

MATHEMATICS – Code No. 041

MARKING SCHEME

CLASS – XII (2025-26)

SECTION-A (MCQs of 1 mark each)		
Sol. N.	Hint / Solution	Marks
1	<p>Clearly from the graph Domain is $\left[-\frac{1}{2}, \frac{1}{2}\right]$ So graph is of the function $\sin^{-1}(2x)$ Answer is (B) $\sin^{-1}(2x)$</p>	1
1 (V.I.)	<p>Domain is $\left[-\frac{1}{3}, \frac{1}{3}\right]$ So the function is $\cos^{-1}(3x)$ Answer is (C) $\cos^{-1}(3x)$</p>	1
2	<p>AB is defined so n=4 AC is defined so p=4 AB and AC are square matrices of same order $m \times 3 = m \times q \Rightarrow q = 3 = m$ Answer is (A) $m = q = 3$ and $n = p = 4$</p>	1
3	<p>As A is skew symmetric So $p = 0, q = 2, r = -3, t = 4$ $\text{So } \frac{q+t}{p+r} = \frac{6}{-3} = -2$ Answer is (A) -2</p>	1
4	<p>$\text{adj } A = 27 \Rightarrow A ^3 = 27 = 3^3 \Rightarrow A = 3$ $A(\text{adj } A) = A I = 3I$ Answer is (C) $3I$</p>	1
5	<p>Inverse of the matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{5} \end{bmatrix}$</p> <p>Answer is (B)</p>	1
6	<p>$\begin{vmatrix} \cos 67^\circ & \sin 67^\circ \\ \sin 23^\circ & \cos 23^\circ \end{vmatrix} = \cos 67^\circ \cos 23^\circ - \sin 67^\circ \sin 23^\circ = \cos(67^\circ + 23^\circ) = \cos 90^\circ = 0$ Answer is (A) 0</p>	1
7	<p>$f(x)$ is continuous at $x = \pi$ $\Rightarrow \lim_{x \rightarrow \pi^-} (kx + 1) = \lim_{x \rightarrow \pi^+} \cos x = f(\pi)$ $\Rightarrow \lim_{h \rightarrow 0} [k(\pi - h) + 1] = \lim_{h \rightarrow 0} \cos(\pi + h) = k\pi + 1$ $\Rightarrow k\pi + 1 = -1 \quad \Rightarrow k = \frac{-2}{\pi}$ Answer is (D) $\frac{-2}{\pi}$</p>	1

8	$f(x) = x \tan^{-1} x$ $f'(x) = 1 \cdot \tan^{-1} x + x \cdot \frac{1}{1+x^2}$ $f'(1) = 1 \cdot \tan^{-1} 1 + \frac{1}{1+1} = \frac{\pi}{4} + \frac{1}{2}$ Answer is (B) $\frac{\pi}{4} + \frac{1}{2}$	1
9	$f(x) = 10 - x - 2x^2$ $\Rightarrow f'(x) = -1 - 4x$ For increasing function $f'(x) \geq 0$ $\Rightarrow -(1 + 4x) \geq 0$ $\Rightarrow (1 + 4x) \leq 0$ $\Rightarrow x \leq -\frac{1}{4}$ $\Rightarrow x \in \left(-\infty, -\frac{1}{4}\right]$ Answer is (A) $\left(-\infty, -\frac{1}{4}\right]$	1
10	$xdx + ydy = 0$ $\Rightarrow \int xdx = -\int ydy$ $\Rightarrow \frac{x^2}{2} = -\frac{y^2}{2} + k$ $\Rightarrow x^2 + y^2 = 2k$ Solution is $x^2 + y^2 = 2k$, k being an arbitrary constant. Answer is (C) Circles	1
11	$I = \int_a^b x f(x) dx = \int_a^b (a + b - x) f(a + b - x) dx$ $\Rightarrow I = \int_a^b (a + b - x) f(x) dx$ (given $f(a + b - x) = f(x)$) $\Rightarrow I = \int_a^b (a + b) f(x) dx - \int_a^b x f(x) dx$ $\Rightarrow 2I = (a + b) \int_a^b f(x) dx$ $\Rightarrow I = \frac{1}{2} (a + b) \int_a^b f(x) dx$ Answer is (D) $\frac{a+b}{2} \int_a^b f(x) dx$	1
12	Let $I = \int x^3 \sin^4(x^4) \cos(x^4) dx$ Let $\sin(x^4) = t \Rightarrow 4x^3 \cos(x^4) dx = dt \Rightarrow x^3 \cos(x^4) = \frac{1}{4} dt$ Thus $I = \int t^4 \left(\frac{1}{4} dt\right) = \frac{1}{20} t^5 + C = \frac{1}{20} \sin^5(x^4) + C$ $\Rightarrow I = \frac{1}{20} \sin^5(x^4) + C = a \sin^5(x^4) + C$ So, $a = \frac{1}{20}$ Answer is (B) $\frac{1}{20}$	1
13	The projection of the vector $\hat{i} + 2\hat{j} + \hat{k}$ on the line $\vec{r} = (3\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$ is $\frac{1x1 + 2x2 + 1x3}{\sqrt{1^2 + 2^2 + 3^2}} = \frac{8}{\sqrt{14}}$ units Answer is (C) $\frac{8}{\sqrt{14}}$ units	1

