Between Simultaneity and Sequentiality

Shao-Heng Ko

Collective Algorithms Lab
National Taiwan University
r04921049@ntu.edu.tw

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Outline

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Simultaneous Game vs. Sequential Game

- Given n players in simultaneous game, each has action space A_i
- The sequential version: order the players from 1 to n
- Strategy is from $A_1 \times A_2 \times ... \times A_{j-1}$ to A_j for player j
- SPNE(Subgame Perfect Nash Equilibrum):
  In all subgames, the player in turn makes a no-regret choice

SPNE vs. NE

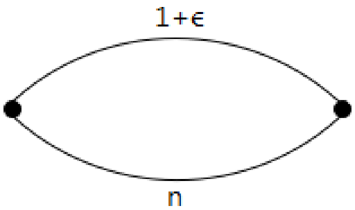
- Perfect information: players fully observe what the predecessors do
- If perfect information, SPNE exists and can be derived using backward-induction
- $NE \subset SPNE$, for the sequential game itself
- In general not a NE for the simultaneous game

Outline

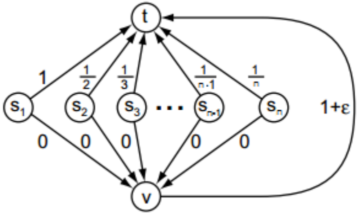
Machine Cost Sharing: Model

- n jobs (players)
- m machines
- R_i: the set of machines that job i can choose
- $\gamma_r(x)$: decreasing cost function for machine r
- Machine $s_i \in R_i$ is chosen by job i (strategies)
- Cost function is given by:
  $c_i(S) = \gamma_{s_i}(n_{s_i})$ where $n_r = |j \in N; s_j = r|$
- Fair cost allocation: $\gamma_r(x) = c_r/x$

Simultaneous Version: PoA and PoS [2]

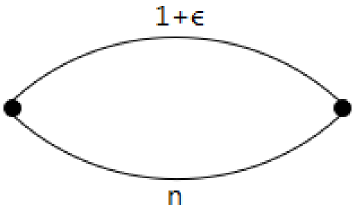


(a) PoA = $O(n)$

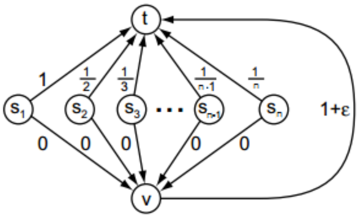


(b) PoS = $O(log(n))$

Sequential Version: SPNE and SPoA [1]



(c) SPoA = 1



(d) SPoA = $O(log(n))$

- $O(log(n))$-optimal SPNE
- Can be found by a simple $O(log(n))$ greedy algorithm
- SPNE is independent of the order of the sequence
- $\Rightarrow SPNE \in NE$

Concept and Result

- Coordination reduces cost
- By choosing "coordination machines", former players can expect latter players to join (cannot be done in simultaneity)
- SPoA also improves (compared to PoA) in the following games:
  - Unrelated Machine Scheduling Games
  - Concensus Games
- In Cut Games(opposition of Concensus Games), SPoA is worse

Outline

Symmetric Atomic Network Routing: Model

- Directed graph $G(V, E)$
- Source and destination $s, t \in V$
- A linear latency function $l_e(x) = d_e(x)$ for each $e \in E$
- n players traveling from s to t (symmetric)
- Each player choosing one $s - t$ path (strategies)
- Cost function is given by: by choosing path A_i,
  $c_i(A) = \sum_{e \in A_i} d_e n_e$,
  where $n_e = \sum_{i=1}^{n} |A_i \bigcap \{e\}|$ is # of players using e.

Previous Work on This Model

- For atomic case:
  $PoA = 5/2$, tight for $n \geq 3$ (Christodoulou, 2005) [4]
- For non-atomic case: (comparison)
  $PoA = 4/3$ (Braess's Paradox!) (Roughgarden, 2001) [5]
- For sequential version of the game:
  $SPoA < PoA$ for small n (de Jong, 2014) [6]
- de Jong and Uetz thought that SPoA is also $5/2$.

