Series : YXW2Z

SET ~ 3

रोल नं.

Roll No.

प्रश्न-पत्र कोड
Q.P. Code 65/2/3

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

नोट

(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।

(II) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।

(III) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।

(IV) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें ।**

(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

NOTE

(I) Please check that this question paper contains **23** printed pages.

(II) Please check that this question paper contains **38** questions.

(III) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(IV) **Please write down the serial number of the question in the answer-book at the given place before attempting it.**

(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

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General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A**, **B**, **C**, **D** and **E**.*

(iii) *In **Section A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) type and Question Number **19** & **20** are Assertion-Reason based questions of 1 mark each.*

(iv) *In **Section B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In **Section E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, 2 questions in Section – **D** and **2** questions in Section – **E**.*

(ix) *Use of calculator is **NOT** allowed.*

SECTION – A

(This section comprises of **20** multiple choice questions (MCQs) of **1** mark each.)

(20 × 1 = 20)

1. If $\vec{p}$ and $\vec{q}$ are unit vectors, then which of the following values of $\vec{p} \cdot \vec{q}$ is not possible ?

   (A) $\frac{-1}{2}$ (B) $\frac{1}{\sqrt{2}}$

   (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{3}$

2. Which of the following can be both a symmetric and skew-symmetric matrix ?

   (A) Unit Matrix (B) Diagonal Matrix

   (C) Null Matrix (D) Row Matrix

3. If $\int_0^a x\, dx \leq \frac{a}{2} + 6$, then

   (A) $-4 \leq a \leq 3$ (B) $a \geq 4, a \leq -3$

   (C) $-3 \leq a \leq 4$ (D) $-3 \leq a \leq 0$

4. If A and B are square matrices of same order, then $(AB^T - BA^T)$ is a

   (A) symmetric matrix (B) skew-symmetric matrix

   (C) null matrix (D) unit matrix

5. The value of $\cos\left(\frac{\pi}{6} + \cot^{-1}(-\sqrt{3})\right)$ is

   (A) -1 (B) $\frac{-\sqrt{3}}{2}$

   (C) 0 (D) 1

6. If p and q are respectively the order and degree of the differential equation $\frac{d}{dx}\left(\frac{dy}{dx}\right)^3 = 0$, then $(p - q)$ is

(A) 0 (B) 1

(C) 2 (D) 3

7. The function $f(x) = x^2 - 4x + 6$ is increasing in the interval

(A) $(0, 2)$ (B) $(-\infty, 2]$

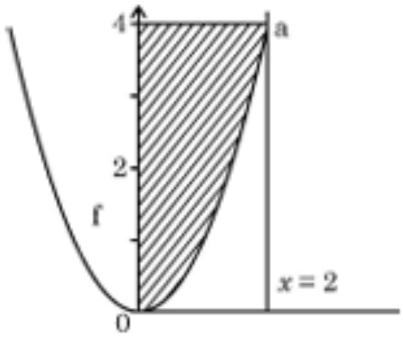
(C) $[1, 2]$ (D) $[2, \infty)$

8. The line $x = 1 + 5\mu$, $y = -5 + \mu$, $z = -6 - 3\mu$ passes through which of the following point ?

(A) $(1, -5, 6)$ (B) $(1, 5, 6)$

(C) $(1, -5, -6)$ (D) $(-1, -5, 6)$

9. The area of the shaded region (figure) represented by the curves $y = x^2$, $0 \leq x \leq 2$ and y-axis is given by



(A) $\int_0^2 x^2 \, dx$ (B) $\int_0^2 \sqrt{y} \, dy$

(C) $\int_0^4 x^2 \, dx$ (D) $\int_0^4 \sqrt{y} \, dy$

10. If E and F are two events such that $P(E) > 0$ and $P(F) \neq 1$, then $P(\overline{E}/\overline{F})$ is

(A) $\dfrac{P(\overline{E})}{P(\overline{F})}$ (B) $1 - P(\overline{E}/F)$

(C) $1 - P(E/F)$ (D) $\dfrac{1 - P(E \cup F)}{P(\overline{F})}$

11. The probability distribution of a random variable X is given by :

X	-4	-3	-2	-1	0
P(X)	0.1	0.2	0.3	0.2	0.2

Then E(X) of distribution is

(A) -1.8 (B) -1

(C) 1 (D) 1.8

12. If projection of $\vec{a} = \alpha\hat{i} + \hat{j} + 4\hat{k}$ on $\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$ is 4 units, then α is

(A) -13 (B) -5

(C) 13 (D) 5

13. The equation of a line parallel to the vector $3\hat{i} + \hat{j} + 2\hat{k}$ and passing through the point $(4, -3, 7)$ is :

