Series : ZXW4Y **SET~1**

प्रश्न-पत्र कोड
Q.P. Code **65/4/1**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* *अधिकतम अंक : **80***
*Time allowed : **3** hours* *Maximum Marks : **80***

नोट	NOTE
(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।	(I) Please check that this question paper contains **23** printed pages.
(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।	(II) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।	(III) Please check that this question paper contains **38** questions.
(IV) **कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।**	(IV) Please write down the Serial Number of the question in the answer-book at the given place before attempting it.
(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।	(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The principal value of $\sin^{-1}\left(\sin\left(-\frac{10\pi}{3}\right)\right)$ is :

(A) $-\frac{2\pi}{3}$ (B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

2. If A and B are square matrices of same order such that AB = A and BA = B, then $A^2 + B^2$ is equal to :

(A) A + B (B) BA

(C) 2 (A + B) (D) 2BA

3. For real x, let $f(x) = x^3 + 5x + 1$. Then :

(A) f is one-one but not onto on R

(B) f is onto on R but not one-one

(C) f is one-one and onto on R

(D) f is neither one-one nor onto on R

4. If $y = \sin^{-1} x$, then $(1 - x^2)\dfrac{d^2y}{dx^2}$ is equal to :

(A) $x\dfrac{dy}{dx}$ (B) $-x\dfrac{dy}{dx}$

(C) $x^2\dfrac{dy}{dx}$ (D) $-x^2\dfrac{dy}{dx}$

5. The values of λ so that $f(x) = \sin x - \cos x - \lambda x + C$ decreases for all real values of x are :

(A) $1 < \lambda < \sqrt{2}$ (B) $\lambda \geq 1$

(C) $\lambda \geq \sqrt{2}$ (D) $\lambda < 1$

6. If P is a point on the line segment joining $(3, 6, -1)$ and $(6, 2, -2)$ and y-coordinate of P is 4, then its z-coordinate is :

(A) $-\dfrac{3}{2}$ (B) 0

(C) 1 (D) $\dfrac{3}{2}$

7. If M and N are square matrices of order 3 such that det (M) = m and MN = mI, then det (N) is equal to :

(A) -1 (B) 1

(C) $-m^2$ (D) m^2

8. If $f(x) = \begin{cases} 3x - 2, & 0 < x \leq 1 \\ 2x^2 + ax, & 1 < x < 2 \end{cases}$ is continuous for $x \in (0, 2)$, then a is equal to :

(A) -4 (B) $-\dfrac{7}{2}$

(C) -2 (D) -1

9. If $f : N \rightarrow W$ is defined as

$$f(n) = \begin{cases} \frac{n}{2}, & \text{if n is even} \\ 0, & \text{if n is odd} \end{cases},$$

then f is :

(A) injective only

(B) surjective only

(C) a bijection

(D) neither surjective nor injective

10. The matrix $\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -7 \\ 2 & 7 & 0 \end{bmatrix}$ is a :

(A) diagonal matrix

(B) symmetric matrix

(C) skew symmetric matrix

(D) scalar matrix

11. If the sides AB and AC of Δ ABC are represented by vectors $\hat{j} + \hat{k}$ and $3\hat{i} - \hat{j} + 4\hat{k}$ respectively, then the length of the median through A on BC is :

(A) $2\sqrt{2}$ units

(B) $\sqrt{18}$ units

(C) $\frac{\sqrt{34}}{2}$ units

(D) $\frac{\sqrt{48}}{2}$ units

12. The function f defined by

$$f(x) = \begin{cases} x, & \text{if } x \leq 1 \\ 5, & \text{if } x > 1 \end{cases}$$

is ***not*** continuous at :

(A) $x = 0$

(B) $x = 1$

(C) $x = 2$

(D) $x = 5$

13. If $f(x) = 2x + \cos x$, then $f(x)$:

(A) has a maxima at $x = \pi$

(B) has a minima at $x = \pi$

(C) is an increasing function

(D) is a decreasing function

14. $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha}\, dx$ is equal to :

