Homework 6

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Question 1.

```
clear;clc
```

(a)

```
s = tf("s");
```

The "true" plant model can be seen as an element in a set of uncertainty models. The nominal model is choose to be a lower-order and non-delay model.

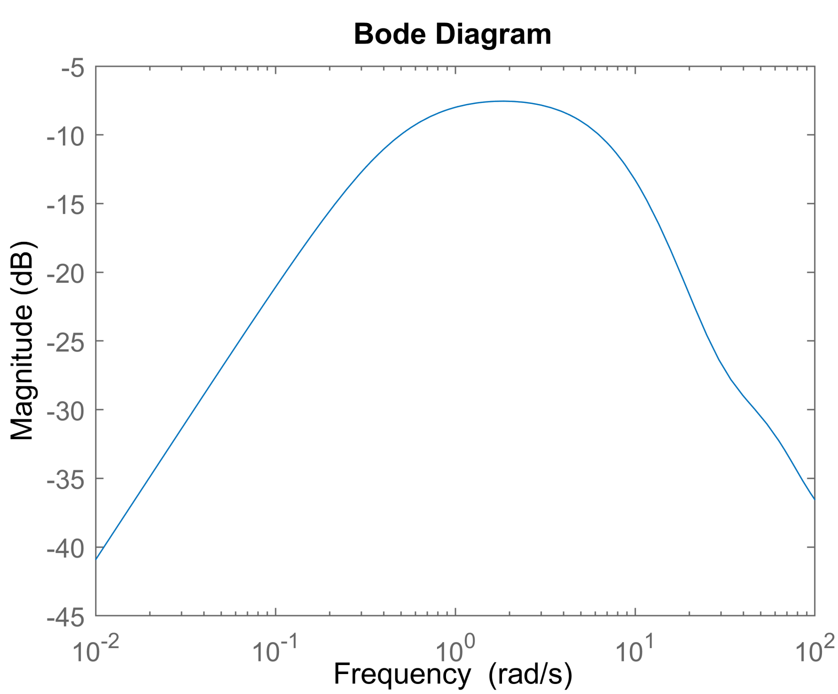
```
Gtrue = 3*exp(-0.1*s)/(2*s+1)/(0.1*s+1)^2;
Gnorm = 3/(2*s+1);
```

Based on additive uncertainty model:

$$G_p = G_{norm} + W_A \Delta_A$$
$$W_A \Delta_A = G_p - G_{norm}$$
$$\|\Delta_A\|_\infty \le 1 \Rightarrow |W_A| \ge G_p - G_{norm}$$

In order to find the minimized possible W_A, then $W_A = G_{true} - G_{norm}$:

```
WA = Gtrue - Gnorm;
bodemag(WA);
```



(b)

```
clear;clc
```

A simplified model is choosed to be the nominal model. Therefore, the actual model can be written as the uncertain model with this nominal model under the largest uncertainty.

```
s = tf('s');
Gactual = 10*(-0.5*s+1)/(6*s+1)/(0.2*s+1)/(20*s+1);
Gnorm = 10/(6*s+1);
```

Based on multiplicative uncertainty assumption:

