Marking Scheme
Strictly Confidential
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Senior Secondary Examination, 2025
SUBJECT NAME MATHEMATICS [Q.P. CODE – 65(B)]

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	**"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and IPC."**
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. **However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.**
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. **This is the most common mistake which evaluators are committing.**
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note **"Extra Question"**.

10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.
11	A full scale of marks_____ (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.
12	Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.
13	Ensure that you do not make the following common types of errors committed by the Examiner in the past: - • Leaving answer or part thereof unassessed in an answer book. • Giving more marks for an answer than assigned to it. • Wrong totaling of marks awarded on an answer. • Wrong transfer of marks from the inside pages of the answer book to the title page. • Wrong question wise totaling on the title page. • Wrong totaling of marks of the two columns on the title page. • Wrong grand total. • Marks in words and figures not tallying/not same. • Wrong transfer of marks from the answer book to online award list. • Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.) • Half or a part of the answer marked correct and the rest as wrong, but no marks
14	While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.
15	Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.
16	The Examiners should acquaint themselves with the guidelines given in the "**Guidelines for Spot Evaluation**" before starting the actual evaluation.
17	Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.
18	The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

1. MARKING SCHEME – 65(B)

Q.No.	EXPECTED ANSWER / VALUE POINTS	Marks
	SECTION-A ***This section comprises multiple choice questions (MCQs) of 1 mark each.***	
1.	Which of the following functions from Z to Z is both one-one and onto ? (A) $f(x) = 2x - 1$ (B) $f(x) = 3x^2 + 5$ (C) $f(x) = x + 5$ (D) $f(x) = 5x^3$	
Ans	**(C) $f(x) = x + 5$**	**1**
2.	Value of $4 \cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]$ is (A) 3 (B) -3 (C) 1 (D) -1	
Ans	**(A) 3**	**1**
3.	If $A = \begin{bmatrix} x & 0 & m \\ y & z & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6I$, where I is a unit matrix, then $x + y + z + m$ is equal to (A) 18 (B) 12 (C) 6 (D) 2	
Ans	**(B) 12**	**1**
4.	If $B = \begin{bmatrix}23 & 41 & 57\end{bmatrix}\begin{bmatrix}31 & 42 \\ 53 & 64 \\ 75 & 86\end{bmatrix}$, then the order of B is : (A) 3×2 (B) 2×2 (C) 1×3 (D) 1×2	
Ans	**(D) 1×2**	**1**
5.	If A and B are square matrices of the same order, then $(A - B)^2 = ?$ (A) $A^2 - 2AB + B^2$ (B) $A^2 - AB - BA + B^2$ (C) $A^2 - 2BA + B^2$ (D) $A^2 - AB + BA + B^2$	
Ans	**(B) $A^2 - AB - BA + B^2$**	**1**

6.	If $\begin{vmatrix} 5 & 3 & -1 \\ -7 & x & 2 \\ 9 & 6 & -2 \end{vmatrix} = 0$, then the value of x is (A) 0 (B) 9 (C) -6 (D) 6	
Ans	**(C) -6**	**1**
7.	If $A^{-1} = \begin{bmatrix} 7 & 2 \\ 8 & 2 \end{bmatrix}$, then matrix A is (A) $\begin{bmatrix} 2 & -2 \\ -8 & 7 \end{bmatrix}$ (B) $\begin{bmatrix} -7 & 8 \\ 2 & -2 \end{bmatrix}$ (C) $\begin{bmatrix} -1 & 1 \\ 4 & -\frac{7}{2} \end{bmatrix}$ (D) $\begin{bmatrix} 1 & -1 \\ -4 & \frac{7}{2} \end{bmatrix}$	
Ans	**(C)** $\begin{bmatrix} \mathbf{-1} & \mathbf{1} \\ \mathbf{4} & \mathbf{\frac{-7}{2}} \end{bmatrix}$	**1**
8.	If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, then $\frac{dy}{dx}$ is (A) $\frac{-\sqrt{x}}{\sqrt{y}}$ (B) $-\frac{1}{2}\frac{\sqrt{y}}{\sqrt{x}}$ (C) $-\frac{\sqrt{y}}{\sqrt{x}}$ (D) $\frac{-2\sqrt{y}}{\sqrt{x}}$	
Ans	**(C)** $\mathbf{-\frac{\sqrt{y}}{\sqrt{x}}}$	**1**
9.	If $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$, then $\frac{dy}{dx}$ is (A) 1 (B) $\frac{1}{2}$ (C) $-\frac{1}{2}$ (D) -1	
Ans	**(B)** $\mathbf{\frac{1}{2}}$	**1**

