PTAS, FPTAS, FPRAS and APX

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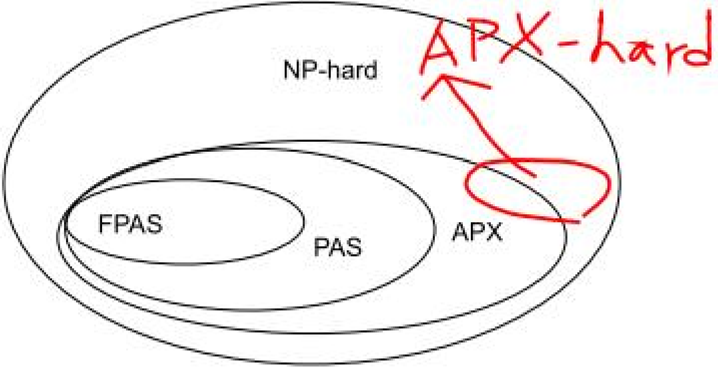

APX-hard
NP-hard
FPAS
PAS
APX

Definition (Approximation Scheme)

An algorithm is an approximation scheme for a minimization (resp. maximization) problem if, given some error parameter $\epsilon > 0$, it acts as a $(1+\epsilon)$-approximation algorithm (resp. $(1-\epsilon)$-approximation algorithm).

Definition (Polynomial Time Approximation Scheme - PTAS)

An approximation scheme is a polynomial time approximation scheme, if for each fixed $\epsilon > 0$, the running time is bounded by a polynomial in the size of the problem instance I.

For example, an approximation scheme with running time

- $O(|I|^{1/\epsilon})$: a PTAS,
- $O(|I|^2 \cdot (1/\epsilon)^3)$: a PTAS,
- $O(2^{|I|} \cdot (1/\epsilon)^3)$: not a PTAS.

Definition (Polynomial Time Approximation Scheme - PTAS)

An approximation scheme is a polynomial time approximation scheme, if for each fixed $\epsilon > 0$, the running time is bounded by a polynomial in the size of the problem instance I.

Definition (Fully Polynomial Time Approximation Scheme - FPTAS)

A fully polynomial time approximation scheme is a PTAS with the modied constraint that the running time is bounded by a polynomial in the size of the problem instance I and $1/\epsilon$.

For example, an approximation scheme with running time

- $O(|I|^{1/\epsilon})$: a PTAS, but not an FPTAS,
- $O(|I|^2 \cdot (1/\epsilon)^3)$: a PTAS, and also an FPTAS.

Randomness

Definition (Polynomial Randomized Approximation Scheme - PRAS)

For cost metric c, an algorithm A is a polynomial randomized approximation scheme if

- For any $\epsilon > 0$, $\Pr[|c(A(I) - c(OPT(I))| \leq \epsilon \cdot c(OPT(I))] > 1/2$,
- A runs in poly$(|I|)$.

Definition (Fully Polynomial Randomized Approximation Scheme - FPRAS)

For cost metric c, an algorithm A is a fully polynomial randomized approximation scheme if

- For any $\epsilon > 0$, $\Pr[|c(A(I) - c(OPT(I))| \leq \epsilon \cdot c(OPT(I))] > 1/2$,
- A runs in poly$(|I|, 1/\epsilon)$.

Definition (Polynomial Time Approximation Scheme - PTAS)

An approximation scheme is a polynomial time approximation scheme, if for each fixed $\epsilon > 0$, the running time is bounded by a polynomial in the size of the problem instance I.

Definition (APX)

APX: class of problems for which approximation algorithms exist with a constant approximation ratio.

Remark

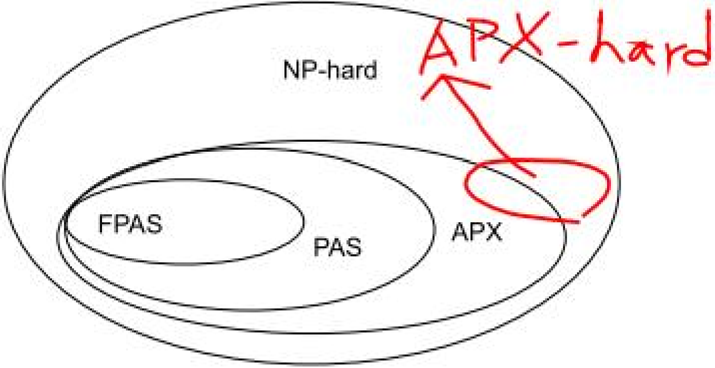
- APX is different from the PTAS / FPTAS setting where we can get arbitrarily good approximations.
- Problems with an (F)PTAS are in APX.
- For example, Minimum Vertex Cover Problem is in APX because it has a 2-approximation algorithm. (Find a maximal matching and collect all endpoints of these edges.)

Definition (APX, APX-hard and APX-complete)

- APX: class of problems for which approximation algorithms exist with a constant approximation ratio.
- APX-hard: class of problems for which there is a constant c such that it is NP-hard to find an approximation algorithm with approximation ratio better than c.
- APX-complete: problems that are APX-hard and also in APX .

Remark

APX-hard problems do not admit PTAS.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