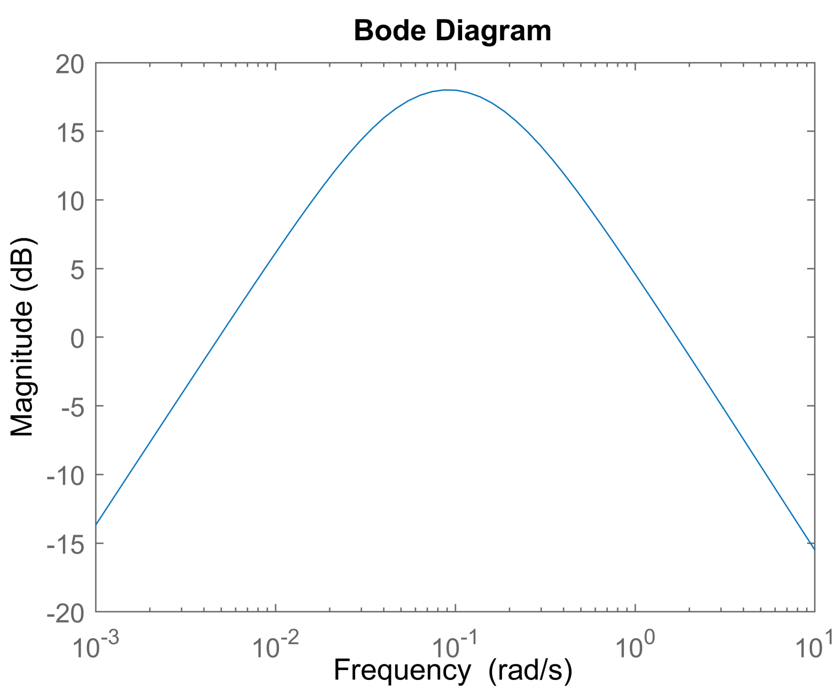
$$G_p = G_{norm}(1 + W_I\Delta)$$

$$W_I\Delta = \frac{G_p}{G_{norm}} - 1$$

Therefore,

$$W_I = \frac{G_{actual}}{G_{norm}} - 1 = \frac{-0.5s + 1}{(0.2s + 1)(20s + 1)} - 1$$

```
WI = Gactual-Gnorm;
bodemag(WI);
```



(c)

a.

$$\begin{cases} z = w_A u \\ y = w + Gu \end{cases} \Rightarrow \begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} 0 & w_A \\ 1 & G \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \Rightarrow H = \begin{bmatrix} 0 & w_A \\ 1 & G \end{bmatrix}$$

b.

$$\begin{cases} z = w_I u \\ y = G(w+u) \end{cases} \Rightarrow \begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} 0 & w_I \\ G & G \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \Rightarrow H = \begin{bmatrix} 0 & w_I \\ G & G \end{bmatrix}$$

c.

$$\begin{cases} z = w_O G u \\ y = w + Gu \end{cases} \Rightarrow \begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} 0 & w_O G \\ 1 & G \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \Rightarrow H = \begin{bmatrix} 0 & w_O G \\ 1 & G \end{bmatrix}$$

d.

$$\begin{cases} z = G w_{iA} w + Gu \\ y = G w_{iA} w + Gu \end{cases} \Rightarrow \begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} G w_{iA} & G \\ G w_{iA} & G \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \Rightarrow H = \begin{bmatrix} G w_{iA} & G \\ G w_{iA} & G \end{bmatrix}$$

e.

$$\begin{cases} z = w_{iI} w + u \\ y = G w_{iI} w + Gu \end{cases} \Rightarrow \begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} w_{iI} & 1 \\ G w_{iI} & G \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \Rightarrow H = \begin{bmatrix} w_{iI} & 1 \\ G w_{iI} & G \end{bmatrix}$$

f.

$$\begin{cases} z = w_{iO} w + Gu \\ y = w_{iO} w + Gu \end{cases} \Rightarrow \begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} w_{iO} & G \\ w_{iO} & G \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \Rightarrow H = \begin{bmatrix} w_{iO} & G \\ w_{iO} & G \end{bmatrix}$$

Question 2

(a)

```
clear;clc
```

Import two uncertain models:

```
HDDModel_DS_Uncertain
```

Find multiplicative uncertainty models that covers all cases:

```
VCMvec = usample(VCM,100);
PZTvec = usample(PZT,100);

[VCMp,VCM_info] = ucover(VCMvec,VCM.NominalValue,2);
[PZTp,PZT_info] = ucover(PZTvec,PZT.NominalValue,2);
```

