Series : Z6YWX **SET~3**

प्रश्न-पत्र कोड
Q.P. Code **65/6/3**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* *अधिकतम अंक : **80***

*Time allowed : **3** hours* *Maximum Marks : **80***

नोट		NOTE	
(I)	कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।	(I)	Please check that this question paper contains **23** printed pages.
(II)	प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।	(II)	Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III)	कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।	(III)	Please check that this question paper contains **38** questions.
(IV)	**कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।**	(IV)	**Please write down the Serial Number of the question in the answer-book at the given place before attempting it.**
(V)	इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।	(V)	15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\frac{dy}{dx}$ is :

(A) $\frac{x}{y}$ (B) $-\frac{x}{y}$

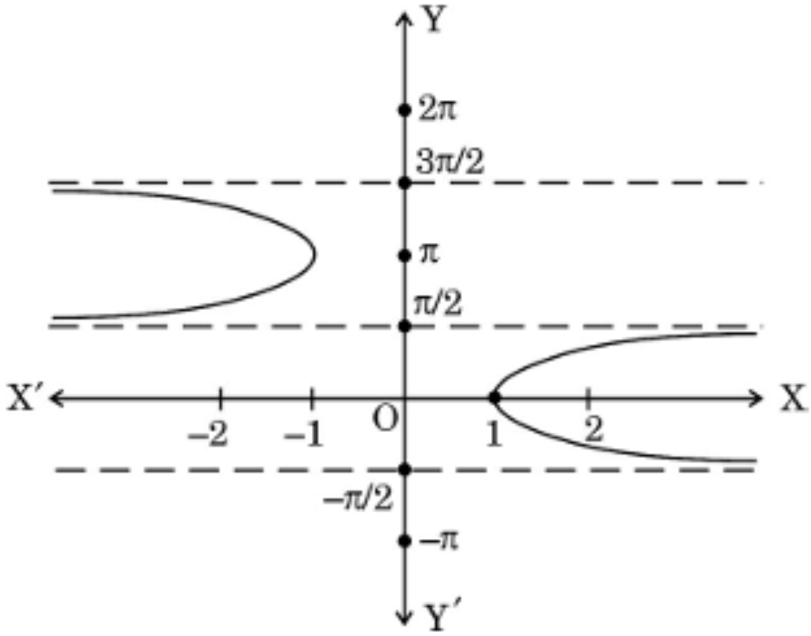
(C) $\frac{a}{x}$ (D) $\frac{a}{y}$

2. If $A = \begin{bmatrix} 0 & 0 & -5 \\ 0 & 3 & 0 \\ 4\cdot3 & 0 & 0 \end{bmatrix}$, then A is a :

(A) skew-symmetric matrix (B) scalar matrix

(C) diagonal matrix (D) square matrix

3. The graph shown below depicts :



(A) $y = \sec^{-1} x$ (B) $y = \sec x$

(C) $y = \mathrm{cosec}^{-1} x$ (D) $y = \mathrm{cosec}\, x$

4. $\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)\right]$ is equal to :

(A) $\dfrac{11\pi}{12}$ (B) $\dfrac{5\pi}{12}$

(C) $-\dfrac{5\pi}{12}$ (D) $\dfrac{7\pi}{12}$

5. Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is :

(A) $n \times n$ (B) $n \times m$

(C) $m \times m$ (D) $m \times n$

6. Sum of two skew-symmetric matrices of same order is always a/an :

(A) skew-symmetric matrix

(B) symmetric matrix

(C) null matrix

(D) identity matrix

7. If $y = a \cos(\log x) + b \sin(\log x)$, then $x^2 y_2 + x y_1$ is :

(A) $\cot(\log x)$ (B) y

(C) $-y$ (D) $\tan(\log x)$

8. If $f(x) = \begin{cases} \dfrac{\log(1+ax) + \log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$

is continuous at $x = 0$, then the value of k is :

(A) a (B) $a + b$

(C) $a - b$ (D) b

9. $f(x) = x^x$ has a critical point at :

