
1: 30 points

Norms and SVD

(a) Consider the matrices $A = \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ 3 & 0 \end{bmatrix}$. Compute the spectral radius, the Frobenius norm, the 1-norm, the 2-norm, and the ∞-norm of A and B.

(b) A generalized matrix norm is norm on a matrix that satisfies all properties other than $\|AB\| \leq \|A\|\|B\|$. Show that $\|A\|_{\max} = \max_{i,j} |a_{ij}|$ is a generalized matrix norm by demonstrating that it satisfies the four fundamental properties and giving examples that show $\|A\|_{\max} < \rho(A)$ and $\|AB\|_{\max} > \|A\|_{\max}\|B\|_{\max}$.

(c) For a SISO system, show that the H_2 and H_∞ norms are invariant to time delays and all-pass filters, i.e. show $\|QG\|_2 = \|G\|_2$ and $\|QG\|_\infty = \|G\|_\infty$ for $Q = e^{-sT}$ and $Q = \frac{s-a}{s+a}$ with $a > 0$.

2: 20 points

MIMO Design

Consider the 2×2 transfer function matrix

$$G(s) = \begin{bmatrix} \frac{10(s+2)}{s^2+0.2s+100} & \frac{1}{s+5} \\ \frac{s+2}{s^2+0.1s+10} & \frac{5(s+1)}{(s+2)(s+3)} \end{bmatrix}.$$

In this problem you will design two DIDO controllers for this system using different approaches.

(a) Dynamic Decoupling. Find a proper approximation to G^{-1}. Use this to design a dynamic decoupling-based controller that achieves (approximately) loop shapes of $L = \frac{100}{s}$ for each of the diagonal elements. Form the loop transfer function $L = GK$ and use the margin command to show the performance of $L_{1,1}$ and plot the Bode magnitude of the 2×2 sensitivity function.

(b) Mixed Sensitivity Synthesis. Using first order weights for W_p and W_T and a constant actuator weight corresponding to a maximum control usage of 100 at each input, design a mixed sensitivity controller that

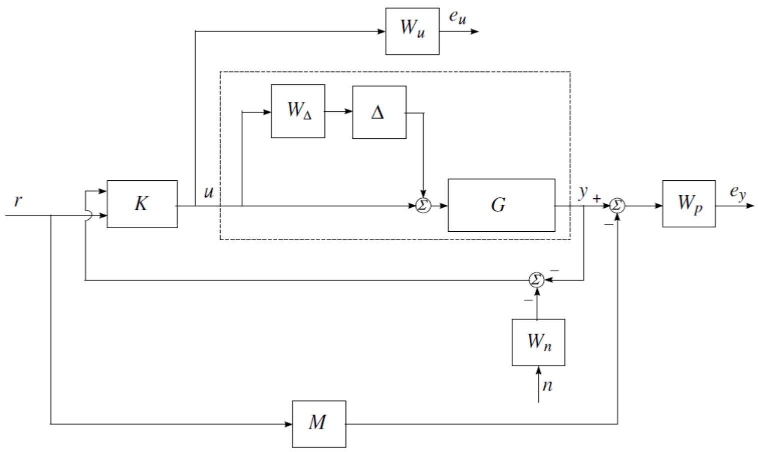
- Maximizes the bandwidth such that $\gamma < 1$.
- Rejects steady state disturbances by a factor of 1000
- Rejects high frequency noise by a factor of 1000
- Has a sensitivity peak of no more than 2 and a complementary sensitivity peak of no more than 1.5
- Has a complementary sensitivity crossover frequency at most $3\times$ the sensitivity crossover frequency.

Plot the magnitudes of W_pS, W_TT, and W_UKS for your final design.

3: 30 points

Generalized Plants

(a) For the block diagram shown below: 1.) Find the generalized plant P, and 2.) Find $N = F_l(P, K)$. For simplicity, feel free to give the system inside the dashed line a name - say $G^\star$.



(b) Use the Matlab command *sysic* to generate the generalized plant for the mixed sensitivity problem in Problem 2.b. NOTE: Use the block diagram from class - it is very different from the one in 3.a. Run the command $K = hinfsyn(P,2,2)$ for your generalized plant and compare the controller to the one you found in Problem 2.b.

(c) The state space description of a transfer function matrix can be written as an LFT. Find H such that

$$F_l\left(H, \frac{1}{s}\right) = C\left(sI - A\right)^{-1} B + D.$$