

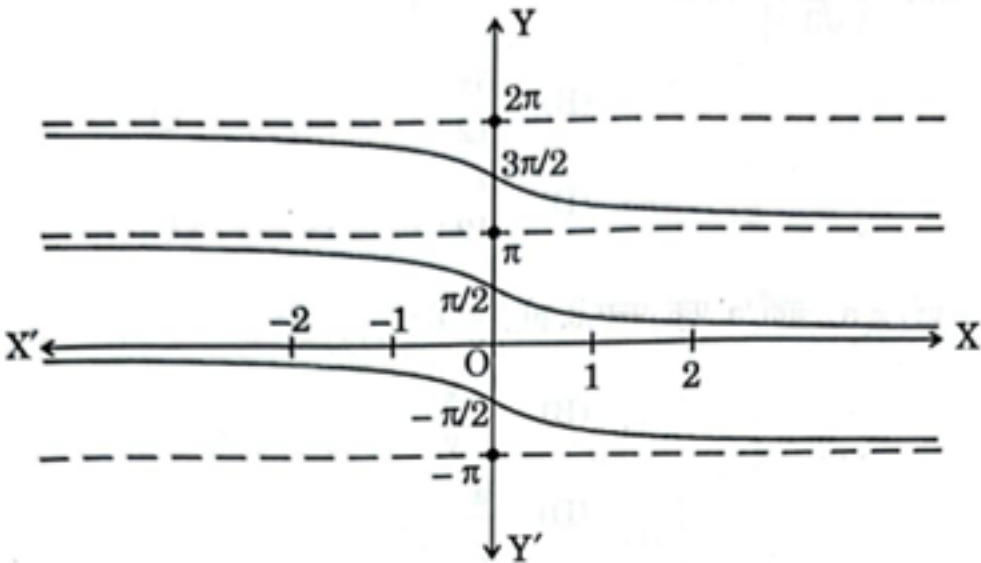
Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary Examination, 2025
SUBJECT NAME MATHEMATICS (Q.P. CODE – 65/6/2)

General Instructions: -

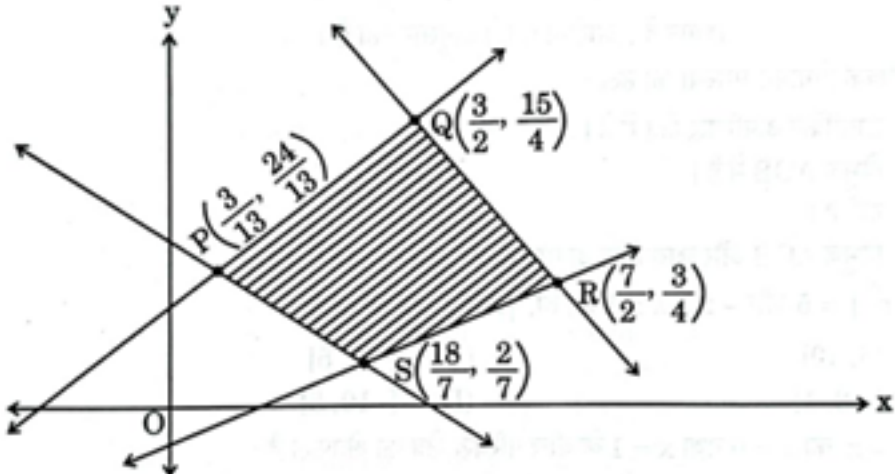
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| 1 | You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully. |
| 2 | “Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC.” |
| 3 | Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded. |
| 4 | The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly. |
| 5 | The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators. |
| 6 | Evaluators will mark (✓) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. This is the most common mistake which evaluators are committing. |
| 7 | If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly. |
| 8 | If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly. |
| 9 | If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note “Extra Question” . |

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| 10 | No marks to be deducted for the cumulative effect of an error. It should be penalized only once. |
| 11 | A full scale of marks_____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it. |
| 12 | Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper. |
| 13 | <p>Ensure that you do not make the following common types of errors committed by the Examiner in the past: -</p> <ul style="list-style-type: none"> • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totaling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totaling on the title page. • Wrong totaling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) • Half or a part of the answer marked correct and the rest as wrong, but no marks |
| 14 | While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks. |
| 15 | Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously. |
| 16 | The Examiners should acquaint themselves with the guidelines given in the " Guidelines for Spot Evaluation " before starting the actual evaluation. |
| 17 | Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words. |
| 18 | The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme. |

