Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior School Examination, 2025
SUBJECT: MATHEMATICS (Q.P. CODE – 65/4/3)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.

10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totaling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totaling on the title page. • Wrong totaling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) • Half or a part of the answer marked correct and the rest as wrong, but no marks
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

MARKING SCHEME

SENIOR SECONDARY SCHOOL EXAMINATION 2024-25

MATHEMATICS (Code–041)

[Paper Code: 65/4/3]

Q. No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION - A Questions no. 1 to 18 are multiple choice questions (MCQs) of **1 mark each**.	
Q1.	Domain of $\sin^{-1}(2x^2-3)$ is : (A) $(-1, 0) \cup (1, \sqrt{2})$ (B) $(-\sqrt{2}, -1) \cup (0, 1)$ (C) $[-\sqrt{2}, -1] \cup [1, \sqrt{2}]$ (D) $(-\sqrt{2}, -1) \cup (1, \sqrt{2})$	
A1.	(C) $[-\sqrt{2}, -1] \cup [1, \sqrt{2}]$	1
Q2.	The matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -7 \\ 2 & 7 & 0 \end{bmatrix}$ is a : (A) diagonal matrix (B) symmetric matrix (C) skew symmetric matrix (D) scalar matrix	
A2.	(C) skew symmetric matrix	1
Q3.	If $f(x) = \begin{cases} 3x-2, & 0 < x \le 1 \\ 2x^2+ax, & 1 < x < 2 \end{cases}$ is continuous for $x \in (0, 2)$, then a is equal to : (A) -4 (B) $-\frac{7}{2}$ (C) -2 (D) -1	
A3.	(D) -1	1
Q4.	If $y = \log_{2x}(\sqrt{2x})$, then $\frac{dy}{dx}$ is equal to : (A) 0 (B) 1 (C) $\frac{1}{x}$ (D) $\frac{1}{\sqrt{2x}}$	
A4.	(A) 0	1

Q5.	If $f : N \to W$ is defined as $$f(n) = \begin{cases} \frac{n}{2}, & \text{if n is even} \\ 0, & \text{if n is odd} \end{cases},$$ then f is : (A) injective only (B) surjective only (C) a bijection (D) neither surjective nor injective	
A5.	(B) surjective only	1
Q6.	The coordinates of the foot of the perpendicular drawn from the point $A(-2, 3, 5)$ on the y-axis is : (A) $(0, 0, 5)$ (B) $(0, 3, 0)$ (C) $(-2, 0, 5)$ (D) $(-2, 0, 0)$	
A6.	(B) $(0, 3, 0)$	1
Q7.	If A and B are invertible matrices of order 3×3 such that $\det(A) = 4$ and $\det[(AB)^{-1}] = \frac{1}{20}$, then $\det(B)$ is equal to : (A) $\frac{1}{20}$ (B) $\frac{1}{5}$ (C) 20 (D) 5	
A7.	(D) 5	1
Q8.	For real x, let $f(x) = x^3 + 5x + 1$. Then : (A) f is one-one but not onto on R (B) f is onto on R but not one-one (C) f is one-one and onto on R (D) f is neither one-one nor onto on R	
A8.	(C) f is one-one and onto on R	1
Q9.	The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are : (A) $1 < \lambda < \sqrt{2}$ (B) $\lambda \geq 1$ (C) $\lambda \geq \sqrt{2}$ (D) $\lambda < 1$	
A9.	(C) $\lambda \geq \sqrt{2}$	1

Q10.	If A and B are square matrices of same order such that AB = A and BA = B, then $A^2 + B^2$ is equal to : (A) $A + B$ (B) BA (C) $2(A + B)$ (D) $2BA$	
A10.	(A) $A + B$	1
Q11.	The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $x = 4$ and x-axis is : (A) $\frac{16}{9}$ sq. units (B) $\frac{32}{9}$ sq. units (C) $\frac{16}{3}$ sq. units (D) $\frac{32}{3}$ sq. units	
Q11.	(C) $\frac{16}{3}$ sq. units	1
Q12.	The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is : (A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\tan^{-1} e - \frac{\pi}{4}$ (D) $\tan^{-1} e$	
A12.	(C) $\tan^{-1} e - \frac{\pi}{4}$	1
Q13.	The corner points of the feasible region of a Linear Programming Problem are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). If $Z = ax + by$; $(a, b > 0)$ be the objective function, and maximum value of Z is obtained at (0, 2) and (3, 0), then the relation between a and b is : (A) $a = b$ (B) $a = 3b$ (C) $b = 6a$ (D) $3a = 2b$	
A13.	(D) $3a = 2b$	1
Q14.	If $\int e^{-3 \log x} dx = f(x) + C$, then f(x) is : (A) $e^{-3 \log x}$ (B) $e^{\log\left(\frac{1}{x^3}\right)}$ (C) $\frac{-1}{2x^2}$ (D) $\frac{-1}{4x^4}$	
A14.	(C) $\frac{-1}{2x^2}$	1