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Two-player Network Routing

- $PoA = 4/3$ (Why?)
- It is proved that $SPoA = 7/5$ in this paper
- Upper bound is proved by linear programming (omitted)
- Lower bound: an example

Lower Bound Example

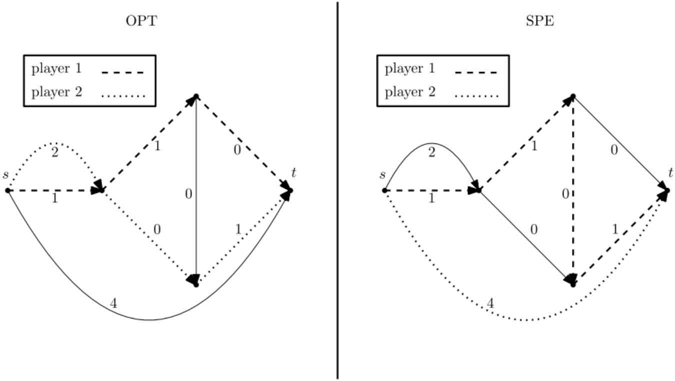

FIGURE 2. Lower bound example for 2 players. Numbers are arc latencies.

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SPoA of N-Player Game is Unbounded

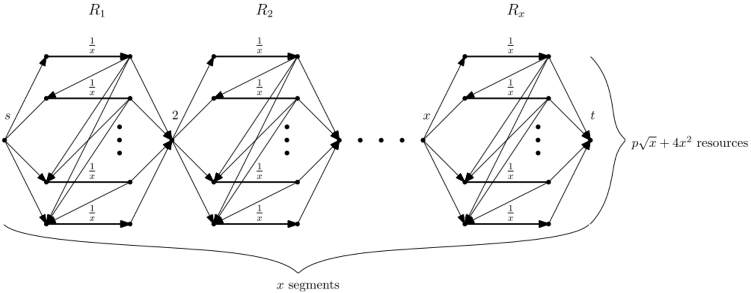

FIGURE 4. A lower bound instance of a network routing game. Players travel from s to t.

There are $n = p\sqrt{x} + 5x^2$ players, while n is large enough (larger than disjoint strategies).

SPoA of N-Player Game is Unbounded

- Optimality: each player chooses 1 arc from each segment (Players share arcs as little as possible)
- Optimal social cost is $p\sqrt{x} + 7x^2$.
- An algorithmetic SPNE is constructed
- Threaten former players and force inefficient strategies
- Threatening cost is low for latter players; high for former
- SPNE social cost is $p\sqrt{x}x + 7x^2$.
- $SPoA \to x$.

Outline

Comparisons

Game	PoA	SPoA	Winner
Machine Cost Sharing	$O(n)$	$O(log(n))$	Sequentiality
Consensus Games	∞	1(SPNE=opt)	Sequentiality
Cut Games	2	$2 \leq SPoA \leq 4$	Simultaneity
2-p Linear Routing	$4/3$	$7/5$	Simultaneity
General Linear Routing	$5/2$	∞	Simultaneity

Q: How to decide the winner?

Revisit NE vs. SPNE

- In NE, everyone decides simultaneously
  $\rightarrow$ some kind of fairness
- In SPNE, the order of players create unfairness
- An important concept of SPNE:
  SPNE is supported by threats on off-equilibrum actions
- How about threats cannot be formed?

On Externality

- Ironically, in Machine Cost Sharing Games
  latter players can do only positive externalities on formers
- $SPNE \subset NE$; in fact they are the best NE's
- While in Linear Routing Games
  latter players can do only negative externalities on formers
- Exist some $SPNE \not\in NE$; they are far more inefficient
- Threats are constructed on negative externalities

Outline

Conclusion

- Insight: from the nature of games, we may see whether simulatneity or sequentiality does better
- However, even in games with negative externalities, sequentiality may help avoid bad (unfortunate) NE.
  Ex: Traffic lights, Consensus games
- My guess: SPoA(sequentiality) is better most of the time, but society needs not only efficiency but also fairness

R. P. Leme, V. Syrgkanis, E. Tardos,
The Curse of Simultaneity,
Proceedings of the 3rd ITCS, 60-67, 2012.

E. Anshelevich, A. Dasgupta, J.M. Kleinberg, E. Tardos, T. Wexler, T. Roughgarden,
The Price of Stability for Network Design with Fair Cost Allocation,
45th Symposium on Foundations of Computer Science (FOCS 2004), 295-304, 2004.

J. Correa, J. de Jong, B. de Keijzer, M. Uetz,
The Curse of Sequentiality in Routing Games,
Web and Internet Economics: 11th International Conference, 258-271, 2015.

G. Christodoulou, E. Koutsoupias,
The Price of Anarchy of Finite Congestion Games,

Proceedings of the 37th STOC, 67-73, 2005.

T. Roughgarden, E. Tardos,
How Bad is Selfish Routing?,
Journal of ACM, vol. 49, no. 2, 236–259, 2002.

J. de Jong, M. Utez,
The Sequential Price of Anarchy for Atomic Congestion Games,
Proceedings of the 10th WINE, 429-434, 2014.