(A) $2(\sin x + x\cos\alpha) + C$ (B) $2(\sin x - x\cos\alpha) + C$

(C) $2(\sin x + 2x\cos\alpha) + C$ (D) $2(\sin x + \sin\alpha) + C$

15. The value of $\int_0^1 \frac{dx}{e^x + e^{-x}}$ is :

(A) $-\frac{\pi}{4}$ (B) $\frac{\pi}{4}$

(C) $\tan^{-1} e - \frac{\pi}{4}$ (D) $\tan^{-1} e$

16. The order and degree of the differential equation

$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x \sin\left(\frac{dy}{dx}\right)$ are :

(A) order 2, degree 2 (B) order 2, degree 1

(C) order 2, degree not defined (D) order 1, degree not defined

17. The area of the region enclosed by the curve $y = \sqrt{x}$ and the lines $x = 0$ and $x = 4$ and x-axis is :

(A) $\frac{16}{9}$ sq. units (B) $\frac{32}{9}$ sq. units

(C) $\frac{16}{3}$ sq. units (D) $\frac{32}{3}$ sq. units

18. The corner points of the feasible region of a Linear Programming Problem are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). If $Z = ax + by$; $(a, b > 0)$ be the objective function, and maximum value of Z is obtained at (0, 2) and (3, 0), then the relation between a and b is :

(A) $a = b$ (B) $a = 3b$

(C) $b = 6a$ (D) $3a = 2b$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* If A and B are two events such that $P(A \cap B) = 0$, then A and B are independent events.

Reason (R) : Two events are independent if the occurrence of one does not effect the occurrence of the other.

20. *Assertion (A) :* In a Linear Programming Problem, if the feasible region is empty, then the Linear Programming Problem has no solution.

Reason (R) : A feasible region is defined as the region that satisfies all the constraints.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. Let A and B be two square matrices of order 3 such that det (A) = 3 and det (B) = – 4. Find the value of det (– 6AB).

22. (a) Find the least value of 'a' so that $f(x) = 2x^2 - ax + 3$ is an increasing function on [2, 4].

OR

(b) If $f(x) = x + \frac{1}{x}$, $x \geq 1$, show that f is an increasing function.

23. (a) Simplify $\sin^{-1}\left(\frac{x}{\sqrt{1 + x^2}}\right)$.

OR

(b) Find domain of $\sin^{-1}\sqrt{x-1}$.

24. Calculate the area of the region bounded by the curve $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the x-axis using integration.

25. For the curve $y = 5x - 2x^3$, if x increases at the rate of 2 units/s, then how fast is the slope of the curve changing when $x = 2$?

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. (a) If $f : R^+ \to R$ is defined as $f(x) = \log_a x$ ($a > 0$ and $a \neq 1$), prove that f is a bijection.

(R^+ is a set of all positive real numbers.)

OR

(b) Let A = {1, 2, 3} and B = {4, 5, 6}. A relation R from A to B is defined as $R = \{(x, y) : x + y = 6, x \in A, y \in B\}$.

(i) Write all elements of R.

(ii) Is R a function ? Justify.

(iii) Determine domain and range of R.

27. (a) Find k so that

$$f(x) = \begin{cases} \dfrac{x^2 - 2x - 3}{x + 1}, & x \neq -1 \\ k, & x = -1 \end{cases}$$

is continuous at $x = -1$.

OR

(b) Check the differentiability of function $f(x) = x|x|$ at $x = 0$.

28. Evaluate :

$$\int_{\pi/2}^{\pi} e^{x}\left(\frac{1-\sin x}{1-\cos x}\right)dx$$

29. (a) Find the probability distribution of the number of boys in families having three children, assuming equal probability for a boy and a girl.

OR

(b) A coin is tossed twice. Let X be a random variable defined as number of heads minus number of tails. Obtain the probability distribution of X and also find its mean.

30. Find the distance of the point (–1, –5, –10) from the point of intersection of the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$.

31. Solve the following Linear Programming Problem using graphical method :

Maximise $Z = 100x + 50y$

subject to the constraints

$$3x + y \leq 600$$

$$x + y \leq 300$$

$$y \leq x + 200$$

$$x \geq 0, y \geq 0$$

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. If A is a 3×3 invertible matrix, show that for any scalar $k \neq 0$, $(kA)^{-1} = \frac{1}{k}A^{-1}$. Hence calculate $(3A)^{-1}$, where

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}.$$

33. The relation between the height of the plant (y cm) with respect to exposure to sunlight is governed by the equation $y = 4x - \frac{1}{2}x^2$, where x is the number of days exposed to sunlight.