MARKING SCHEME – 65/6/2

| Q.No. | EXPECTED ANSWER / VALUE POINTS | Marks |
|--|---|-------|
| SECTION-A | | |
| This section comprises multiple choice questions (MCQs) of 1 mark each. | | |
| 1. | Sum of two skew-symmetric matrices of same order is always a/an : (A) skew-symmetric matrix (B) symmetric matrix (C) null matrix (D) identity matrix | |
| Ans | (A) skew-symmetric matrix | 1 |
| 2. | If $A = \begin{bmatrix} 0 & -3 & 8 \\ 3 & 0 & 5 \\ -8 & -5 & 0 \end{bmatrix}$, then A is a : (A) null matrix (B) symmetric matrix (C) skew-symmetric matrix (D) diagonal matrix | |
| Ans | (C) skew-symmetric matrix | 1 |
| 3. | The graph shown below depicts :  (A) $y = \cot x$ (B) $y = \cot^{-1} x$ (C) $y = \tan x$ (D) $y = \tan^{-1} x$ | |
| Ans | (B) $y = \cot^{-1} x$ | 1 |
| 4. | Let both AB' and $B'A$ be defined for matrices A and B. If order of A is $n \times m$, then the order of B is : (A) $n \times n$ (B) $n \times m$ (C) $m \times m$ (D) $m \times n$ | |
| Ans | (B) $n \times m$ | 1 |

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| 5. | <p>If $f(x) = \begin{cases} \frac{\log(1+ax) + \log(1-bx)}{x}, & \text{for } x \neq 0 \\ k, & \text{for } x = 0 \end{cases}$</p> <p>is continuous at $x = 0$, then the value of k is :</p> <p>(A) a (B) $a + b$ (C) $a - b$ (D) b</p> | |
| Ans | (C) $a - b$ | 1 |
| 6. | <p>If $y = a \cos(\log x) + b \sin(\log x)$, then $x^2 y_2 + xy_1$ is :</p> <p>(A) $\cot(\log x)$ (B) y (C) $-y$ (D) $\tan(\log x)$</p> | |
| Ans | (C) $-y$ | 1 |
| 7. | <p>$\left[\sec^{-1}(-\sqrt{2}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) \right]$ is equal to :</p> <p>(A) $\frac{11\pi}{12}$ (B) $\frac{5\pi}{12}$ (C) $-\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$</p> | |
| Ans | (D) $\frac{7\pi}{12}$ | 1 |
| 8. | <p>If $\tan^{-1}(x^2 - y^2) = a$, where 'a' is a constant, then $\frac{dy}{dx}$ is :</p> <p>(A) $\frac{x}{y}$ (B) $-\frac{x}{y}$ (C) $\frac{a}{x}$ (D) $\frac{a}{y}$</p> | |
| Ans | (A) $\frac{x}{y}$ | 1 |
| 9. | <p>Let $f(x) = x^2$, $x \in \mathbb{R}$. Then, which of the following statements is <i>incorrect</i> ?</p> <p>(A) Minimum value of f does not exist. (B) There is no point of maximum value of f in \mathbb{R}. (C) f is continuous at $x = 0$. (D) f is differentiable at $x = 0$.</p> | |
| Ans | (A) Minimum value of f does not exist | 1 |
| 10. | <p>$\int \frac{x+5}{(x+6)^2} e^x dx$ is equal to :</p> <p>(A) $\log(x+6) + C$ (B) $e^x + C$ (C) $\frac{e^x}{x+6} + C$ (D) $\frac{-1}{(x+6)^2} + C$</p> | |

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| Ans | (C) $\frac{e^x}{x+6} + C$ | 1 |
| 11. | <p>Let $f'(x) = 3(x^2 + 2x) - \frac{4}{x^3} + 5$, $f(1) = 0$. Then, $f(x)$ is :</p> <p>(A) $x^3 + 3x^2 + \frac{2}{x^2} + 5x + 11$ (B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$</p> <p>(C) $x^3 + 3x^2 - \frac{2}{x^2} + 5x - 11$ (D) $x^3 - 3x^2 - \frac{2}{x^2} + 5x - 11$</p> | |
| Ans | (B) $x^3 + 3x^2 + \frac{2}{x^2} + 5x - 11$ | 1 |
| 12. | <p>The order and degree of the differential equation $\frac{d^2y}{dx^2} + 4\left(\frac{dy}{dx}\right) = x \log\left(\frac{d^2y}{dx^2}\right)$ are respectively :</p> <p>(A) 0, 3 (B) 2, 1</p> <p>(C) 2, not defined (D) 1, not defined</p> | |
| Ans | (C) 2, not defined | 1 |
| 13. | <p>For a Linear Programming Problem (LPP), the given objective function is $Z = x + 2y$. The feasible region PQRS determined by the set of constraints is shown as a shaded region in the graph.</p>  <p>(Note : The figure is not to scale)</p> <p>$P = \left(\frac{3}{13}, \frac{24}{13}\right)$, $Q = \left(\frac{3}{2}, \frac{15}{4}\right)$, $R = \left(\frac{7}{2}, \frac{3}{4}\right)$, $S = \left(\frac{18}{7}, \frac{2}{7}\right)$</p> <p>Which of the following statements is correct ?</p> <p>(A) Z is minimum at $S\left(\frac{18}{7}, \frac{2}{7}\right)$</p> <p>(B) Z is maximum at $R\left(\frac{7}{2}, \frac{3}{4}\right)$</p> <p>(C) (Value of Z at P) > (Value of Z at Q)</p> <p>(D) (Value of Z at Q) < (Value of Z at R)</p> | |

