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Series PQ3RS/3 | Set – 3

प्रश्न-पत्र कोड
Q.P. Code **65/3/3**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न–पत्र में **38** प्रश्न हैं ।*
- *प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए प्रश्न–पत्र कोड को परीक्षार्थी उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर–पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न–पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न–पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न–पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर–पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*
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General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The value of $\begin{vmatrix} 8 & 2 & 7 \\ 12 & 3 & 5 \\ 16 & 4 & 3 \end{vmatrix}$ is :

(A) 0 (B) 2

(C) 7 (D) – 2

2. If $y = \sin^{-1} x$, then $\dfrac{d^2y}{dx^2}$ is :

(A) sec y (B) sec y tan y

(C) $\sec^2 y \tan y$ (D) $\tan^2 y \sec y$

3. If $|\vec{a}| = 2$ and $-3 \le k \le 2$, then $|k\vec{a}| \in$:

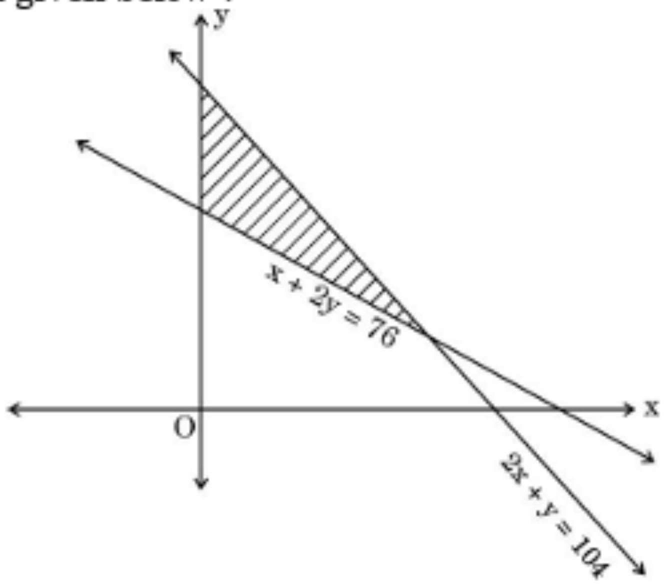
(A) [– 6, 4] (B) [0, 4]

(C) [4, 6] (D) [0, 6]

4. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of both x-axis and z-axis, then the angle which it makes with the positive direction of y-axis is :

(A) 0 (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) π

5. Of the following, which group of constraints represents the feasible region given below ?



(A) $x + 2y \leq 76, 2x + y \geq 104, x, y \geq 0$

(B) $x + 2y \leq 76, 2x + y \leq 104, x, y \geq 0$

(C) $x + 2y \geq 76, 2x + y \leq 104, x, y \geq 0$

(D) $x + 2y \geq 76, 2x + y \geq 104, x, y \geq 0$

6. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then A^{-1} is :

(A) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$ (B) $30\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

(C) $\frac{1}{30}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ (D) $\frac{1}{30}\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$

7. If $A = [a_{ij}]$ is an identity matrix, then which of the following is true ?

(A) $a_{ij} = \begin{cases} 0, & \text{if } i = j \\ 1, & \text{if } i \neq j \end{cases}$

(B) $a_{ij} = 1, \forall\ i, j$

(C) $a_{ij} = 0, \forall\ i, j$

(D) $a_{ij} = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}$

8. Let Z denote the set of integers, then function $f : Z \to Z$ defined as $f(x) = x^3 - 1$ is :

(A) both one-one and onto

(B) one-one but not onto

(C) onto but not one-one

(D) neither one-one nor onto

9. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a square matrix such that adj A = A. Then, $(a + b + c + d)$ is equal to :

(A) 2a

(B) 2b

(C) 2c

(D) 0

10. A function $f(x) = |1 - x + |x||$ is :

(A) discontinuous at $x = 1$ only

(B) discontinuous at $x = 0$ only

(C) discontinuous at $x = 0, 1$

(D) continuous everywhere

11. The rate of change of surface area of a sphere with respect to its radius 'r', when r = 4 cm, is :

(A) $64\pi\ cm^2/cm$

(B) $48\pi\ cm^2/cm$

(C) $32\pi\ cm^2/cm$

(D) $16\pi\ cm^2/cm$

12. $\int_{-a}^{a} f(x)\, dx = 0$, if :

(A) $f(-x) = f(x)$

(B) $f(-x) = -f(x)$

(C) $f(a - x) = f(x)$

(D) $f(a - x) = -f(x)$

13. $x \log x \frac{dy}{dx} + y = 2 \log x$ is an example of a :

(A) variable separable differential equation.

(B) homogeneous differential equation.

(C) first order linear differential equation.

(D) differential equation whose degree is not defined.

14. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$, then $\vec{a}$ and $\vec{b}$ are :

(A) collinear vectors which are not parallel

(B) parallel vectors

(C) perpendicular vectors

(D) unit vectors

15. If α, β and γ are the angles which a line makes with positive directions of x, y and z axes respectively, then which of the following is ***not*** true ?