14	The distance of the point (a, b, c) from the y-axis is $\sqrt{a^2 + c^2}$ So, the distance is $\sqrt{3^2 + 5^2} = \sqrt{34}$ units. Answer is (B) $\sqrt{34}$ units	1
15	$(2\vec{a} \cdot \hat{i})\hat{i} - (\vec{b} \cdot \hat{j})\hat{j} + (\vec{c} \cdot \hat{k})\hat{k} = (2 \times 3)\hat{i} - (1)\hat{j} + (2)\hat{k}$ $= 6\hat{i} - \hat{j} + 2\hat{k} = \vec{c}$ Answer is (D) \vec{c}	1
16	The points (1,0) and (0,2) satisfy the equation $2x + y = 2$ And shaded region shows that (0,0) doesn't lie in the feasible solution region So, the inequality is $2x + y \geq 2$ Answer is (B) $2x + y \geq 2$	1
16 (V.I.)	(4,0) and (0,3) gives maximum value so $Z_{(4,0)} = Z_{(0,3)} \Rightarrow 4a + c = 3b + c \Rightarrow 4a = 3b$ Answer is (A) $4a = 3b$	1
17	The student may read the point (2,9) from the line on the graph. The student may find the equation $3x + y = 15$ joining (5,0) and (0,15) and then verify the point (2,9) satisfies it. Answer is (A) (2,9)	1
17 (V.I.)	Answer is (C) Open Half plane that contains origin, but not the points of the line $3x + 5y = 10$	1
18	Answer is (B) $\frac{1}{100}$ The person knows the first 4 digits. So the person has to guess the remaining two digits. $P(\text{guessing the PIN}) = 1 \times 1 \times 1 \times 1 \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$	1
19	$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \tan^{-1} 1 - \sec^{-1}(\sqrt{2}) = \frac{\pi}{3} + \frac{\pi}{4} - \frac{\pi}{4} = \frac{\pi}{3} \neq \frac{\pi}{4}$ So, A is false. Principal Value branch of $\sin^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and that of $\sec^{-1} x$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$. So, R is true Answer is (D) Assertion is false, but Reason is true	1
20	C. $\vec{r} \times (\vec{a} + \vec{b}) = \vec{0} \Rightarrow \vec{r}$ is parallel to $(\vec{a} + \vec{b})$ and $(\vec{a} + \vec{b})$ lies on the plane of \vec{a} and \vec{b} . So, \vec{r} is parallel to the plane of \vec{a} and $\vec{b} \Rightarrow \vec{r}$ is perpendicular to $(\vec{a} \times \vec{b})$. So, Assertion is true But $(\vec{a} + \vec{b})$ lies on the plane of \vec{a} and \vec{b} , so $(\vec{a} + \vec{b})$ is not perpendicular to the plane of \vec{a} and \vec{b} Therefore, Reason is false. Answer is (C) Assertion is true, but Reason is false	1

SECTION B
(VSA type questions of 2 marks each)