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| Ans | (A) Z is minimum at S ($\frac{18}{7}, \frac{2}{7}$) | 1 |
| 14. | <p>The area of the region bounded by the curve $y^2 = x$ between $x = 0$ and $x = 1$ is :</p> <p>(A) $\frac{3}{2}$ sq units (B) $\frac{2}{3}$ sq units</p> <p>(C) 3 sq units (D) $\frac{4}{3}$ sq units</p> | |
| Ans | (D) $\frac{4}{3}$ sq units | 1 |
| 15. | <p>Let $\vec{a} = 5$ and $-2 \leq \lambda \leq 1$. Then, the range of $\lambda \vec{a}$ is :</p> <p>(A) [5, 10] (B) [-2, 5]</p> <p>(C) [-2, 1] (D) [-10, 5]</p> | |
| Ans | 1 mark for any attempt as correct answer is not given in any option | 1 |
| 16. | <p>The solution for the differential equation $\log \left(\frac{dy}{dx} \right) = 3x + 4y$ is :</p> <p>(A) $3e^{4y} + 4e^{-3x} + C = 0$ (B) $e^{3x+4y} + C = 0$</p> <p>(C) $3e^{-3y} + 4e^{4x} + 12C = 0$ (D) $3e^{-4y} + 4e^{3x} + 12C = 0$</p> | |
| Ans | (D) $3e^{-4y} + 4e^{3x} + 12C = 0$ | 1 |
| 17. | <p>In a Linear Programming Problem (LPP), the objective function $Z = 2x + 5y$ is to be maximised under the following constraints :</p> <p>$x + y \leq 4$, $3x + 3y \geq 18$, $x, y \geq 0$</p> <p>Study the graph and select the correct option.</p> <p>(Note : The figure is not to scale)</p> <p>The solution of the given LPP :</p> <p>(A) lies in the shaded unbounded region.</p> <p>(B) lies in ΔAOB.</p> <p>(C) does not exist.</p> <p>(D) lies in the combined region of ΔAOB and unbounded shaded region.</p> | |
| Ans | (C) does not exist | 1 |

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| 18. | <p>Chances that three persons A, B, and C go to the market are 30%, 60% and 50% respectively. The probability that at least one will go to the market is :</p> <p>(A) $\frac{14}{10}$ (B) $\frac{43}{50}$ (C) $\frac{9}{100}$ (D) $\frac{7}{50}$</p> | |
| Ans | (B) $\frac{43}{50}$ | 1 |
| <p>Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.</p> <p>(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.</p> | | |
| 19. | <p>Assertion (A) : If $\vec{a} \times \vec{b} ^2 + \vec{a} \cdot \vec{b} ^2 = 256$ and $\vec{b} = 8$, then $\vec{a} = 2$.</p> <p>Reason (R) : $\sin^2 \theta + \cos^2 \theta = 1$ and $\vec{a} \times \vec{b} = \vec{a} \vec{b} \sin \theta$ and $\vec{a} \cdot \vec{b} = \vec{a} \vec{b} \cos \theta$.</p> | |
| Ans | (A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A). | 1 |
| 20. | <p>Assertion (A) : Let $f(x) = e^x$ and $g(x) = \log x$. Then $(f + g) x = e^x + \log x$ where domain of $(f + g)$ is \mathbb{R}.</p> <p>Reason (R) : $\text{Dom}(f + g) = \text{Dom}(f) \cap \text{Dom}(g)$.</p> | |
| Ans | (D) Assertion (A) is false but, Reason (R) is true. | 1 |
| <p style="text-align: center;">SECTION-B This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.</p> | | |