The final weight for each model:

```
tf(VCM_info.W1)
```

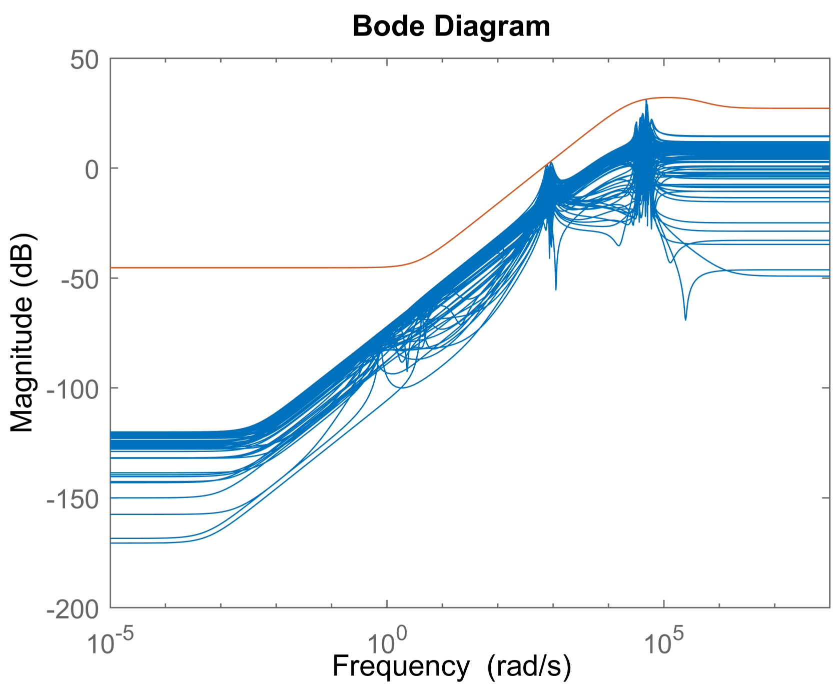
```
ans =

  22.89 s^2 + 1.674e07 s + 5.801e07
  ---------------------------------
     s^2 + 4.192e05 s + 1.067e10

Continuous-time transfer function.
```

```
tf(PZT_info.W1)
```

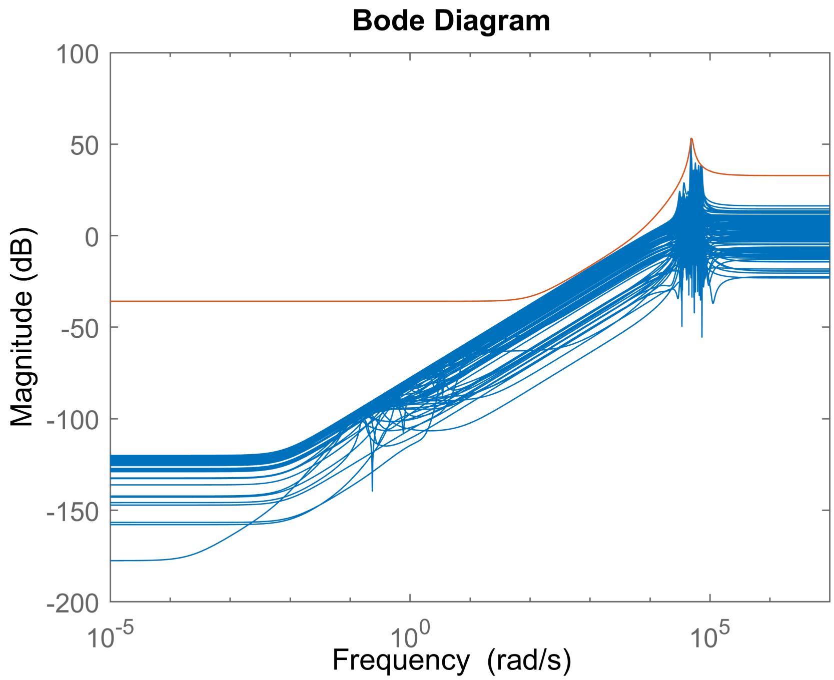
```
ans =

  43.89 s^2 + 3.649e05 s + 3.975e07
  ---------------------------------
       s^2 + 4801 s + 2.455e09

Continuous-time transfer function.
```

The uncertainty weight performance:

```
bodemag((VCMvec-VCM.NominalValue)/VCM.NominalValue,VCM_info.W1);
```




```
bodemag((PZTvec-PZT.NominalValue)/PZT.NominalValue,PZT_info.W1);
```



As shown in the plotted result, the weight is too conservative in the lower frequency zone.

