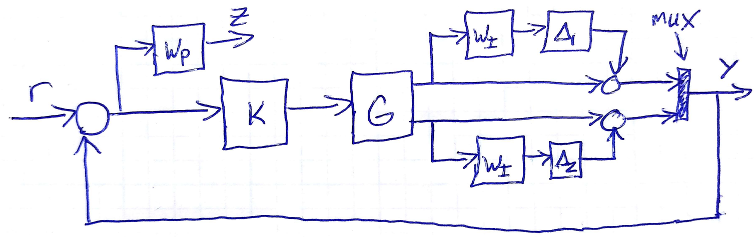

1: 20 points

Spinning Satellite Control

In a prior problem we considered a system with plant model

$$G(s) = \frac{1}{s^2 + \frac{a}{50}s + a^2} \begin{bmatrix} s - a^2 & a(s+1) \\ -a(s+1) & s - a^2 \end{bmatrix}$$

These dynamics model the motion of a cylindrical satellite spinning at constant rate Ω about the z-axis with $a = (1 - I_{zz}/I_{xx})\Omega$. The controls affect the spin rates about x and y; as a becomes large, the system becomes increasingly sensitive to uncertainty. Assume $a = 100$ throughout.



(a) Assuming multiplicative output uncertainty of $W_I(s) = \frac{20s+10}{s+100}$ in the configuration shown and a performance weight of $W_P(s) = \frac{0.5s+20}{s+0.2} I_{2\times 2}$, design a controller using the command *mixsyn*. Plot the achieved sensitivity function. Check whether you met your RP / RS goals.

(b) Assuming the same uncertainty model, design a controller using μ synthesis. Plot the achieved sensitivity function, check whether RP / RS are met, and compare the results with the H_∞ design.

2: 30 points

Robust Stability and Performance

(a) Consider the feedback system shown with a scalar plant having both multiplicative and additive uncertainty, i.e.

$$P_p = P(1 + W_1\Delta_1) + W_2\Delta_2$$

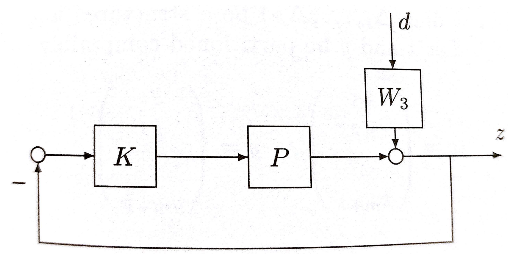
and $\|\Delta_i\|_\infty < 1$. Assume that W_1 and W_2 are stable and show the following.

1. The feedback system is robustly stable $\iff$ K stabilizes P and

$$\||W_1T| + |W_2KS|\|_\infty \leq 1.$$

2. The feedback system has robust performance $\iff$ K stabilizes P and

$$\||W_3S| + |W_1T| + |W_2KS|\|_\infty \leq 1.$$

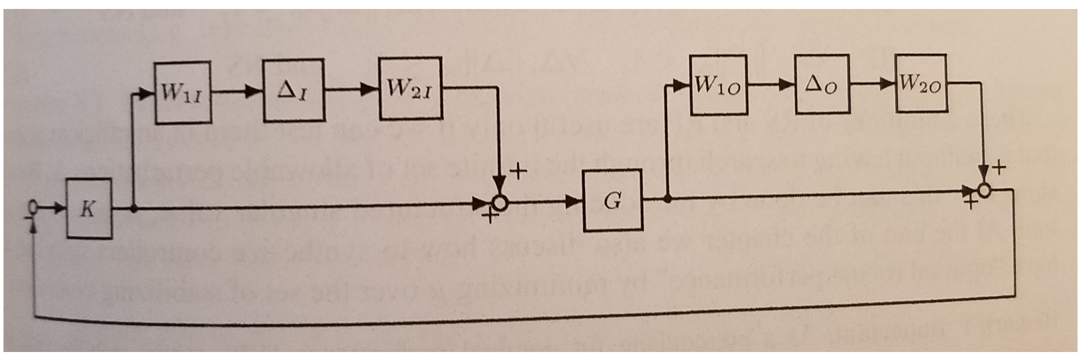


(b) Consider the block diagram shown where we have both input and output multiplicative uncertainty blocks. The set of possible plants is given by

$$G_p = (I + W_{2O}\Delta_O W_{1O})G(I + W_{2I}\Delta_I W_{1I}),$$

where $\|\Delta_I\|_\infty \leq 1$ and $\|\Delta_O\|_\infty \leq 1$. Collect the perturbations into $\Delta = \mathrm{diag}\{\Delta_I, \Delta_O\}$ and rearrange the system into the $M - \Delta$ structure. Show that

$$M = \begin{bmatrix} W_{1I} & 0 \\ 0 & W_{1O} \end{bmatrix} \begin{bmatrix} -T_I & -KS \\ SG & -T \end{bmatrix} \begin{bmatrix} W_{2I} & 0 \\ 0 & W_{2O} \end{bmatrix}$$



(c) Consider the plant model

$$G(s) = \frac{1}{75s+1}\begin{bmatrix} -87.8 & 1.4 \\ -108.2 & -1.4 \end{bmatrix},$$

which is ill-conditioned with $\gamma(G) = 70.8$ at all frequencies. With an inverse-based controller $K(s) = \frac{0.7}{s}G^{-1}(s)$ and uncertainty and performance weights $w_I = \frac{s+0.2}{0.5s+1}$ and $w_P = \frac{s/2+0.05}{s}$, compute μ for RP with both diagonal and full-block input uncertainty.

3: 20 points

Norm Computations

(a) Write your own function to compute the ∞-norm of an arbitrary system (assumed in transfer function matrix form). Test your performance on the system

$$G(s) = \begin{bmatrix} \frac{1}{s+5} & \frac{s-2}{s+10} \\ \frac{10}{s^2+4s+15} & \frac{3s}{4s+3} \end{bmatrix}$$

(b) Repeat Part b for the 2-norm. Because the 2-norm is finite only for strictly proper systems, test it on

$$G(s) = \begin{bmatrix} \frac{1}{s+5} & \frac{1}{s+10} \\ \frac{10}{s^2+4s+15} & \frac{3}{4s+3} \end{bmatrix}$$

or some other suitably modified version of the plant from Part b.