Series PQ2RS/2

Set – 2

प्रश्न-पत्र कोड
Q.P. Code **65/2/2**

अनुक्रमांक
Roll No.

परीक्षार्थी प्रश्न-पत्र कोड को उत्तर-पुस्तिका के मुख-पृष्ठ पर अवश्य लिखें ।
Candidates must write the Q.P. Code on the title page of the answer-book.

- *कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ **23** हैं ।*
- *कृपया जाँच कर लें कि इस प्रश्न–पत्र में **38** प्रश्न हैं ।*
- *प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए प्रश्न–पत्र कोड को परीक्षार्थी उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें ।*
- ***कृपया प्रश्न का उत्तर लिखना शुरू करने से पहले, उत्तर–पुस्तिका में प्रश्न का क्रमांक अवश्य लिखें ।***
- *इस प्रश्न–पत्र को पढ़ने के लिए 15 मिनट का समय दिया गया है । प्रश्न–पत्र का वितरण पूर्वाह्न में 10.15 बजे किया जाएगा । 10.15 बजे से 10.30 बजे तक छात्र केवल प्रश्न–पत्र को पढ़ेंगे और इस अवधि के दौरान वे उत्तर–पुस्तिका पर कोई उत्तर नहीं लिखेंगे ।*
- *Please check that this question paper contains **23** printed pages.*
- *Please check that this question paper contains **38** questions.*
- *Q.P. Code given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.*
- ***Please write down the serial number of the question in the answer-book before attempting it.***
- *15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer-book during this period.*

गणित
MATHEMATICS

निर्धारित समय : 3 घण्टे — *अधिकतम अंक : 80*

Time allowed : 3 hours — *Maximum Marks : 80*

General Instructions :

Read the following instructions very carefully and strictly follow them :

(i) *This question paper contains **38** questions. **All** questions are **compulsory**.*

(ii) *This question paper is divided into **five** Sections – **A, B, C, D** and **E**.*

(iii) *In **Section A,** Questions no. **1** to **18** are multiple choice questions (MCQs) and questions number **19** and **20** are Assertion-Reason based questions of **1** mark each.*

(iv) *In **Section B,** Questions no. **21** to **25** are very short answer (VSA) type questions, carrying **2** marks each.*

(v) *In **Section C,** Questions no. **26** to **31** are short answer (SA) type questions, carrying **3** marks each.*

(vi) *In **Section D,** Questions no. **32** to **35** are long answer (LA) type questions carrying **5** marks each.*

(vii) *In **Section E,** Questions no. **36** to **38** are case study based questions carrying **4** marks each.*

(viii) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.*

(ix) *Use of calculators is **not** allowed.*

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. The number of solutions of differential equation $\frac{dy}{dx} - y = 1$, given that $y(0) = 1$, is :

(A) 0 (B) 1

(C) 2 (D) infinitely many

2. For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ?

(A) $\vec{a} \cdot \vec{b} \geq |\vec{a}|\,|\vec{b}|$ (B) $\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}|$

(C) $\vec{a} \cdot \vec{b} \leq |\vec{a}|\,|\vec{b}|$ (D) $\vec{a} \cdot \vec{b} < |\vec{a}|\,|\vec{b}|$

3. The coordinates of the foot of the perpendicular drawn from the point (0, 1, 2) on the x-axis are given by :

(A) (1, 0, 0) (B) (2, 0, 0)

(C) $(\sqrt{5}, 0, 0)$ (D) (0, 0, 0)

4. The common region determined by all the constraints of a linear programming problem is called :

(A) an unbounded region (B) an optimal region

(C) a bounded region (D) a feasible region

5. Let E be an event of a sample space S of an experiment, then $P(S|E) =$

(A) $P(S \cap E)$ (B) $P(E)$

(C) 1 (D) 0

6. The number of all scalar matrices of order 3, with each entry – 1, 0 or 1, is :

(A) 1 (B) 3

(C) 2 (D) 3^9

7. $\frac{d}{dx}[\cos(\log x + e^x)]$ at $x = 1$ is :