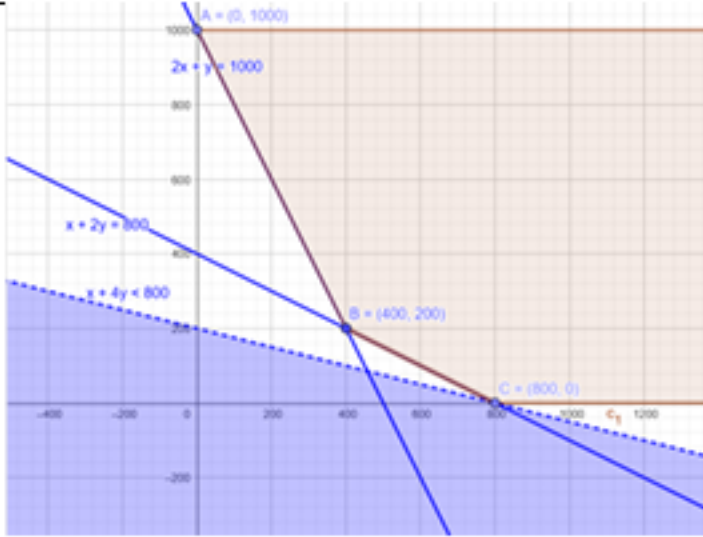
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| 21. | <p>(a) Differentiate $\sqrt{e^{\sqrt{2x}}}$ with respect to $e^{\sqrt{2x}}$ for $x > 0$.</p> <p style="text-align: center;">OR</p> <p>(b) If $(x)^y = (y)^x$, then find $\frac{dy}{dx}$.</p> | |
| Ans | <p>(a) $u = \sqrt{e^{\sqrt{2x}}}$ and $v = e^{\sqrt{2x}}$</p> <p>Derivative of \sqrt{v} wrt $v = \frac{1}{2\sqrt{v}}$</p> <p>Required derivative = $\frac{1}{2\sqrt{e^{\sqrt{2x}}}}$</p> <p style="text-align: center;">OR</p> <p>Taking log on both sides, we get $y \log x = x \log y$</p> <p>Differentiating both sides w.r.t. x, we get</p> $\frac{y}{x} + \log x \frac{dy}{dx} = \frac{x}{y} \frac{dy}{dx} + \log y$ $\Rightarrow \frac{dy}{dx} = \frac{y(x \log y - y)}{x(y \log x - x)}$ | <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> |
| 22. | <p>(a) If \vec{a} and \vec{b} are position vectors of point A and point B respectively, find the position vector of point C on BA produced such that $BC = 3BA$.</p> <p style="text-align: center;">OR</p> <p>(b) Vector \vec{r} is inclined at equal angles to the three axes x, y and z. If magnitude of \vec{r} is $5\sqrt{3}$ units, then find \vec{r}.</p> | |
| Ans | <p>(a) C divides BA in the ratio 3 : 2 externally</p> <p>Required vector = $\vec{c} = \frac{3\vec{a} - 2\vec{b}}{3 - 2} = 3\vec{a} - 2\vec{b}$</p> <p style="text-align: center;">OR</p> <p>(b) Unit vector equally inclined to coordinate axis is $\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}$</p> <p>$\vec{r} = 5\sqrt{3}(\frac{\hat{i}}{\sqrt{3}} + \frac{\hat{j}}{\sqrt{3}} + \frac{\hat{k}}{\sqrt{3}}) = 5\hat{i} + 5\hat{j} + 5\hat{k}$ or $-5\hat{i} - 5\hat{j} - 5\hat{k}$</p> | <p>1</p> <p>1</p> <p>1</p> <p>1</p> |
| 23. | Determine those values of x for which $f(x) = \frac{2}{x} - 5$, $x \neq 0$ is increasing or decreasing. | |
| Ans | <p>$f'(x) = \frac{-2}{x^2} < 0$</p> <p>Hence f is decreasing in its domain.</p> | <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> |

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| 24. | Find the domain of $f(x) = \sin^{-1}(-x^2)$. | |
| Ans | (a) $-1 \leq -x^2 \leq 1 \Rightarrow -1 \leq -x^2 \leq 0$ | 1 |
| | $\Rightarrow 0 \leq x^2 \leq 1 \Rightarrow -1 \leq x \leq 1$ | 1 |
| 25. | Find the value of λ if the following lines are perpendicular to each other : $l_1: \frac{1-x}{-3} = \frac{3y-2}{2\lambda} = \frac{z-3}{3}$ $l_2: \frac{x-1}{3\lambda} = \frac{1-y}{1} = \frac{2z-5}{3}$ | |
| Ans | $l_1: \frac{x-1}{3} = \frac{y-\frac{2}{3}}{\frac{2}{3}\lambda} = \frac{z-3}{3}$ $l_2: \frac{x-1}{3\lambda} = \frac{y-1}{-1} = \frac{z-\frac{5}{2}}{\frac{3}{2}}$ lines are perpendicular $\Rightarrow 3(3\lambda) + \frac{2}{3}\lambda(-1) + 3 \cdot \frac{3}{2} = 0$ $\lambda = \frac{-27}{50}$ | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ |

SECTION-C


This section comprises 6 Short Answer (SA) type questions of 3 marks each.

