Wireless Information Flow: A Deterministic Approach

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Introduction

Two main distinguishing features of wireless communication: Broadcast and Superposition.

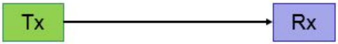
Because of complex interaction of signals, we know much less about information flow over wireless networks than wired networks.

We'd like to ask such questions:

- How to model the interaction of signals?
- How much information can we transmit over wireless networks?
- How to achieve?

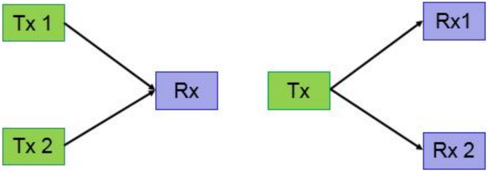
What We Know

- Point-to-point Channel



Shannon 48

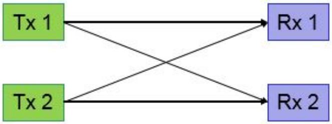
- Multiple Access Channel & Broadcast Channel



Alshwede, Liao 70's Cover, Bergmans 70's

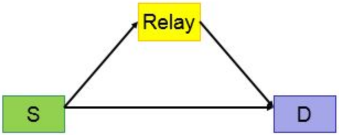
What We Don't Know

- Interference Channel



Best known achievable region: Han & Kobayashi 81

- Relay Network



Best known achievable region: El Gamal & Cover 79

Focus

Even popular Gaussian channels, we still know little.

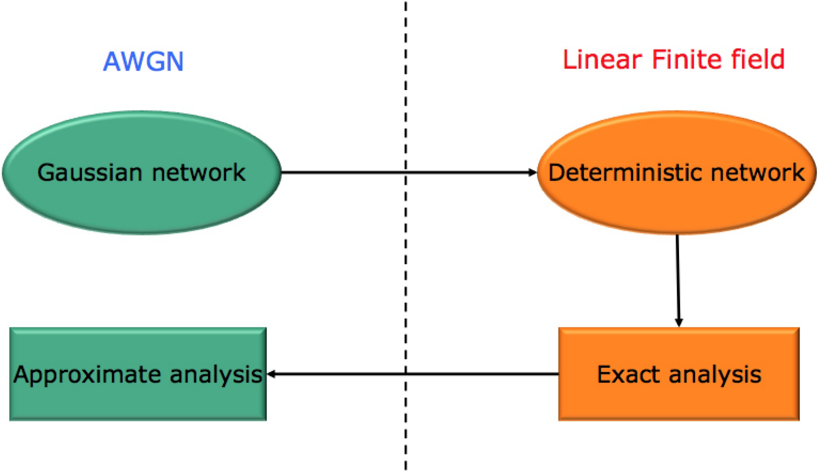
We start from point-to-point real scalar Gaussian channels.

$$y = \sqrt{SNR}x + z \quad \text{where} \quad z \sim \mathcal{N}(0, 1)$$

- Most literature studied the effect of noise.
- However, with coding scheme, we can reduce noise effect.
- It's how we exploit information from interference.

Avestimehr, Diggavi, and Tse [1], introduces deterministic model.

General Methodology



How to Downplay the Noise

- Quantize the signal as a sequence of bits at multiple signal levels.
- Limit the peak transmission and noise power to 1.
- $n = \log \sqrt{SNR}$ bits are received without noise.

$$x = 0.11101100......, \sqrt{SNR} = 2^5$$
$$z = 0.11101011......$$
$$y = 11101.11101011......$$

Point-to-point deterministic channel

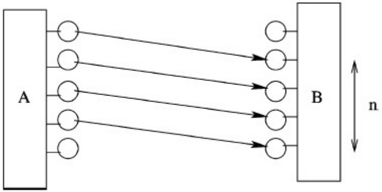
Given SNR, we describe the capacity as

$$n \longleftrightarrow \lceil \frac{1}{2} log SNR \rceil^+.$$

The gap between n and $C_{AWGN} = \frac{1}{2} log(1 + SNR)$ is at most 1 bit.

How to Model Interaction of Signals

- When 2 bits at the same signal level collide, instead of dropping both bits, the receiver hears the modulo 2 summation of them.
- We ignore the carry-overs and operates in $\mathbb{F}_2$.

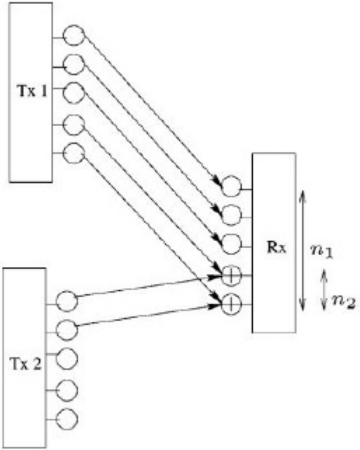


Pictorial representation of the point-to-point model

Now we take a look on MAC and BC network.