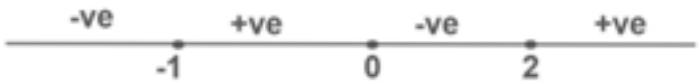
10.	When x is positive, the minimum value of x^x is (A) e^e (B) $\frac{1}{e}$ (C) $e^{\frac{1}{e}}$ (D) $e^{\frac{-1}{e}}$	
Ans	**(D) $e^{-\frac{1}{e}}$**	**1**
11.	$\int \frac{2x^3}{4+x^8} dx$ is equal to (A) $\frac{1}{4}\tan^{-1}\frac{x^4}{2}+C$ (B) $\frac{1}{2}\tan^{-1}\frac{x^4}{2}+C$ (C) $\frac{1}{4}\tan^{-1}\frac{x^4}{4}+C$ (D) $\frac{1}{4}\tan^{-1}x^4+C$	
Ans	**(A) $\frac{1}{4}tan^{-1}\frac{x^4}{2}+C$**	1
12.	. $\int e^x \cdot \frac{x}{(1+x)^2} dx$ is equal to (A) $e^x \cdot \frac{x}{1+x}+C$ (B) $e^x \cdot \frac{1}{1+x}+C$ (C) $e^x \cdot \frac{1}{x}+C$ (D) $e^x \cdot \frac{1}{(1+x)^2}+C$	
Ans	**(B) $e^x.\frac{1}{1+x}+C$**	**1**
13.	The area of the region bounded by the lines y = x + 1, x = 1, x = 3 and x-axis is (A) 6 sq units (B) 8 sq units (C) 7.5 sq units (D) 2 sq units	
Ans	**(A) 6 sq units**	**1**
14.	The integrating factor for solving the differential equation $x \cdot \frac{dy}{dx} - y = 2x^2$ is (A) x (B) $\frac{1}{x}$ (C) e^{-x} (D) $-\log x$	

Ans	**(B)** $\frac{1}{x}$	**1**
15.	The number of vector(s) of unit length perpendicular to the vectors $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$ is (are) : (A) one (B) two (C) three (D) infinite	
Ans	**(B) two**	**1**
16.	Of all the points of the feasible region, for maximum or minimum values of the objective function, the point lies : (A) inside the feasible region (B) at the boundary line of the feasible region (C) at the corner points of the feasible region (D) at the coordinate axes	
Ans	**(C) at the corner points of the feasible region**	**1**
17.	The common region for the inequalities $x \geq 0$, $x + y \leq 1$ and $y \geq 0$, lies in (A) IV Quadrant (B) II Quadrant (C) III Quadrant (D) I Quadrant	
Ans	**(D) I Quadrant**	**1**
18.	A and B appeared for an interview for two vacancies. The probability of A's selection is $\frac{1}{5}$ and that of B's selection is $\frac{1}{3}$. The probability that none of them is selected is : (A) $\frac{11}{15}$ (B) $\frac{7}{15}$ (C) $\frac{8}{15}$ (D) $\frac{1}{5}$	
Ans	**(C)** $\frac{8}{15}$	**1**