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| 26. | If $A = \begin{bmatrix} 1 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 3 & 4 \\ 0 & 5 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, are three matrices, then find ABC. | |
| Ans | Required product = $[2 + 1 + 0 \quad 0 - 3 + 0 \quad 1 - 4 + 0] \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ $= [3 \quad -3 \quad -3] \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ $= [-15]$ | 1 1 1 |
| 27. | 29. Consider the Linear Programming Problem, where the objective function $Z = (x + 4y)$ needs to be minimized subject to constraints $2x + y \geq 1000$ $x + 2y \geq 800$ $x, y \geq 0$. Draw a neat graph of the feasible region and find the minimum value of Z. | |

| Ans |  <p>Correct Graph and shading:</p> <table><tr><th>Corner points</th><th>Value of Z</th></tr><tr><td>(800, 0)</td><td>800</td></tr><tr><td>(400, 200)</td><td>1200</td></tr><tr><td>(0, 1000)</td><td>4000</td></tr></table> <p>$x + 4y < 800$ has no region common with feasible region, hence 800 is minimum</p> | Corner points | Value of Z | (800, 0) | 800 | (400, 200) | 1200 | (0, 1000) | 4000 | <p>1 ½</p> <p>1</p> <p>½</p> |
|---------------|---|---|------------|----------|-----|------------|------|-----------|------|------------------------------|
| Corner points | Value of Z | | | | | | | | | |
| (800, 0) | 800 | | | | | | | | | |
| (400, 200) | 1200 | | | | | | | | | |
| (0, 1000) | 4000 | | | | | | | | | |
| 28. | <p>(a) Find the distance of the point P(2, 4, -1) from the line</p> $\frac{x+5}{1} = \frac{y+3}{4} = \frac{z-6}{-9}$ <p>OR</p> <p>(b) Let the position vectors of the points A, B and C be $3\hat{i} - \hat{j} - 2\hat{k}$, $\hat{i} + 2\hat{j} - \hat{k}$ and $\hat{i} + 5\hat{j} + 3\hat{k}$ respectively. Find the vector and cartesian equations of the line passing through A and parallel to line BC.</p> | | | | | | | | | |
| Ans | <p>(a) Let $\vec{a}_2 = 2\hat{i} + 4\hat{j} - \hat{k}$, $\vec{a}_1 = -5\hat{i} - 3\hat{j} + 6\hat{k}$ and $\vec{b} = \hat{i} + 4\hat{j} - 9\hat{k}$</p> <p>Distance between point and line is given by $d = \frac{ (\vec{a}_2 - \vec{a}_1) \times \vec{b} }{ \vec{b} }$</p> <p>Here $(\vec{a}_2 - \vec{a}_1) = 7\hat{i} + 7\hat{j} - 7\hat{k}$</p> $(\vec{a}_2 - \vec{a}_1) \times \vec{b} = -35\hat{i} + 56\hat{j} + 21\hat{k}$ $d = \frac{49\sqrt{2}}{7\sqrt{2}} = 7$ <p>OR</p> <p>(b) Direction vector of line = $3\hat{j} + 4\hat{k}$</p> <p>Vector equation is $\vec{r} = 3\hat{i} - \hat{j} - 2\hat{k} + \mu(3\hat{j} + 4\hat{k})$</p> <p>Cartesian equation is $\frac{x-3}{0} = \frac{y+1}{3} = \frac{z+2}{4}$</p> | <p>½</p> <p>1 ½</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> | | | | | | | | |