(i) Find the rate of growth of the plant with respect to sunlight. *2*

(ii) In how many days will the plant attain its maximum height ? What is the maximum height ? *3*

34. (a) Find :

$$\int \frac{\cos x}{(4+\sin^2 x)(5-4\cos^2 x)}\, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi} \frac{dx}{a^2\cos^2 x + b^2 \sin^2 x}$$

35. (a) Show that the area of a parallelogram whose diagonals are represented by $\vec{a}$ and $\vec{b}$ is given by $\frac{1}{2}|\vec{a} \times \vec{b}|$. Also find the area of a parallelogram whose diagonals are $2\hat{i} - \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} - \hat{k}$.

OR

(b) Find the equation of a line in vector and cartesian form which passes through the point (1, 2, – 4) and is perpendicular to the lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$, and

$\vec{r} = 15\hat{i} + 29\hat{j} + 5\hat{k} + \mu(3\hat{i} + 8\hat{j} - 5\hat{k})$.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Some students are having a misconception while comparing decimals. For example, a student may mention that $78{\cdot}56 > 78{\cdot}9$ as $7856 > 789$. In order to assess this concept, a decimal comparison test was administered to the students of class VI through the following question : In the recently held Sports Day in the school, 5 students participated in a javelin throw competition. The distances to which they have thrown the javelin are shown below in the table :

Name of student	Distance of javelin (in meters)
Ajay	47·7
Bijoy	47·07
Kartik	43·09
Dinesh	43·9
Devesh	45·2

The students were asked to identify who has thrown the javelin the farthest.

Based on the test attempted by the students, the teacher concludes that 40% of the students have the misconception in the concept of decimal comparison and the rest do not have the misconception. 80% of the students having misconception answered Bijoy as the correct answer in the paper. 90% of the students who are identified with not having misconception, did not answer Bijoy as their answer.

On the basis of the above information, answer the following questions :

(i) What is the probability of a student not having misconception but still answers Bijoy in the test ? *1*

(ii) What is the probability that a randomly selected student answers Bijoy as his answer in the test ? *1*

(iii) (a) What is the probability that a student who answered as Bijoy is having misconception ? *2*

OR

(iii) (b) What is the probability that a student who answered as Bijoy is amongst students who do not have the misconception ? *2*

Case Study – 2

37. An engineer is designing a new metro rail network in a city.

Initially, two metro lines, Line A and Line B, each consisting of multiple stations are designed. The track for Line A is represented by $l_1 : \dfrac{x-2}{3} = \dfrac{y+1}{-2} = \dfrac{z-3}{4}$, while the track for Line B is represented by $l_2 : \dfrac{x-1}{2} = \dfrac{y-3}{1} = \dfrac{z+2}{-3}$.

Based on the above information, answer the following questions :

(i) Find whether the two metro tracks are parallel. *1*

(ii) Solar panels are to be installed on the rooftop of the metro stations. Determine the equation of the line representing the placement of solar panels on the rooftop of Line A's stations, given that panels are to be positioned parallel to Line A's track (l_1) and pass through the point $(1, -2, -3)$. *1*

(iii) (a) To connect the stations, a pedestrian pathway perpendicular to the two metro lines is to be constructed which passes through point (3, 2, 1). Determine the equation of the pedestrian walkway. *2*

OR

(iii) (b) Find the shortest distance between Line A and Line B. *2*

Case Study – 3

38. During a heavy gaming session, the temperature of a student's laptop processor increases significantly. After the session, the processor begins to cool down, and the rate of cooling is proportional to the difference between the processor's temperature and the room temperature (25°C). Initially the processor's temperature is 85°C. The rate of cooling is defined by the equation $\frac{d}{dt}(T(t)) = -k(T(t) - 25)$,

where T(t) represents the temperature of the processor at time t (in minutes) and k is a constant.

Based on the above information, answer the following questions :

(i) Find the expression for temperature of processor, T(t) given that $T(0) = 85°C$. *2*

(ii) How long will it take for the processor's temperature to reach 40°C ? Given that $k = 0{\cdot}03$, $\log_e 4 = 1{\cdot}3863$. *2*