21A	$\begin{aligned} \tan\left(\tan^{-1}(-1) + \frac{\pi}{3}\right) &= \tan\left(-\frac{\pi}{4} + \frac{\pi}{3}\right) \\ &= \frac{\tan\frac{\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{\pi}{3}\tan\frac{\pi}{4}} \\ &= \frac{\sqrt{3}-1}{1+\sqrt{3}} \text{ or } 2 - \sqrt{3} \end{aligned}$ <p style="text-align: center;">OR</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
21B	<p>For domain, $-1 \leq 3x - 2 \leq 1$</p> $\begin{aligned} \Rightarrow 1 &\leq 3x \leq 3 \\ \Rightarrow \frac{1}{3} &\leq x \leq 1 \end{aligned}$ <p>So, domain of $\cos^{-1}(3x - 2)$ is $\left[\frac{1}{3}, 1\right]$</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
22	$y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$ <p>Differentiating with respect to x</p> $\begin{aligned} \frac{dy}{dx} &= \frac{1}{\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)} \cdot \sec^2\left(\frac{\pi}{4} + \frac{x}{2}\right) \cdot \frac{1}{2} \\ &= \frac{\cos\left(\frac{\pi}{4} + \frac{x}{2}\right)}{\sin\left(\frac{\pi}{4} + \frac{x}{2}\right)} \cdot \frac{1}{\cos^2\left(\frac{\pi}{4} + \frac{x}{2}\right)} \cdot \frac{1}{2} \\ &= \frac{1}{2 \sin\left(\frac{\pi}{4} + \frac{x}{2}\right) \cos\left(\frac{\pi}{4} + \frac{x}{2}\right)} = \frac{1}{\sin\left(\frac{\pi}{2} + x\right)} = \frac{1}{\cos x} \\ \Rightarrow \frac{dy}{dx} - \sec x &= 0 \end{aligned}$	$\frac{1}{2}$ 1 $\frac{1}{2}$
23A	$\begin{aligned} \int \frac{(x-3)e^x}{(x-1)^3} dx &= \int \frac{(x-1-2)e^x}{(x-1)^3} dx \\ &= \int \left(\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3} \right) e^x dx = \int \left(\frac{1}{(x-1)^2} + \frac{d}{dx} \left(\frac{1}{(x-1)^2} \right) \right) e^x dx \\ &= \frac{e^x}{(x-1)^2} + c \quad (\text{as } \int (f(x) + f'(x))e^x dx = e^x f(x) + c) \end{aligned}$	1 1
23B	<p style="text-align: center;">OR</p> $\begin{aligned} A &= \int_0^4 x dy = \int_0^4 \sqrt{y} dy \\ &= \frac{2}{3} \times y^{3/2} \Big _{y=0}^{y=4} = \frac{16}{3} \text{ sq. units} \end{aligned}$	1 1
23B	<p>For Visually Impaired:</p> $\begin{aligned} A &= \int_0^3 y dx = \int_0^3 \sqrt{x} dx \\ &= \frac{2}{3} \times x^{3/2} \Big _{x=0}^{x=3} = 2\sqrt{3} \text{ sq. units} \end{aligned}$	1 1

24	<p>Given $f(x+y) = f(x)f(y)$</p> $f(0+5) = f(0)f(5)$ $\Rightarrow f(0) = 1$ $f'(5) = \lim_{h \rightarrow 0} \frac{f(5+h)-f(5)}{h} = \lim_{h \rightarrow 0} \frac{f(5)f(h)-f(5)}{h} \quad [\because f(x+y) = f(x)f(y)]$ $= \lim_{h \rightarrow 0} \frac{2f(h)-2}{h} \quad [\because f(5) = 2]$ $= 2 \lim_{h \rightarrow 0} \frac{f(h)-1}{h} = 2 \lim_{h \rightarrow 0} \frac{f(h)-f(0)}{h} = 2 f'(0)$ $= 2(3) \quad [\because f'(0) = 3]$ $= 6$	$\frac{1}{2}$ 1 $\frac{1}{2}$
25	<p>The vector $\overrightarrow{OP} = \frac{1}{2}(4\hat{i} + 4\hat{k}) = 2\hat{i} + 2\hat{k}$</p> <p>Area of the parallelogram formed by the two adjacent sides as OA and OP</p> $= (\overrightarrow{OA} \times \overrightarrow{OP}) = \left \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 0 & 2 \end{vmatrix} \right $ $= 2\hat{i} - 2\hat{k} $ $= 2\sqrt{2} \text{ square units.}$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
SECTION C (SA type questions of 3 marks each)		
26A	<p>$x^y = e^{x-y}$</p> <p>Taking log of both sides</p> $y \log x = (x-y) \log e$ $y \log x + y = x \quad (\text{since } \log e = 1)$ $\Rightarrow y = \frac{x}{1+\log x}$ <p>Differentiating with respect to x</p> $\frac{dy}{dx} = \frac{(1+\log x).1 - x \cdot \frac{1}{x}}{(1+\log x)^2}$ $= \frac{\log x}{(\log e + \log x)^2}$ $= \frac{\log x}{(\log(xe))^2}$ <p>Now $\frac{dy}{dx} \Big _{x=e} = \frac{\log e}{(\log e^2)^2} = \frac{1}{(2\log e)^2} = \frac{1}{2^2} = \frac{1}{4} \quad (\text{as } \log e = 1)$</p>	1 1 1