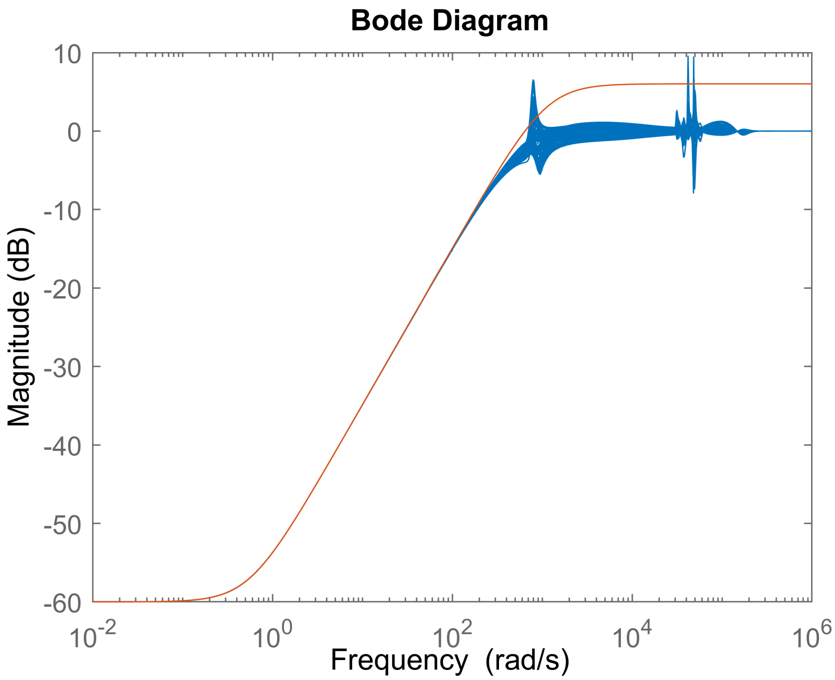
(b)

Design controller for VCM:

```
wh = 1000;
wl = 0;
w_try = wh;
w_new = 1/2*(wh+wl);
while(abs(w_new-w_try)>.001)  %Stopping criterion
    w_try = w_new;
    Wp = makeweight(1000,w_try,1/2);
    [K_VCM,CL,GAM] = mixsyn(VCM,Wp,[],VCM_info.W1);
    if GAM<1
        wl = w_try;  %We're not agressive enough
    else
        wh = w_try;  %We're too agressive
    end
    w_new = 1/2*(wh+wl);
end
```

Plot the uncertain sensitivity function vs. the performance weight:

```
S_VCM = 1-feedback(VCM*K_VCM,1);
S_VCMvec = usample(S_VCM,100);
bodemag(S_VCMvec,1/Wp);
```



As the plot showed, the uncertain sensitivity function with $\gamma < 1$ didn't make sure RP.