(A) $x = e$ (B) $x = e^{-1}$

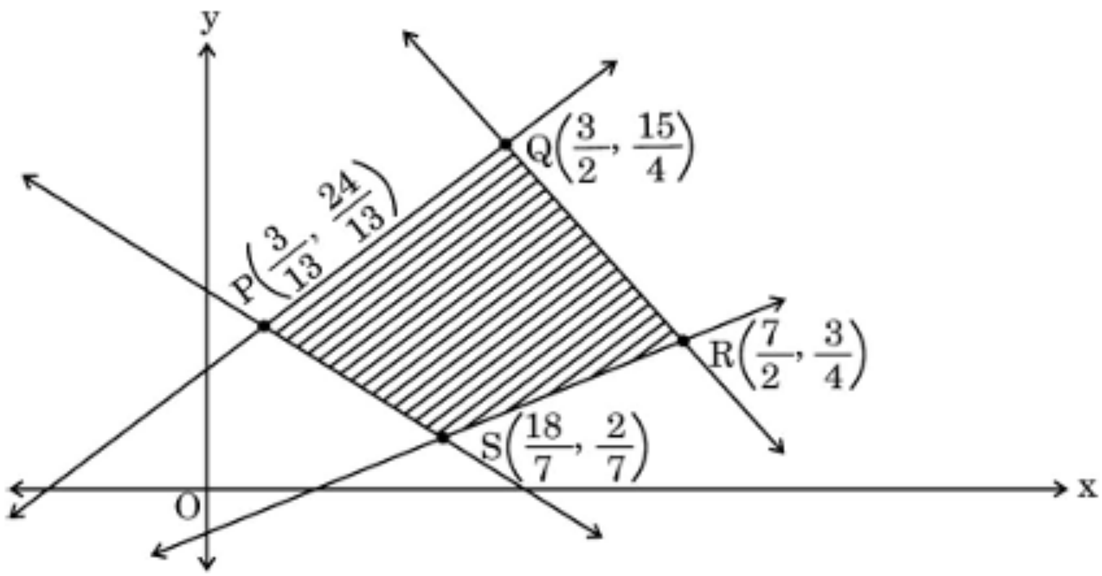
(C) $x = 0$ (D) $x = 1$

10. The solution for the differential equation $\log\left(\dfrac{dy}{dx}\right) = 3x + 4y$ is :

(A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$

(C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$

11. For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.



(Note : The figure is not to scale)

$$P \equiv \left(\frac{3}{13}, \frac{24}{13}\right), Q \equiv \left(\frac{3}{2}, \frac{15}{4}\right), R \equiv \left(\frac{7}{2}, \frac{3}{4}\right), S \equiv \left(\frac{18}{7}, \frac{2}{7}\right)$$

Which of the following statements is correct ?

(A) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$

(B) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$

(C) (Value of Z at P) > (Value of Z at Q)

(D) (Value of Z at Q) < (Value of Z at R)

12. The order and degree of the differential equation $\left[\frac{d^2y}{dx^2} - 1\right]^2 = \frac{dy}{dx}$ are, respectively :

(A) 2, 2 (B) 2, not defined

(C) 1, 2 (D) 1, not defined

13. Let $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, $f(1) = 0$. Then, $f(x)$ is :

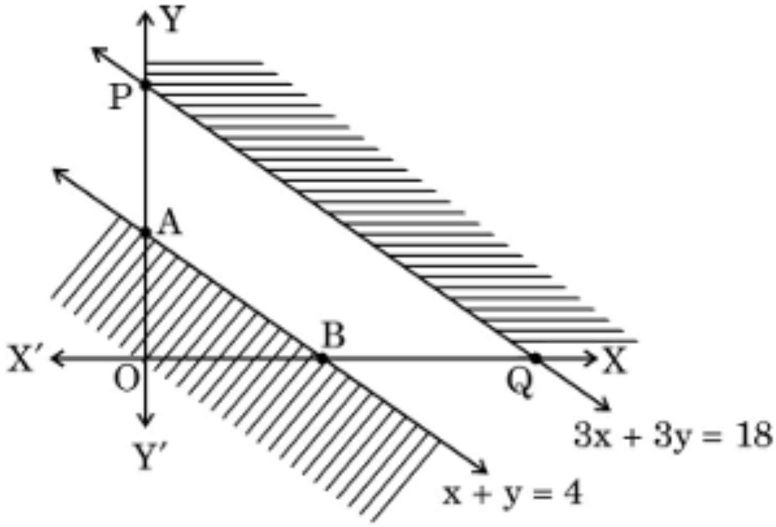
(A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$ (B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$

(C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$

14. In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints :

$$x + y \leq 4,\ 3x + 3y \geq 18,\ x, y \geq 0$$

Study the graph and select the correct option.



(Note : The figure is not to scale)

The solution of the given LPP :

(A) lies in the shaded unbounded region.

(B) lies in Δ AOB.

(C) does not exist.

(D) lies in the combined region of Δ AOB and unbounded shaded region.