	*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.* (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.	
19.	**Assertion (A) :** The vectors $\vec{a} = 4\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + 3\hat{j} - 5\hat{k}$ are mutually perpendicular vectors. **Reason (R) :** Two vectors $\vec{a}$ and $\vec{b}$ are perpendicular to each other, if $\vec{a} \cdot \vec{b} = 0$.	
Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**
20.	**Assertion (A) :** $x^2dy = (2xy + y^2)dx$ is a homogeneous differential equation. **Reason (R) :** A differential equation of the form $\frac{dy}{dx} = F\left(\frac{y}{x}\right)$ is a homogeneous differential equation.	
Ans	**(A) Both Assertion (A) and Reason (R) are true, and Reason (R) is the correct explanation of the Assertion (A).**	**1**
	SECTION-B ***This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.***	
21.	Evaluate : $\tan^{-1}(\sqrt{3}) - \sec^{-1}(-2)$	
Ans	$\boldsymbol{tan^{-1}\sqrt{3} - sec^{-1}(-2)}$ $= \frac{\pi}{3} - \left[\pi - \frac{\pi}{3}\right]$ $= \frac{-\pi}{3}$	**1** **1**

22.	(a) Show that the function $f(x) = (x-1)^{\frac{1}{3}}$ is not differentiable at $x = 1$. **OR** (b) Differentiate $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ w.r.t. x.	
Ans	**(a)** $f'(x) = \frac{1}{3}(x-1)^{\frac{-2}{3}}$ $f'(1)$ **is not defined** **OR** **(b)** $y = log\left(x + \sqrt{x^2 + a^2}\right)$ $\frac{dy}{dx} = \frac{1 + \frac{x}{\sqrt{x^2+a^2}}}{x + \sqrt{x^2+a^2}}$ $= \frac{1}{\sqrt{x^2+a^2}}$	**1** **1** **1** **1**
23.	If $y = 7x - x^3$ and x increases at the rate of 2 units per second, then how fast is the slope of the curve changing, when $x = 5$?	
Ans	$y = 7x - x^3$ $m = 7 - 3x^2$ $\frac{dm}{dt} = -6x\frac{dx}{dt}$ $\left[\frac{dm}{dt}\right]_{x=5} = -30(2) = -60$	**1** **1**
24.	(a) If $\lvert\vec{a} + \vec{b}\rvert = 60$, $\lvert\vec{a} - \vec{b}\rvert = 40$ and $\lvert\vec{b}\rvert = 46$, then find $\lvert\vec{a}\rvert$. **OR** (b) Using vectors, find the value of K such that the points (K, –11, 2), (0, –2, 2) and (2, 4, 2) are collinear.	
Ans	**(a)** $\lvert\vec{a} + \vec{b}\rvert^2 + \lvert\vec{a} - \vec{b}\rvert^2 = 2\left(\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2\right)$ $(60)^2 + (40)^2 = 2(\lvert\vec{a}\rvert^2 + (46)^2)$ $\frac{(3600 + 1600)}{2} = (\lvert\vec{a}\rvert^2 + (46)^2)$ $2600 - 2116 = \lvert\vec{a}\rvert^2$ $484 = \lvert\vec{a}\rvert^2$	**1** **$\frac{1}{2}$**

	$\|\vec{a}\| = 22$	$\frac{1}{2}$
	OR	
	(b)Let A, B, C be the points $(k, -11, 2), (0, -2, 2)$ *and* $(2, 4, 2)$***respectively***	
	$\overrightarrow{AB} = -k\hat{i} + 9\hat{j}$	1
	$\overrightarrow{BC} = 2\hat{i} + 6\hat{j}$	
	Since $\overrightarrow{AB}$ ***is parallel to*** $\overrightarrow{BC}$	$\frac{1}{2}$
	$\frac{-k}{2} = \frac{9}{6}$	
	$\Rightarrow k = -3$	$\frac{1}{2}$
25.	Find the angle between the two lines whose equations are $2x = 3y = -z$ and $6x = -y = -4z$.	
Ans	$2x = 3y = -z \quad \Rightarrow \frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$	$\frac{1}{2}$
	$6x = -y = -4z \Rightarrow \frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$	$\frac{1}{2}$
	$\cos\theta = \frac{3\times2+2\times(-12)-6\times(-3)}{\sqrt{3^2+2^2+(-6)^2}\sqrt{2^2+(-12)^2+(-3)^2}}$	$\frac{1}{2}$
	$\because\ 3\times2+2\times(-12)-6\times(-3)=0 \therefore \theta = \frac{\pi}{2}$	$\frac{1}{2}$
	SECTION-C ***This section comprises 6 Short Answer (SA) type questions of 3 marks each.***	
26.	Find the intervals in which the function $f(x) = 3x^4 - 4x^3 - 12x^2 + 5$ is (a) strictly increasing (b) strictly decreasing	
Ans	$f(x) = 3x^4 - 4x^3 - 12x^2 + 5$	
	$f'(x) = 12x^3 - 12x^2 - 24x$	
	$f'(x) = 12x(x-2)(x+1)$	1