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| 29. | <p>(a) Differentiate $y = \sin^{-1}(3x - 4x^3)$ w.r.t. x, if $x \in \left[-\frac{1}{2}, \frac{1}{2}\right]$.</p> <p style="text-align: center;">OR</p> <p>(b) Differentiate $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ with respect to x, when $x \in (0, 1)$.</p> | |
| Ans | <p>(a) $x = \sin t$ gives $y = \sin^{-1}(\sin 3t) = 3t = 3\sin^{-1}x$</p> $\frac{dy}{dx} = \frac{3}{\sqrt{1-x^2}}$ <p>Aliter: $\frac{dy}{dx} = \frac{3-12x^2}{\sqrt{1-(3x-4x^3)^2}}$</p> <p style="text-align: center;">OR</p> <p>(b) $x = \tan t$ gives $y = \cos^{-1}(\cos 2t) = 2t = 2\tan^{-1}x$</p> $\frac{dy}{dx} = \frac{2}{1+x^2}$ <p>Aliter: $\frac{dy}{dx} = \frac{-1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-4x}{(1+x^2)^2}$</p> | $\frac{1}{2} + 1 + \frac{1}{2}$ 1 3 $\frac{1}{2} + 1 + \frac{1}{2}$ 1 3 |
| 30. | <p>(a) A student wants to pair up natural numbers in such a way that they satisfy the equation $2x + y = 41$, $x, y \in \mathbb{N}$. Find the domain and range of the relation. Check if the relation thus formed is reflexive, symmetric and transitive. Hence, state whether it is an equivalence relation or not.</p> <p style="text-align: center;">OR</p> <p>(b) Show that the function $f : \mathbb{N} \rightarrow \mathbb{N}$, where \mathbb{N} is a set of natural numbers, given by $f(n) = \begin{cases} n-1, & \text{if } n \text{ is even} \\ n+1, & \text{if } n \text{ is odd} \end{cases}$ is a bijection.</p> | |
| Ans | <p>(a) $R = \{(1,39), (2,37), \dots, (20,1)\}$</p> <p>Domain = $\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20\}$</p> <p>Range = $\{1,3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37,39\}$</p> <p>(1, 1) does not belong to R hence not reflexive</p> <p>(1, 39) belongs to R but (39, 1) does not belong to R hence not symmetric</p> <p>(11, 19) and (19, 3) belong to R but (11, 3) does not belong to R hence not transitive</p> <p>Hence R is not an equivalence relation.</p> | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 |

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|-------|---|---|-----------------|----------------|---|---|------|-----------------|-----------------|----------------|----------------|-------|---|-----------------|-----------------|----------------|--|
| | <p style="text-align: center;">OR</p> <p>(b) Let $f(x) = f(y)$ Let x and y are both odd or both even Then either $x+1 = y + 1$ or $x-1 = y-1$ gives $x = y$ x odd and y even is rejected as $x + 1 = y - 1$ gives $x - y = -2$ not possible as odd number and even number cannot differ by 2 Hence f is one-one</p> <p>For onto: Let $f(x) = y$ gives $x = y + 1$ or $x = y - 1$ If y is odd, x is even and if y is even, x is odd Range = N = co-domain, hence onto As f is both one-one and onto hence bijective</p> | <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> | | | | | | | | | | | | | | | |
| 31. | A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed three times, find the probability distribution of number of tails. Hence, find the mean of the distribution. | | | | | | | | | | | | | | | | |
| Ans | <p>$P(H) = \frac{3}{4}, P(T) = \frac{1}{4}$</p> <table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>P(X)</td><td>$\frac{27}{64}$</td><td>$\frac{27}{64}$</td><td>$\frac{9}{64}$</td><td>$\frac{1}{64}$</td></tr><tr><td>XP(X)</td><td>0</td><td>$\frac{27}{64}$</td><td>$\frac{18}{64}$</td><td>$\frac{3}{64}$</td></tr></table> <p>Mean = $\frac{3}{4}$</p> | X | 0 | 1 | 2 | 3 | P(X) | $\frac{27}{64}$ | $\frac{27}{64}$ | $\frac{9}{64}$ | $\frac{1}{64}$ | XP(X) | 0 | $\frac{27}{64}$ | $\frac{18}{64}$ | $\frac{3}{64}$ | <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> |
| X | 0 | 1 | 2 | 3 | | | | | | | | | | | | | |
| P(X) | $\frac{27}{64}$ | $\frac{27}{64}$ | $\frac{9}{64}$ | $\frac{1}{64}$ | | | | | | | | | | | | | |
| XP(X) | 0 | $\frac{27}{64}$ | $\frac{18}{64}$ | $\frac{3}{64}$ | | | | | | | | | | | | | |
| | SECTION-D | | | | | | | | | | | | | | | | |
| | This section comprises 4 Long Answer (LA) type questions of 5 marks each. | | | | | | | | | | | | | | | | |
| 32. | <p>(a) Solve the differential equation : $x^2y \, dx - (x^3 + y^3) \, dy = 0$.</p> <p style="text-align: center;">OR</p> <p>(b) Solve the differential equation $(1 + x^2) \frac{dy}{dx} + 2xy - 4x^2 = 0$ subject to initial condition $y(0) = 0$.</p> | | | | | | | | | | | | | | | | |
| Ans | <p>(a) Given differential equation can be written as $\frac{dy}{dx} = \frac{yx^2}{x^3+y^3}$----- (i)</p> <p>Let $y = vx \Rightarrow \frac{dv}{dx} = v + x \frac{dv}{dx}$ substituting in (i) we get</p> $v + x \frac{dv}{dx} = \frac{vx^3}{x^3+v^3x^3} \cdot \frac{v}{1+v^3}$ $x \frac{dv}{dx} = \frac{-v^4}{1+v^3}$ | <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>1</p> | | | | | | | | | | | | | | | |