(A) $x = 4t + 3, y = -3t + 1, z = 7t + 2$

(B) $x = 3t + 4, y = t + 3, z = 2t + 7$

(C) $x = 3t + 4, y = t - 3, z = 2t + 7$

(D) $x = 3t + 4, y = -t + 3, z = 2t + 7$

14. If a line makes angles of $\dfrac{3\pi}{4}, \dfrac{\pi}{3}$ and θ with the positive directions of x, y and z-axis respectively, then θ is

(A) $\dfrac{-\pi}{3}$ only (B) $\dfrac{\pi}{3}$ only

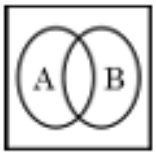
(C) $\dfrac{\pi}{6}$ (D) $\pm\dfrac{\pi}{3}$

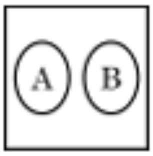
15. A factory produces two products X and Y. The profit earned by selling X and Y is represented by the objective function $Z = 5x + 7y$, where x and y are the number of units of X and Y respectively sold. Which of the following statement is correct ?
   (A) The objective function maximizes the difference of the profit earned from products X and Y.
   (B) The objective function measures the total production of products X and Y.
   (C) The objective function maximizes the combined profit earned from selling X and Y.
   (D) The objective function ensures the company produces more of product X than product Y.

16. If A denotes the set of continuous functions and B denotes set of differentiable functions, then which of the following depicts the correct relation between set A and B ?

(A) 

(B) 

(C) 

(D) 

17. Four friends Abhay, Bina, Chhaya and Devesh were asked to simplify $4\,AB + 3(AB + BA) - 4\,BA$, where A and B are both matrices of order 2×2. It is known that $A \neq B \neq I$ and $A^{-1} \neq B$.
Their answers are given as :
Abhay : 6 AB
Bina : 7 AB – BA
Chhaya : 8 AB
Devesh : 7 BA – AB
Who answered it correctly ?
   (A) Abhay
   (B) Bina
   (C) Chhaya
   (D) Devesh

18. If A and B are square matrices of order m such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following is always correct ?
   (A) $A = B$
   (B) $AB = BA$
   (C) $A = 0$ or $B = 0$
   (D) $A = I$ or $B = I$

ASSERTION – REASON BASED QUESTIONS

Direction : Question number **19** and **20** are Assertion (A) and Reason (R) based questions. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below :

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** Every point of the feasible region of a Linear Programming Problem is an optimal solution.

    Reason (R) : The optimal solution for a Linear Programming Problem exists only at one or more corner point(s) of the feasible region.

20. **Assertion (A) :** A = diag [3 5 2] is a scalar matrix of order 3 × 3.

    Reason (R) : If a diagonal matrix has all non-zero elements equal, it is known as a scalar matrix.

SECTION – B

(This section comprises of **5** Very Short Answer (VSA) type questions of **2** marks each.) **(5 × 2 = 10)**

21. Find the values of 'a' for which $f(x) = \sin x - ax + b$ is increasing on R.

22. Find : $\int 2x^3 e^{x^2} dx$.

23. (a) If $x = e^{\frac{x}{y}}$, then prove that $\frac{dy}{dx} = \frac{x - y}{x \log x}$.

    OR

    (b) If $f(x) = \begin{cases} 2x - 3 & , \ -3 \le x \le -2 \\ x + 1 & , \ -2 < x \le 0 \end{cases}$

    Check the differentiability of $f(x)$ at $x = -2$.

24. If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, then evaluate $|\vec{a} + 2\vec{b}|$.

25. (a) A vector $\vec{a}$ makes equal angles with all the three axes. If the magnitude of the vector is $5\sqrt{3}$ units, then find $\vec{a}$.

OR

(b) If $\vec{\alpha}$ and $\vec{\beta}$ are position vectors of two points P and Q respectively, then find the position vector of a point R in QP produced such that $QR = \frac{3}{2} QP$.

SECTION – C

(This section comprises of **6** Short Answer (SA) type questions of **3** marks each.) **(6 × 3 = 18)**

26. (a) If $y = \log\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$, then show that $x(x+1)^2 y_2 + (x+1)^2 y_1 = 2$.

OR

(b) If $x\sqrt{1+y} + y\sqrt{1+x} = 0, -1 < x < 1, x \neq y$, then prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$.

27. Let R be a relation on set of real numbers $\mathbb{R}$ defined as $\{(x, y) : x - y + \sqrt{3}$ is an irrational number, $x, y \in \mathbb{R}\}$ Verify R for reflexivity, symmetry and transitivity.

