Series PQ1RS/1

Set – 3

प्रश्न-पत्र कोड
Q.P. Code **65/1/3**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ* ***23*** *हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न–पत्र में* ***38*** *प्रश्न हैं ।*
- *प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए प्रश्न–पत्र कोड को परीक्षार्थी उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर–पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न–पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न–पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न–पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर–पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains* ***23*** *printed pages.*
- *Please check that this question paper contains* ***38*** *questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे | *अधिकतम अंक : 80*

Time allowed : 3 hours | *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory****.*

(ii) *This question paper is divided into* ***five*** *Sections –* ***A, B, C, D*** *and* ***E****.*

(iii) *In* ***Section A,*** *Questions no.* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and questions number* ***19*** *and* ***20*** *are Assertion-Reason based questions of* ***1*** *mark each.*

(iv) *In* ***Section B,*** *Questions no.* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*

(v) *In* ***Section C,*** *Questions no.* ***26*** *to* ***31*** *are short answer (SA) type questions, carrying* ***3*** *marks each.*

(vi) *In* ***Section D,*** *Questions no.* ***32*** *to* ***35*** *are long answer (LA) type questions carrying* ***5*** *marks each.*

(vii) *In* ***Section E,*** *Questions no.* ***36*** *to* ***38*** *are case study based questions carrying* ***4*** *marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is* ***not*** *allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $x = at,\ y = \frac{a}{t}$, then $\frac{dy}{dx}$ is :

(A) t^2 (B) $-t^2$

(C) $\frac{1}{t^2}$ (D) $-\frac{1}{t^2}$

2. The solution of the differential equation $\frac{dy}{dx} = \frac{1}{\log y}$ is :

(A) $\log y = x + c$ (B) $y \log y - y = x + c$

(C) $\log y - y = x + c$ (D) $y \log y + y = x + c$

3. The vector with terminal point A (2, – 3, 5) and initial point B (3, – 4, 7) is :

(A) $\hat{i} - \hat{j} + 2\hat{k}$ (B) $\hat{i} + \hat{j} + 2\hat{k}$

(C) $-\hat{i} - \hat{j} - 2\hat{k}$ (D) $-\hat{i} + \hat{j} - 2\hat{k}$

4. The distance of point P(a, b, c) from y-axis is :

(A) b (B) b^2

(C) $\sqrt{a^2 + c^2}$ (D) $a^2 + c^2$

5. The number of corner points of the feasible region determined by constraints $x \geq 0, y \geq 0, x + y \geq 4$ is :

(A) 0 (B) 1

(C) 2 (D) 3

6. If matrices A and B are of order 1×3 and 3×1 respectively, then the order of $A'B'$ is :

(A) 1×1 (B) 3×1

(C) 1×3 (D) 3×3

7. A relation R defined on a set of human beings as

$$R = \{(x, y) : x \text{ is 5 cm shorter than } y\}$$

is :

(A) reflexive only

(B) reflexive and transitive

(C) symmetric and transitive

(D) neither transitive, nor symmetric, nor reflexive

8. If a matrix has 36 elements, the number of possible orders it can have, is :

(A) 13 (B) 3

(C) 5 (D) 9

9. Which of the following statements is true for the function $f(x) = \begin{cases} x^2 + 3, & x \neq 0 \\ 1 \quad, & x = 0 \end{cases}$?

(A) f(x) is continuous and differentiable $\forall x \in \mathbb{R}$

(B) f(x) is continuous $\forall x \in \mathbb{R}$

(C) f(x) is continuous and differentiable $\forall x \in \mathbb{R} - \{0\}$

(D) f(x) is discontinuous at infinitely many points

10. Let f(x) be a continuous function on [a, b] and differentiable on (a, b). Then, this function f(x) is strictly increasing in (a, b) if

(A) $f'(x) < 0, \forall x \in (a, b)$

(B) $f'(x) > 0, \forall x \in (a, b)$

(C) $f'(x) = 0, \forall x \in (a, b)$

(D) $f(x) > 0, \forall x \in (a, b)$

11. If $\begin{bmatrix} x+y & 2 \\ 5 & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then the value of $\left(\frac{24}{x} + \frac{24}{y}\right)$ is :