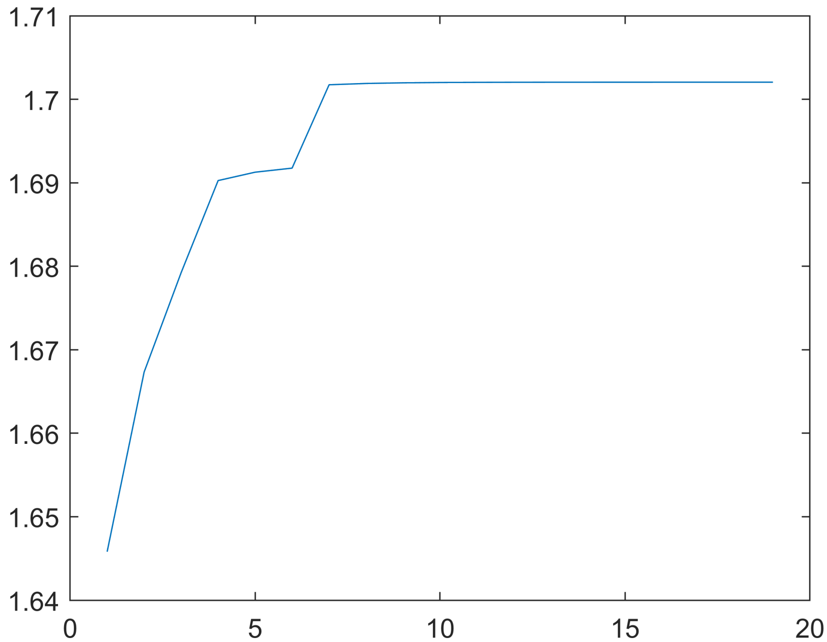
(c)

```
VCMn = VCM.NominalValue;
PZTn = PZT.NominalValue;
Wvcm = VCM_info.W1;
Wpzt = PZT_info.W1;

wh = 1000;
wl = 0;
w_try = wh;
w_new = 1/2*(wh+wl);
k = 1;
while(abs(w_new-w_try)>.001)  %Stopping criterion
    w_try = w_new;
    Wp = makeweight(1000,w_try,1/2);
    systemnames = 'VCMn PZTn Wvcm Wpzt Wp';
    inputvar = '[udv;udp;r;uv;up]';
    outputvar = '[Wvcm;Wpzt;Wp;udv+udp+VCMn+PZTn;r-udv-udp-VCMn-PZTn]';

    input_to_VCMn = '[uv]';
    input_to_PZTn = '[up]';
    input_to_Wp = '[r-udv-udp-VCMn-PZTn]';
    input_to_Wvcm = '[VCMn]';
    input_to_Wpzt = '[PZTn]';

    cleanupsysic = 'yes';
    P = sysic;
    [K,CL,GAM(k)] = hinfsyn(P,1,2);

    if GAM(k)<3.5
```

```
        wl = w_try;  %We're not agressive enough
    else
        wh = w_try;  %We're too agressive
    end
    w_new = 1/2*(wh+wl);
    k = k+1;
end
```

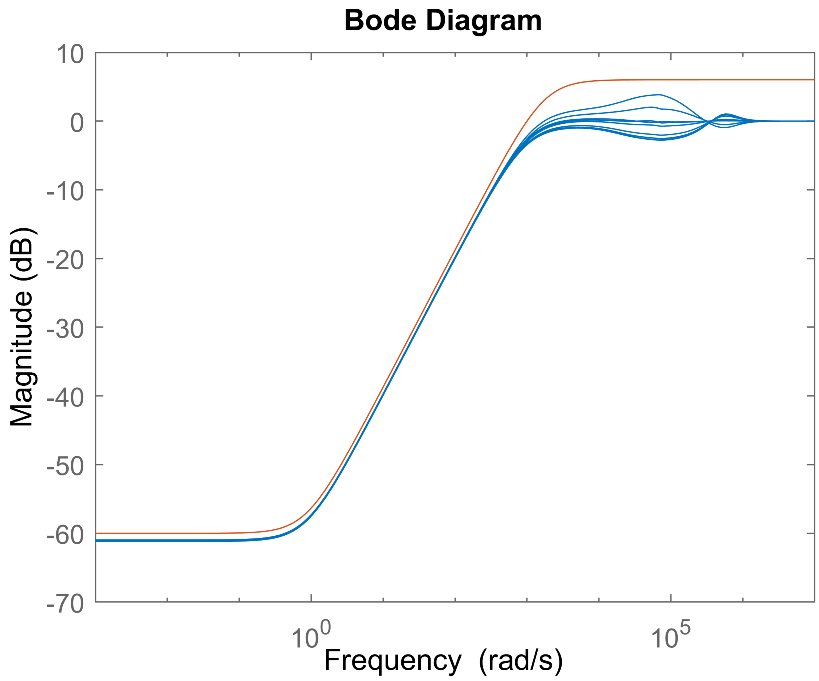
Plot values of γ through iteration:

```
plot(1:length(GAM),GAM);
```



