Series : ZWY7X

SET~2

प्रश्न-पत्र कोड
Q.P. Code **65/7/2**

रोल नं.
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।
Candidates must write the Q.P. Code on the title page of the answer-book.

गणित
MATHEMATICS

*निर्धारित समय : **3** घण्टे* *अधिकतम अंक : **80***

*Time allowed : **3** hours* *Maximum Marks : **80***

नोट	NOTE
(I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ **23** हैं।	(I) Please check that this question paper contains **23** printed pages.
(II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।	(II) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
(III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में **38** प्रश्न हैं।	(III) Please check that this question paper contains **38** questions.
(IV) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।	(IV) Please write down the Serial Number of the question in the answer-book at the given place before attempting it.
(V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।	(V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

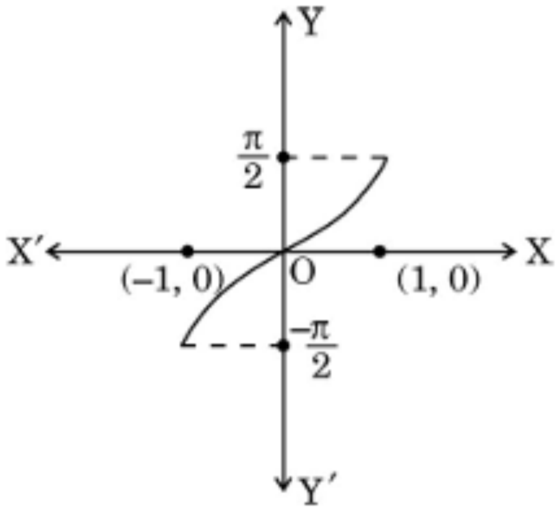
(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculator is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. Study the given graph. It illustrates :



(A) $y = \tan^{-1} x$ (B) $y = \cos^{-1} x$

(C) $y = \sec^{-1} x$ (D) $y = \sin^{-1} x$

2. If $\begin{bmatrix} 2x-1 & 3x \\ 0 & y^2-1 \end{bmatrix} = \begin{bmatrix} x+3 & 12 \\ 0 & 35 \end{bmatrix}$, then the value of $(x - y)$ is :

(A) 2 or 10 (B) – 2 or 10

(C) 2 or – 10 (D) – 2 or – 10

3. Let A be a square matrix of order 3. If $|A| = 5$, then $|\text{adj } A|$ is :

(A) 5 (B) 125

(C) 25 (D) – 5

4. If A and B are two square matrices each of order 3 with $|A| = 3$ and $|B| = 5$, then $|2AB|$ is :

(A) 30 (B) 120

(C) 15 (D) 225

5. The matrix $A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$ is a/an :

(A) scalar matrix (B) identity matrix

(C) null matrix (D) symmetric matrix

6. What is the total number of possible matrices of order 3×3 with each entry as $\sqrt{2}$ or $\sqrt{3}$?

(A) 9 (B) 512

(C) 615 (D) 64

7. Domain of $f(x) = \cos^{-1} x + \sin x$ is :

(A) R (B) $(-1, 1)$

(C) $[-1, 1]$ (D) ϕ

8. If $f(x) = \begin{cases} \dfrac{\sin^2 ax}{x^2} & , \text{if } x \neq 0 \\ 1 & , \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of 'a' is :

(A) ± 1 (B) -1

(C) 0 (D) 1

9. If $f : R \rightarrow R$ is defined as $f(x) = 2x - \sin x$, then f is :

(A) a decreasing function (B) an increasing function

(C) maximum at $x = \frac{\pi}{2}$ (D) maximum at $x = 0$

10. If R be a relation defined as $a R b$ iff $|a - b| > 0$ $a, b \in \mathbb{R}$ then R is :

(A) reflexive (B) symmetric

(C) transitive (D) symmetric and transitive

11. If $f(x) = -2x^8$, then the correct statement is :

(A) $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$ (B) $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$

(C) $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$ (D) $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$

12. For a function f(x), which of the following holds true ?

(A) $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

(B) $\int_{-a}^{a} f(x)\,dx = 0$, if f is an even function

(C) $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$, if f is an odd function

(D) $\int_0^{2a} f(x)\,dx = \int_0^a f(x)\,dx - \int_0^a f(2a+x)\,dx$

13. $\int \frac{e^{9\log x} - e^{8\log x}}{e^{6\log x} - e^{5\log x}}\, dx$ is equal to :