	$f'(x) = 0 \Rightarrow x = -1, 0, 2$	$\frac{1}{2}$
	 **Sign of f(x)**	$\frac{1}{2}$
	f is strictly increasing in (-1,0) as well as (2, ∞) or [-1,0] as well as [2, ∞)	$\frac{1}{2}$
	and strictly decreasing in (-∞, -1) as well as (0,2) or (-∞, -1] as well as [0,2]	$\frac{1}{2}$
27.	Find : $\int \frac{x^2 - x + 1}{(x-1)(x^2+1)}\,dx$ **OR** Evaluate : $\int_1^4 (\|x\| + \|3-x\|)\,dx$	
Ans	**(a)** $\int \frac{x^2-x+1}{(x-1)(x^2+1)}\,dx$	
	Let $\frac{x^2-x+1}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$	$\frac{1}{2}$
	Getting $A = \frac{1}{2}, B = \frac{1}{2}, C = \frac{-1}{2}$	1
	$\frac{1}{2}\int \frac{1}{x-1}dx + \frac{1}{2}\int \frac{x}{x^2+1}dx - \frac{1}{2}\int \frac{1}{x^2+1}dx$	
	$= \frac{1}{2}log\|x-1\| + \frac{1}{4}log\|x^2+1\| - \frac{1}{2}tan^{-1}x + C$	$1\frac{1}{2}$
	OR **(b)** $\int_1^4 (\|x\| + \|3-x\|)\,dx$	
	$\int_1^4 \|x\|\,dx + \int_1^4 \|3-x\|\,dx$	
	$\int_1^4 x\,dx + \int_1^3 (3-x)\,dx - \int_3^4 (3-x)\,dx$	$1\frac{1}{2}$

	$= \frac{1}{2}[x^2]_1^4 + \left[3x - \frac{x^2}{2}\right]_1^3 - \left[3x - \frac{x^2}{2}\right]_3^4$ $= \frac{15}{2} + 2 + \frac{1}{2} = 10$	1 $\frac{1}{2}$
28.	(a) Find the particular solution of the differential equation, $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$, given that y = 1 when $x = 0$. **OR** (b) Solve the differential equation : $2xy\frac{dy}{dx} = x^2 + 3y^2$.	
Ans	**(a)** $\frac{dy}{dx} = 1 + x^2 + y^2 + x^2y^2$ $\frac{dy}{dx} = (1 + x^2)(1 + y^2)$ $\int \frac{dy}{1 + y^2} = \int (1 + x^2)\, dx$ $\Rightarrow tan^{-1}y = x + \frac{x^3}{3} + C$ **y=1 when x= 0** $\Rightarrow C = \frac{\pi}{4}$ **Particular solution is :** $tan^{-1}y = x + \frac{x^3}{3} + \frac{\pi}{4}$ **OR** **(b) The given differential equation can be written as** $\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$ **putting y= vx , we get** $v + x\frac{dv}{dx} = \frac{x^2 + 3v^2x^2}{2xvx}$ $\int \frac{2v}{1+v^2}\, dv = \int \frac{dx}{x}$ $\Rightarrow log(1 + v^2) = log\lvert x\rvert + logC$ $\Rightarrow 1 + v^2 = Cx$ $\Rightarrow 1 + \frac{y^2}{x^2} = Cx$ **or** $x^2 + y^2 = Cx^3$	$\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 1
29.	If $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j}$ and $\vec{c} = 3\hat{i} - 4\hat{j} - 5\hat{k}$, then find a unit vector perpendicular to both the vectors $(\vec{a} - \vec{b})$ and $(\vec{c} - \vec{b})$.	
Ans	let $\vec{p} = \vec{a} - \vec{b} = -\hat{i} + \hat{j} + \hat{k}$	$\frac{1}{2}$

