

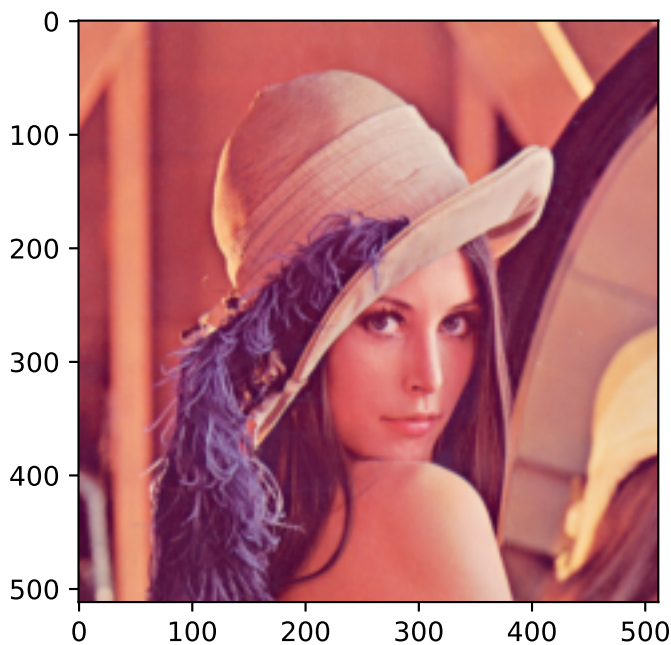
作业目的：理解图像灰度变换的基本原理，掌握灰度变换的实现方法

作业内容：选取一张灰度图像，用给定的变换函数对图像进行灰度变换，对比灰度变换效果

```
In [133... #导入包
import numpy as np
import matplotlib.pyplot as plt
import math

#选择图像
lenna = plt.imread("images/lenna.jpg") # 加载当前文件夹的图片
plt.imshow(lenna)
```

Out[133... <matplotlib.image.AxesImage at 0x126da47c0>



作业要求：（1）在（0，1）范围内随机设定5对(x,y)值（包含(0,0)和（1，1）），并利用其构建4阶拟合多项式（2）利用拟合多项式曲线作为变换函数对灰度图像进行处理，给出处理结果

作业提交：（1）PDF格式（2）绘制离散点及拟合的多项式曲线，同时给原始图像和用该曲线对应函数进行灰度变换的输出图像（3）从数学角度推导多项式拟合参数计算方法，利用Numpy线性代数工具箱求解多项式系数，与Numpy的polyfit函数的计算结果进行对比，看看两者是否一致。（4）给出算法实现的Python代码

```
In [134... # (2) 绘制离散点及拟合的多项式曲线，同时给原始图像和用该曲线对应函数进行灰度变换的输出图像

# 模拟生成10个离散的点
xp = np.linspace(0, 1, 10)
y1 = np.log10(1+xp) #像素值取对数
y2 = xp ** 2        #像素值二次方
y3 = xp ** 3        #像素值三次方
y4 = np.sqrt(xp)    #像素值开方
y5 = np.arctan(xp)  #像素值
print(y1)

# 通过多项式拟合得到拟合曲线
z_1 = np.poly1d(np.polyfit(xp, y1, 4))
z_2 = np.poly1d(np.polyfit(xp, y2, 4))
z_3 = np.poly1d(np.polyfit(xp, y3, 4))
```

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z_4 = np.polyld(np.polyfit(xp, y4, 4))
z_5 = np.polyld(np.polyfit(xp, y5, 4))