(A) $x + C$ (B) $\frac{x^2}{2} + C$

(C) $\frac{x^4}{4} + C$ (D) $\frac{x^3}{3} + C$

14. $\int \frac{a^x}{\sqrt{1 - a^{2x}}}\, dx$ is equal to :

(A) $\frac{\sin^{-1}(a^x)}{\log_e a} + C$ (B) $\log_e(1 - a^{2x}) + C$

(C) $\frac{\cos^{-1}(a^x)}{\log_e a} + C$ (D) $\frac{\sin^{-1}(a^x)}{a^x} + C$

15. A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is :

(A) $\frac{2}{13}$ (B) $\frac{3}{26}$

(C) $\frac{19}{26}$ (D) $\frac{3}{13}$

16. A student tries to tie ropes, parallel to each other from one end of the wall to the other. If one rope is along the vector $3\hat{i} + 15\hat{j} + 6\hat{k}$ and the other is along the vector $2\hat{i} + 10\hat{j} + \lambda\hat{k}$, then the value of λ is :

(A) 6 (B) 1

(C) $\frac{1}{4}$ (D) 4

17. If $P(A) = \frac{1}{5}$, $P(B) = \frac{3}{5}$ and $P\left(\frac{A}{B}\right) = \frac{2}{5}$, then $P(A' \cap B')$ is :

(A) $\frac{11}{25}$ (B) $\frac{19}{25}$

(C) $\frac{8}{25}$ (D) $\frac{6}{25}$

18. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$ for any two vectors, then vectors $\vec{a}$ and $\vec{b}$ are :

(A) orthogonal vectors
(B) parallel to each other
(C) unit vectors
(D) collinear vectors

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* $f(x) = \begin{cases} x \sin \dfrac{1}{x} & , \ x \neq 0 \\ 0 & , \ x = 0 \end{cases}$ is continuous at $x = 0$.

Reason (R) : When $x \to 0$, $\sin \dfrac{1}{x}$ is a finite value between -1 and 1.

20. *Assertion (A) :* Set of values of $\sec^{-1}\left(\dfrac{\sqrt{3}}{2}\right)$ is a null set.

Reason (R) : $\sec^{-1} x$ is defined for $x \in R - (-1, 1)$.

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.

OR

(b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village ?

22. If $\begin{bmatrix} 3 & -1 \\ 0 & 1 \\ 2 & -3 \end{bmatrix} A = \begin{bmatrix} 2 \\ -5 \\ -17 \end{bmatrix}$, then find matrix A.

23. Let $f : A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$, where $A = R - \{3\}$ and $B = R - \{1\}$.

Discuss the bijectivity of the function.

24. In a Linear Programming Program (LPP) for objective function $Z = 14x - 10y$

subject to constraints

$$x + y \leq 8$$
$$3x - 2y \geq -6$$
$$x, y \geq 0$$

shade the feasible region and mark the corner points in a neatly drawn graph.

25. (a) Differentiate $\left(\frac{5^x}{x^5}\right)$ with respect to x.

OR

(b) If $-2x^2 - 5xy + y^3 = 76$, then find $\frac{dy}{dx}$.

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. (a) Let $2x + 5y - 1 = 0$ and $3x + 2y - 7 = 0$ represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.

OR

(b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹ 150 (Chemistry), ₹ 175 (Physics) and ₹ 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹ 35,000, what profit did he earn after the sale of two days ?

27. (a) Show that the function $f : R \rightarrow R$ defined by $f(x) = 4x^3 - 5$, $\forall\, x \in R$ is one-one and onto.

OR

(b) Let R be a relation defined on a set N of natural numbers such that $R = \{(x, y) : xy$ is a square of a natural number, $x, y \in N\}$. Determine if the relation R is an equivalence relation.

28. Show that the derivative of $\tan^{-1}(\sec x + \tan x)$, $\left[-\frac{\pi}{2} < x < \frac{\pi}{2}\right]$ with respect to x is equal to $\frac{1}{2}$.

29. Find dimensions of a rectangle of perimeter 12 cm which will generate maximum volume when swept along a circular rotation keeping the shorter side fixed as the axis.

30. (a) The scalar product of the vector $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$ with a unit vector along sum of vectors $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$ is equal to 1. Find the value of λ.