28. Solve the following linear programming problem graphically :

Minimise $Z = 2x + y$

subject to the constraints :

$3x + y \geq 9$

$x + y \geq 7$

$x + 2y \geq 8$

$x, y \geq 0$

29. (a) A die with number 1 to 6 is biased such that $P(2) = \frac{3}{10}$ and probability of other numbers is equal. Find the mean of the number of times number 2 appears on the dice, if the dice is thrown twice.

OR

(b) Two dice are thrown. Defined are the following two events A and B :
$A = \{(x, y) : x + y = 9\}$, $B = \{(x, y) : x \neq 3\}$, where (x, y) denote a point in the sample space.
Check if events A and B are independent or mutually exclusive.

30. (a) Solve the differential equation $2(y + 3) - xy \frac{dy}{dx} = 0$; given $y(1) = -2$.

OR

(b) Solve the following differential equation :

$$(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2.$$

31. If $\int_a^b x^3\, dx = 0$ and $\int_a^b x^2\, dx = \frac{2}{3}$, then find the values of a and b.

SECTION – D

(This section comprises of **4** Long Answer (LA) type questions of **5** marks each.) **(4 × 5 = 20)**

32. (a) Find the shortest distance between the lines :

$$\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3} \text{ and}$$

$$\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}.$$

OR

(b) Find the image A′ of the point A(2, 1, 2) in the line $l : \vec{r} = 4\hat{i} + 2\hat{j} + 2\hat{k} + \lambda\,(\hat{i} - \hat{j} - \hat{k})$. Also, find the equation of line joining AA′. Find the foot of perpendicular from point A on the line l.

33. Find : $\int \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx.$

34. Using integration, find the area of the region bounded by the line $y = 5x + 2$, the $x-$ axis and the ordinates $x = -2$ and $x = 2$.

35. (a) Given $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find AB. Hence, solve the system of linear equations :

$$x - y + z = 4$$
$$x - 2y - 2z = 9$$
$$2x + y + 3z = 1$$

OR

(b) If $A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & -1 & -2 \\ 0 & -1 & 1 \end{bmatrix}$, then find A^{-1}.

Hence, solve the system of linear equations :

$$x - 2y = 10$$
$$2x - y - z = 8$$
$$-2y + z = 7$$

SECTION – E

(This section comprises of **3** case study based questions of **4** marks each.)

(3 × 4 = 12)

36. A school is organizing a debate competition with participants as speakers $S = \{S_1, S_2, S_3, S_4\}$ and these are judged by judges $J = \{J_1, J_2, J_3\}$. Each speaker can be assigned one judge. Let R be a relation from set S to J defined as $R = \{(x, y) : \text{speaker } x \text{ is judged by judge } y, x \in S, y \in J\}$.

Based on the above, answer the following :

(i) How many relations can be there from S to J ? **1**

(ii) A student identifies a function from S to J as f = {(S_1, J_1), (S_2, J_2), (S_3, J_2), (S_4, J_3)} Check if it is bijective. **1**

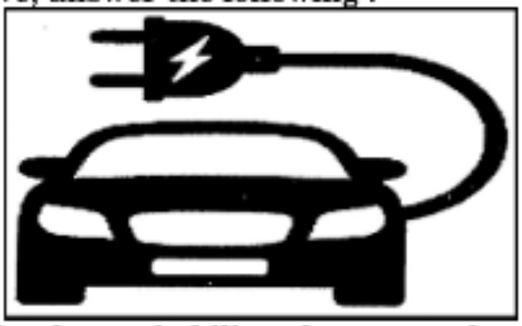
(iii) (a) How many one-one functions can be there from set S to set J ? **2**

OR

(iii) (b) Another student considers a relation R_1 = {(S_1, S_2), {S_2, S_4)} in set S. Write minimum ordered pairs to be included in R_1 so that R_1 is reflexive but not symmetric. **2**

37. Three persons viz. Amber, Bonzi and Comet are manufacturing cars which run on petrol and on battery as well. Their production share in the market is 60%, 30% and 10% respectively. Of their respective production capacities, 20%, 10% and 5% cars respectively are electric (or battery operated).

Based on the above, answer the following :



(i) (a) What is the probability that a randomly selected car is an electric car ? **2**

OR

(i) (b) What is the probability that a randomly selected car is a petrol car ? **2**

(ii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Comet ? **1**

(iii) A car is selected at random and is found to be electric. What is the probability that it was manufactured by Amber or Bonzi ? **1**

38. 

A small town is analyzing the pattern of a new street light installation. The lights are set up in such a way that the intensity of light at any point x metres from the start of the street can be modelled by $f(x) = e^x \sin x$, where x is in metres.

Based on the above, answer the following :

(i) Find the intervals on which the $f(x)$ is increasing or decreasing, $x \in [0, \pi]$. **2**

(ii) Verify, whether each critical point when $x \in [0, \pi]$ is a point of local maximum or local minimum or a point of inflexion. **2**