(A) $-\sin e$ (B) $\sin e$

(C) $-(1 + e)\sin e$ (D) $(1 + e)\sin e$

8. The degree of the differential equation $(y'')^2 + (y')^3 = x \sin(y')$ is :

(A) 1 (B) 2

(C) 3 (D) not defined

9. The unit vector perpendicular to both vectors $\hat{i} + \hat{k}$ and $\hat{i} - \hat{k}$ is :

(A) $2\hat{j}$ (B) $\hat{j}$

(C) $\frac{\hat{i} - \hat{k}}{\sqrt{2}}$ (D) $\frac{\hat{i} + \hat{k}}{\sqrt{2}}$

10. Direction ratios of a vector parallel to line $\frac{x-1}{2} = -y = \frac{2z+1}{6}$ are :

(A) 2, – 1, 6 (B) 2, 1, 6

(C) 2, 1, 3 (D) 2, – 1, 3

11. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $[F(x)]^2 = F(kx)$, then the value of k is :

(A) 1 (B) 2

(C) 0 (D) – 2

12. If a line makes an angle of 30° with the positive direction of x-axis, 120° with the positive direction of y-axis, then the angle which it makes with the positive direction of z-axis is :

(A) 90° (B) 120°

(C) 60° (D) 0°

13. If the sum of all the elements of a 3 × 3 scalar matrix is 9, then the product of all its elements is :

(A) 0 (B) 9

(C) 27 (D) 729

14. Which of the following statements is ***not*** true about equivalence classes A_i ($i = 1, 2, \ldots n$) formed by an equivalence relation R defined on a set A ?

(A) $\bigcup_{i=1}^{n} A_i = A$

(B) $A_i \cap A_j \neq \phi,\ i \neq j$

(C) $x \in A_i$ and $x \in A_j \Rightarrow A_i = A_j$

(D) All elements of A_i are related to each other, for all i

15. If $\begin{vmatrix} -a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix} = kabc$, then the value of k is :

(A) 0 (B) 1

(C) 2 (D) 4

16. The number of points of discontinuity of $f(x) = \begin{cases} |x|+3, & \text{if } x \leq -3 \\ -2x, & \text{if } -3<x<3 \\ 6x+2, & \text{if } x \geq 3 \end{cases}$ is :

(A) 0 (B) 1

(C) 2 (D) infinite

17. The function $f(x) = x^3 - 3x^2 + 12x - 18$ is :

(A) strictly decreasing on R

(B) strictly increasing on R

(C) neither strictly increasing nor strictly decreasing on R

(D) strictly decreasing on $(-\infty, 0)$

18. If $\int_0^2 2\,e^{2x}\,dx = \int_0^a e^x\,dx$, the value of 'a' is :

(A) 1 (B) 2

(C) 4 (D) $\frac{1}{2}$

*Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.*

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).

(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is ***not*** the correct explanation of the Assertion (A).

(C) Assertion (A) is true, but Reason (R) is false.

(D) Assertion (A) is false, but Reason (R) is true.

19. *Assertion (A) :* For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$.

Reason (R) : For two non-zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$.

20. *Assertion (A) :* For any symmetric matrix A, B′AB is a skew-symmetric matrix.

Reason (R) : A square matrix P is skew-symmetric if $P' = -P$.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. If M and m denote the local maximum and local minimum values of the function $f(x) = x + \frac{1}{x}$ $(x \neq 0)$ respectively, find the value of $(M - m)$.

22. Evaluate :

$$\int_0^{a^3} \frac{x^2}{x^6 + a^6}\, dx$$

23. Show that $f(x) = e^x - e^{-x} + x - \tan^{-1} x$ is strictly increasing in its domain.

24. (a) Find the value of $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left[\sin\left(-\frac{\pi}{2}\right)\right]$.

OR

(b) Find the domain of the function $f(x) = \sin^{-1}(x^2 - 4)$. Also, find its range.

25. (a) Check the differentiability of $f(x) = |\cos x|$ at $x = \frac{\pi}{2}$.

OR

(b) If $y = A \sin 2x + B \cos 2x$ and $\frac{d^2y}{dx^2} - ky = 0$, find the value of k.

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. (a) Find the particular solution of the differential equation given by $2xy + y^2 - 2x^2 \frac{dy}{dx} = 0$; $y = 2$, when $x = 1$.