Alternative Solution:

$$x^y = e^{x-y}$$

Taking log of both sides

 $y \log x = (x-y) \log e$
 $y \log x + y = x \quad (\text{since } \log e = 1)$

Differentiating both sides w.r.t. x

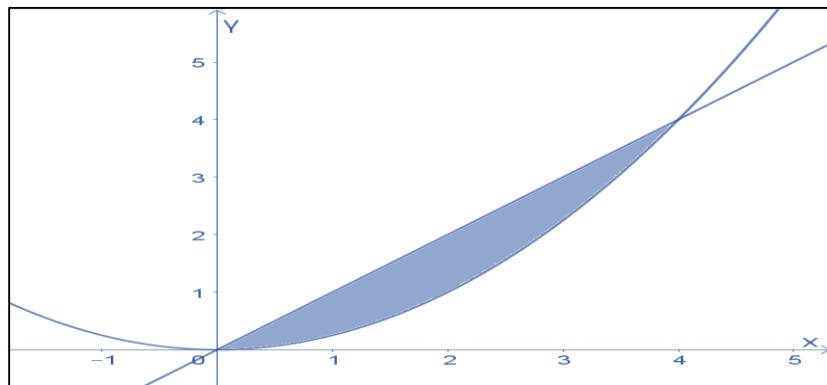
 $\log x \frac{dy}{dx} + \frac{y}{x} + \frac{dy}{dx} = 1$
 $\Rightarrow \frac{dy}{dx} (1 + \log x) = 1 - \frac{y}{x}$
 $\Rightarrow \frac{dy}{dx} = \frac{x-y}{x(1+\log x)} = \frac{x-\frac{x}{1+\log x}}{x(1+\log x)} = \frac{x(1+\log x)-x}{x(1+\log x)^2} = \frac{x(1+\log x-1)}{x(\log e + \log x)^2} = \frac{\log x}{(\log(xe))^2}$

Now $\frac{dy}{dx} \Big|_{x=e} = \frac{\log e}{(\log e^2)^2} = \frac{1}{(2\log e)^2} = \frac{1}{2^2} = \frac{1}{4} \quad (\text{as } \log e = 1)$

	OR	OR
26B	$\frac{dx}{d\theta} = a(1 - \cos \theta), \frac{dy}{d\theta} = a(0 + \sin \theta),$ $\Rightarrow \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a \sin \theta}{a(1 - \cos \theta)}$ $= \frac{2 \sin(\frac{\theta}{2}) \cos(\frac{\theta}{2})}{2 \sin^2(\frac{\theta}{2})} = \cot \frac{\theta}{2}$ $\Rightarrow \frac{d^2y}{dx^2} = -\frac{1}{2} \operatorname{cosec}^2\left(\frac{\theta}{2}\right) \frac{d\theta}{dx}$ $= -\frac{1}{2a} \operatorname{cosec}^2\left(\frac{\theta}{2}\right) \frac{1}{2 \sin^2\left(\frac{\theta}{2}\right)}$ $= -\frac{1}{4a} \operatorname{cosec}^4\left(\frac{\theta}{2}\right)$	1 1 1 1
27	<p>Let r be the radius of ice ball at time t.</p> $V = \frac{4}{3} \pi r^3 \dots\dots\dots (1)$ $S = 4\pi r^2 \dots\dots\dots (2)$ <p>Given $\frac{dV}{dt} \propto S$</p> $\Rightarrow \frac{dV}{dt} = -k S \text{ (where } k \text{ is some positive constant)} \dots\dots\dots (3)$ <p>Differentiating (1) w.r.t. t, we get</p> $\frac{dV}{dt} = \frac{4}{3} \pi \cdot (3r^2) \frac{dr}{dt}$ $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \dots\dots\dots (4)$ $\Rightarrow -k S = 4\pi r^2 \frac{dr}{dt} \quad (\text{from (3) and (4)})$ $\Rightarrow -k S = S \frac{dr}{dt} \quad (\text{using (2)})$ $\Rightarrow \frac{dr}{dt} = -k$ <p>\Rightarrow radius of the ice-ball decreases at a constant rate</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
28A		$\frac{1}{2}$
	$\int_{-4}^2 x + 1 dx = \int_{-4}^{-1} (-x - 1) dx + \int_{-1}^2 (x + 1) dx$ $= -\frac{(x+1)^2}{2} \Big _{-4}^{-1} + \frac{(x+1)^2}{2} \Big _{-1}^2$ $= -\left(0 - \frac{9}{2}\right) + \left(\frac{9}{2} - 0\right) = 9$ <p>It represents the area of shaded region bounded by the curve $y = x + 1$, x-axis and the lines $x = -4$ and $x = 2$</p>	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

