



प्रश्न-पत्र कोड  
Q.P. Code

**65/7/3**

रोल नं.

Roll No.



परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें।

Candidates must write the Q.P. Code on the title page of the answer-book.



## गणित

### MATHEMATICS



निर्धारित समय : 3 घण्टे

Time allowed : 3 hours

अधिकतम अंक : 80

Maximum Marks : 80

**नोट**

- (I) कृपया जाँच कर लें कि इस प्रश्न-पत्र में मुद्रित पृष्ठ 23 हैं।
- (II) प्रश्न-पत्र में दाहिने हाथ की ओर दिए गए प्रश्न-पत्र कोड को परीक्षार्थी उत्तर-पुस्तिका के मुख-पृष्ठ पर लिखें।
- (III) कृपया जाँच कर लें कि इस प्रश्न-पत्र में 38 प्रश्न हैं।
- (IV) कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर-पुस्तिका में यथा स्थान पर प्रश्न का क्रमांक अवश्य लिखें।
- (V) इस प्रश्न-पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है। प्रश्न-पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा। 10.15 बजे से 10.30 बजे तक परीक्षार्थी केवल प्रश्न-पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर-पुस्तिका पर कोई उत्तर नहीं लिखेंगे।

**NOTE**

- (I) Please check that this question paper contains 23 printed pages.
- (II) Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.
- (III) Please check that this question paper contains 38 questions.
- (IV) Please write down the Serial Number of the question in the answer-book at the given place before attempting it.
- (V) 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the candidates will read the question paper only and will not write any answer on the answer-book during this period. #



### ***General Instructions :***

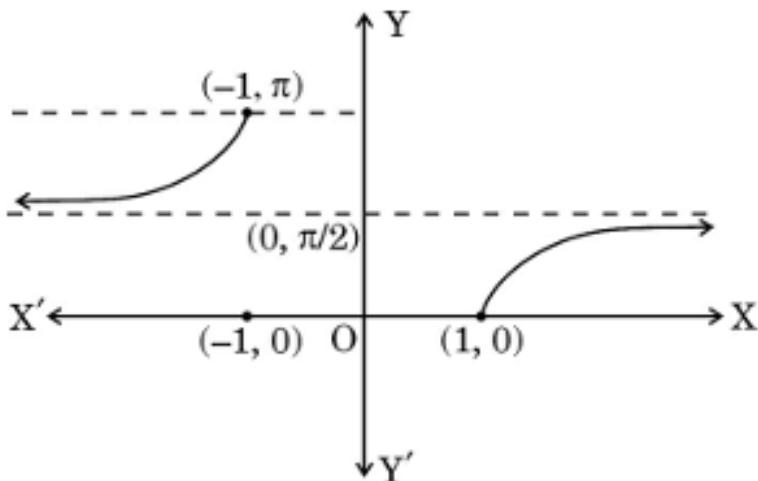
*Read the following instructions very carefully and strictly follow them :*

- (i) This question paper contains 38 questions. All questions are compulsory.
  - (ii) This question paper is divided into five Sections – A, B, C, D and E.
  - (iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
  - (iv) In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
  - (v) In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.
  - (vi) In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.
  - (vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
  - (viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.
  - (ix) Use of calculator is not allowed.

## **SECTION A**

*This section comprises multiple choice questions (MCQs) of 1 mark each.*

- 1.** The given graph illustrates :



- (A)  $y = \sec^{-1} x$       (B)  $y = \cot^{-1} x$   
(C)  $y = \tan^{-1} x$       (D)  $y = \operatorname{cosec}^{-1} x$

- 2.** Let A be a square matrix of order 3. If  $|A| = 5$ , then  $|\text{adj } A|$  is :



3. If A and B are two square matrices each of order 3 with  $|A| = 3$  and  $|B| = 5$ , then  $|2AB|$  is :

(A) 30 (B) 120  
 (C) 15 (D) 225

4. What is the total number of possible matrices of order  $3 \times 3$  with each entry as  $\sqrt{2}$  or  $\sqrt{3}$  ?