	$\vec{q} = \vec{c} - \vec{b} = \hat{t} - 5\hat{j} - 5\hat{k}$	$\frac{1}{2}$
	Vector perpendicular to $\vec{p}$ and $\vec{q} = \vec{p} \times \vec{q} = \begin{vmatrix} \hat{t} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -5 & -5 \end{vmatrix} = -4\hat{j} + 4\hat{k}$	
	Unit Vector perpendicular to $\vec{p}$ and $\vec{q}$ $= \frac{-4\hat{j}+4\hat{k}}{\sqrt{4^2+4^2}}$	$1\frac{1}{2}$
	$= \frac{-\hat{j}+\hat{k}}{\sqrt{2}}$ **or** $\frac{-1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$	$\frac{1}{2}$
30.	The corner points of the feasible region determined by some system of linear inequations, are (0, 0), (5, 0), (3, 4) and (0, 5). Let Z = ax + by, where a, b > 0. Find the condition on a and b so that the maximum of Z occurs at both points (3, 4) and (0, 5).	
Ans	**Z= ax + by**	
	At (3 ,4), Z=3a+4b	**1**
	At (0 ,5), Z=5b	**1**
	3a+4b=0+5b	
	3a=b	**1**
31.	(a) Find the probability distribution of the number of doublets in three throws of a pair of dice. **OR** (b) If E and F are two independent events with P(E) = p, P(F) = 2p and P(exactly one of E, F) = $\frac{5}{9}$, then find the value of p.	
Ans	**(a) Let X denote the number of doublets**	
	X= 0, 1, 2, 3	$\frac{1}{2}$
	P(doublet) = $\frac{1}{6}$, P(not a doublet) = $\frac{5}{6}$	$\frac{1}{2}$

X	0	1	2	3
P(X)	$\left(\frac{5}{6}\right)^3 = \frac{125}{216}$	$3 \times \frac{1}{6} \times \left(\frac{5}{6}\right)^2 = \frac{75}{216}$	$3 \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right) = \frac{15}{216}$	$\left(\frac{1}{6}\right)^3 = \frac{1}{216}$

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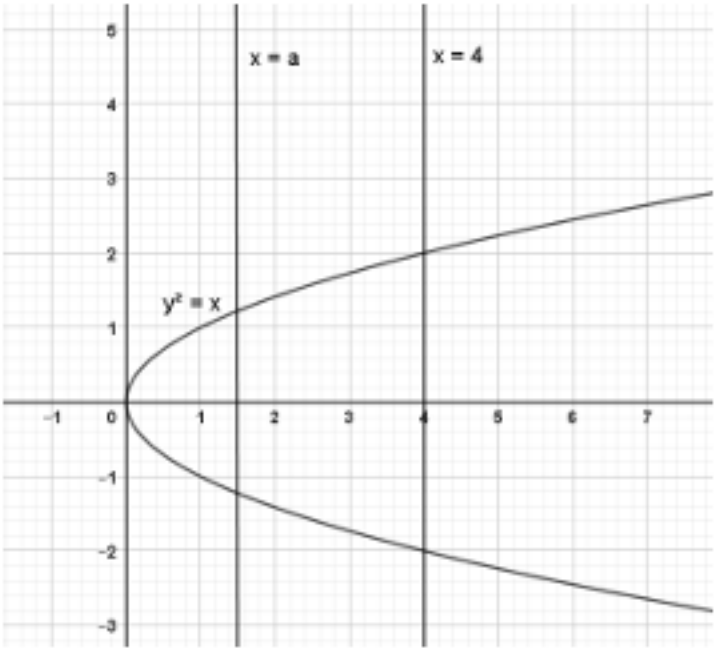
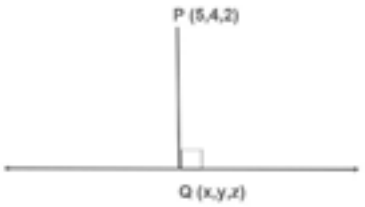
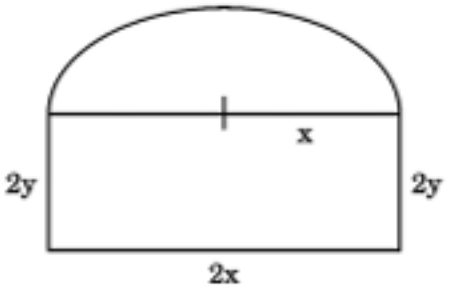
OR