Plot uncertain sensitivity function vs. performance weight:

```
G = [VCMp,PZTp];
S = 1/(1+G*K);
Svec = usample(S,10);
bodemag(Svec,1/Wp);
```



Question 3

(a)

```
clear;clc
```

Load a vector of responses:

```
load("responses.mat");
s = tf('s');
```

The nominal plant model is:

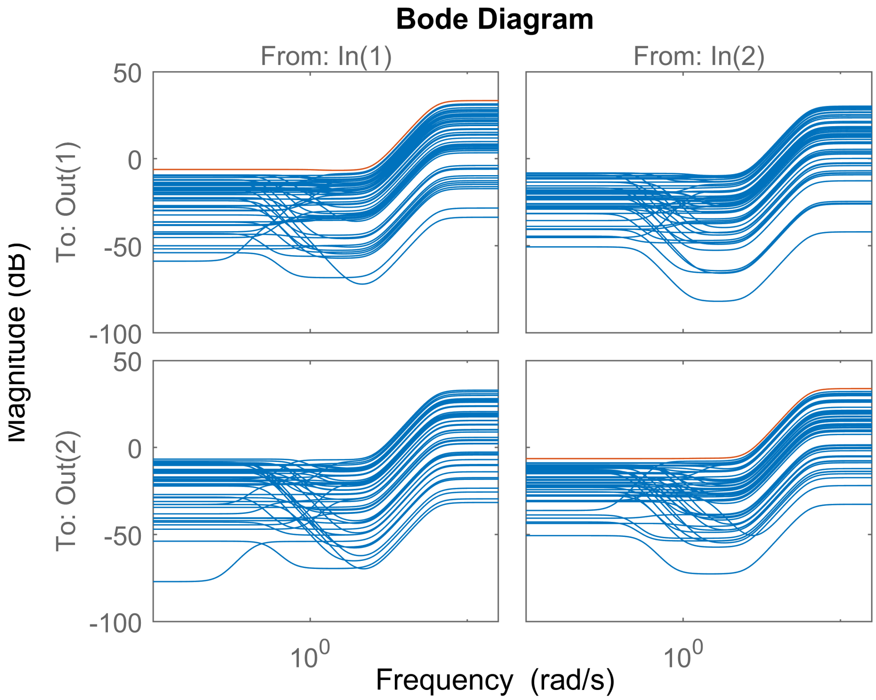
```
A = [-0.0226,-36.6,-18.9,-32.1;0,-1.9,0.983,0;0.0123,-11.7,-2.63,0;0,0,1,0];
B = [0,0;-0.414,0;-77.8,22.4;0,0];
C = [0,57.3,0,0;0,0,0,57.3];
D = [0,0;0,0];
Gnorm = ss(A,B,C,D);
```

Fit the sampling responses as an uncertain model:

```
[Gp,info] = ucover(Gp_samples,Gnorm,[2,2]);
Wdel = info.W1;
```

Plot the uncertainty weight vs. the sampling relative error:

```
relerr = Gnorm\Gp_samples-eye(2);
bodemag(relerr,Wdel);
```



As shown in the plot, ucover assume only diagnol weight for uncertainty. But it fit well with the diagonal terms.