(A) 9 (B) 512  
 (C) 615 (D) 64

5. Domain of  $f(x) = \cos^{-1} x + \sin x$  is :

(A) R (B)  $(-1, 1)$   
 (C)  $[-1, 1]$  (D)  $\emptyset$

6. The matrix  $A = \begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{5} \end{bmatrix}$  is a/an :

(A) scalar matrix (B) identity matrix  
 (C) null matrix (D) symmetric matrix

7. If  $f(x) = -2x^8$ , then the correct statement is :

(A)  $f'\left(\frac{1}{2}\right) = f'\left(-\frac{1}{2}\right)$  (B)  $f'\left(\frac{1}{2}\right) = -f'\left(-\frac{1}{2}\right)$   
 (C)  $-f'\left(\frac{1}{2}\right) = f\left(-\frac{1}{2}\right)$  (D)  $f\left(\frac{1}{2}\right) = -f\left(-\frac{1}{2}\right)$

8. If  $f(x) = \begin{cases} 3ax - b & , x > 1 \\ 11 & , x = 1 \\ -5ax - 2b & , x < 1 \end{cases}$





14.  $\int \frac{e^{-x}}{16 + 9e^{-2x}} dx$  is equal to :

- (A)  $\frac{16}{9} \tan^{-1}(e^{-x}) + C$       (B)  $-\frac{1}{12} \tan^{-1}\left(\frac{3e^{-x}}{4}\right) + C$   
(C)  $\tan^{-1}\left(\frac{e^{-x}}{4}\right) + C$       (D)  $-\frac{1}{3} \tan^{-1}\left(\frac{e^{-x}}{4}\right) + C$

15. If  $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$  for any two vectors, then vectors  $\vec{a}$  and  $\vec{b}$  are :

- (A) orthogonal vectors      (B) parallel to each other  
(C) unit vectors      (D) collinear vectors

16. A coin is tossed and a card is selected at random from a well shuffled pack of 52 playing cards. The probability of getting head on the coin and a face card from the pack is :

- (A)  $\frac{2}{13}$       (B)  $\frac{3}{26}$   
(C)  $\frac{19}{26}$       (D)  $\frac{3}{13}$

17. If A and B are two events such that  $P(B) = \frac{1}{5}$ ,  $P(A | B) = \frac{2}{3}$  and  $P(A \cup B) = \frac{3}{5}$ , then  $P(A)$  is :

- (A)  $\frac{10}{15}$       (B)  $\frac{2}{15}$   
(C)  $\frac{1}{5}$       (D)  $\frac{8}{15}$



18. For a function  $f(x)$ , which of the following holds true ?

- (A)  $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$
- (B)  $\int_{-a}^a f(x) dx = 0$ , if  $f$  is an even function
- (C)  $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$ , if  $f$  is an odd function
- (D)  $\int_0^{2a} f(x) dx = \int_0^a f(x) dx - \int_0^a f(2a+x) dx$

Questions number 19 and 20 are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.

- (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
- (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A).
- (C) Assertion (A) is true, but Reason (R) is false.
- (D) Assertion (A) is false, but Reason (R) is true.

19. Assertion (A) :  $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$  is continuous at  $x = 0$ .

Reason (R) : When  $x \rightarrow 0$ ,  $\sin \frac{1}{x}$  is a finite value between  $-1$  and  $1$ .

20. Assertion (A) : Set of values of  $\sec^{-1} \left( \frac{\sqrt{3}}{2} \right)$  is a null set.

Reason (R) :  $\sec^{-1} x$  is defined for  $x \in \mathbb{R} - (-1, 1)$ .



## SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21. (a) Differentiate  $\left(\frac{5^x}{x^5}\right)$  with respect to x.

**OR**

- (b) If  $-2x^2 - 5xy + y^3 = 76$ , then find  $\frac{dy}{dx}$ .

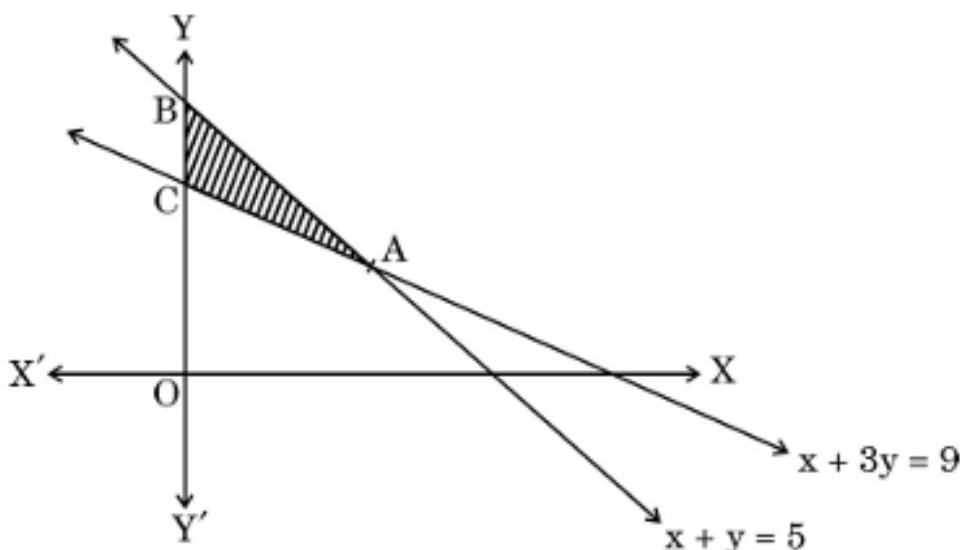
22. If  $A = \begin{bmatrix} 1 & 0 \\ -1 & 5 \end{bmatrix}$ , then find the value of K if  $A^2 = 6A + KI_2$ , where  $I_2$  is an identity matrix.

