Series : W1XZY

SET ~ 5

प्रश्न-पत्र कोड
Q.P. Code 65(B)

रोल नं.

Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।

Candidates must write the Q.P. Code on the title page of the answer-book.

गणित

(केवल दृष्टिबाधित परीक्षार्थियों के लिए)

MATHEMATICS

(FOR VISUALLY IMPAIRED CANDIDATES ONLY)

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

नोट

(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं ।

(II) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं ।

(III) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें ।

(IV) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें ।**

(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।

NOTE

(I) Please check that this question paper contains **23** printed pages.

(II) Please check that this question paper contains **38** questions.

(III) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

(IV) **Please write down the serial number of the question in the answer-book at the given place before attempting it.**

(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period.

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General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This Question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *Question paper is divided into **FIVE** Sections – Section **A, B, C, D** and **E**.*

(iii) *In **Section A** – Question Number **1** to **18** are Multiple Choice Questions (MCQs) type and Question Number **19** & **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B** – Question Number **21** to **25** are Very Short Answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C** – Question Number **26** to **31** are Short Answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D** – Question Number **32** to **35** are Long Answer (LA) type questions, carrying **5** marks each.*

(vii) *In **Section E** – Question Number **36** to **38** are case study based questions, carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in **2** questions in Section – **B**, **3** questions in Section – **C**, 2 questions in Section – **D** and **2** questions in Section – **E**.*

(ix) *Use of calculator is **NOT** allowed.*

SECTION – A

This section consists of **20** multiple choice questions, each of **1** mark.

1. Which of the following functions from Z to Z is both one-one and onto ?

(A) $f(x) = 2x - 1$ (B) $f(x) = 3x^2 + 5$

(C) $f(x) = x + 5$ (D) $f(x) = 5x^3$

2. Value of $4\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$ is

(A) 3
(B) -3
(C) 1
(D) -1

3. If $A = \begin{bmatrix} x & 0 & m \\ y & z & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$, where I is a unit matrix, then $x + y + z + m$ is equal to

(A) 18
(B) 12
(C) 6
(D) 2

4. If $B = \begin{bmatrix} 23 & 41 & 57 \end{bmatrix} \begin{bmatrix} 31 & 42 \\ 53 & 64 \\ 75 & 86 \end{bmatrix}$, then the order of B is :

(A) 3×2
(B) 2×2
(C) 1×3
(D) 1×2

5. If A and B are square matrices of the same order, then $(A - B)^2 = ?$

(A) $A^2 - 2AB + B^2$
(B) $A^2 - AB - BA + B^2$
(C) $A^2 - 2BA + B^2$
(D) $A^2 - AB + BA + B^2$

6. If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is

(A) 0
(B) 9
(C) -6
(D) 6

7. If $A^{-1} = \begin{bmatrix} 7 & 2 \\ 8 & 2 \end{bmatrix}$, then matrix A is

(A) $\begin{bmatrix} 2 & -2 \\ -8 & 7 \end{bmatrix}$

(B) $\begin{bmatrix} -7 & 8 \\ 2 & -2 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 1 \\ 4 & -\frac{7}{2} \end{bmatrix}$

(D) $\begin{bmatrix} 1 & -1 \\ -4 & \frac{7}{2} \end{bmatrix}$

8. If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, then $\frac{dy}{dx}$ is

(A) $\frac{-\sqrt{x}}{\sqrt{y}}$

(B) $-\frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}}$

(C) $-\frac{\sqrt{y}}{\sqrt{x}}$

(D) $\frac{-2\sqrt{y}}{\sqrt{x}}$

9. If $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$, then $\frac{dy}{dx}$ is

(A) 1

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) -1

10. When x is positive, the minimum value of x^x is

(A) e^e

(B) $\frac{1}{e}$

(C) $e^{\frac{1}{e}}$

(D) $e^{\frac{-1}{e}}$

11. $\int \frac{2x^3}{4+x^8} dx$ is equal to

(A) $\frac{1}{4} \tan^{-1} \frac{x^4}{2} + C$

(B) $\frac{1}{2} \tan^{-1} \frac{x^4}{2} + C$

(C) $\frac{1}{4} \tan^{-1} \frac{x^4}{4} + C$

(D) $\frac{1}{4} \tan^{-1} x^4 + C$

12. $\int e^x \cdot \frac{x}{(1+x)^2}$ dx is equal to

(A) $e^x \cdot \frac{x}{1+x} + C$

(B) $e^x \cdot \frac{1}{1+x} + C$

(C) $e^x \cdot \frac{1}{x} + C$

(D) $e^x \cdot \frac{1}{(1+x)^2} + C$

13. The area of the region bounded by the lines $y = x + 1$, $x = 1$, $x = 3$ and x-axis is

(A) 6 sq units

(B) 8 sq units

(C) 7.5 sq units

(D) 2 sq units

14. The integrating factor for solving the differential equation $x \cdot \frac{dy}{dx} - y = 2x^2$ is

(A) x

(B) $\frac{1}{x}$

(C) e^{-x}

(D) $-\log x$

15. The number of vector(s) of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is (are) :

(A) one (B) two

(C) three (D) infinite

16. Of all the points of the feasible region, for maximum or minimum values of the objective function, the point lies :

(A) inside the feasible region

(B) at the boundary line of the feasible region

(C) at the corner points of the feasible region

(D) at the coordinate axes

17. The common region for the inequalities $x \geq 0$, $x + y \leq 1$ and $y \geq 0$, lies in

(A) IV Quadrant (B) II Quadrant

(C) III Quadrant (D) I Quadrant

18. A and B appeared for an interview for two vacancies. The probability of A's selection is $\frac{1}{5}$ and that of B's selection is $\frac{1}{3}$. The probability that none of them is selected is :

(A) $\frac{11}{15}$ (B) $\frac{7}{15}$

(C) $\frac{8}{15}$ (D) $\frac{1}{5}$

Assertion – Reason Based Questions

Question numbers **19** and **20** are Assertion (A) and Reason (R) based questions carrying **1** mark each. Two statements are given, one labelled Assertion (A) and other labelled Reason (R). Select the correct answer from the options (A), (B), (C) and (D) as given below.