28B

OR



$$\begin{aligned}\text{Required Area} &= \int_0^4 x \, dx - \int_0^4 \frac{x^2}{4} \, dx \\ &= \left[\frac{x^2}{2} \right]_0^4 - \frac{1}{12} [x^3]_0^4 \\ &= \frac{1}{2}(16 - 0) - \frac{1}{12}(64 - 0) = 8 - \frac{16}{3} = \frac{8}{3} \text{ sq. units}\end{aligned}$$

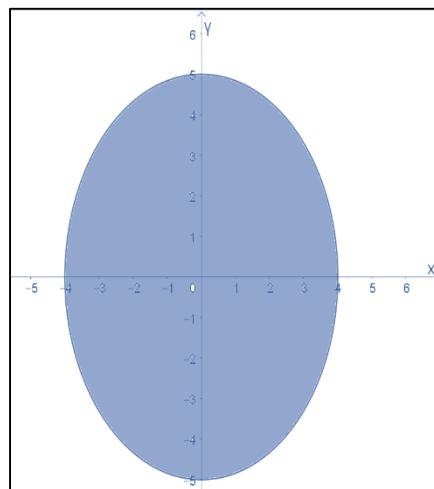
For Visually Impaired:

$$y = |x + 1| = f(x) = \begin{cases} -x - 1, & x < -1 \\ x + 1, & x \geq -1 \end{cases}$$

$$\begin{aligned}\int_{-4}^2 |x + 1| \, dx &= \int_{-4}^{-1} (-x - 1) \, dx + \int_{-1}^2 (x + 1) \, dx \\ &= -\frac{(x+1)^2}{2} \Big|_{-4}^{-1} + \frac{(x+1)^2}{2} \Big|_{-1}^2 \\ &= -\left(0 - \frac{9}{2}\right) + \left(\frac{9}{2} - 0\right) = 9\end{aligned}$$

It represent the area of shaded region bounded by the curve $y = |x + 1|$,
x – axis and the lines $x = -4$ and $x = 2$

OR



	$25x^2 + 16y^2 = 400 \Rightarrow \frac{x^2}{16} + \frac{y^2}{25} = 1 \Rightarrow \frac{x^2}{4^2} + \frac{y^2}{5^2} = 1 \Rightarrow y = \frac{5}{4}\sqrt{4^2 - x^2}$ $\text{Required Area} = 4 \int_0^4 \frac{5}{4}\sqrt{4^2 - x^2} dx$ $= 5 \left[\frac{x\sqrt{4^2 - x^2}}{2} + \frac{4^2}{2} \sin^{-1} \left(\frac{x}{4} \right) \right]_0^4$ $= 5[0 + 8 \sin^{-1}(1) - 0]$ $= 40 \times \frac{\pi}{2} = 20\pi \text{ sq. units}$	1 1 1 1
29A	<p>The line through $(2, -1, 3)$ parallel to the z-axis is given by $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{k})$</p> <p>Any point on this line is $P(2, -1, 3 + \lambda)$</p> <p>Any point on the given line $\vec{r} = (2\hat{i} - \hat{j} + 2\hat{k}) + \mu(3\hat{i} + 6\hat{j} + 2\hat{k})$ is $Q(2 + 3\mu, -1 + 6\mu, 2 + 2\mu)$</p> <p>For the intersection point $Q(2 + 3\mu, -1 + 6\mu, 2 + 2\mu) = P(2, -1, 3 + \lambda) \Rightarrow 2 = 2 + 3\mu \Rightarrow \mu = 0$</p> <p>The point of intersection is $(2, -1, 2)$</p> <p>The distance from $(2, -1, 3)$ to $(2, -1, 2)$ is clearly 1 unit.</p>	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
	<p>Alternative Solution:</p> <p>Any point on the line through $(2, -1, 3)$ parallel to the z-axis is $(2, -1, \lambda)$</p> <p>Any point on the given line is $(2 + 3\mu, -1 + 6\mu, 2 + 2\mu)$</p> <p>Therefore, $2 = 2 + 3\mu \Rightarrow \mu = 0$</p> <p>The point of intersection is $(2, -1, 2)$</p> <p>The distance from $(2, -1, 3)$ to $(2, -1, 2)$ is clearly 1 unit.</p>	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
29B	<p style="text-align: center;">OR</p> <p>The line through $(2, -1, 1)$ parallel to the z-axis is $\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{k})$</p> <p>Any point on this line is $P(2, -1, 1 + \lambda)$</p> <p>Any point on the given line is $A(3 + \mu, \mu, 1 + \mu)$</p> <p>$A(3 + \mu, \mu, 1 + \mu) = P(2, -1, 1 + \lambda) \Rightarrow \mu = -1$</p> <p>The point of intersection is $(2, -1, 0)$</p> <p>The distance of $(2, -1, 0)$ from the z-axis is $\sqrt{2^2 + (-1)^2} = \sqrt{5}$ units.</p>	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
30	<p>Sketching the graph</p>	$1\frac{1}{2}$