(A) 7 (B) 6

(C) 8 (D) 18

12. If f(x) is an odd function, then $\int_{-\pi/2}^{\pi/2} f(x) \cos^3 x \, dx$ equals :

(A) $2\int_{0}^{\pi/2} f(x) \cos^3 x \, dx$ (B) 0

(C) $2\int_{0}^{\pi/2} f(x) \, dx$ (D) $2\int_{0}^{\pi/2} \cos^3 x \, dx$

13. Let θ be the angle between two unit vectors $\hat{a}$ and $\hat{b}$ such that $\sin \theta = \frac{3}{5}$. Then, $\hat{a} \cdot \hat{b}$ is equal to :

(A) $\pm \frac{3}{5}$ (B) $\pm \frac{3}{4}$

(C) $\pm \frac{4}{5}$ (D) $\pm \frac{4}{3}$

14. The integrating factor of the differential equation $(1 - x^2) \frac{dy}{dx} + xy = ax$, $-1 < x < 1$, is :

(A) $\frac{1}{x^2 - 1}$ (B) $\frac{1}{\sqrt{x^2 - 1}}$

(C) $\frac{1}{1 - x^2}$ (D) $\frac{1}{\sqrt{1 - x^2}}$

15. If the direction cosines of a line are $\sqrt{3}\,k$, $\sqrt{3}\,k$, $\sqrt{3}\,k$, then the value of k is :

(A) ± 1 (B) $\pm \sqrt{3}$

(C) ± 3 (D) $\pm \frac{1}{3}$

16. A linear programming problem deals with the optimization of a/an :

(A) logarithmic function (B) linear function

(C) quadratic function (D) exponential function

17. If $P(A|B) = P(A'|B)$, then which of the following statements is true ?

(A) $P(A) = P(A')$ (B) $P(A) = 2\,P(B)$

(C) $P(A \cap B) = \frac{1}{2}\,P(B)$ (D) $P(A \cap B) = 2\,P(B)$

18. $\begin{vmatrix} x+1 & x-1 \\ x^2+x+1 & x^2-x+1 \end{vmatrix}$ is equal to :

(A) $2x^3$ (B) 2

(C) 0 (D) $2x^3 - 2$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* For matrix $A = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$, where $\theta \in [0, 2\pi]$, $|A| \in [2, 4]$.

Reason (R) : $\cos\theta \in [-1, 1], \forall\, \theta \in [0, 2\pi]$.

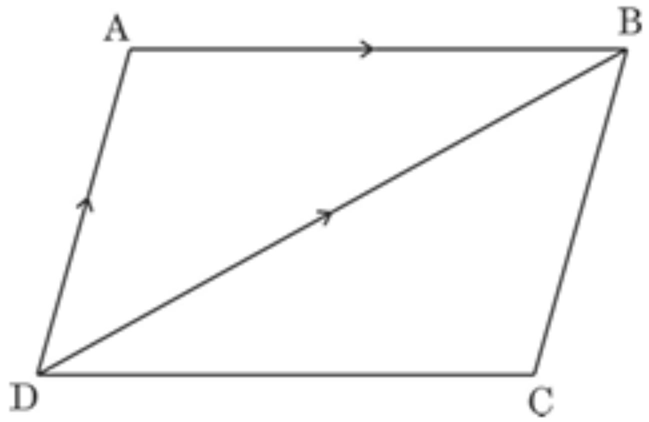
20. *Assertion (A) :* A line in space cannot be drawn perpendicular to x, y and z axes simultaneously.

Reason (R) : For any line making angles, α, β, γ with the positive directions of x, y and z axes respectively, $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. In the given figure, ABCD is a parallelogram. If $\vec{AB} = 2\hat{i} - 4\hat{j} + 5\hat{k}$ and $\vec{DB} = 3\hat{i} - 6\hat{j} + 2\hat{k}$, then find $\vec{AD}$ and hence find the area of parallelogram ABCD.



22. (a) Check the differentiability of function $f(x) = [x]$ at $x = -3$, where $[\cdot]$ denotes greatest integer function.