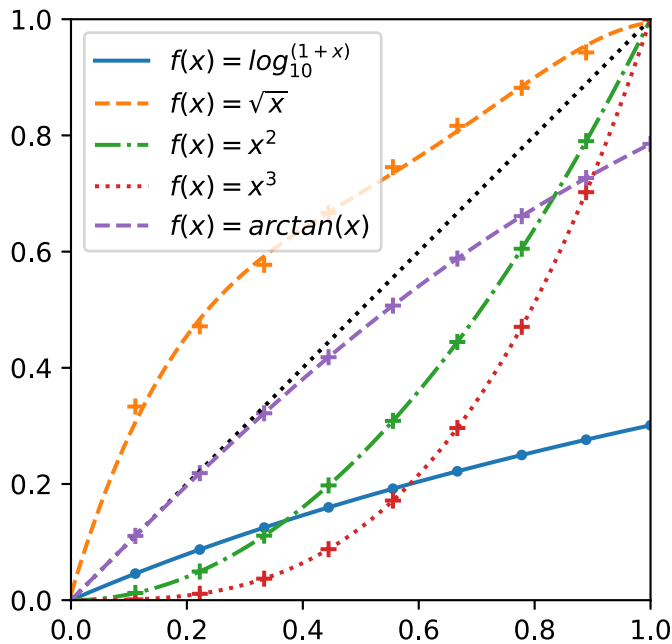
# 绘制拟合曲线
# 这里也展示了显示单张图片的一种方法
fig = plt.figure()
ax = fig.add_subplot(111)
ax.set_aspect('equal')
x = np.linspace(0, 1, 100)
ax.plot(x, x, linestyle=':', color = 'black')
ax.scatter(xp, y1, marker='.')
ax.plot(x, z_1(x), linestyle='-', label='$f(x)=\log_{10}\{(1+x)\}$')
ax.scatter(xp, y4, marker='+')
ax.plot(x, z_4(x), linestyle='--', label='$f(x)=\sqrt{x}$')
ax.scatter(xp, y2, marker='+')
ax.plot(x, z_2(x), linestyle='-.', label='$f(x)=x^2$')
ax.scatter(xp, y3, marker='+')
ax.plot(x, z_3(x), linestyle=':', label='$f(x)=x^3$')
ax.scatter(xp, y5, marker='+')
ax.plot(x, z_5(x), linestyle='dashed', label='$f(x)=\arctan(x)$')
plt.ylim(0,1)
plt.xlim(0,1)
plt.legend()
plt.show()

```

```

[0.          0.04575749 0.08715018 0.12493874 0.15970084 0.19188553
 0.22184875 0.24987747 0.27620641 0.30103      ]

```



In [135...

```

# 根据拟合曲线构建映射表
map_1 = np.zeros(256)
map_2 = np.zeros(256)
map_3 = np.zeros(256)
map_4 = np.zeros(256)
map_5 = np.zeros(256)
x = np.linspace(0, 1, 256)
for i in np.arange(256):
    temp = z_1(x[i])
    if temp > 1:
        map_1[i] = 1
    elif temp < 0:
        map_1[i] = 0
    else:
        map_1[i] = temp

```

```

for i in np.arange(256):
    temp = z_2(x[i])
    if temp > 1:
        map_2[i] = 1
    elif z_2(x[i]) < 0:
        map_2[i] = 0
    else:
        map_2[i] = temp
for i in np.arange(256):
    temp = z_3(x[i])
    if temp > 1:
        map_3[i] = 1
    elif z_3(x[i]) < 0:
        map_3[i] = 0
    else:
        map_3[i] = temp
for i in np.arange(256):
    temp = z_4(x[i])
    if temp > 1:
        map_4[i] = 1
    elif z_3(x[i]) < 0:
        map_4[i] = 0
    else:
        map_4[i] = temp
for i in np.arange(256):
    temp = z_5(x[i])
    if temp > 1:
        map_5[i] = 1
    elif z_5(x[i]) < 0:
        map_5[i] = 0
    else:
        map_5[i] = temp

```

In [136...