Apply to Multiple Access Channel

Applying deterministic model to the Gaussian MAC channel yields



Deterministic MAC model

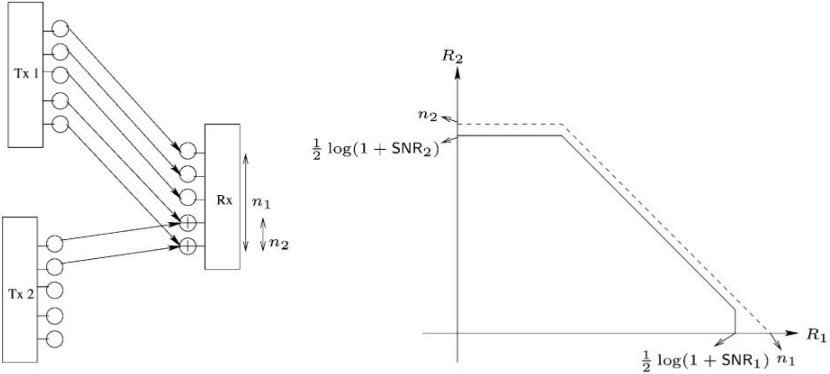
The capacity region is given by

$$R_2 \leq n_2$$
$$R_1 + R_2 \leq n_1$$

where $n_i = \lceil \frac{1}{2} log SNR_i \rceil^+, i = 1, 2.$

Apply to Multiple Access Channel

The capacity regions of wireless and deterministic MACs are within 1 bit/user of each other.



Pictorial representation of the deterministic MAC

Apply to Broadcast Channel

- Assume $SNR_1 \geq SNR_2$, i.e., receiver 1 is the stronger user.
- Utilize successive interference cancellation decoding.

Applying deterministic model to the Gaussian BC channel yields

Deterministic BC model

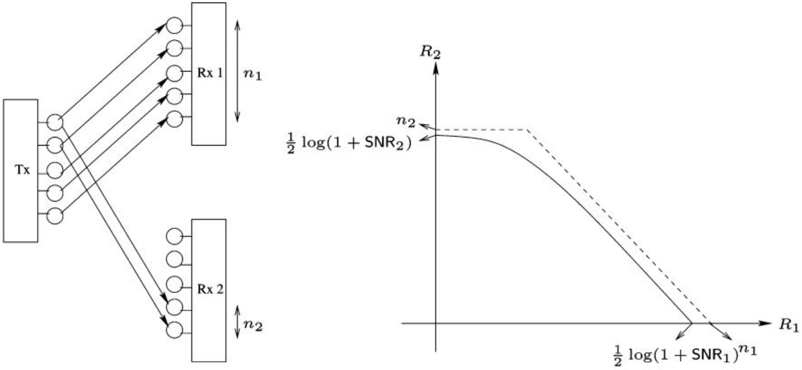
The capacity region is given by

$$
\begin{aligned}
R_2 &\leq n_2 \\
R_1 + R_2 &\leq n_1
\end{aligned}
$$

where $n_i = \lceil \frac{1}{2} log SNR_i{}^{+} \rceil, i = 1, 2.$

Apply to Broadcast Channel

The capacity regions of wireless and deterministic BCs are within 1 bit/user of each other.



Pictorial representation of the deterministic BC

Cut-set Upper Bound

We try to approach wireless networks with unknown capacity regions.

For single-unicast, assume the networks to be synchronized, we have a cut-set bound for such networks. (Cover & Thomas [5])

Cut-set upper bound for synchronized relay network

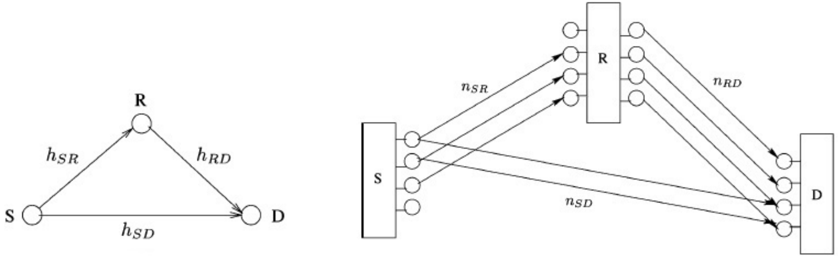
The capacity C is upper bounded by

$$\bar{C} = \max_{p(\{\mathbf{x}_j\}_{j \in V})} \min_{\Omega \in \Lambda_D} I(\mathbf{y}_{\Omega^c}; \mathbf{x}_\Omega | \mathbf{x}_{\Omega^c})$$

We will compare the cut-set bound to our achievable capacity in the following analysis, to assess the approximation.

