,1]);
D2 = ultidyn('D2',[1,1]);
D3 = ultidyn('D3',[1,1]);
D4 = ultidyn('D4',[1,1]);
D_full = [D1,D2;D3,D4];
Gp_full = G*(eye(2)+D_full*WI);
S_full = eye(2)-feedback(Gp_full*K,eye(2));
[perfmarg_full,~] = robustperf(WP*S_full);
mu_full = 1/perfmarg_full.LowerBound
```

```
mu_full = 4.6265
```

Therefore, close-loop system with given controller has RP for diagonal case but not for full-block case.

Question 3

```
clear;clc
```

(a) H-inf norm

```
s = tf('s');
G1 = [1/(s+5),(s-2)/(s+10);10/(s^2+4*s+15),3*s/(4*s+3)];
calNorm_inf(G1)
```

```
ans = 1.2505
```

```
norm(G1,'inf')
```

```
ans = 1.2500
```

(b) H-2 norm

```
G2 = [1/(s+5),1/(s+10);10/(s^2+4*s+15),3/(4*s+3)];
calNorm_2(G2)
```

```
ans = 1.1655
```

```
norm(G2,2)
```

```
ans = 1.1655
```

The results from custom function and matlab function are the same for both H-inf and H-2 norm.

```
function norm = calNorm_inf(G)
Gss = ss(G);
A = Gss.A;
B = Gss.B;
C = Gss.C;
D = Gss.D;
eps = 1e-6;

gamh = 100;
gaml = 0;
gam_try = gamh;
gam_new = 1/2*(gamh+gaml);
while(abs(gam_new-gam_try)>1e-3)  %Stopping criterion
    gam_try = gam_new;
    R = gam_try^2*eye(size(D,2))-D'*D;
    Ham = [A+B/R*D'*C,B/R*B';-C'*(eye(size(D,1))+D/R*D')*C,-(A+B/R*D'*C)'];
    e = eig(Ham);
    if any(abs(real(e))<eps)
        gaml = gam_try;
    else
        gamh = gam_try;
    end
    gam_new = 1/2*(gamh+gaml);
end
norm = gam_new;
```

```
end
```

```
function norm = calNorm_2(G)
Gss = ss(G);
B = Gss.B;
Q = gram(Gss,"o");
norm = sqrt(trace(B'*Q*B));
end
```