```

def f_1(x):
    return map_1[x]
def f_2(x):
    return map_2[x]
def f_3(x):
    return map_3[x]
def f_4(x):
    return map_4[x]
def f_5(x):
    return map_5[x]

```

In [137...

f_1仅对单个标量进行操作, frompyfunc函数让其支持对图像每个像素的处理

```

im_1 = np.frompyfunc(f_1,1,1)(lenna).astype(np.float)
im_2 = np.frompyfunc(f_2,1,1)(lenna).astype(np.float)
im_3 = np.frompyfunc(f_3,1,1)(lenna).astype(np.float)
im_4 = np.frompyfunc(f_4,1,1)(lenna).astype(np.float)
im_4 = np.frompyfunc(f_4,1,1)(lenna).astype(np.float)

```

In [138...

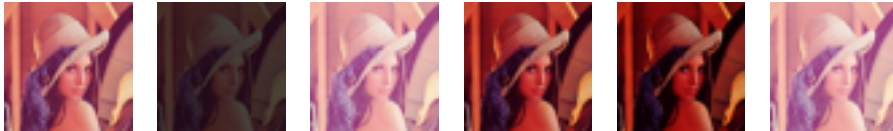
```

plt.subplot(161)
plt.imshow(lenna)
plt.axis('off')
plt.title('Lenna_Orig')
plt.subplot(162)
plt.imshow(im_1)
plt.axis('off')
plt.title('$\log^{(1+x)}$')
plt.subplot(163)
plt.imshow(im_4)
plt.axis('off')
plt.title('$\sqrt{x}$')

```

```
plt.subplot(164)
plt.imshow(im_2)
plt.axis('off')
plt.title('$x^2$')
plt.subplot(165)
plt.imshow(im_3)
plt.axis('off')
plt.title('$x^3$')
plt.subplot(166)
plt.imshow(im_5)
plt.axis('off')
plt.title('$\arctan(x)$')
plt.show()
```

Lenna_Orig $\log^{(1+x)}$ \sqrt{x} x^2 x^3 $\arctan(x)$



In [139...

(3) 从数学角度推导多项式拟合参数计算方法, 利用Numpy线性代数工具箱求解多项式系数, 与Numpy.

```
#Numpy的polyfit结果
print("z_1:\n",z_1,"\n")
print("z_2:\n",z_2,"\n")
print("z_3:\n",z_3,"\n")
print("z_4:\n",z_4,"\n")
print("z_5:\n",z_5,"\n")

# y1 = np.log10(1+xp)
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import math
import random

fig = plt.figure()
ax = fig.add_subplot(121)

#阶数为9阶
order=9

#生成曲线上的各个点
x = np.arange(-1,1,0.02)
y = [np.log10(1+a) for a in x]

# 生成的曲线上的各个点偏移一下, 并放入到xa,ya中去
i = 0
xa = []
ya = []
for xx in x:
    yy = y[i]
    d = float(random.randint(60, 140)) / 100
    # ax.plot([xx*d],[yy*d],color='m',linestyle='',marker='.')
    i += 1
    xa.append(xx * d)
    ya.append(yy * d)

'''for i in range(0,5):
    xx=float(random.randint(-100,100))/100
    yy=float(random.randint(-60,60))/100
    xa.append(xx)
```

```

ya.append(yy)'''

ax.plot(xa, ya, color='m', linestyle='', marker='.')

# 求出等式左边的矩阵A

matA=[]
for i in range(0,order+1):
    mat=[]
    for j in range(0+i,order+1+i):
        sumA=0
        for xx in xa:
            sumA=sumA+xx**j
        mat.append(sumA)
    matA.append(mat)
A=np.array(matA)

# 求出右边的等式B
matB=[]
for j in range(0,order+1):
    sumB=0
    for xx,yy in zip(xa,ya):
        sumB=sumB+xx**j*yy
    matB.append(sumB)
B=np.array(matB)
# 另外一种该方法求A
# 求出等式左边的矩阵A
A=[]
for xx in xa:
    matA = []
    for i in range(0,order+1):
        mat = []
        for j in range(0+i,order+1+i):
            mat.append(xx**j)
        matA.append(mat)
    A.append(matA)
# 求和
A=sum(np.array(A))