Q15.	The function f defined by $$f(x) = \begin{cases} x, & \text{if } x \le 1 \\ 5, & \text{if } x > 1 \end{cases}$$ is ***not*** continuous at : (A) $x = 0$ (B) $x = 1$ (C) $x = 2$ (D) $x = 5$	
A15.	(B) $x = 1$	1
Q16.	The solution of the differential equation $\frac{dy}{dx} = \frac{-x}{y}$ represents family of : (A) parabolas (B) circles (C) ellipses (D) hyperbolas	
A16.	(B) circles	1
Q17.	If the sides AB and AC of Δ ABC are represented by vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ respectively, then the length of the median through A on BC is : (A) $2\sqrt{2}$ units (B) $\sqrt{18}$ units (C) $\frac{\sqrt{34}}{2}$ units (D) $\frac{\sqrt{48}}{2}$ units	
A17.	(C) $\frac{\sqrt{34}}{2}$ units	1
Q18.	If $f(x) = 2x + \cos x$, then f(x) : (A) has a maxima at $x = \pi$ (B) has a minima at $x = \pi$ (C) is an increasing function (D) is a decreasing function	
A18.	(C) is an increasing function	1

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

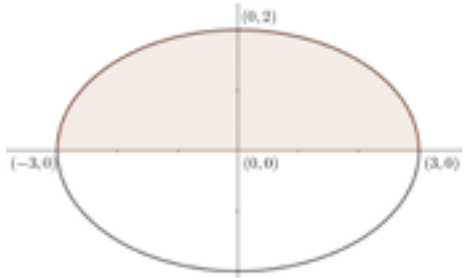
Q19.	*Assertion (A) :* If A and B are two events such that $P(A \cap B) = 0$, then A and B are independent events. *Reason (R) :* Two events are independent if the occurrence of one does not effect the occurrence of the other.	
A19.	(D) Assertion (A) is false, but Reason (R) is true.	1
Q20.	*Assertion (A) :* In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution. *Reason (R) :* A feasible region is defined as the region that satisfies all the constraints.	
A20.	(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).	1

SECTION B

This section comprises very short answer (VSA) type questions of **2 marks each**.

Q21.	If $A = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$ and $A^2 = kA$, then find the value of k.	
A21.	$A=\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \Rightarrow A^2=\begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}$	1
	$A^2 = kA \Rightarrow \begin{bmatrix} 8 & -8 \\ -8 & 8 \end{bmatrix}=\begin{bmatrix} 2k & -2k \\ -2k & 2k \end{bmatrix}$	½
	$\Rightarrow k=4$	½
Q22.	(a) Simplify $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$. **OR** (b) Find domain of $\sin^{-1}\sqrt{x-1}$.	
A22.(a)	Put $x=\tan\theta \Rightarrow \theta=\tan^{-1}x$	½
	Now $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$	
	$=\sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right)=\sin^{-1}(\sin\theta)$	1
	$=\theta=\tan^{-1}x$	½

OR

A22.(b)	Here $-1 \le \sqrt{x-1} \le 1$ $\Rightarrow 0 \le x-1 \le 1 \Rightarrow 1 \le x \le 2$ Hence, domain is $x \in [1, 2]$	1 1
Q23.	Calculate the area of the region bounded by the curve $\frac{x^2}{9}+\frac{y^2}{4}=1$ and the x-axis using integration.	
A23.	 $A = 2 \times \frac{2}{3}\int_0^3 \sqrt{9-x^2}\,dx$ $= \frac{4}{3}\left[\frac{x}{2}\sqrt{9-x^2}+\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right]_0^3$ $= \frac{4}{3}\left[\left(0+\frac{9}{2}\sin^{-1}1\right)-0\right]$ $= 3\pi$	**(½ for correct figure)** ½ ½ ½
Q24.	(a) Find the least value of 'a' so that $f(x) = 2x^2 - ax + 3$ is an increasing function on [2, 4]. **OR** (b) If $f(x) = x + \frac{1}{x}$, $x \ge 1$, show that f is an increasing function.	
A24.(a)	$f(x) = 2x^2 - ax + 3 \Rightarrow f'(x) = 4x - a$ Now $2 \le x \le 4 \Rightarrow 8 - a \le 4x - a \le 16 - a$ For f to be an increasing function, $f'(x) \ge 0$ $\Rightarrow 8 - a \ge 0 \Rightarrow a \le 8$ $\therefore$ Least value of a does not exist.	½ 1 ½
	OR	
A24.(b)	$f(x) = x + \frac{1}{x} \Rightarrow f'(x) = 1 - \frac{1}{x^2} = \frac{x^2-1}{x^2}$ Now $\frac{x^2-1}{x^2} \ge 0$ for all $x \ge 1$ $\Rightarrow f'(x) \ge 0 \Rightarrow f$ is an increasing function.	1 ½ ½
Q25.	A cylindrical water container has developed a leak at the bottom. The water is leaking at the rate of 5 cm^3/s from the leak. If the radius of the container is 15 cm, find the rate at which the height of water is decreasing inside the container, when the height of water is 2 metres.	