23. (a) 10 identical blocks are marked with '0' on two of them, '1' on three of them, '2' on four of them and '3' on one of them and put in a box. If X denotes the number written on the block, then write the probability distribution of X and calculate its mean.

**OR**

- (b) In a village of 8000 people, 3000 go out of the village to work and 4000 are women. It is noted that 30% of women go out of the village to work. What is the probability that a randomly chosen individual is either a woman or a person working outside the village?

24. For a Linear Programming Problem, find  $\min Z = 5x + 3y$  (where Z is the objective function) for the feasible region shaded in the given figure.



(Note : The figure is not to scale)



25. Let  $f : A \rightarrow B$  be defined by  $f(x) = \frac{x-2}{x-3}$ , where  $A = \mathbb{R} - \{3\}$  and  $B = \mathbb{R} - \{1\}$ .

Discuss the bijectivity of the function.

## SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26. In the Linear Programming Problem for objective function  $Z = 18x + 10y$  subject to constraints

$$4x + y \geq 20$$

$$2x + 3y \geq 30$$

$$x, y \geq 0$$

find the minimum value of  $Z$ .

27. (a) The scalar product of the vector  $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$  with a unit vector along sum of vectors  $\vec{b} = 2\hat{i} - 4\hat{j} + 5\hat{k}$  and  $\vec{c} = \lambda\hat{i} - 2\hat{j} - 3\hat{k}$  is equal to 1. Find the value of  $\lambda$ .

OR

- (b) Find the shortest distance between the lines :

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} - 2\hat{j} + 3\hat{k})$$

$$\vec{r} = (\hat{i} + 4\hat{k}) + \mu(3\hat{i} - 6\hat{j} + 9\hat{k}).$$

28. Differentiate  $\log(x^x + \operatorname{cosec}^2 x)$  with respect to  $x$ .

29. Show that of all the rectangles with a fixed perimeter, the square has the greatest area.

30. (a) Show that the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 4x^3 - 5$ ,  $\forall x \in \mathbb{R}$  is one-one and onto.

OR

- (b) Let  $R$  be a relation defined on a set  $N$  of natural numbers such that  $R = \{(x, y) : xy \text{ is a square of a natural number, } x, y \in N\}$ . Determine if the relation  $R$  is an equivalence relation.



- 31.** (a) Let  $2x + 5y - 1 = 0$  and  $3x + 2y - 7 = 0$  represent the equations of two lines on which the ants are moving on the ground. Using matrix method, find a point common to the paths of the ants.

**OR**

- (b) A shopkeeper sells 50 Chemistry, 60 Physics and 35 Maths books on day I and sells 40 Chemistry, 45 Physics and 50 Maths books on day II. If the selling price for each such subject book is ₹ 150 (Chemistry), ₹ 175 (Physics) and ₹ 180 (Maths), then find his total sale in two days, using matrix method. If cost price of all the books together is ₹ 35,000, what profit did he earn after the sale of two days?

### **SECTION D**

*This section comprises 4 Long Answer (LA) type questions of 5 marks each.*

- 32.** (a) Find :

$$\int \frac{3x+1}{(x-2)^2(x+2)} dx$$

**OR**

- (b) Evaluate :

$$\int_0^{\pi/2} \frac{x}{\cos x + \sin x} dx$$

- 33.** (a) Find the point Q on the line  $\frac{2x+4}{6} = \frac{y+1}{2} = \frac{-2z+6}{-4}$  at a distance of  $3\sqrt{2}$  from the point P(1, 2, 3).

**OR**

- (b) Find the image of the point (-1, 5, 2) in the line  $\frac{2x-4}{2} = \frac{y}{2} = \frac{2-z}{3}$ . Find the length of the line segment joining the points (given point and the image point).