(A) Both Assertion (A) and Reason (R) are true and the Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).

(C) Assertion (A) is true and Reason (R) is false.

(D) Assertion (A) is false and Reason (R) is true.

19. **Assertion (A)** : The vectors $\vec{a} = 4\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$ are mutually perpendicular vectors.

    Reason (R) : Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, if $\vec{a} \cdot \vec{b} = 0$.

20. **Assertion (A)** : $x^2 dy = (2xy + y^2) dx$ is a homogeneous differential equation.

    Reason (R) : A differential equation of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is a homogeneous differential equation.

SECTION – B

This section consists of **5** very short answer type questions, each of **2** marks.

21. Evaluate : $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$

22. (a) Show that the function $f(x) = (x-1)^{\frac{1}{3}}$ is not differentiable at $x = 1$.

OR

(b) Differentiate $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ w.r.t. x.

23. If $y = 7x - x^3$ and x increases at the rate of 2 units per second, then how fast is the slope of the curve changing, when $x = 5$?

24. (a) If $|\vec{a} + \vec{b}| = 60$, $|\vec{a} - \vec{b}| = 40$ and $|\vec{b}| = 46$, then find $|\vec{a}|$.

OR

(b) Using vectors, find the value of K such that the points (K, –11, 2), (0, –2, 2) and (2, 4, 2) are collinear.

25. Find the angle between the two lines whose equations are $2x = 3y = -z$ and $6x = -y = -4z$.

SECTION – C

In this section there are **6** short answer type questions, each of **3** marks.

26. Find the intervals in which the function

$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is

(a) strictly increasing

(b) strictly decreasing

27. (a) Find : $\int \frac{x^2 - x + 1}{(x-1)(x^2+1)} dx$

OR

(b) Evaluate : $\int_1^4 (|x| + |3 - x|) \, dx$

28. (a) Find the particular solution of the differential equation, $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that y = 1 when $x = 0$.

OR

(b) Solve the differential equation : $2xy \frac{dy}{dx} = x^2 + 3y^2$.

29. If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k}$, then find a unit vector perpendicular to both the vectors $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$.

30. The corner points of the feasible region determined by some system of linear inequations, are (0, 0), (5, 0), (3, 4) and (0, 5). Let $Z = ax + by$, where a, b > 0. Find the condition on a and b so that the maximum of Z occurs at both points (3, 4) and (0, 5).

31. (a) Find the probability distribution of the number of doublets in three throws of a pair of dice.

OR

(b) If E and F are two independent events with P(E) = p, P(F) = 2p and P(exactly one of E, F) = $\frac{5}{9}$, then find the value of p.

SECTION – D

This section consists of **4** long answer type questions, each of **5** marks.

32. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Using A^{-1}, solve the system of equations :

$2x - 3y + 5z = 11$

$3x + 2y - 4z = -5$

$x + y - 2z = -3$

33. (a) Differentiate $x^{\sin x} + (\sin x)^x$ w.r.t. x.

OR

(b) If $y = x + \tan x$, then prove that

$$\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$$

34. The region enclosed between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$. Find the value of a.

35. (a) Find the shortest distance between the lines given by

$\vec{r} = (4\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and

$\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} + 2\hat{j} - 4\hat{k})$

OR

(b) Find the coordinates of the foot of the perpendicular and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$.

SECTION – E

In this section there are **3** case-study based questions of **4** marks each.

36. An architect designs a building for a Company. The design of window on the ground floor is proposed to be different than at the other floors. The window is in the shape of a rectangle whose top length is surmounted by a semi-circular opening. This window has a perimeter of 10 m.

Based on the above information, answer the following :

(i) If $2x$ and $2y$ represent the length and breadth of the rectangular portion of the window, then establish a relation between x and y.

(ii) Find the total area of the window in terms of x.

(iii) (a) Find the values of x and y for the maximum area of the window.

OR

(iii) (b) If x and y represent the length and breadth of the rectangle, then establish the expression for the area of the window in terms of x only.

37. There are three categories of students in a class of 60 students : A : Very hardworking students, B : Regular but not so hard working, C : Careless and irregular students. It is known that 6 students in category A, 26 in category B and the rest in category C. It is also found that probability of students of category A, unable to get good marks in the final year examination, is 0.002, of category B it is 0.02 and of category C, this probability is 0.20.

Based on the above information, answer the following :

(i) Find the probability that a student selected at random is unable to get good marks in the final examination. **2**

(ii) A student selected at random was found to be one who could not get good marks in the final examination. Find the probability, that this student is NOT of category A. **2**

38. Rajesh, a student of Class-XII, visited an exhibition with his family. There he saw a huge swing and found that it traced the path of a parabola $y = x^2$. The following questions came to his mind. Answer the questions :

(i) Let $f : R \to R$ be a function defined as $f(x) = x^2$. Find whether f is one-one function.

(ii) Let $f : R \to R$ be defined as $f(x) = x^2$. Find whether f is an onto function.

(iii) (a) Let $f : N \to N$ be defined as $f(x) = x^2$. Find whether f is one-one function. Also, find if it is an onto function.

OR

(iii) (b) Let $f : N \to \{1, 4, 9, 16, ...\}$ defined as $f(x) = x^2$, find where f is one-one function. Also, find if it is an onto function.