OR

(b) Find the shortest distance between the lines :

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

$$\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k}).$$

31. In the Linear Programming Problem for objective function $Z = 18x + 10y$ subject to constraints

$$4x + y \geq 20$$

$$2x + 3y \geq 30$$

$$x, y \geq 0$$

find the minimum value of Z.

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32. (a) Find :

$$\int \frac{x}{(x-1)(x^2+4)}\,dx$$

OR

(b) Evaluate :

$$\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\,dx$$

33. (a) Find the point Q on the line $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$ at a distance of $3\sqrt{2}$ from the point P(1, 2, 3).

OR

(b) Find the image of the point $(-1, 5, 2)$ in the line $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$. Find the length of the line segment joining the points (given point and the image point).

34. Solve the differential equation $(x^2 + y^2)\, dx + xy\, dy = 0,\ y(1) = 1$.

35. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle $\frac{\pi}{4}$ anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let A_1 : People with good health,

A_2 : People with average health,

and A_3 : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category A_1, A_2 and A_3 are 25%, 35% and 50%, respectively.

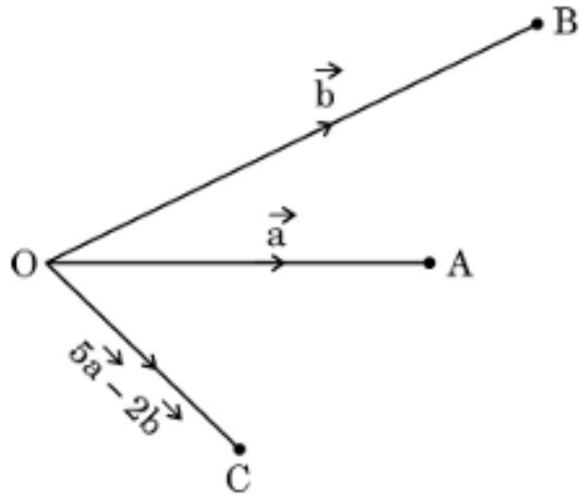
Based upon the above information, answer the following questions :

(i) A person was tested randomly. What is the probability that he/she has contracted the disease ? *2*

(ii) Given that the person has not contracted the disease, what is the probability that the person is from category A_2 ? *2*

Case Study – 2

37. Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ and $\overrightarrow{OC} = 5\vec{a} - 2\vec{b}$ respectively.



Based upon the above information, answer the following questions :

(i) Complete the given figure to explain their entire movement plan along the respective vectors. *1*

(ii) Find vectors $\overrightarrow{AC}$ and $\overrightarrow{BC}$. *1*

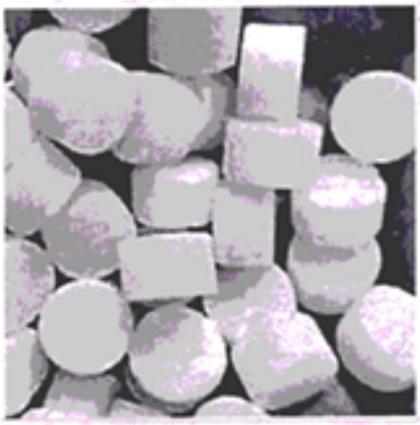
(iii) (a) If $\vec{a} \cdot \vec{b} = 1$, distance of O to A is 1 km and that from O to B is 2 km, then find the angle between $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Also, find $|\vec{a} \times \vec{b}|$. *2*

OR

(iii) (b) If $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$, then find a unit vector perpendicular to $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$. *2*

Case Study – 3

38. Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.



(Cylindrical-shaped Camphor tablets)

A cylindrical camphor tablet whose height is equal to its radius (r) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus, $\frac{dV}{dt} = kS$ is the differential equation, where V is the volume, S is the surface area and t is the time in hours.

Based upon the above information, answer the following questions :

(i) Write the order and degree of the given differential equation. *1*

(ii) Substituting $V = \pi r^3$ and $S = 2\pi r^2$, we get the differential equation $\frac{dr}{dt} = \frac{2}{3}k$. Solve it, given that r(0) = 5 mm. *1*

(iii) (a) If it is given that r = 3 mm when t = 1 hour, find the value of k. Hence, find t for r = 0 mm. *2*

OR

(iii) (b) If it is given that r = 1 mm when t = 1 hour, find the value of k. Hence, find t for r = 0 mm. *2*