a=np.linalg.solve(A,B)
# 定义拟合函数
def fun_solve(x,a):
    y=0
    for i in range(len(a)):
        y+=a[i]*x**i
    return y

xxa= np.arange(-1,1.06,0.01)
yya=[]
for xxaa in xxa:
    yya.append(fun_solve(xxaa,a))

ax2 = fig.add_subplot(122)

ax2.plot(x, z_1(x),linestyle='-', label='$f(x)=\log_{10}\{(1+x)\}$')

ax.plot(xxa,yya,color='g',linestyle='-',marker='')

```

```

z_1:
          4          3          2
-0.02411 x + 0.0951 x - 0.2027 x + 0.4328 x + 1.436e-05

```

```

z_2:
          4          3          2

```

$$-3.967\text{e-}15\ x + 9.573\text{e-}15\ x + 1\ x + 2.575\text{e-}15\ x - 3.774\text{e-}16$$

z_3:

$$2.9\text{e-}15\ x^4 + 1\ x^3 + 2.324\text{e-}15\ x^2 - 6.511\text{e-}16\ x - 1.931\text{e-}16$$

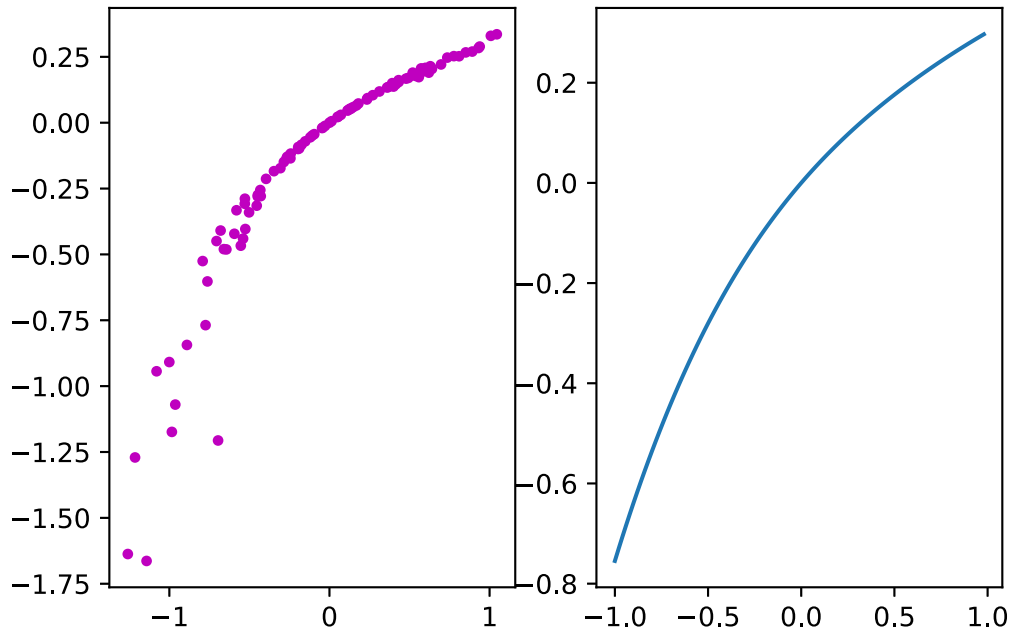
z_4:

$$-3.33\ x^4 + 8.181\ x^3 - 7.241\ x^2 + 3.374\ x + 0.009951$$

z_5:

$$0.1401\ x^4 - 0.3425\ x^3 - 0.01506\ x^2 + 1.003\ x - 4.859\text{e-}05$$

Out[139... [<matplotlib.lines.Line2D at 0x12890aee0>]



由图像观察发现在 $[-1, -0.5]$ 部分拟合的不是很好但是 $[0, 1]$ 处则点比较密集,相对于Numpy的polyfit函数还是做的不够好, 这方法使用的是最小二乘法多项式曲线拟合原理

作业提示: 建设离散点位数 N , 拟合多项式阶次 $N-1$, 求解多项式系数过程就是解线性组问题