	OR (b) $P(E \cup F) = P(E) + P(F) - P(E \cap F)$ $= p + 2p - p \times 2p$	1
	$P(E \cup F) = P(exactly\ one\ of E, F) + P(E \cap F)$ $= \frac{5}{9} + p \times 2p$ $\Rightarrow 3p - 2p^2 = \frac{5}{9} + 2p^2$ $\Rightarrow 36p^2 - 27p + 5 = 0$ $\Rightarrow (3p - 1)(12p - 5) = 0$ $\Rightarrow p = \frac{1}{3}\ or\ \frac{5}{12}$	1 1
	SECTION-D *This section comprises 4 Long Answer (LA) type questions of 5 marks each.*	
32.	If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find A^{-1}. Using A^{-1}, solve the system of equations : $2x - 3y + 5z = 11$ $3x + 2y - 4z = -5$ $x + y - 2z = -3$	
Ans	$adjA = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$ $\lvert A \rvert = 2 \times 0 - 3 \times 2 + 5 \times 1 = -6 + 5 = -1 \neq 0 \Rightarrow A^{-1}$ exists $A^{-1} = \frac{1}{\lvert A \rvert}(adjA) = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$ $\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$ $AX = B \Rightarrow X = A^{-1}B$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$	$1\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$

	$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix}$ $\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\Rightarrow$x=1, y=2, z=3	$1\frac{1}{2}$
33.	(a) Differentiate $x^{\sin x} + (\sin x)^x$ w.r.t. x. **OR** (b) If $y = x + \tan x$, then prove that $\cos^2 x \frac{d^2y}{dx^2} - 2y + 2x = 0$	
Ans	(a) $Let\ y = x^{sinx} + (sinx)^x = u + v \Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$ $u = x^{sinx}$ $\Rightarrow logu = sinx\ logx$ $\Rightarrow \frac{du}{dx} = x^{sinx}\left[\frac{sinx}{x} + cosx\ logx\right]$ $v = (sinx)^x$ $\Rightarrow logv = x\ log(sinx)$ $\Rightarrow \frac{dv}{dx} = (sinx)^x[xcotx + log(sinx)]$ $\Rightarrow \frac{dy}{dx} = x^{sinx}\left[\frac{sinx}{x} + cosx\ logx\right] + (sinx)^x[xcotx + log(sinx)]$ **OR** (b) $y = x + tanx$ $\Rightarrow \frac{dy}{dx} = 1 + sec^2x$ $\Rightarrow \frac{d^2y}{dx^2} = 2sec^2xtanx$ $\Rightarrow cos^2x\frac{d^2y}{dx^2} = 2tanx$ $\Rightarrow cos^2x\frac{d^2y}{dx^2} = 2x + 2tanx - 2x$ $\Rightarrow cos^2x\frac{d^2y}{dx^2} = 2y - 2x$ $\Rightarrow cos^2x\frac{d^2y}{dx^2} - 2y + 2x = 0$	1 2 $1\frac{1}{2}$ $\frac{1}{2}$ 2 1 1 1

34.	The region enclosed between $x = y^2$ and $x = 4$ is divided into two equal parts by the line $x = a$. Find the value of a.	
Ans		
	$2\int_0^a \sqrt{x}\,dx = \int_0^4 \sqrt{x}\,dx$	**2**
	$\Rightarrow 2 \times \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^a = \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^4$	**2**
	$\Rightarrow \frac{4}{3}\left[a^{\frac{3}{2}}\right] = \frac{2}{3}[8]$	
	$\Rightarrow a^{\frac{3}{2}} = 4$	
	$\Rightarrow a = 4^{\frac{2}{3}}$	**1**
35.	(a) Find the shortest distance between the lines given by $\vec{r} = (4\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} + 2\hat{j} - 4\hat{k})$ **OR** (b) Find the coordinates of the foot of the perpendicular and the length of the perpendicular drawn from the point P(5, 4, 2) to the line $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$.	