OR

(b) If $x^{1/3} + y^{1/3} = 1$, find $\frac{dy}{dx}$ at the point $\left(\frac{1}{8}, \frac{1}{8}\right)$.

23. Find local maximum value and local minimum value (whichever exists) for the function $f(x) = 4x^2 + \frac{1}{x}$ $(x \neq 0)$.

24. (a) Find :

$$\int x \sqrt{1 + 2x}\, dx$$

OR

(b) Evaluate :

$$\int_0^{\frac{\pi^2}{4}} \frac{\sin \sqrt{x}}{\sqrt{x}}\, dx$$

25. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $(\vec{a} + \vec{b}) \perp \vec{a}$ and $(2\vec{a} + \vec{b}) \perp \vec{b}$, then prove that $|\vec{b}| = \sqrt{2}\, |\vec{a}|$.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Solve the following linear programming problem graphically :

Minimise $z = 5x - 2y$

subject to the constraints

$$x + 2y \leq 120$$
$$x + y \geq 60$$
$$x - 2y \geq 0$$
$$x, y \geq 0$$

27. E and F are two independent events such that $P(\bar{E}) = 0{\cdot}6$ and $P(E \cup F) = 0{\cdot}6$. Find P(F) and $P(\bar{E} \cup \bar{F})$.

28. (a) A relation R on set A = {1, 2, 3, 4, 5} is defined as $R = \{(x, y) : |x^2 - y^2| < 8\}$. Check whether the relation R is reflexive, symmetric and transitive.

OR

(b) A function f is defined from $R \to R$ as $f(x) = ax + b$, such that $f(1) = 1$ and $f(2) = 3$. Find function f(x). Hence, check whether function f(x) is one-one and onto or not.

29. (a) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$.

OR

(b) If $y = (\tan x)^x$, then find $\frac{dy}{dx}$.

30. (a) Find :

$$\int \frac{x^2}{(x^2+4)(x^2+9)}\, dx$$

OR

(b) Evaluate :

$$\int_1^3 (|x-1|+|x-2|+|x-3|)\, dx$$

31. Solve the following differential equation :

$$(\tan^{-1} y - x)\, dy = (1 + y^2)\, dx$$

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. Find the equation of a line l_2 which is the mirror image of the line l_1 with respect to line $l : \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$, given that line l_1 passes through the point P(1, 6, 3) and parallel to line l.

33. (a) If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & -1 & -1 \\ 0 & -2 & 1 \end{bmatrix}$, find A^{-1} and use it to solve the following system of equations :

$x - 2y = 10,\ 2x - y - z = 8,\ -2y + z = 7$

OR

(b) If $A = \begin{bmatrix} -1 & a & 2 \\ 1 & 2 & x \\ 3 & 1 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ -8 & 7 & -5 \\ b & y & 3 \end{bmatrix}$,

find the value of $(a + x) - (b + y)$.

34. (a) Find :

$$\int \frac{(3 \cos x - 2) \sin x}{5 - \sin^2 x - 4 \cos x} \, dx$$

OR

(b) Evaluate :

$$\int_{-2}^{2} \frac{x^3 + |x| + 1}{x^2 + 4|x| + 4} \, dx$$

35. Using integration, find the area of the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$, included between the lines $x = -2$ and $x = 2$.