A25.	Let V, r, h be the volume, radius and height of cylindrical container. Given $\frac{dV}{dt} = -5 \text{ cm}^3/\text{s}$ $V = \pi r^2 h = \pi (15)^2 h = 225\pi h$ $\therefore \frac{dV}{dt} = 225\pi \frac{dh}{dt} \Rightarrow -5 = 225\pi \frac{dh}{dt}$ $\Rightarrow \frac{dh}{dt} = -\frac{5}{225\pi} = -\frac{1}{45\pi}$ $\therefore$ Height of the water is decreasing at the rate of $\frac{1}{45\pi}$ cm/s	½ 1 ½

SECTION C

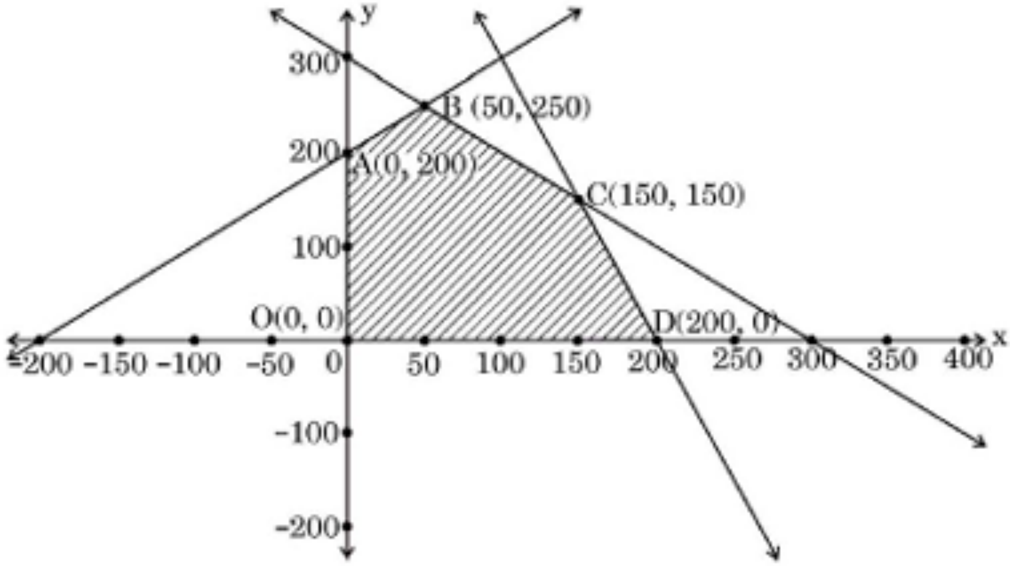
This section comprises short answer (SA) type questions of **3 marks each**.

Q26.	Find : $\int \frac{\sqrt{x}}{1+\sqrt{x^{3/2}}} dx$	
A26.	$I = \int \frac{\sqrt{x}}{1+\sqrt{x^{3/2}}} dx$ Put $x^{3/2} = t \Rightarrow \frac{3}{2}\sqrt{x} dx = dt$ $\Rightarrow I = \frac{2}{3}\int \frac{dt}{1+\sqrt{t}}$ Put $\sqrt{t} = z \Rightarrow \frac{1}{2\sqrt{t}} dt = dz$ $\Rightarrow I = \frac{2}{3}\int \frac{2z}{1+z} dz$ $= \frac{4}{3}\left[\int 1\, dz - \int \frac{1}{1+z} dz\right]$ $= \frac{4}{3}\left[z - \log\lvert 1+z\rvert\right] + C$ $= \frac{4}{3}\left[\sqrt{t} - \log\left\lvert 1+\sqrt{t}\right\rvert\right] + C$ $= \frac{4}{3}\left[\sqrt{x^{3/2}} - \log\left\lvert 1+\sqrt{x^{3/2}}\right\rvert\right] + C$	½ ½ ½ 1 ½
Q27.	Find the distance of the point (–1, –5, –10) from the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.	