OR

(b) Find the general solution of the differential equation :

$$y\,dx = (x + 2y^2)\,dy$$

27. If vectors $\vec{a}$, $\vec{b}$ and $2\vec{a} + 3\vec{b}$ are unit vectors, then find the angle between $\vec{a}$ and $\vec{b}$.

28. A pair of dice is thrown simultaneously. If X denotes the absolute difference of the numbers appearing on top of the dice, then find the probability distribution of X.

29. Find :

$$\int x^2 \,.\, \sin^{-1}(x^{3/2})\,dx$$

30. (a) If $x^{30}\,y^{20} = (x + y)^{50}$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

OR

(b) Find $\frac{dy}{dx}$, if $5^x + 5^y = 5^{x+y}$.

31. (a) Evaluate :

$$\int_{-2}^{2} \sqrt{\frac{2 - x}{2 + x}}\,dx$$

OR

(b) Find :

$$\int \frac{1}{x\,[(\log x)^2 - 3\log x - 4]}\,dx$$

SECTION D

This section comprises long answer (LA) type questions of 5 marks each.

32. Find the value of p for which the lines

$$\vec{r} = \lambda\hat{i} + (2\lambda + 1)\hat{j} + (3\lambda + 2)\hat{k} \text{ and}$$
$$\vec{r} = \hat{i} - 3\mu\hat{j} + (p\mu + 7)\hat{k}$$

are perpendicular to each other and also intersect. Also, find the point of intersection of the given lines.

33. (a) If $A = \begin{bmatrix} 5 & 0 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ and $B^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, find $(AB)^{-1}$.

Also, find $|(AB)^{-1}|$.

OR

(b) Given $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & 1 & 2 \end{bmatrix}$, find A^{-1}. Use it to solve the following system of equations :

$$x + y + z = 1$$
$$2x + 3y + 2z = 2$$
$$x + y + 2z = 4$$

34. If A_1 denotes the area of region bounded by $y^2 = 4x$, $x = 1$ and x-axis in the first quadrant and A_2 denotes the area of region bounded by $y^2 = 4x$, $x = 4$, find $A_1 : A_2$.

35. (a) Show that a function $f : R \to R$ defined by $f(x) = \frac{2x}{1+x^2}$ is neither one-one nor onto. Further, find set A so that the given function $f : R \to A$ becomes an onto function.

OR

(b) A relation R is defined on $\mathbb{N} \times \mathbb{N}$ (where $\mathbb{N}$ is the set of natural numbers) as :

$$(a, b)\ R\ (c, d) \Leftrightarrow a - c = b - d$$

Show that R is an equivalence relation.

SECTION E

This section comprises 3 case study based questions of 4 marks each.

Case Study – 1

36. The month of September is celebrated as the Rashtriya Poshan Maah across the country. Following a healthy and well-balanced diet is crucial in order to supply the body with the proper nutrients it needs. A balanced diet also keeps us mentally fit and promotes improved level of energy.