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| | $\left(\frac{1}{v^4} + \frac{1}{v}\right)dv = \frac{-dx}{x}$ <p>Integrating we get</p> $\frac{-1}{3v^3} + \log v = -\log x + C$ $\frac{-x^3}{3y^3} + \log y = C$ <p style="text-align: center;">OR</p> <p>(b) Given D.E. is $\frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{4x^2}{1+x^2}$</p> <p>Integrating factor is $e^{\int \frac{2x}{1+x^2} dx} = e^{\log(1+x^2)} = (1+x^2)$</p> <p>Solution is $y(1+x^2) = \int 4x^2 dx + C$</p> $y(1+x^2) = \frac{4x^3}{3} + C$ <p>$y(0) = 0$ gives $C = 0$, hence solution is $y(1+x^2) = \frac{4x^3}{3}$</p> | <p>1</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> |
| 33. | Use integration to find the area of the region enclosed by curve $y = -x^2$ and the straight lines $x = -3$, $x = 2$ and $y = 0$. Sketch a rough figure to illustrate the bounded region. | |
| Ans |  <p>Correct Graph:</p> <p>Required area = $\left \int_{-3}^2 -x^2 dx \right$</p> $= \left -\frac{1}{3} x^3 \right _{-3}^2$ $= \left -\frac{1}{3} (8 - (-27)) \right $ $= \frac{35}{3}$ | <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> |

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| 34. | <p>(a) Find :</p> $\int \frac{x^2 + 1}{(x-1)^2 (x+3)} dx$ <p>OR</p> <p>(b) Evaluate :</p> $\int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$ | |
| Ans | <p>(a) $\frac{x^2+1}{(x-1)^2(x+3)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+3} = \frac{3/8}{x-1} + \frac{1/2}{(x-1)^2} + \frac{5/8}{x+3}$</p> $I = \frac{3}{8} \log x-1 - \frac{1}{2(x-1)} + \frac{5}{8} \log x+3 + C$ <p>OR</p> <p>(b) Let $I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x} dx$</p> $I = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{\sin x + \cos x} dx \quad \text{using property}$ $2I = \int_0^{\pi/2} \frac{\frac{\pi}{2}}{\sin x + \cos x} dx$ $= \frac{\pi}{2\sqrt{2}} \int_0^{\pi/2} \frac{1}{\sin(\frac{\pi}{4} + x)} dx$ $= \frac{\pi}{2\sqrt{2}} \log \left \operatorname{cosec} \left(\frac{\pi}{4} + x \right) - \cot \left(\frac{\pi}{4} + x \right) \right _0^{\pi/2}$ $= \frac{\pi}{2\sqrt{2}} \log \frac{\sqrt{2}+1}{\sqrt{2}-1}$ | <p>1 + 2</p> <p>2</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> |
| 35. | Find the foot of the perpendicular drawn from point (2, -1, 5) to the line $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11}$. Also, find the length of the perpendicular. | |
| Ans | <p>Let $\frac{x-11}{10} = \frac{y+2}{-4} = \frac{z+8}{-11} = \lambda$</p> <p>Coordinates of any point on l are $x = 10\lambda + 11, y = -4\lambda - 2, z = -11\lambda - 8$</p> <p>Drs of perpendicular line are $(10\lambda + 9, -4\lambda - 1, -11\lambda - 13)$</p> <p>Drs of given line are $10, -4, -11$</p> <p>As lines are perpendicular, so</p> $(10\lambda + 9)10 + (-4\lambda - 1)(-4) + (-11\lambda - 13)(-11) = 0$ $\Rightarrow \lambda = -1$ <p>Hence coordinates of point are $(1, 2, 3)$ which is the foot of the \perp from P to l</p> <p>length of $\perp = \sqrt{(1-2)^2 + (2+1)^2 + (3-5)^2} = \sqrt{1+9+4} = \sqrt{14}$</p> | <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> <p>1</p> <p>1</p> <p>$\frac{1}{2}$</p> <p>$\frac{1}{2}$</p> |
| | SECTION-E | |
| | This section comprises 3 case study-based questions of 4 marks each | |
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| 36. | <p>A shop selling electronic items sells smartphones of only three reputed companies A, B and C because chances of their manufacturing a defective smartphone are only 5%, 4% and 2% respectively. In his inventory he has 25% smartphones from company A, 35% smartphones from company B and 40% smartphones from company C.</p> <p>A person buys a smartphone from this shop.</p> <p>(i) Find the probability that it was defective. 2</p> <p>(ii) What is the probability that this defective smartphone was manufactured by company B? 2</p> | |
| Ans | <p>(i) $P(\text{Defective}) = 0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02$</p> <p>$= 0.0345$</p> <p>(ii) $P(B/\text{Defective}) = \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02}$</p> <p>$= \frac{140}{345} \text{ or } \frac{28}{69}$</p> | $1\frac{1}{2}$ $1\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$ |
| 37. | <p>Three students, Neha, Rani and Sam go to a market to purchase stationery items. Neha buys 4 pens, 3 notepads and 2 erasers and pays ₹ 60. Rani buys 2 pens, 4 notepads and 6 erasers for ₹ 90. Sam pays ₹ 70 for 6 pens, 2 notepads and 3 erasers.</p> <p>Based upon the above information, answer the following questions :</p> <p>(i) Form the equations required to solve the problem of finding the price of each item, and express it in the matrix form $AX = B$. 1</p> <p>(ii) Find A and confirm if it is possible to find A^{-1}. 1</p> <p>(iii) (a) Find A^{-1}, if possible, and write the formula to find X. 2</p> <p style="text-align: center;">OR</p> <p>(iii) (b) Find $A^2 - 8I$, where I is an identity matrix. 2</p> | |
| Ans | <p>(i) Let the price of each pen, notepad, eraser be ₹x, ₹y and ₹z respectively</p> <p>Given system in the form $AX = B$ is $\begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 60 \\ 90 \\ 70 \end{pmatrix}$</p> <p>(ii) $A = 50 \neq 0$, hence A^{-1} exists</p> <p>(iii) (a) $A^{-1} = \frac{\text{adj}A}{ A } = \frac{1}{50} \begin{pmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{pmatrix}$</p> <p>$X = A^{-1}B$</p> <p style="text-align: center;">OR</p> | 1 1 $1\frac{1}{2}$ $\frac{1}{2}$ |