A27.	$l_1: \frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$ Any point on l_1 is $(2\lambda+1, 3\lambda+2, 4\lambda+3)$ $l_2: \frac{x-4}{5}=\frac{y-1}{2}=\frac{z-0}{1}=\mu$ Any point on l_2 is $(5\mu+4, 2\mu+1, \mu)$	1
	for point of intersection, $2\lambda+1=5\mu+4, 3\lambda+2=2\mu+1$ solving, $\lambda=\mu=-1$	1
	since, $\lambda=\mu=-1$ satisfy $4\lambda+3=\mu$ $\therefore$ Point of intersection is $(-1,-1,-1)$	½
	Now distance of $(-1,-5,-10)$ from $(-1,-1,-1)$ is: $\sqrt{(-1+1)^2+(-1+5)^2+(-1+10)^2}=\sqrt{97}$ units	½
Q28.	(a) If $f: R^+ \to R$ is defined as $f(x) = \log_a x$ ($a > 0$ and $a \neq 1$), prove that f is a bijection. (R^+ is a set of all positive real numbers.) **OR** (b) Let A = {1, 2, 3} and B = {4, 5, 6}. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$. (i) Write all elements of R. (ii) Is R a function ? Justify. (iii) Determine domain and range of R.	
A28.(a)	$f(x)=\log_a x \quad (a>0, a\neq1)$ Let $x_1, x_2 \in R^+$ such that $f(x_1)=f(x_2)$ $\Rightarrow \log_a x_1 = \log_a x_2$ $\Rightarrow x_1 = x_2 \Rightarrow f$ is one-one.	1½
	Let $f(x)=y \Rightarrow \log_a x = y \Rightarrow a^y = x$ $\therefore$ there exists $y \in R$ for all $x \in R^+$ $\therefore f$ is onto. f is a bijection.	1½
OR		
A28.(b)	$(i)\ R=\{(1,5),(2,4)\}$	1
	$(ii)\ R$ is not a function as $3 \in A$ do not have an image in co-domain.	1
	(iii) Domain of $R=\{1,2\}$, Range of $R=\{4,5\}$	1

Q29.	(a) The probability distribution of a random variable X is given by :	
	<table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(X)</td><td>p</td><td>$\frac{p}{3}$</td><td>$\frac{p}{6}$</td><td>$\frac{p}{12}$</td></tr></table>	
	(i) Determine the value of p.	*1*
	(ii) Calculate $P(X \geq 1)$.	*1*
	(iii) Calculate expectation of X, i.e. E(X)	*1*
	OR	
	(b) In a city, a survey was conducted among residents about their preferred mode of commuting. It was found that 50% people preferred using public transport, 35% preferred using a bicycle and 20% use both public transport and a bicycle. If a person is selected at random, find the probability that :	
	(i) The person uses only public transport.	*1*
	(ii) The person uses a bicycle, given that they also use the public transport.	*1*
	(iii) The person uses neither public transport nor a bicycle.	*1*
A29.(a)	$(i)\ p+\frac{p}{3}+\frac{p}{6}+\frac{p}{12}=1$	½
	$\Rightarrow p=\frac{12}{19}$	½
	$(ii)\ P(X\geq 1)=1-P(X=1)$	½
	$=1-\frac{p}{3}=1-\frac{12}{19}=\frac{7}{19}$	½
	$(iii)\ E(X)=\sum X.P(X)$	
	$=0(p)+1\left(\frac{p}{3}\right)+2\left(\frac{p}{6}\right)+3\left(\frac{p}{12}\right)$	½
	$=\frac{11}{12}p=\frac{11}{12}\times\frac{12}{19}=\frac{11}{19}$	½
	OR	

