1: 30 Points

For the system $L(s) = \frac{100(0.5s+1)}{s(0.2s+1)(s+10)}$:

- (a) Sketch the Bode plot using asymptotic approximations and the Nyquist plot based on your Bode plot.
- (b) Find the gain margin and phase margin based on your plots, clearly indicating where these were measured. Compare the approximate results from your graphs to the values obtained using the *margin* command in Matlab.
- (c) Based on the phase margin, calculate the delay margin of the system. Simulate the step response of the closed loop system.

2: 20 points

For the plant $G(s) = \frac{100}{s(s+1)(s+10)}$:

- (a) Design a continuous time lead compensator to achieve a phase margin of 60 degrees at a frequency of 5 rad / s. Show that your design meets the specifications with the margin command, and plot the step response of the system.
- (b) Find a state space realization for the plant G(s) (can use Matlab). Design a state space output feedback controller (observer + state feedback) that achieves (roughly) 20% overshoot and a settling time of 6 s. Build the closed loop system in Matlab (the reg command is useful remember positive feedback!) or Simulink and plot an initial condition response (set one of the plant states to 1 initially the initial command will do this in Matlab).

3: 30 points

The file HDD_freqresp.mat includes a (fake) frequency response model of a single-stage hard disk drive. The model's lowest frequency mode is due to compliance in the bearing, whereas the higher frequency modes are due to flexibility in the structure between the actuator and the recording head. A "good" controller for this system will achieve an open loop crossover frequency of 1 kHz, a phase margin of at least 40 degrees, and a gain margin of at least 6 dB.

- (a) Modern hard disk drives have relatively high sampling rates. Assuming the sampling rate is $F_s = 50$ kHz (you need to add this to your plant model with a delay approximation it is not present in the measured data), design a (continuous time) feedback controller that includes integral action and meets the specifications. Use margin to demonstrate that the goals were acheived.
- (b) Plot the sensitivity and complementary sensitivity function for your design. What are their peak values over frequency?
- (c) FRD-based design can be quite effective in practice, but Matlab will not allow you to use an

FRD model in simulink or to simulate performance in Matlab. For simulation we will need to find a state space approximation to the data. Read about the command fitfrd, and use that command to fit an approximation to the complementary sensitivity function T. Plot the Bode plot of the FRD and its approximation on the same axes, and use the approximation to plot the step response.