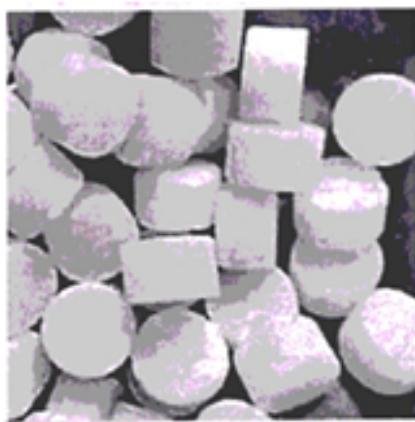
34. Solve the differential equation  $(x - \sin y) dy + (\tan y) dx = 0$ , given  $y(0) = 0$ .
35. A woman discovered a scratch along a straight line on a circular table top of radius 8 cm. She divided the table top into 4 equal quadrants and discovered the scratch passing through the origin inclined at an angle  $\frac{\pi}{4}$  anticlockwise along the positive direction of x-axis. Find the area of the region enclosed by the x-axis, the scratch and the circular table top in the first quadrant, using integration.

## SECTION E

*This section comprises 3 case study based questions of 4 marks each.*

### Case Study – 1

36. Camphor is a waxy, colourless solid with strong aroma that evaporates through the process of sublimation, if left in the open at room temperature.



(Cylindrical-shaped Camphor tablets)

A cylindrical camphor tablet whose height is equal to its radius ( $r$ ) evaporates when exposed to air such that the rate of reduction of its volume is proportional to its total surface area. Thus,  $\frac{dV}{dt} = kS$  is the differential equation, where  $V$  is the volume,  $S$  is the surface area and  $t$  is the time in hours.



Based upon the above information, answer the following questions :

- (i) Write the order and degree of the given differential equation. 1
- (ii) Substituting  $V = \pi r^3$  and  $S = 2\pi r^2$ , we get the differential equation  $\frac{dr}{dt} = \frac{2}{3}k$ . Solve it, given that  $r(0) = 5$  mm. 1
- (iii) (a) If it is given that  $r = 3$  mm when  $t = 1$  hour, find the value of  $k$ . Hence, find  $t$  for  $r = 0$  mm. 2

**OR**

- (iii) (b) If it is given that  $r = 1$  mm when  $t = 1$  hour, find the value of  $k$ . Hence, find  $t$  for  $r = 0$  mm. 2

**Case Study – 2**

- 37.** Based upon the results of regular medical check-ups in a hospital, it was found that out of 1000 people, 700 were very healthy, 200 maintained average health and 100 had a poor health record.

Let  $A_1$  : People with good health,  
 $A_2$  : People with average health,  
and  $A_3$  : People with poor health.

During a pandemic, the data expressed that the chances of people contracting the disease from category  $A_1$ ,  $A_2$  and  $A_3$  are 25%, 35% and 50%, respectively.

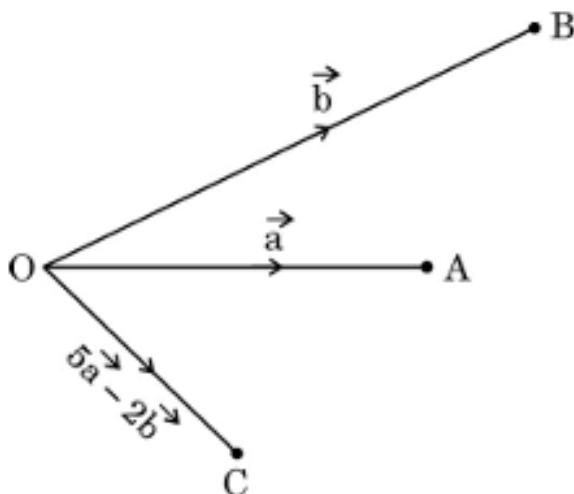
Based upon the above information, answer the following questions :

- (i) A person was tested randomly. What is the probability that he/she has contracted the disease ? 2
- (ii) Given that the person has not contracted the disease, what is the probability that the person is from category  $A_2$  ? 2



### Case Study – 3

38. Three friends A, B and C move out from the same location O at the same time in three different directions to reach their destinations. They move out on straight paths and decide that A and B after reaching their destinations will meet up with C at his predecided destination, following straight paths from A to C and B to C in such a way that  $\vec{OA} = \vec{a}$ ,  $\vec{OB} = \vec{b}$  and  $\vec{OC} = 5\vec{a} - 2\vec{b}$  respectively.



Based upon the above information, answer the following questions :

- Complete the given figure to explain their entire movement plan along the respective vectors. 1
  - Find vectors  $\vec{AC}$  and  $\vec{BC}$ . 1
  - (a) If  $\vec{a} \cdot \vec{b} = 1$ , distance of O to A is 1 km and that from O to B is 2 km, then find the angle between  $\vec{OA}$  and  $\vec{OB}$ . Also, find  $|\vec{a} \times \vec{b}|$ . 2
- OR**
- (b) If  $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$  and  $\vec{b} = \hat{j} - \hat{k}$ , then find a unit vector perpendicular to  $(\vec{a} + \vec{b})$  and  $(\vec{a} - \vec{b})$ . 2