	Corner points A(600,0), B(1200,0), C(800,400), D(400,200) Values of Z: $Z_A = 1200, Z_B = 2400, Z_C = 2000, Z_D = 1000$ Maximum Z = 2400 when $x = 1200$ and $y = 0$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
30	For Visually Impaired: At Corner points A(600,0), B(1200,0), C(800,400), D(400,200) Values of Z are $Z_A = 1800, Z_B = 3600, Z_C = 3200, Z_D = 1600$ Maximum Value of Z = 3600 at B(1200,0) And Minimum Value of Z= 1600 at D(400,200)	1 1 1
31	Let the events be: A: Mehul is selected B: Rashi is selected Then according to the question, A and B are independent events and $P(A) = 0.4, P(A \cap \bar{B}) + P(B \cap \bar{A}) = 0.5$ Let $P(B) = x$ Then $P(A \cap \bar{B}) + P(B \cap \bar{A}) = 0.5$ $\Rightarrow P(A)P(\bar{B}) + P(B)P(\bar{A}) = 0.5$ $\Rightarrow 0.4(1 - x) + x(1 - 0.4) = 0.5$ $\Rightarrow 0.4 - 0.4x + 0.6x = 0.5$ $\Rightarrow 0.2x = 0.5 - 0.4 = 0.1$ $\Rightarrow x = \frac{0.1}{0.2} = \frac{1}{2} = 0.5$ So, probability of selection of Rashi = 0.5 Probability of selection of at least one of them $= 1 - P(\bar{A} \cap \bar{B})$ $= 1 - P(\bar{A})P(\bar{B})$ $= 1 - 0.6 \times 0.5$ $= 1 - 0.3 = 0.7$	1 1 1
	SECTION D (LA type questions of 5 marks each)	
32	$AB = \begin{bmatrix} 3 & -6 & -1 \\ 2 & -5 & -1 \\ -2 & 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$ So, $A^{-1} = B$ and $B^{-1} = A$ Given system of equations is $3x - 6y - z = 3, 2x - 5y - z + 2 = 0, -2x + 4y + z = 5$ In matrix form it can be written as: $AX = C$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $C = \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix}$ Here $ A = -3 - 0 + 2 = -1 \neq 0$ So, the system is consistent and has unique solution given by the expression $X = A^{-1}C = BC$ $\Rightarrow X = \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & -1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 5 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 21 \end{bmatrix}$ Thus $x = 2, y = -3, z = 21$	$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