(b)

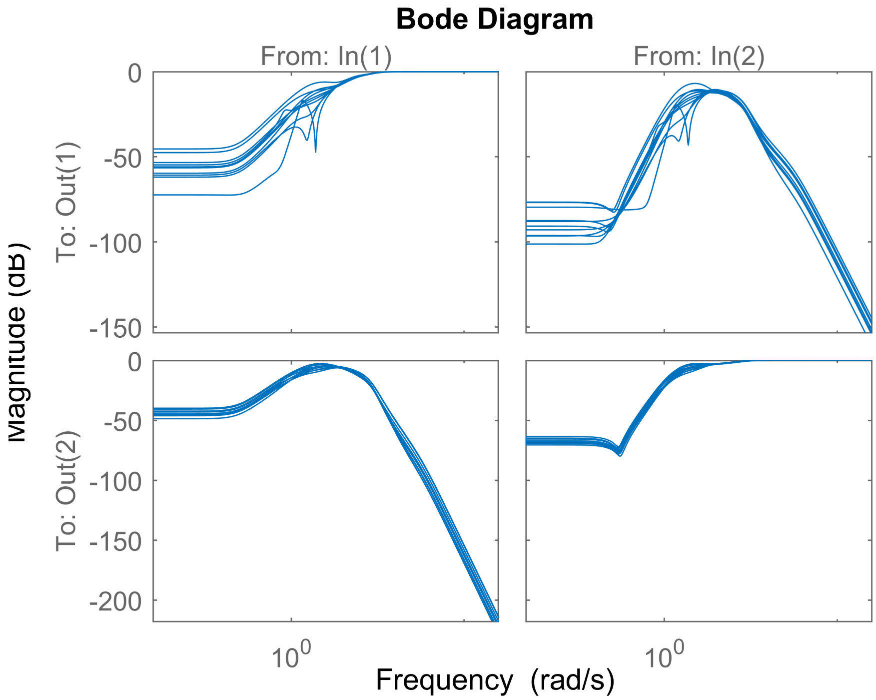
```
Wp = [(s+3)/(s+0.03),0;0,0.5*(s+3)/(s+0.03)];
Wn = [2*(s+1.28)/(s+320),0;0,2*(s+1.28)/(s+320)];
```

Use the synthesis form to do controller design:

```
systemnames = 'Gnorm Wdel Wp Wn';
inputvar = '[p{2};d{2};n{2};u{2}]';
outputvar = '[Wdel;Wp;Gnorm+d+Wn]';

input_to_Gnorm = '[p+u]';
input_to_Wdel = '[u]';
input_to_Wp = '[d+Gnorm]';
input_to_Wn = '[n]';
cleanupsysic = 'yes';
P = sysic;
[K,CL,GAM] = hinfsyn(P,2,2);
```

Plot the sensitivity function:

```
S = eye(2)-feedback(Gp*K,eye(2));
Svec = usample(S,10);
bodemag(Svec);
```



First check nominal stability:

```
isNS = isstable(P)
```

```
isNS = logical
   1
```

Use the analysis form to check NP:

```
N = lft(P,K);
isNP = norm(N(2,2),'inf')<1
```

```
isNP = logical
   1
```

Rebuild generalized plant with uncertain model:

```
systemnames = 'Gp Wp Wn';
inputvar = '[d{2};n{2};u{2}]';
outputvar = '[Wp;Gp+d+Wn]';

input_to_Gp = '[u]';
input_to_Wp = '[d+Gp]';
input_to_Wn = '[n]';
cleanupsysic = 'yes';
Phat = sysic;

Nhat = lft(Phat,K);
[perfmarg, robg_wcu] = robgain(Nhat,1);
isRP = 1/perfmarg.LowerBound<1
```

```
isRP = logical
   0
```

```
[stabmarg,robs_wcu] = robstab(Nhat);
isRS = 1/stabmarg.LowerBound<1
```

```
isRS = logical
   1
```

```
isRS_con = norm(N(1,1),'inf')<1  % conservative
```

```
isRS_con = logical
   1
```

Therefore, the robust performance is not met, but the robust stability is met.