A29.(b)	Let T: Person uses public transport B: Person uses a bicycle Given $P(T)=\frac{50}{100}, P(B)=\frac{35}{100}, P(T\cap B)=\frac{20}{100}$ $(i)\ P(\text{only } T)=P(T)-P(T\cap B)$ $=\frac{50}{100}-\frac{20}{100}=\frac{30}{100}$ $(ii)\ P(B\mid T)=\frac{P(T\cap B)}{P(T)}$ $=\frac{\frac{20}{100}}{\frac{50}{100}}=\frac{2}{5}$ $(iii)\ P(T'\cap B')=1-P(T\cup B)$ $=1-\left[P(T)+P(B)-P(T\cap B)\right]$ $=1-\left[\frac{50}{100}+\frac{35}{100}-\frac{20}{100}\right]$ $=1-\frac{65}{100}=\frac{35}{100}$	 ½ ½ ½ ½ ½ ½
Q30.	(a) Find k so that $$f(x)=\begin{cases}\frac{x^2-2x-3}{x+1}, & x\neq -1\\ k, & x=-1\end{cases}$$ is continuous at x = – 1. **OR** (b) Check the differentiability of function $f(x) = x\|x\|$ at $x = 0$.	
A30.(a)	$\lim\limits_{x\to -1}\frac{x^2-2x-3}{x+1}=\lim\limits_{x\to -1}\frac{(x-3)(x+1)}{x+1}=\lim\limits_{x\to -1}(x-3)=-4$ Also, $f(-1)=k$ as f is continuous, $k=-4$	2 ½ ½
	OR	

A30.(b)	$f(x)=x\|x\|=\begin{cases}-x^2 & ,x\le 0\\ x^2 & ,x>0\end{cases}$	1
	$\text{LHD}=\lim\limits_{h\to 0}\dfrac{f(0-h)-f(0)}{-h}=\lim\limits_{h\to 0}\dfrac{-h^2-0}{-h}=0$	1
	$\text{RHD}=\lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim\limits_{h\to 0}\dfrac{h^2-0}{h}=0$	½
	Since LHD = RHD, f is differentiable at $x=0$	½
Q31.	 For the given graph of a Linear Programming Problem, write all the constraints satisfying the given feasible region.	
A31.	Equation of AB: $y-200=\dfrac{250-200}{50-0}(x-0)$ i.e. $-x+y=200$ Equation of BC: $y-250=\dfrac{150-250}{150-50}(x-50)$ i.e. $x+y=300$ Equation of CD: $y-0=\dfrac{0-150}{200-150}(x-200)$ i.e. $3x+y=600$ Required constraints are:	
	$-x+y\le 200$	
	$x+y\le 300$	1
	$3x+y\le 600$	1
	$x\ge 0,\ y\ge 0$	1

SECTION D

This section comprises long answer (LA) type questions of **5 marks each**.

Q32. The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight.

(i) Find the rate of growth of the plant with respect to sunlight. *2*

(ii) In how many days will the plant attain its maximum height ? What is the maximum height ? *3*

A32.

$(i)\ y = 4x - \frac{1}{2}x^2 \Rightarrow \frac{dy}{dx} = (4 - x)$ cm/day — 2

(ii) For maximum height, $\frac{dy}{dx} = 0 \Rightarrow x = 4$ days

as $\frac{d^2y}{dx^2} < 0$, number of days $= 4$ — 2

Now, Maximum height $= y(4) = 16 - \frac{1}{2}(16) = 8$ cm — 1

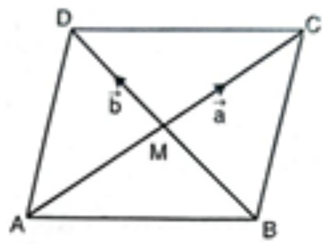
Q33. (a) Show that the area of a parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is given by $\frac{1}{2}|\vec{a} \times \vec{b}|$. Also find the area of a parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

OR

(b) Find the equation of a line in vector and cartesian form which passes through the point (1, 2, – 4) and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$, and

$\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

A33.(a) Let $ABCD$ be the parallelogram with diagonals $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{BD} = \vec{b}$.



$\therefore \overrightarrow{AB} = \frac{1}{2}(\vec{a} - \vec{b})$ and $\overrightarrow{AD} = \frac{1}{2}(\vec{a} + \vec{b})$ — ½

Area of $ABCD$

$= |\overrightarrow{AB} \times \overrightarrow{AD}|$

$= \left|\frac{1}{2}(\vec{a} - \vec{b}) \times \frac{1}{2}(\vec{a} + \vec{b})\right|$ — ½

$= \frac{1}{4}|\vec{a} \times \vec{a} + \vec{a} \times \vec{b} - \vec{b} \times \vec{a} - \vec{b} \times \vec{b}|$ — ½

