Definition 0.1. Connected components of a graph are its maximal connected subgraphs.

For connected components problem (CCP), it is well-known that we can apply BFS or DFS to find connected components.

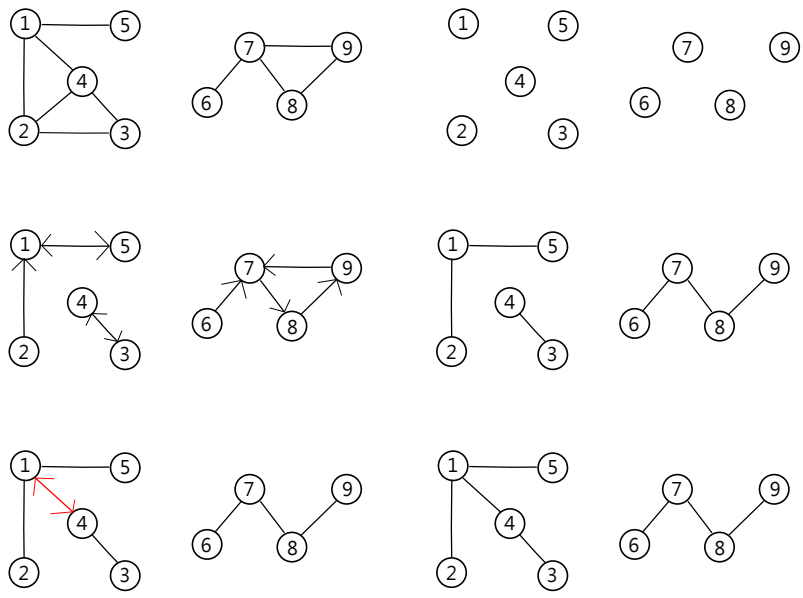
Question: For SCCP, how to find connected components?

Answer: Apply Boruvka's algorithm to construct a maximal forest.

Boruvka's algorithm:

1. Initially, view each vertex as a "part",
2. for each part, find an "out-going edge",
3. combine some parts by these out-going edges (delete cycle edge if necessary),
4. repeat 2. & 3. until there is no out-going edge for any part.

Example:



Question: For SCCP, how to find out-going edges?

Definition 0.2. Consider a graph $G = (V, E)$, for $A \subsetneq V$, $A \neq \emptyset$, a cut in G is defined as the set of edges: $[A, V \setminus A] = \{(u, v) : u \in A, v \in V \setminus A\}$.

Observation:

- Each non-trivial proper subset A corresponds to a cut $[A, V \setminus A]$; therefore, each part (Boruvka's algorithm) corresponds to a cut,
- for each part, each cut-edge in the corresponding cut can be viewed as an out-going edge.

Strategy: Try to find a cut-edge in the corresponding cut. How to find?