33A	<p>Let $x = \tan \theta \Rightarrow dx = \sec^2 \theta d\theta$</p> $I = \int_0^{\frac{\pi}{4}} \frac{\log(1+\tan\theta)}{1+\tan^2\theta} \cdot \sec^2\theta d\theta$ $I = \int_0^{\frac{\pi}{4}} \log(1+\tan\theta) d\theta = \int_0^{\frac{\pi}{4}} \log \left[1 + \tan \left(\frac{\pi}{4} - \theta \right) \right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log \left[1 + \frac{1-\tan\theta}{1+\tan\theta} \right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log \left[\frac{1+\tan\theta+1-\tan\theta}{1+\tan\theta} \right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log \left[\frac{2}{1+\tan\theta} \right] d\theta$ $= \int_0^{\frac{\pi}{4}} \log 2 d\theta - \int_0^{\frac{\pi}{4}} \log[1+\tan\theta] d\theta$ $= \log 2 \times x \Big _0^{\frac{\pi}{4}} - I$ $\Rightarrow 2I = \frac{\pi}{4} \log 2$ $\Rightarrow I = \frac{\pi}{8} \log 2$	$\frac{1}{2}$ 1 1 1 1 1 $\frac{1}{2}$
33B	OR	OR
	$I = \int \frac{(3 \sin \theta - 2) \cos \theta}{5 - \cos^2 \theta - 4 \sin \theta} d\theta = \int \frac{(3 \sin \theta - 2) \cos \theta}{5 - (1 - \sin^2 \theta) - 4 \sin \theta} d\theta$ <p>Let $\sin \theta = t \Rightarrow \cos \theta d\theta = dt$</p> $I = \int \frac{(3t-2)}{5-(1-t^2)-4t} dt$ $= \int \frac{(3t-2)}{t^2-4t+4} dt = \int \frac{3t-2}{(t-2)^2} dt$ <p>Let $\frac{3t-2}{(t-2)^2} = \frac{A}{(t-2)} + \frac{B}{(t-2)^2}$</p> $3t-2 = A(t-2) + B$ <p>Comparing the coefficients of t and constant terms on both sides</p> $A = 3, -2A + B = -2, B = 4$ $\int \frac{(3 \sin \theta - 2) \cos \theta}{5 - \cos^2 \theta - 4 \sin \theta} d\theta = \int \frac{3}{t-2} dt + \int \frac{4}{(t-2)^2} dt$ $= 3 \log t-2 - \frac{4}{t-2} + C$ $= 3 \log \sin \theta - 2 - \frac{4}{\sin \theta - 2} + C$	$\frac{1}{2}$ 1 $\frac{1}{2}$ $\frac{1}{2}$ +$\frac{1}{2}$ 1+1 $\frac{1}{2}$
34A	$y + \frac{d}{dx}(xy) = x(\sin x + x)$ $\Rightarrow y + (x \frac{dy}{dx} + y) = x(\sin x + x)$ $\Rightarrow 2y + x \frac{dy}{dx} = x(\sin x + x)$ $\Rightarrow \frac{dy}{dx} + \frac{2y}{x} = (\sin x + x)$ <p>This a linear differential equation of the form $\frac{dy}{dx} + Py = Q$</p> $P = \frac{2}{x}, Q = (\sin x + x)$ $I.F = e^{\int \frac{2}{x} dx} = e^{2 \log x} = e^{\log x^2} = x^2$ <p>Solution will be $y \cdot I.F = \int Q \cdot I.F dx$</p> $yx^2 = \int (\sin x + x)x^2 dx$ $yx^2 = \int \sin x \cdot x^2 dx + \int x^3 dx$	1 1 1 1

	$\Rightarrow yx^2 = -x^2 \cos x + 2 \int x \cos x dx + \frac{x^4}{4} + C$ $\Rightarrow yx^2 = -x^2 \cos x + 2(x \sin x + \cos x) + \frac{x^4}{4} + C$ Which is the required solution	1
	OR	
34B	$2y e^{x/y} dx + (y - 2x e^{x/y}) dy = 0$ $\Rightarrow \frac{dx}{dy} = \frac{2x e^{x/y} - y}{2y e^{x/y}} = \frac{2\frac{x}{y} e^{\frac{x}{y}} - 1}{2e^{\frac{x}{y}}}$ It is a homogeneous differential equation. Let $x = vy \Rightarrow \frac{dx}{dy} = v + y \frac{dv}{dy}$ $v + y \frac{dv}{dy} = \frac{2ve^v - 1}{2e^v}$ $\Rightarrow y \frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} - v = \frac{2ve^v - 1 - 2ve^v}{2e^v}$ $\Rightarrow y \frac{dv}{dy} = \frac{-1}{2e^v}$ $\Rightarrow 2e^v dv = -\frac{dy}{y}$ $\int 2e^v dv = -\int \frac{dy}{y}$ $\Rightarrow 2e^v = -\log y + C$ $\Rightarrow 2e^{\frac{x}{y}} + \log y = C$ When $x = 0, y = 1, C = 2$ Required solution $2e^{\frac{x}{y}} + \log y = 2$	1 1 1 1 1 1 1 1 1 1
35	Let $\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda \Rightarrow$ Any point on it is $(3\lambda+1, -\lambda, -1)$ For the point where $y = 1 \Rightarrow \lambda = -1$ \Rightarrow The point is $(-2, 1, -1)$ The directions of the two lines are $\vec{m} = 3\hat{i} - \hat{j}$ $\text{and } \vec{n} = -2\hat{i} + 2\hat{j} + \hat{k}$ $\vec{m} \times \vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 0 \\ -2 & 2 & 1 \end{vmatrix} = -\hat{i} - 3\hat{j} + 4\hat{k}$ The equation of the required line is $\vec{r} = (-2\hat{i} + \hat{j} - \hat{k}) + \mu(-\hat{i} - 3\hat{j} + 4\hat{k})$	1/2 1 1/2 1 1/2 1 1 1 1/2 1 1/2
	Alternative Solution: Let $\frac{x-1}{3} = \frac{y-0}{-1} = \frac{z+1}{0} = \lambda \Rightarrow$ Any point on it is $(3\lambda+1, -\lambda, -1)$ For the point where $y = 1 \Rightarrow \lambda = -1$ \Rightarrow The point is $(-2, 1, -1)$ Let the direction ratios of the required line be a, b, c Then $3a - b = 0$ And $-2a + 2b + c = 0$ Solving we get $\frac{a}{-1} = \frac{-b}{3} = \frac{c}{4} \Rightarrow \frac{a}{-1} = \frac{b}{-3} = \frac{c}{4}$ The required line is $\frac{x+2}{-1} = \frac{y-1}{-3} = \frac{z+1}{4} = \mu$ In vector form $\vec{r} = (-2\hat{i} + \hat{j} - \hat{k}) + \mu(-\hat{i} - 3\hat{j} + 4\hat{k})$	1/2 1 1/2 1 1 1 1 1 1 1 1/2 1 1/2