	$=\frac{1}{4}\left\|\vec{a}\times\vec{b}+\vec{a}\times\vec{b}\right\| \quad (\because \vec{a}\times\vec{a}=\vec{0})$	
	$=\frac{1}{4}\left\|2\left(\vec{a}\times\vec{b}\right)\right\|$	
	$=\frac{1}{2}\left\|\vec{a}\times\vec{b}\right\|$	½
	Given $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$, $\vec{b}=\hat{i}+3\hat{j}-\hat{k}$	
	Area of parallelogram $=\frac{1}{2}\left\|\vec{a}\times\vec{b}\right\|$	
	Now $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & 1\\ 1 & 3 & -1\end{vmatrix}=-2\hat{i}+3\hat{j}+7\hat{k}$	2
	$\left\|\vec{a}\times\vec{b}\right\|=\sqrt{62}$	½
	Area of parallelogram $=\frac{1}{2}\sqrt{62}$	½
	OR	
A33.(b)	Given lines are $\frac{x-8}{3}=\frac{y+19}{-16}=\frac{z-10}{7}$	
	and $\vec{r}=\left(15\hat{i}+29\hat{j}+5\hat{k}\right)+\mu\left(3\hat{i}+8\hat{j}-5\hat{k}\right)$	
	The first line in vector form is $\vec{r}=\left(8\hat{i}-19\hat{j}+10\hat{k}\right)+\lambda\left(3\hat{i}-16\hat{j}+7\hat{k}\right)$	1
	$\vec{a}_1=8\hat{i}-19\hat{j}+10\hat{k}, \vec{a}_2=15\hat{i}+29\hat{j}+5\hat{k}$	
	$\vec{b}_1=3\hat{i}-16\hat{j}+7\hat{k}$, $\vec{b}_2=3\hat{i}+8\hat{j}-5\hat{k}$	1
	$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -16 & 7\\ 3 & 8 & -5\end{vmatrix}=24\hat{i}+36\hat{j}+72\hat{k}$	1
	$\therefore$ Equation of line passing through $(1,2,-4)$ and parallel to $\vec{b}$ is	
	$\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+t\left(24\hat{i}+36\hat{j}+72\hat{k}\right)$ or $\vec{r}=\left(\hat{i}+2\hat{j}-4\hat{k}\right)+t'\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$	1
	Cartesian form of line is $\frac{x-1}{24}=\frac{y-2}{36}=\frac{z+4}{72}$ or $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$	1

Q34.	(a) Evaluate : $$\int_0^{3/2} \lvert x\cos \pi x \rvert \, dx$$ **OR** (b) Find : $$\int \frac{dx}{\sin x + \sin 2x}$$	
A34.(a)	$I=\int_0^{3/2} \lvert x\cos \pi x \rvert dx$	
	$=\int_0^{1/2} x\cos \pi x\, dx - \int_{1/2}^{3/2} x\cos \pi x\, dx \quad \ldots(1)$	1
	Consider $\int x\cos \pi x\, dx$	
	$=\frac{x\sin \pi x}{\pi} - \int \frac{\sin \pi x}{\pi} dx$	1
	$=\frac{x\sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2} \quad \ldots(2)$	1
	using (2) in (1),	
	$\left.\frac{x\sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right]_0^{1/2} - \left.\frac{x\sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right]_{1/2}^{3/2}$	
	$=\left(\frac{1}{2\pi} - \frac{1}{\pi^2}\right) - \left(-\frac{3}{2\pi} - \frac{1}{2\pi}\right)$	1
	$=\frac{5}{2\pi} - \frac{1}{\pi^2}$	1
OR		

A34.(b)	$I=\int\frac{dx}{\sin x+\sin 2x}$ $=\int\frac{dx}{\sin x(1+2\cos x)}$ $=\int\frac{\sin x}{\sin^2 x(1+2\cos x)}dx$ $=\int\frac{\sin x}{(1-\cos x)(1+\cos x)(1+2\cos x)}dx$ Put $\cos x=t\Rightarrow\sin x\,dx=dt$ $I=-\int\frac{dt}{(1-t)(1+t)(1+2t)}$ $=-\frac{1}{6}\int\frac{dt}{1-t}+\frac{1}{2}\int\frac{dt}{1+t}-\frac{4}{3}\int\frac{dt}{1+2t}$ $=\frac{1}{6}\log\lvert 1-t\rvert+\frac{1}{2}\log\lvert 1+t\rvert-\frac{2}{3}\log\lvert 1+2t\rvert+C$ $=\frac{1}{6}\log\lvert 1-\cos x\rvert+\frac{1}{2}\log\lvert 1+\cos x\rvert-\frac{2}{3}\log\lvert 1+2\cos x\rvert+C$	 ½ ½ ½ ½ 1 1½ ½
Q35.	If A is a 3 × 3 invertible matrix, show that for any scalar k ≠ 0, $(kA)^{-1}=\frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where $A=\begin{bmatrix}2&-1&1\\-1&2&-1\\1&-1&2\end{bmatrix}$.	
A35.	Consider $(kA)\left(\frac{1}{k}A^{-1}\right)=k.\frac{1}{k}\left(A.A^{-1}\right)=1.I=I$ $\Rightarrow kA$ and $\frac{1}{k}A^{-1}$ are inverse of each other. $\therefore(kA)^{-1}=\frac{1}{k}A^{-1}$ $\therefore(3A)^{-1}=\frac{1}{3}A^{-1}$	1