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| | $(iii) (b) A^2 = \begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 34 & 28 & 32 \\ 52 & 34 & 46 \\ 46 & 32 & 33 \end{pmatrix}$ $A^2 - 8I = \begin{pmatrix} 26 & 28 & 32 \\ 52 & 26 & 46 \\ 46 & 32 & 25 \end{pmatrix}$ | $1\frac{1}{2}$ $\frac{1}{2}$ |
| 38. |  <p>A ladder of fixed length 'h' is to be placed along the wall such that it is free to move along the height of the wall.</p> <p>Based upon the above information, answer the following questions :</p> <p>(i) Express the distance (y) between the wall and foot of the ladder in terms of 'h' and height (x) on the wall at a certain instant. Also, write an expression in terms of h and x for the area (A) of the right triangle, as seen from the side by an observer. 1</p> <p>(ii) Find the derivative of the area (A) with respect to the height on the wall (x), and find its critical point. 1</p> <p>(iii) (a) Show that the area (A) of the right triangle is maximum at the critical point. 2</p> <p style="text-align: center;">OR</p> <p>(iii) (b) If the foot of the ladder whose length is 5 m, is being pulled towards the wall such that the rate of decrease of distance (y) is 2 m/s, then at what rate is the height on the wall (x) increasing, when the foot of the ladder is 3 m away from the wall ? 2</p> | |
| Ans | <p>(i) $y^2 = h^2 - x^2$</p> <p>$A = \frac{1}{2}xy = \frac{1}{2}x\sqrt{h^2 - x^2}$</p> <p>ii) $\frac{dA}{dx} = \frac{1}{2}\sqrt{h^2 - x^2} + \frac{1}{2}x \frac{-x}{\sqrt{h^2 - x^2}}$</p> <p>$\frac{dA}{dx} = 0$ gives $x = \frac{h}{\sqrt{2}}$</p> <p>(iii) (a) $A'' = \frac{1}{2} \frac{-4x\sqrt{h^2 - x^2} - (h^2 - 2x^2) \frac{-x}{\sqrt{h^2 - x^2}}}{h^2 - x^2}$ is < 0 at $x = \frac{h}{\sqrt{2}}$</p> | $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$ |

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| | Hence A is maximum at critical point | $\frac{1}{2}$ |
| | OR | |
| | (iii)(b) $y^2 = 25 - x^2$ hence $y = 3$ gives $x = 4$ | $\frac{1}{2}$ |
| | $2y \frac{dy}{dt} = -2x \frac{dx}{dt}$ | 1 |
| | $\frac{dx}{dt} = 1.5 \text{ m/s}$ | $\frac{1}{2}$ |