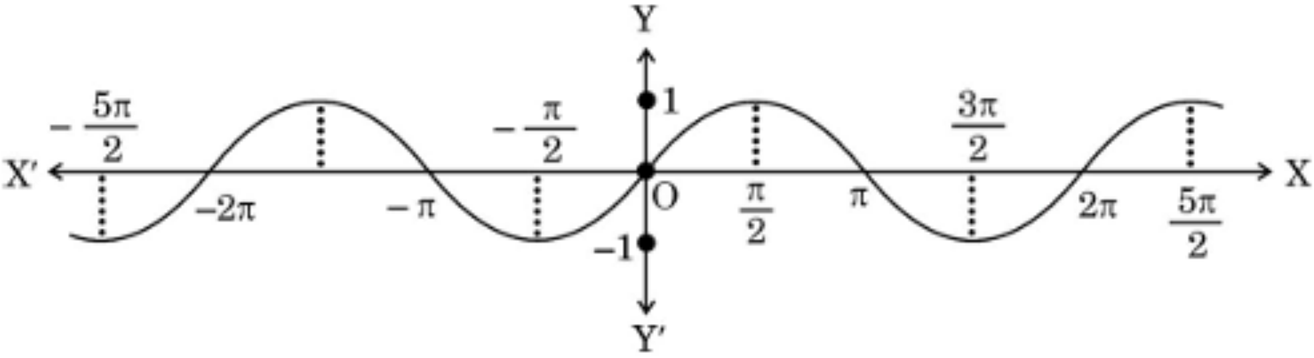
SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. If a function $f : X \to Y$ defined as $f(x) = y$ is one-one and onto, then we can define a unique function $g : Y \to X$ such that $g(y) = x$, where $x \in X$ and $y = f(x)$, $y \in Y$. Function g is called the inverse of function f.

The domain of sine function is R and function sine : $R \to R$ is neither one-one nor onto. The following graph shows the sine function.



$y = \sin x$

Let sine function be defined from set A to $[-1, 1]$ such that inverse of sine function exists, i.e., $\sin^{-1} x$ is defined from $[-1, 1]$ to A.

On the basis of the above information, answer the following questions :

(i) If A is the interval other than principal value branch, give an example of one such interval. *1*

(ii) If $\sin^{-1}(x)$ is defined from $[-1, 1]$ to its principal value branch, find the value of $\sin^{-1}\left(-\frac{1}{2}\right) - \sin^{-1}(1)$. *1*

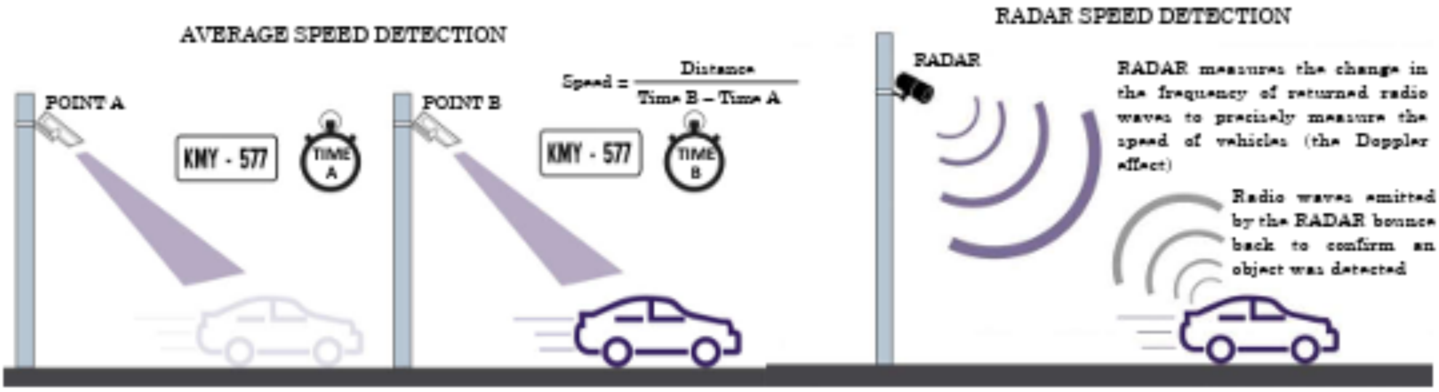
(iii) (a) Draw the graph of $\sin^{-1} x$ from $[-1, 1]$ to its principal value branch. *2*

OR

(iii) (b) Find the domain and range of $f(x) = 2\sin^{-1}(1 - x)$. *2*

Case Study – 2

37. The traffic police has installed Over Speed Violation Detection (OSVD) system at various locations in a city. These cameras can capture a speeding vehicle from a distance of 300 m and even function in the dark.



A camera is installed on a pole at the height of 5 m. It detects a car travelling away from the pole at the speed of 20 m/s. At any point, x m away from the base of the pole, the angle of elevation of the speed camera from the car C is θ.