(A) $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

(B) $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$

(C) $\cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$

(D) $\cos \alpha + \cos \beta + \cos \gamma = 1$

16. The restrictions imposed on decision variables involved in an objective function of a linear programming problem are called :

(A) feasible solutions (B) constraints

(C) optimal solutions (D) infeasible solutions

17. Let E and F be two events such that $P(E) = 0{\cdot}1$, $P(F) = 0{\cdot}3$, $P(E \cup F) = 0{\cdot}4$, then $P(F \mid E)$ is :

(A) 0·6 (B) 0·4 (C) 0·5 (D) 0

18. If A and B are two skew symmetric matrices, then (AB + BA) is :

(A) a skew symmetric matrix (B) a symmetric matrix

(C) a null matrix (D) an identity matrix

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* For any non-zero unit vector $\vec{a}$, $\vec{a} \cdot (-\vec{a}) = (-\vec{a}) \cdot \vec{a} = -1$.

Reason (R) : Angle between $\vec{a}$ and $(-\vec{a})$ is $\frac{\pi}{2}$.

20. *Assertion (A) :* Every scalar matrix is a diagonal matrix.

Reason (R) : In a diagonal matrix, all the diagonal elements are 0.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors. If θ is the angle between $\vec{a}$ and $(2\vec{a} + 3\vec{b} + 6\vec{c})$, find the value of $\cos \theta$.

22. Evaluate :

$$\cot^2 \left\{\text{cosec}^{-1} 3\right\} + \sin^2 \left\{\cos^{-1}\left(\frac{1}{3}\right)\right\}$$

23. (a) If $x = e^{x/y}$, prove that $\frac{dy}{dx} = \frac{\log x - 1}{(\log x)^2}$

OR

(b) Check the differentiability of $f(x) = \begin{cases} x^2 + 1, & 0 \leq x < 1 \\ 3 - x, & 1 \leq x \leq 2 \end{cases}$ at $x = 1$.

24. (a) Evaluate :

$$\int_0^{\pi/2} \sin 2x \cos 3x \, dx$$

OR

(b) Given $\frac{d}{dx} F(x) = \frac{1}{\sqrt{2x-x^2}}$ and $F(1) = 0$, find $F(x)$.

25. Find the position vector of point C which divides the line segment joining points A and B having position vectors $\hat{i} + 2\hat{j} - \hat{k}$ and $-\hat{i} + \hat{j} + \hat{k}$ respectively in the ratio 4 : 1 externally. Further, find $|\overrightarrow{AB}| : |\overrightarrow{BC}|$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Solve the following linear programming problem graphically :

Maximize $z = x + y$

subject to constraints

$$2x + 5y \leq 100$$
$$8x + 5y \leq 200$$
$$x \geq 0, y \geq 0.$$

27. The chances of P, Q and R getting selected as CEO of a company are in the ratio 4 : 1 : 2 respectively. The probabilities for the company to increase its profits from the previous year under the new CEO, P, Q or R are 0·3, 0·8 and 0·5 respectively. If the company increased the profits from the previous year, find the probability that it is due to the appointment of R as CEO.

28. (a) If x cos (p + y) + cos p sin (p + y) = 0, prove that

$\cos p \dfrac{dy}{dx} = -\cos^2 (p + y)$, where p is a constant.

OR

(b) Find the value of a and b so that function f defined as :

$$f(x) = \begin{cases} \dfrac{x-2}{|x-2|} + a, & \text{if } x < 2 \\ a + b, & \text{if } x = 2 \\ \dfrac{x-2}{|x-2|} + b, & \text{if } x > 2 \end{cases}$$

is a continuous function.

29. (a) Find the intervals in which the function $f(x) = \dfrac{\log x}{x}$ is strictly increasing or strictly decreasing.

OR

(b) Find the absolute maximum and absolute minimum values of the function f given by $f(x) = \dfrac{x}{2} + \dfrac{2}{x}$, on the interval [1, 2].

30. Find :

$$\int \frac{\sqrt{x}}{(x+1)(x-1)} dx$$

31. (a) Find :

$$\int \frac{2+\sin 2x}{1+\cos 2x} e^x \, dx$$

OR

(b) Evaluate :

$$\int_0^{\pi/4} \frac{1}{\sin x + \cos x} dx$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. (a) Find the equation of the line passing through the point of intersection of the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ and $\frac{x-1}{0} = \frac{y}{-3} = \frac{z-7}{2}$ and perpendicular to these given lines.

OR

(b) Two vertices of the parallelogram ABCD are given as A(–1, 2, 1) and B(1, –2, 5). If the equation of the line passing through C and D is $\frac{x-4}{1} = \frac{y+7}{-2} = \frac{z-8}{2}$, then find the distance between sides AB and CD. Hence, find the area of parallelogram ABCD.

33. Let A = R – {3} and B = R – {a}. Find the value of 'a' such that the function $f : A \to B$ defined by $f(x) = \frac{x-2}{x-3}$ is onto. Also, check whether the given function is one-one or not.

34. (a) It is given that function $f(x) = x^4 - 62x^2 + ax + 9$ attains local maximum value at x = 1. Find the value of 'a', hence obtain all other points where the given function f(x) attains local maximum or local minimum values.