Ans	**(a)** $\overrightarrow{a_1} = 4\hat{t} - \hat{f} + 2\hat{k} \quad \overrightarrow{b_1} = \hat{t} + 2\hat{f} - 3\hat{k}$ $\overrightarrow{a_2} = 2\hat{t} + \hat{f} - \hat{k} \quad \overrightarrow{b_2} = 3\hat{t} + 2\hat{f} - 4\hat{k}$	**1**
	$\overrightarrow{a_2} - \overrightarrow{a_1} = -2\hat{t} + 2\hat{f} - 3\hat{k}$	$\frac{1}{2}$
	$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{t} & \hat{f} & \hat{k} \\ 1 & 2 & -3 \\ 3 & 2 & -4 \end{vmatrix} = -2\hat{t} - 5\hat{f} - 4\hat{k}$	**2**
	$\left\|\overrightarrow{b_1} \times \overrightarrow{b_2}\right\| = \sqrt{4 + 25 + 16} = \sqrt{45} = 3\sqrt{5}$	$\frac{1}{2}$
	$(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2}) = 4 - 10 + 12 = 6$	$\frac{1}{2}$
	$SD = \left\|\frac{(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2})}{\left\|\overrightarrow{b_1} \times \overrightarrow{b_2}\right\|}\right\| = \frac{6}{3\sqrt{5}} = \frac{2}{\sqrt{5}}$	$\frac{1}{2}$
	OR	
	(b)  	
	Let coordinates of Q be $(2\lambda - 1, 3\lambda + 3, 1 - \lambda)$	**1**
	D. Ratios of PQ $\langle 2\lambda - 6, 3\lambda - 1, -1 - \lambda \rangle$	**1**
	PQ $\perp$ **line** $\Rightarrow 2(2\lambda - 6) + 3(3\lambda - 1) - (-1 - \lambda) = 0$ $\Rightarrow \lambda = 1$	$1\frac{1}{2}$
	Foot of perpendicular is Q (1, 6, 0)	**1**
	Length of perpendicular $= \sqrt{(4)^2 + (-2)^2 + (2)^2}$ $= \sqrt{16 + 4 + 4} = \sqrt{24} = 2\sqrt{6}$	$\frac{1}{2}$
	SECTION-E ***This section comprises 3 case study-based questions of 4 marks each***	

36.	An architect designs a building for a Company. The design of window on the ground floor is proposed to be different than at the other floors. The window is in the shape of a rectangle whose top length is surmounted by a semi-circular opening. This window has a perimeter of 10 m. Based on the above information, answer the following : (i) If $2x$ and $2y$ represent the length and breadth of the rectangular portion of the window, then establish a relation between x and y. (ii) Find the total area of the window in terms of x. (iii) (a) Find the values of x and y for the maximum area of the window. **OR** (iii) (b) If x and y represent the length and breadth of the rectangle, then establish the expression for the area of the window in terms of x only.	
Ans	**(i)** $2x + 4y + \pi x = 10$  **(ii)** $A = (2x)(2y) + \frac{1}{2}\pi x^2$ $y = \frac{10 - 2x - \pi x}{4}$ $A = 4x\frac{(10 - 2x - \pi x)}{4} + \frac{1}{2}\pi x^2$ $A = 10x - 2x^2 - \frac{1}{2}\pi x^2$ **(iii) (a)** $\frac{dA}{dx} = 10 - 4x - \pi x = 0$ $\Rightarrow 10 = (\pi + 4)x$ $\Rightarrow x = \frac{10}{\pi + 4}$ $\frac{d^2y}{dx^2} = -4 - \pi < 0$ $\Rightarrow y = \frac{1}{4}\left[10 - 2\left(\frac{10}{\pi + 4}\right) - \pi\left(\frac{10}{\pi + 4}\right)\right]$ $\Rightarrow y = \frac{1}{4}\frac{[10\pi + 40 - 20 - 10\pi]}{\pi + 4}$ $y = \frac{1}{4}\left[\frac{20}{\pi + 4}\right] = \frac{5}{\pi + 4}$	**1** $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