On the basis of the above information, answer the following questions :

(i) Express θ in terms of height of the camera installed on the pole and x. *1*

(ii) Find $\frac{d\theta}{dx}$. *1*

(iii) (a) Find the rate of change of angle of elevation with respect to time at an instant when the car is 50 m away from the pole. *2*

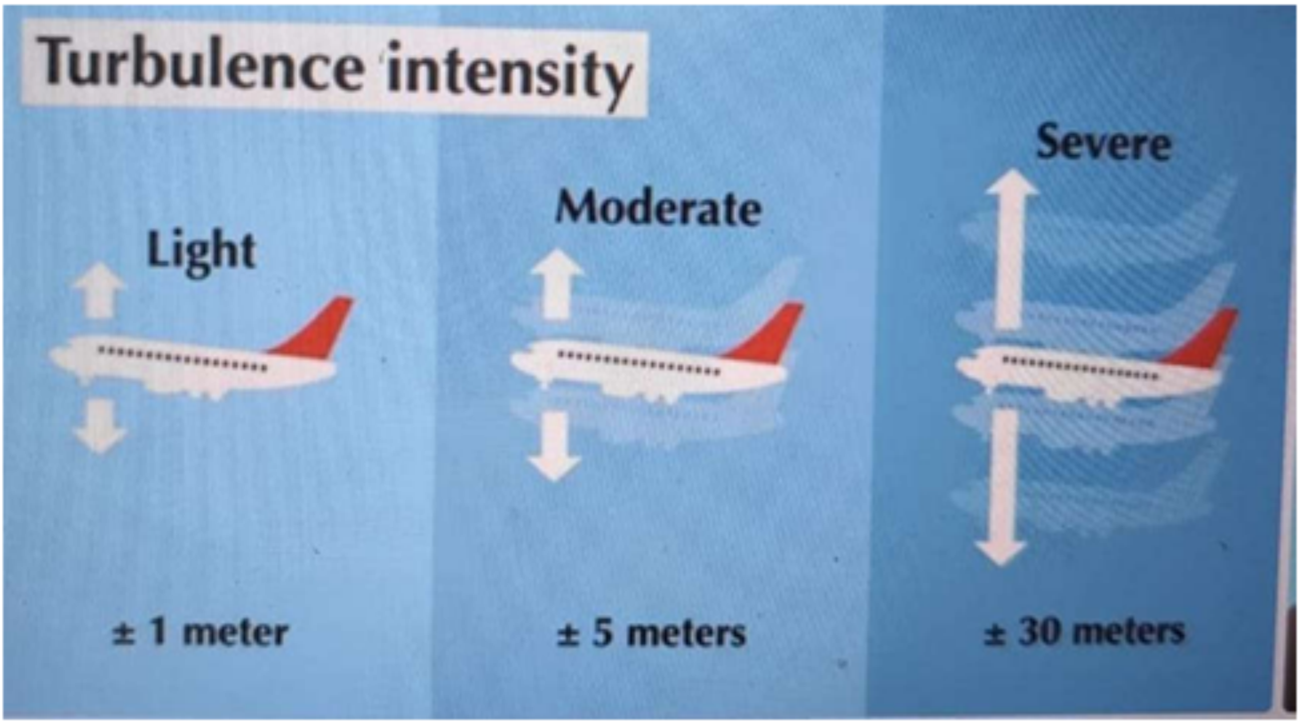
OR

(iii) (b) If the rate of change of angle of elevation with respect to time of another car at a distance of 50 m from the base of the pole is $\frac{3}{101}$ rad/s, then find the speed of the car. *2*

Case Study – 3

38. According to recent research, air turbulence has increased in various regions around the world due to climate change. Turbulence makes flights bumpy and often delays the flights.

Assume that, an airplane observes severe turbulence, moderate turbulence or light turbulence with equal probabilities. Further, the chance of an airplane reaching late to the destination are 55%, 37% and 17% due to severe, moderate and light turbulence respectively.



On the basis of the above information, answer the following questions :

(i) Find the probability that an airplane reached its destination late. *2*

(ii) If the airplane reached its destination late, find the probability that it was due to moderate turbulence. *2*