OR

(b) The perimeter of a rectangular metallic sheet is 300 cm. It is rolled along one of its sides to form a cylinder. Find the dimensions of the rectangular sheet so that volume of cylinder so formed is maximum.

35. Using integration, find the area of the region enclosed between the curve $y = \sqrt{4-x^2}$ and the lines x = –1, x = 1 and the x-axis.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. A scholarship is a sum of money provided to a student to help him or her pay for education. Some students are granted scholarships based on their academic achievements, while others are rewarded based on their financial needs.

Every year a school offers scholarships to girl children and meritorious achievers based on certain criteria. In the session 2022 – 23, the school offered monthly scholarship of ₹ 3,000 each to some girl students and ₹ 4,000 each to meritorious achievers in academics as well as sports.

In all, 50 students were given the scholarships and monthly expenditure incurred by the school on scholarships was ₹ 1,80,000.

Based on the above information, answer the following questions :

(i) Express the given information algebraically using matrices. *1*

(ii) Check whether the system of matrix equations so obtained is consistent or not. *1*

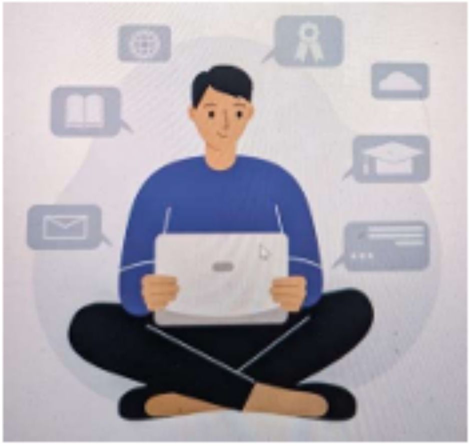
(iii) (a) Find the number of scholarships of each kind given by the school, using matrices. *2*

OR

(iii) (b) Had the amount of scholarship given to each girl child and meritorious student been interchanged, what would be the monthly expenditure incurred by the school ? *2*

Case Study – 2

37. Self-study helps students to build confidence in learning. It boosts the self-esteem of the learners. Recent surveys suggested that close to 50% learners were self-taught using internet resources and upskilled themselves.



A student may spend 1 hour to 6 hours in a day in upskilling self. The probability distribution of the number of hours spent by a student is given below :

$$P(X = x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3 \\ 2kx, & \text{for } x = 4, 5, 6 \\ 0, & \text{otherwise} \end{cases}$$

where x denotes the number of hours.

Based on the above information, answer the following questions :

(i) Express the probability distribution given above in the form of a probability distribution table. *1*

(ii) Find the value of k. *1*

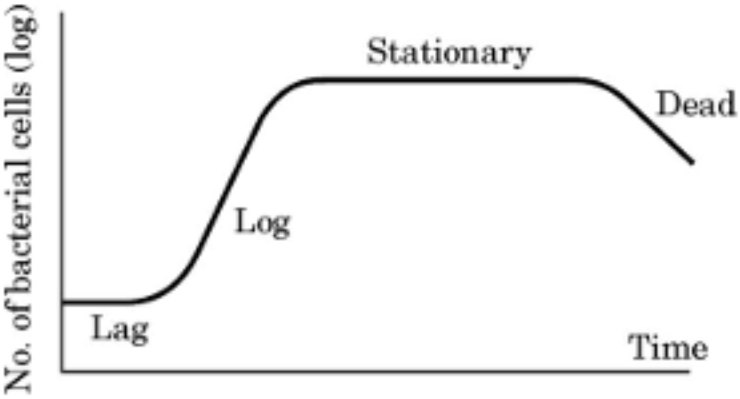
(iii) (a) Find the mean number of hours spent by the student. *2*

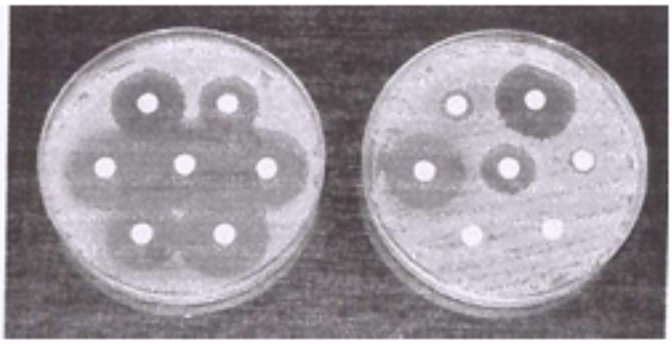
OR

(iii) (b) Find $P(1 < X < 6)$. *2*

Case Study – 3

38. A bacteria sample of certain number of bacteria is observed to grow exponentially in a given amount of time. Using exponential growth model, the rate of growth of this sample of bacteria is calculated.





The differential equation representing the growth of bacteria is given as :

$\frac{dP}{dt} = kP$, where P is the population of bacteria at any time 't'.

Based on the above information, answer the following questions :

(i) Obtain the general solution of the given differential equation and express it as an exponential function of 't'. *2*

(ii) If population of bacteria is 1000 at $t = 0$, and 2000 at $t = 1$, find the value of k. *2*