Single-Relay Deterministic Model

The single-relay Gaussian network and its deterministic model:



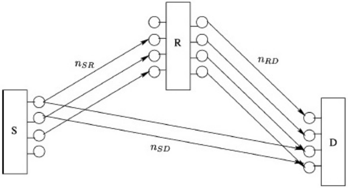
Best known achievable region: El Gamal & Cover 79

Single-Relay Deterministic Model

Capacity for single relay network

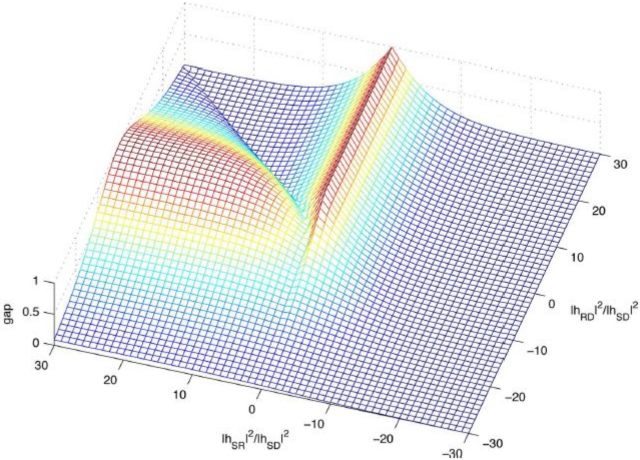
The capacity C is given by

$$C^{d}_{relay} \leq \min\{\max(n_{SR}, n_{SD}), \max(n_{RD}, n_{SD})\}$$
$$= \begin{cases} n_{SD}, & \text{if } n_{SD} > \min(n_{SR}, n_{RD}) \\ \min(n_{SR}, n_{RD}), & \text{otherwise.} \end{cases}$$



which is again proved to be within 1 bit/s/Hz of the cut-set bound.

Single-Relay Deterministic Model



Value of gap between the cut-set bound and the achievable rate in bits/sec/Hz

Main Results

Deterministic model introduces new transmission strategy: quantize-map-and-forward, and the following results:

- Capacity of linear finite-field deterministic relay networks
- Upper and lower bound of Gaussian relay networks
- Connections between truncated deterministic model and Gaussian relay networks

Linear Finite-field Deterministic Relay Networks

Single-unicast linear finite-field deterministic relay network

The capacity C is given by

$$C = \min_{\Omega \in \Lambda_D} \operatorname{rank}(\mathbf{G}_{\Omega,\Omega^c})$$

where $\Lambda_D = \{\Omega : S \in \Omega, D \in \Omega^c\}$: all source-destination cuts.

Achievability proof major illustration:

1. Paths from source to destination are all equal (layered networks)
   - Source maps each message into a random codeword.
   - Each relay j uses random linear mapping $f_j : \mathbf{x}_j \to \mathbf{y}_j$
   - Idea: messages can be distinguished
2. Generalize to arbitrary networks via time-expansion

For multicast, take $\min_{D \in \mathcal{D}} C$, $\mathcal{D}$: the set of destinations.

Gaussian Relay Networks

Let node $j \in \mathcal{V}$ have M_j transmit, N_j receive antennas.

Single-unicast Gaussian relay network (Approximation Theorem)

The capacity C satisfies

$$\bar{C} - \kappa \leq C \leq \bar{C}$$

where $\bar{C}$ is the cut-set upper bound, and κ is a constant and upper bounded by $12\sum_{i=1}^{|\mathcal{V}|} N_i + 3\sum_{i=1}^{|\mathcal{V}|} M_i$

Achievability proof based on layered networks to arbitrary ones:

1. Outer code: encode message into N inner code symbols u_i's
2. Inner code: At each relay j, random mapping F_j of quantized received signal onto i.i.d. $\mathcal{CN}(0,1)$ random vector of length T

For multicast, apply multicast cut-set bound $\bar{C}_{multi}$.

Connections Between Models

The linear finite-field channel model captures certain high SNR behaviors of the Gaussian channel, but its capacity is not within a constant gap to the Gaussian capacity for all MIMO channels.

Q: Can we extend deterministic model to approximate the Gaussian relay network capacity to within a constant gap?

Try "quantization" with MIMO matrix $\mathbf{H}_{ji}$: $\mathbf{y}_j[t] = \left[\sum_{i\in\mathcal{V}} \mathbf{H}_{ji}\mathbf{x}_i[t]\right]$
We call it the "truncated deterministic model."