15. The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is :

(A) $\frac{3}{2}$ sq units (B) $\frac{2}{3}$ sq units

(C) 3 sq units (D) $\frac{4}{3}$ sq units

16. $\int \frac{x+5}{(x+6)^2} e^x \, dx$ is equal to :

(A) $\log(x+6) + C$ (B) $e^x + C$

(C) $\frac{e^x}{x+6} + C$ (D) $\frac{-1}{(x+6)^2} + C$

17. Let $|\vec{a}| = 5$ and $-2 \le \lambda \le 1$. Then, the range of $|\lambda \vec{a}|$ is :

(A) [5, 10] (B) [– 2, 5]

(C) [– 2, 1] (D) [–10, 5]

18. A meeting will be held only if all three members A, B and C are present. The probability that member A does not turn up is 0·10, member B does not turn up is 0·20 and member C does not turn up is 0·05. The probability of the meeting being cancelled is :

(A) 0·35 (B) 0·316

(C) 0·001 (D) 0·65

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If $|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 256$ and $|\vec{b}| = 8$, then $|\vec{a}| = 2$.

Reason (R) : $\sin^2\theta + \cos^2\theta = 1$ and

$|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin\theta$ and $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$.

20. *Assertion (A) :* Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g)\, x = e^x + \log x$ where domain of $(f + g)$ is R.

Reason (R) : $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) If $\vec{a}$ and $\vec{b}$ are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that BC = 3BA.

OR

(b) Vector $\vec{r}$ is inclined at equal angles to the three axes x, y and z. If magnitude of $\vec{r}$ is $5\sqrt{3}$ units, then find $\vec{r}$.

22. Find the domain of $f(x) = \sin^{-1}(-x^2)$.

23. Find the interval in which $f(x) = x + \frac{1}{x}$ is always increasing, $x \neq 0$.

24. (a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$.

OR

(b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.

25. Find the angle at which the given lines are inclined to each other :

$$l_1 : \frac{x-5}{2} = \frac{y+3}{1} = \frac{z-1}{-3}$$

$$l_2 : \frac{x}{3} = \frac{y-1}{2} = \frac{z+5}{-1}$$

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. Find the value of x, if $\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = O$.

27. (a) Find the distance of the point P(2, 4, –1) from the line $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$.

OR

(b) Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.

28. Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints

$$2x + y \geq 1000$$

$$x + 2y \geq 800$$

$$x, y \geq 0.$$

Draw a neat graph of the feasible region and find the minimum value of Z.

29. (a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in N$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.

OR

(b) Show that the function $f : N \rightarrow N$, where N is a set of natural numbers, given by $f(n) = \begin{cases} n-1, & \text{if n is even} \\ n+1, & \text{if n is odd} \end{cases}$ is a bijection.

30. (a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.

OR

(b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.

31. Bag I contains 4 white and 5 black balls. Bag II contains 6 white and 7 black balls. A ball drawn randomly by from bag I is transferred to bag II and then a ball is drawn randomly from bag II. Find the probability that the ball drawn is white.

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Solve the differential equation : $x^2y\,dx - (x^3 + y^3)\,dy = 0$.

OR

(b) Solve the differential equation $(1 + x^2)\,\frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.

33. Using integration, find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ bounded between the lines $x = -\frac{a}{2}$ to $x = \frac{a}{2}$.

34. (a) Find :

$$\int \frac{x^2+1}{(x-1)^2\,(x+3)}\,dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\,dx$$

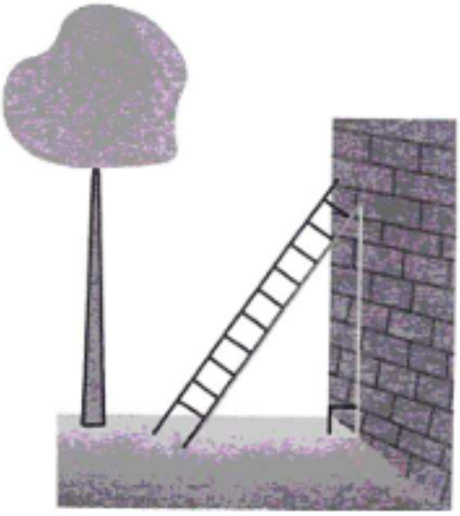
35. Show that the line passing through the points A (0, –1, –1) and B (4, 5, 1) intersects the line joining points C (3, 9, 4) and D (– 4, 4, 4).

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36.



A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.

Based upon the above information, answer the following questions :

(i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. *1*

(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point. *1*

(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. *2*

OR

(iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ? *2*

Case Study – 2

37. A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.

A person buys a smartphone from this shop.

(i) Find the probability that it was defective. *2*

(ii) What is the probability that this defective smartphone was manufactured by company B ? *2*

Case Study – 3

38. Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.

Based upon the above information, answer the following questions :

(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form AX = B. *1*

(ii) Find $|A|$ and confirm if it is possible to find A^{-1}. *1*

(iii) (a) Find A^{-1}, if possible, and write the formula to find X. *2*

OR

(iii) (b) Find $A^2 - 8I$, where I is an identity matrix. *2*