SECTION- E
(3 case-study/passage-based questions of 4 marks each)

36	<p>I. Traffic flow is not reflexive as $(A, A) \notin R$ (or no major spot is connected with itself)</p> <p>II. Traffic flow is not transitive as $(A, B) \in R$ and $(B, E) \in R$, but $(A, E) \notin R$</p> <p>III A. $R = \{(A, B), (A, C), (A, D), (B, C), (B, E), (C, E), (D, E), (D, C)\}$</p> <p>Domain = $\{A, B, C, D\}$</p> <p>Range = $\{B, C, D, E\}$</p> <p style="text-align: center;">OR</p> <p>III B. No, the traffic flow doesn't represent a function as A has three images.</p>	1 1 1 $\frac{1}{2} +$ $\frac{1}{2}$ 1+1
37	<p>I. $P(x) = R(x) - C(x) = -0.3x^2 + 20x - (0.5x^2 - 10x + 150)$ $= -0.8x^2 + 30x - 150$</p> <p>II. For critical points $P'(x) = 0 \Rightarrow -1.6x + 30 = 0$ $\Rightarrow x = \frac{30}{1.6} = \frac{300}{16} = 18.75$</p> <p>III A. Now $P''(x) = -1.6$ In particular $P''(18.75) = -1.6 < 0$ So, critical value $x = 18.75$ corresponds to a maximum profit.</p> <p style="text-align: center;">OR</p> <p>III B. As x is the number of bulbs, so practically 18 bulbs correspond to a maximum profit. Maximum profit is $P(18) = -0.8 \times 18^2 + 30 \times 18 - 150$ $= -259.2 + 540 - 150$ $= 540 - 409.2 = ₹130.80$</p>	1 1 1 1 1 1
38	<p>Let the events be</p> <p>E_1: the student is in the first group (time spent on screen is more than 4 hours)</p> <p>E_2: the student is in the second group (time spent on screen is 2 to 4 hours)</p> <p>E_3: the student is in third group (time spent on screen is less than 2 hours)</p> <p>A: the event of the student showing symptoms of anxiety and low retention</p> <p>$P(E_1) = \frac{60}{100}$ $P(E_2) = \frac{30}{100}$ and $P(E_3) = \frac{10}{100}$</p> <p>$P(A E_1) = \frac{80}{100}$ $P(A E_2) = \frac{70}{100}$ and $P(A E_3) = \frac{30}{100}$</p> <p>I. $P(A) = P(E_1) \times P(A E_1) + P(E_2) \times P(A E_2) + P(E_3) \times P(A E_3)$ $= \frac{60}{100} \times \frac{80}{100} + \frac{30}{100} \times \frac{70}{100} + \frac{10}{100} \times \frac{30}{100} = \frac{72}{100} = 72\%$</p> <p>II. $P(E_1 A) = \frac{P(E_1 \cap A)}{P(A)}$ $= \frac{\left(\frac{60}{100} \times \frac{80}{100}\right)}{\left(\frac{72}{100}\right)} = \frac{48}{72} = \frac{2}{3}$</p>	2 2