Anti-Coordination Games and its Inefficiency

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Outline

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Motivation

- Coordination with others
  - Choosing communication providers
  - Deciding where to eat
- Avoid collision with others
  - Frequency of radio stations
  - Collaborating super-hard HW
- These scenarios share two common properties:
  - Utility is only locally affected
  - Utility is decided by whether strategies meet

Clustering Game

Definition

A Clustering Game is defined by the tuple $\langle G = (V, E), (w_e)_{e \in E}, (b_e \in \{0, 1\})_{e \in E}, (\Sigma_i)_{i \in V} \rangle$

- G is a simple graph
- Each $v \in V$ corresponds to a player
- w_e is the weight of edge e
- b_e is the type of edge e where 0 implies anti-coordination (vice versa)
- Σ_i is the strategy space of player i.
- If all players share the same strategy space $\{1, ..., k\}$ then abuse notation, denote the space $\Sigma = k$

Clustering Game

Utility

- A coordination edge $e = \{i, j\}$ is satisfied iff $\sigma_i = \sigma_j$ (vice versa)
- Let S_e^σ be the indicator of whether edge e is satisfied in outcome σ
- The utility of player i is defined to be the weighted sum $u_i(\sigma) = \Sigma_{e:i\in e} w_e \dot{S}_e^\sigma$

Classifications

- Max-Cut games: all $b_e = 0$, $\Sigma = 2$
- 2-NAE-SAT games: $\Sigma = 2$
- Max-k-cut / Anti-Coordination Games: $b_e = 0$, $\Sigma = k$
- Coordination games: $b_e = 1$
- Symmetric Coordination games: $b_e = 1$, $\Sigma = k$

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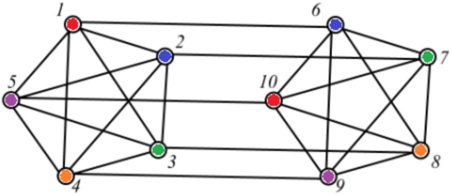
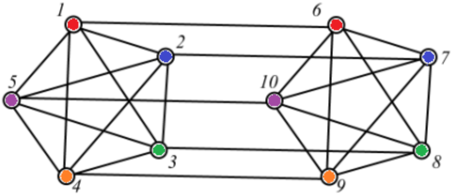
Graph Coloring

- A coloring of a graph $G = (V, E)$
  is a function that maps V to $\{1, ..., k\}$.
- In graph theory we are interested in proper colorings
- A stable coloring corresponds to NE in a coloring game
- A strictly stable coloring corresponds to
  "NE with no alternatives"
- NE, PoA definitions are intuitive

PoA Bound

PoA $= \frac{k}{k-1}$**[1]**





PoA Bound

PoA $= \frac{k}{k-1}$

- Let the potential function be $\Phi =$ (# of satisfied edges)
- Let any "unhappy node" deviate to its locally best choice
- Φ increases each round
- Φ is bounded by $|E|$, so the process will halt
- By pigeonhole principle,
  every "happy node" v achieves at least $\frac{k-1}{k} d(v)$

$\Rightarrow PoA \leq \frac{k}{k-1}$

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Complexity of Graph Coloring

- Determine k-proper-colorability is NP-complete for $k \geq 3$.
- Stable k-coloring for undirected graph:
  - Always exists
  - The greedy algorithm above suffices
- Strictly stable k-coloring for undirected graph:
  NP-complete
- Stable k-coloring for directed graph:
  NP-complete

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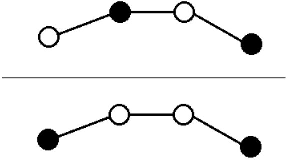
Strong Equilibrum

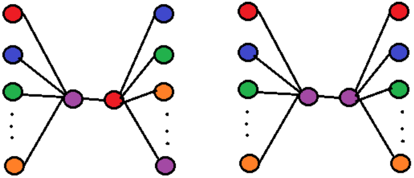
- Recall that a NE is an outcome that no single player strictly wants to deviate
- A q-strong equilibrum (q-SE) is an outcome that no q players can deviate together and strictly(!) improve
- For $q = n$ it is called SE
- SPoA (Strong PoA) and q-SPoA follows ($\neq$ Sequential PoA)

SPoA For Anti-coordination Game

SPoA $\geq \frac{3}{2}$ for $k = 2$ [3]



SPoA $\geq \frac{2k-1}{2k-2}$ [4]



Outline

Clustering Games
Motivation and Classifications

Inefficiency in Anti-Coordination Games
PoA Bounds
Finding Equilibrum and Complexity

Strong PoA and Inefficiency in Clustering Games
Strong Equilibrum and SPoA
Unified Framework for SPoA Bounds in Clustering Games

Future Work

Unified Framework for SPoA bounds

- Feldman and Friedler [2] proposed such framework:
- Reorder all agents(nodes) such that each node doesn't benefit when deviating together with following nodes
- Utilize potential function to obtain lower bound of welfare (function of objects like cuts and interior edges)
- Specific analysis for various game types

Result (Upper Bounds)

For the coordination factor $z(q) = \frac{q-1}{n-1}$,

Class Name	Case Description			Result		
	+/−	# of Str.	Sym	PoA	SPoA	q-SPoA
Max-Cut	−	2	√	1/2	2/3	$\frac{2}{4-z(q)}$ ⋆
2-NAE-SAT	+/−	2	√	1/2	2/3	$\frac{2}{4-z(q)}$ ⋆
Max-k-Cut	−	k	√	$\frac{k-1}{k}$	$\frac{k-1}{k-\frac{1}{2(k-1)}}$ ⋆	$\frac{k-1}{k-\frac{1}{2(k-1)}\cdot z(q)}$ ⋆
SCGGs	+	k	√	$1/k$ ⋆	$\frac{k}{2k-1}$ ⋆	$\frac{2+(k-2)\cdot z(q)}{2k-z(q)}$ ⋆
CGGs	+	k	×	0	1/2	$\frac{z(q)}{2}$
SCGs	+/−	k	√	$1/k$	$\frac{1}{2-\frac{1}{k(k-1)}}$ ⋆	$\frac{2+(k-2)\cdot z(q)}{2k-\frac{1}{k-1}\cdot z(q)}$ ⋆
CGs	+/−	k	×	0	1/2	$\frac{z(q)}{2}$ ⋆

- (S)CGGs = (Symmetric) Coordination Games on Graphs
- (S)CGs = (Symmetric) Clustering Games on Networks
- (Definition of PoA/SPoA is the opposite)

Future Work

For anti-coordination games

- Other payoff functions
- Random graphs
- Weighted-edge graphs

For SE and SPoA for clustering games

- existence of SE
- Gap of q-SPoA on max-k-cut game

J. Kun, B. Powers, L. Reyzin,
Anti-Coordination Games and Stable Graph Colorings,
Algorithmic Game Theory: 6th International Symposium, SAGT 2013, 122-133, 2013.

M. Feldman, P. Friedler,
A Unified Framework for Strong Price of Anarchy in Clustering Games,
International Colloquium of Automata, Languages and Programming, ICALP 2015, 601-613, 2015.

L. Gourves, J.Monnot,
On Strong Equilibria in the Max Cut Game,
WINE 2009, 608-615, 2009.

L. Gourves, J.Monnot,
The Max K-cut Game and its Strong Equilibria,
TAMC 2010, 234-246, 2010.