	(iii) (b) $A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2$	1
	$x + 2y + \pi\left(\frac{x}{2}\right) = 10$	
	$\Rightarrow y = \frac{20 - 2x - \pi x}{4}$	$\frac{1}{2}$
	$\therefore A = x\left(\frac{20 - 2x - \pi x}{4}\right) + \frac{1}{8}\pi x^2$	
	$A = 5x - \frac{1}{2}x^2 - \frac{\pi}{8}x^2$	$\frac{1}{2}$
37.	There are three categories of students in a class of 60 students : A : Very hardworking students, B : Regular but not so hard working, C : Careless and irregular students. It is known that 6 students in category A, 26 in category B and the rest in category C. It is also found that probability of students of category A, unable to get good marks in the final year examination, is 0.002, of category B it is 0.02 and of category C, this probability is 0.20. Based on the above information, answer the following : (i) Find the probability that a student selected at random is unable to get good marks in the final examination. (ii) A student selected at random was found to be one who could not get good marks in the final examination. Find the probability, that this student is NOT of category A.	
Ans	**Let G be an event that student is not able to get good marks** **We have** $P(A) = \frac{6}{60}$ $\qquad$ $P(G\|A) = 0.002$	

	$P(B) = \frac{26}{60}$ $\quad P(G\|B) = 0.02$ $P(C) = \frac{28}{60}$ $\quad P(G\|C) = 0.20$ (i) $P(G) = P(A)P(G\|A) + P(B)P(G\|B) + P(C)P(G\|C)$ $= \frac{6}{60} \times 0.002 + \frac{26}{60} \times 0.02 + \frac{28}{60} \times 0.2$ $= \frac{511}{5000}$ (ii) $1 - P(A\|G)$ $= 1 - \frac{P(A)P(G\|A)}{P(A)P(G\|A)+P(B)P(G\|B)+P(C)P(G\|C)}$ $= 1 - \frac{\frac{1}{10}\times 0.002}{\frac{511}{5000}}$ $= 1 - \frac{1}{511}$ $= \frac{510}{511}$	$1\frac{1}{2}$ $\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$
38.	Rajesh, a student of Class-XII, visited an exhibition with his family. There he saw a huge swing and found that it traced the path of a parabola $y = x^2$. The following questions came to his mind. Answer the questions : (i) Let f : R → R be a function defined as $f(x) = x^2$. Find whether f is one-one function. (ii) Let f : R → R be defined as $f(x) = x^2$. Find whether f is an onto function. (iii) (a) Let f : N → N be defined as $f(x) = x^2$. Find whether f is one-one function. Also, find if it is an onto function. **OR** (iii) (b) Let f : N → {1, 4, 9, 16, ...} defined as $f(x) = x^2$, find where f is one-one function. Also, find if it is an onto function.	
Ans	(i) $f: R \to R$ $f(x) = x^2$ For $x_1 = -1$ *and* $x_2 = 1$ *we have* $f(x_1) = f(x_2) = 1$ Hence f is not a one -one function (ii) $f: R \to R$ $f(x) = x^2$	1

	$for, y = -4 \in R(Codomain)$ there does not exist any $x \in R$ such that f(x) = -4 So f is not onto	1
	(iii) (a) $f: N \to N$ $f(x) = x^2$ $Let, f(x_1) = f(x_2)\ for\ some\ x_1, x_2 \in N$ $\Rightarrow {x_1}^2 = {x_2}^2$ $\Rightarrow x_1 = x_2$ Hence , f is one- one $For,\ y = 5 \in N\ (codomain)$ there does not exist any $x(domain)$ such that f(x)=5 So f is not onto	1 1
	OR	
	(iii)(b) $f: N \to \{1, 4, 9, 16 \ldots\}$ $f(x) = x^2$ Let $f(x_1) = f(x_2)$ $\Rightarrow {x_1}^2 = {x_2}^2$ $\Rightarrow x_1 = x_2$ Hence f is one- one $\forall y \in \{1, 4, 9, 16, \ldots\}$ there exist $x \in N$ such that f(x)=y $\therefore$ f is onto	1 1