	Here, $\lvert A\rvert = 4 \neq 0 \therefore A^{-1}$ exists.	1
	$adjA = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$	2
	$\therefore A^{-1} = \frac{1}{\lvert A\rvert}.adjA = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$	½
	$\therefore (3A)^{-1} = \frac{1}{12}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$	½

SECTION E

This section comprises 3 case study-based questions of **4 marks each**.

Q36. Some students are having a misconception while comparing decimals. For example, a student may mention that 78·56 > 78·9 as 7856 > 789. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question : In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table :

Name of student	Distance of javelin (in meters)
Ajay	47·7
Bijoy	47·07
Kartik	43·09
Dinesh	43·9
Devesh	45·2

The students were asked to identify who has thrown the javelin the farthest.

Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions :

(i) What is the probability of a student not having misconception but still answers Bijoy in the test ? *1*

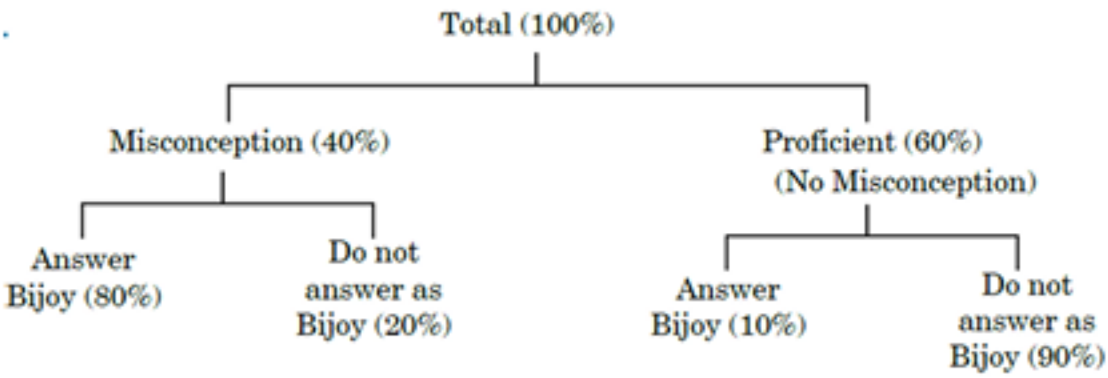
(ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test ? *1*

(iii) (a) What is the probability that a student who answered as Bijoy is having misconception ? *2*

OR

(iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception ? *2*

A36.



Let E_1 : Student has a misconception

E_2 : Student does not have misconception

A: Student answer Bijoy

$$\therefore P(E_1)=\frac{40}{100}, P(E_2)=\frac{60}{100}$$

$$P(A|E_1)=\frac{80}{100}, P(A|E_2)=\frac{10}{100}$$

$$P(\bar{A}|E_1)=\frac{20}{100}, P(\bar{A}|E_2)=\frac{90}{100}$$

	$(i)\ P(A\mid E_2)=\frac{10}{100}\text{ or }\frac{1}{10}$	1
	$(ii)\ P(A)=P(E_1)P(A\mid E_1)+P(E_2)P(A\mid E_2)$	
	$=\frac{40}{100}\times\frac{80}{100}+\frac{60}{100}\times\frac{10}{100}$	
	$=\frac{38}{100}=0.38$	1
	$(iii)(a)\ P(E_1\mid A)=\frac{P(E_1)P(A\mid E_1)}{P(A)}$	
	$=\frac{\frac{40}{100}\times\frac{80}{100}}{\frac{38}{100}}=\frac{16}{19}$	2
	OR	
	$(iii)(b)\ P(E_2\mid A)=\frac{P(E_2)P(A\mid E_2)}{P(A)}$	
	$=\frac{\frac{60}{100}\times\frac{10}{100}}{\frac{38}{100}}=\frac{3}{19}$	2
Q37.	An engineer is designing a new metro rail network in a city.  Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by $l_1:\ \frac{x-2}{3}=\frac{y+1}{-2}=\frac{z-3}{4}$, while the track for Line B is represented by $l_2:\ \frac{x-1}{2}=\frac{y-3}{1}=\frac{z+2}{-3}$.	