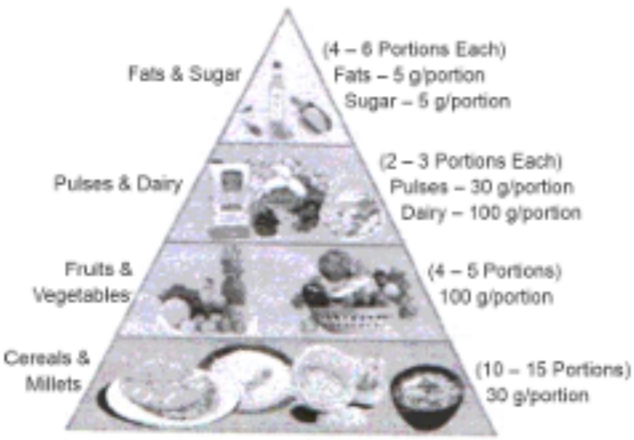


Figure-1

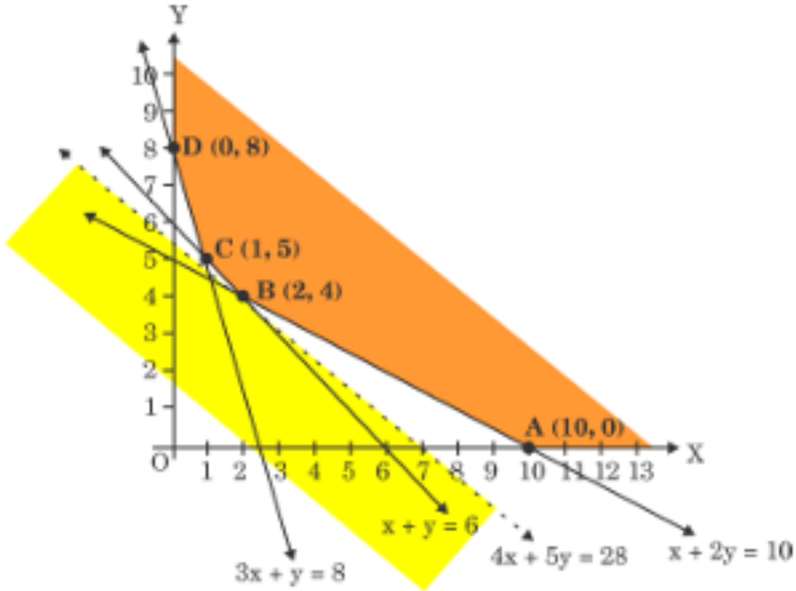


Figure-2

A dietician wishes to minimize the cost of a diet involving two types of foods, food X (x kg) and food Y (y kg) which are available at the rate of ₹ 16/kg and ₹ 20/kg respectively. The feasible region satisfying the constraints is shown in Figure-2.

On the basis of the above information, answer the following questions :

(i) Identify and write all the constraints which determine the given feasible region in Figure-2. *2*

(ii) If the objective is to minimize cost Z = 16x + 20y, find the values of x and y at which cost is minimum. Also, find minimum cost assuming that minimum cost is possible for the given unbounded region. *2*

Case Study – 2

37. Airplanes are by far the safest mode of transportation when the number of transported passengers are measured against personal injuries and fatality totals.

Previous records state that the probability of an airplane crash is 0·00001%. Further, there are 95% chances that there will be survivors after a plane crash. Assume that in case of no crash, all travellers survive.

Let E_1 be the event that there is a plane crash and E_2 be the event that there is no crash. Let A be the event that passengers survive after the journey.

On the basis of the above information, answer the following questions :

(i) Find the probability that the airplane will not crash. *1*

(ii) Find $P(A \mid E_1) + P(A \mid E_2)$. *1*

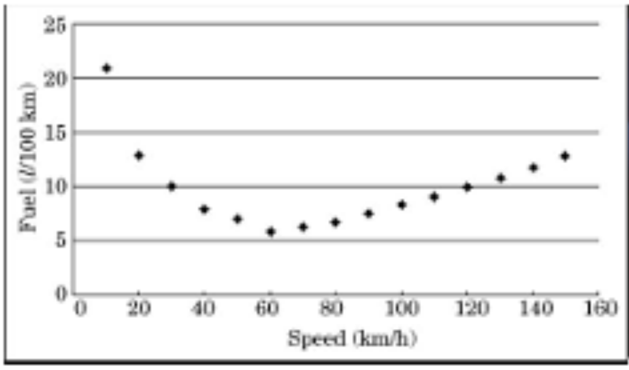
(iii) (a) Find P(A). *2*

OR

(iii) (b) Find $P(E_2 \mid A)$. *2*

Case Study – 3

38. Overspeeding increases fuel consumption and decreases fuel economy as a result of tyre rolling friction and air resistance. While vehicles reach optimal fuel economy at different speeds, fuel mileage usually decreases rapidly at speeds above 80 km/h.



The relation between fuel consumption F (l/100 km) and speed V (km/h) under some constraints is given as $F = \dfrac{V^2}{500} - \dfrac{V}{4} + 14$.

On the basis of the above information, answer the following questions :

(i) Find F, when V = 40 km/h. *1*

(ii) Find $\dfrac{dF}{dV}$. *1*

(iii) (a) Find the speed V for which fuel consumption F is minimum. *2*

OR

(iii) (b) Find the quantity of fuel required to travel 600 km at the speed V at which $\dfrac{dF}{dV} = -0{\cdot}01$. *2*