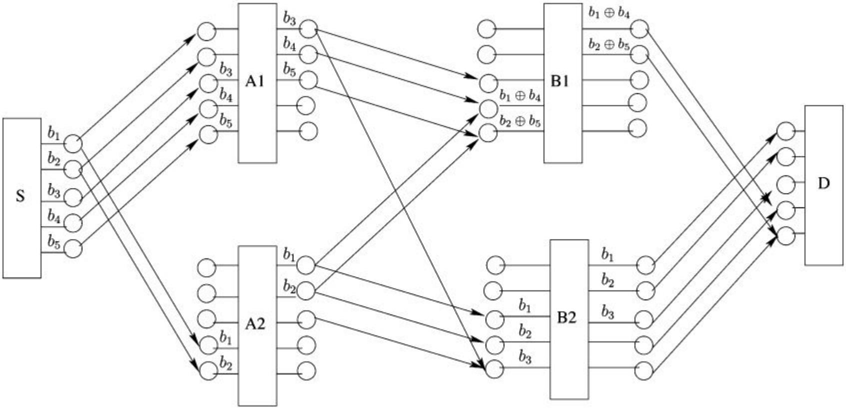
Gap between truncated deterministic model and Gaussian model

The gap is bounded as:

$$|C_{\text{Gaussian}} - C_{\text{Truncated}}| \leq 33|\mathcal{V}|$$

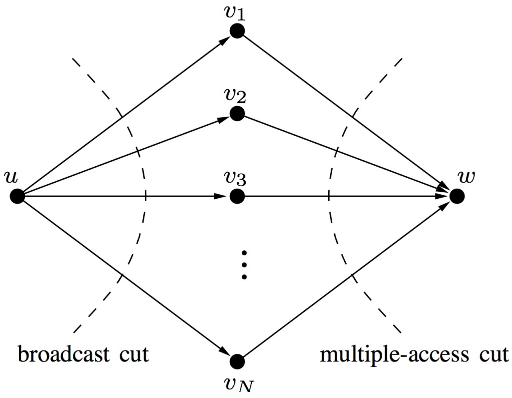
Applications

- Four-relay Gaussian Networks



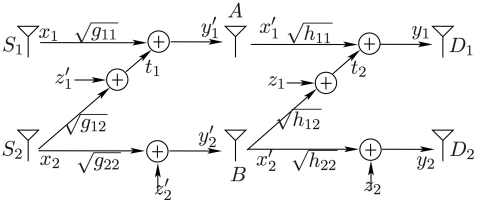
Applications

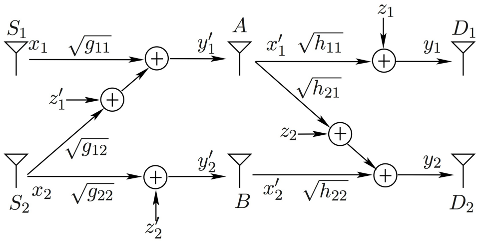
- Niesen & Diggavi [2]: One-stage N-relay Diamond



Applications

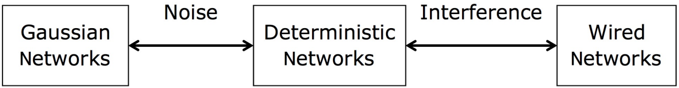
- Mohajer et al. [3]: ZZ, ZS Two-stage Interference Network





Conclusions

1. Insight: if something can't be computed exactly, approximate.
2. Quantize-map-and-forward for achievability
   - Gap to cut-set bound universal with *SNR*'s.
   - It varies with # of transmit and receive antennas.
3. Role of deterministic model



4. Gap between C_{Gaussian} and $C_{\text{Truncated}}$ grows with network size.
5. Q: is there any scheme that circumvents the growth of network size, or this is an inherent feature?
6. Gaussian Interference channels

References

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[2] U. Niesen, S. Diggavi, "The Approximate Capacity of the Gaussian N-Relay Diamond Network," *IEEE Trans. on Information Theory* 59 (2), 2013, 845-859.

[3] S. Mohajer, S. N. Diggavi, C. Fragouli, and D. N. C. Tse,"Approximate capacity of a class of Gaussian interference-relay networks" *IEEE Trans. Information Theory*, vol. 57, no. 5, pp. 2837-2864, May 2011.

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[5] T. M. Cover and J. A. Thomas, *Elements of Information Theory*, ser. Wiley Series in Telecommunications and Signal Processing, 2nd ed. Hoboken, NJ: Wiley, 2006.

[6] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation" *IEEE Trans. Information Theory*, vol. 50, no. 5, pp. 748-767, May 2004.