	Based on the above information, answer the following questions : (i) Find whether the two metro tracks are parallel. *1* (ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point $(1, -2, -3)$. *1* (iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point (3, 2, 1). Determine the equation of the pedestrian walkway. *2* **OR** (iii) (b) Find the shortest distance between Line A and Line B. *2*	
A37.	$l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-3}{4}$; $l_2 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z+2}{-3}$ (i) Drs of l_1 are $<3,-2,4>$, drs of l_2 are $<2,1,-3>$ as drs are not proportional, hence l_1 is not parallel to l_2. (ii) Equations of line parallel to l_1 and passing through $(1,-2,-3)$ is $\frac{x-1}{3} = \frac{y+2}{-2} = \frac{z+3}{4}$ or $\vec{r} = \left(\hat{i} - 2\hat{j} - 3\hat{k}\right) + \lambda\left(3\hat{i} - 2\hat{j} + 4\hat{k}\right)$ $(iii)(a)$ The pathway is perpendicular to l_1 and l_2. $\therefore$ It is parallel to $\vec{b_1} \times \vec{b_2}$ $\vec{b} = \vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -2 & 4 \\ 2 & 1 & -3 \end{vmatrix} = 2\hat{i} + 17\hat{j} + 7\hat{k}$ $\therefore$ Equation of pathway is $\vec{r} = \left(3\hat{i} + 2\hat{j} + \hat{k}\right) + \lambda\left(2\hat{i} + 17\hat{j} + 7\hat{k}\right)$ **OR** $(iii)(b)$ $\vec{a}_1 = 2\hat{i} - \hat{j} + 3\hat{k}$, $\vec{a}_2 = \hat{i} + 3\hat{j} - 2\hat{k}$ $\vec{b_1} = 3\hat{i} - 2\hat{j} + 4\hat{k}$, $\vec{b_2} = 2\hat{i} + \hat{j} - 3\hat{k}$ $d = \left\| \frac{(\vec{a}_2 - \vec{a}_1).(\vec{b_1} \times \vec{b_2})}{\|\vec{b_1} \times \vec{b_2}\|} \right\|$ $= \left\| \frac{\left(-\hat{i} + 4\hat{j} - 5\hat{k}\right).\left(2\hat{i} + 17\hat{j} + 7\hat{k}\right)}{\sqrt{4 + 289 + 49}} \right\|$ $= \frac{31}{\sqrt{342}}$	 1 1 1 1 1 1

Q38. During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature (25°C). Initially the processor's temperature is 85°C. The rate of cooling is defined by the equation $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$,

where T(t) represents the temperature of the processor at time t (in minutes) and k is a constant.

Based on the above information, answer the following questions :

(i) Find the expression for temperature of processor, T(t) given that $T(0) = 85°C$. *2*

(ii) How long will it take for the processor's temperature to reach 40°C ? Given that $k = 0{\cdot}03$, $\log_e 4 = 1{\cdot}3863$. *2*

A38.	$(i)\dfrac{dT}{dt}=-k(T-25)$	
	$\Rightarrow\dfrac{dT}{T-25}=-k\,dt$	½
	$\Rightarrow\int\dfrac{dT}{T-25}=-k\int dt$	
	$\Rightarrow\log\lvert T-25\rvert=-kt+C \quad ...(a)$	1
	When $t=0, T=85$	
	$\Rightarrow \log 60=C$	
	Using in equation (a), $\log\lvert T-25\rvert=-kt+\log 60 \quad ...(b)$	½
	(ii) When $k=0.03, \log\lvert T-25\rvert=-0.03t+\log 60$	
	$\Rightarrow\log\left\lvert\dfrac{T-25}{60}\right\rvert=-0.03t$	
	$\Rightarrow T-25=60.e^{-0.03t}$	1
	When $T=40, t=t_1$	
	$\Rightarrow\dfrac{15}{60}=e^{-0.03t_1}$	½
	$\Rightarrow e^{-0.03t_1}=\dfrac{1}{4}\Rightarrow-0.03t_1=-\log 4$	
	$\Rightarrow t_1=\dfrac{\log 4}{0.03}=\dfrac{1.3863}{0.03}=46.21\text{m}$	½