Soccer features a variety of models for ranking teams and predicting match outcomes. Here, we look at a network-based ranking model called the Offense-Defense Model (ODM) by Govan, Langville and Meyer (2009). We derive a new ranking model from ODM that improves on its strengths. We apply the rankings to predict game outcomes in soccer tournaments. The goal is to outperform the betting markets and other models.

Model 1 (ODM)

ODM posits that teams have positive attack and defense ratings porportional to points scored and points conceded. It assumes that margin of victory and strength of schedule are important in ranking teams. A brief mathematical description follows. Define P_{ij} as the points scored by team j against team i. Attack and defense ratings of team j are:

$$a_j = \sum_{i=1}^{n} P_{ij} \frac{1}{d_i} \tag{Attack}$$

$$d_j = \sum_{j=1}^{n} P_{ji} \frac{1}{a_i} \tag{Defense}$$

Ratings for teams 1, 2, 3, ..., n in compact form are:

$$a = \begin{bmatrix} a_1 & a_2 & a_3 & ... & a_n \end{bmatrix}^T = P^T \frac{1}{d} \tag{Attack}$$

$$d = \begin{bmatrix} d_1 & d_2 & d_3 & ... & d_n \end{bmatrix}^T = P \frac{1}{a} \tag{Defense}$$

Since a and d are mutually dependent, they're computed iteratively in the algorithm below:

Algorithm 1 (ODM Update)
Given a nonnegative matrix P :
$d^{(0)} \leftarrow$ column vector of ones
$a^{(0)} \leftarrow P^T \dfrac{1}{d^{(0)}}$
for k = 1, 2, 3, ..., until convergence of $a^{(k)}$, $d^{(k)}$:
    $d^{(k+1)} \leftarrow P \dfrac{1}{a^{(k)}}$
    $a^{(k+1)} \leftarrow P^T \dfrac{1}{d^{(k)}}$
end for

To guarantee convergence, P is replaced with $P + cee^T$ where c is positive and e is a column vector of ones.

Model 2 (S-ODM)

While ODM effectively distinguishes attack and defense strengths, it is difficult to work with for three reasons:

> (1) ODM can only assign teams a default rating of 1. This works when the network of games between teams is dense but in soccer, teams are organized into confederations that rarely play each other.
>
> (2) ODM ratings are non-linear and poorly-scaled. This makes them difficult to interpret and linearize.
>
> (3) ODM ratings are unstable. For instance, if Germany plays Ukraine but Ukraine plays another at later date, then Germany's ratings are affected by Ukraine's recomputed rating.

To solve these challenges, we propose the Sequential Offense-Defense Model (S-ODM). S-ODM starts by assigning all teams default attack and defense ratings. It then applies ODM to rate teams match-by-match.

In each game, a team has three sets of ratings: they are **pre-game ratings**, which infers the strength of the team before the game, **in-game ratings**, which infers how well it played during the game and **post-game ratings**, which are pre-game ratings of its next game. Only pre-game ratings of the participants in a match are used to compute their in-game ratings. To compute in-game ratings from pre-game ratings, S-ODM abstracts Algorithm 1 to accept as input an attack or defense rating:

Algorithm 2 (Scale Rating)
Given a nonnegative matrix P and positive vector x:
$x^{(0)} \leftarrow x$
$y^{(0)} \leftarrow P^T \dfrac{1}{x^{(0)}}$
for $k = 1, 2, 3, ...,$ until convergence of $x^{(k)}$, $y^{(k)}$:
    $x^{(k+1)} \leftarrow P \dfrac{1}{y^{(k)}}$
    $y^{(k+1)} \leftarrow P^T \dfrac{1}{x^{(k)}}$
end for

Similar to Algorithm 1, P is replaced by $\hat{P}$ to guarantee convergence. S-ODM employs Algorithm 2 to rate teams in the following manner. Define teams 1 and 2 with pre-game attack and defense ratings a_1, d_1 and a_2, d_2. The scoring matrix P is defined as it is in ODM with the only difference being that its size is 2-by-2. The in-game ratings of a match are computed as follows:

Algorithm 3 (S-ODM Update)
Given a nonnegative 2-by-2 matrix P and positive a_1, a_2, d_1, d_2 :
$a^{(0)} \leftarrow \begin{bmatrix} a_1 & a_2 \end{bmatrix}^T$
$d^{(0)} \leftarrow \begin{bmatrix} d_1 & d_2 \end{bmatrix}^T$
for $k = 1, 2, 3, ...,$ until convergence of $a^{(k)}$, $y^{(k)}$:
Define $a_A^{(k)}$, $d_A^{(k)}$ as x_k, y_k from Algorithm 2 with inputs P and $a^{(k)}$.

Define $d_D^{(k)}$, $a_D^{(k)}$ as x_k, y_k from Algorithm 2 with inputs P and $d^{(k)}$.

$$a_{k+1} \leftarrow \frac{a_A^{(k)} + a_D^{(k)}}{2}$$

$$d_{k+1} \leftarrow \frac{d_A^{(k)} + d_D^{(k)}}{2}$$

end for

Intuitively, Algorithm 3 averages in-game ratings relative to pre-game attack and defense ratings.

Algorithm 3 affords S-ODM three important advantages over its ODM counterpart Algorithm 1. Firstly, S-ODM ratings can accept default ratings for each team. Secondly, S-ODM ratings are stable as ratings only change from participation in a game. Finally, S-ODM ratings are well-scaled. They can be linearized by taking the log of the ratings.

Probabilistic S-ODM

We detail an approach for making S-ODM ratings predictive and for making the model probabilistic. There are three challenges.

First is adjusting for home advantage. Each goal scored by a home team is multiplied by the ratio of away vs. home goals in the corresponding contest. This allows the average goals scored by home and away teams to be equal.

The second challenge is compute a team's post-game ratings in each match. Our approach is to assign it a weighted sum of pre-game and in-game ratings. To determine the weight of pre-game ratings, we look at how much evidence there is for a team's performance. When a team plays many recent games, there is more evidence of how well it plays so its pre-game ratings play a greater role in its post-game rating. Thus, the weights assigned to in-game and pre-game ratings are α and $1 - \alpha$ where

$$\alpha = \frac{1}{1 + Ee^{-tk}},$$

E is the evidence for the team's rating, t is the number of days since its previous match and k is an exponential decay constant. The variable E becomes Ee^{-tk} and is incremented by 1 to represent the gain in evidence for the team's performance.

The final challenge is to translate ratings to game outcome probabilities. This requires an appropriate probability distribution. The diagonal inflated bivariate Poisson model (DIBP) by Karlis and Ntzoufras (2003) is chosen for three reasons: it allows correlation between variables of interest (number of goals scored by each team), corrects over-dispersion and removes bias against ties. Its probability function is

$$f_{DIBP}(x,\, y) = \begin{cases} (1-p) f_{BP}(x,\, y \,|\, \lambda_1,\, \lambda_2,\, \lambda_3) + p f_D(x \,|\, \theta) & x = y \\ (1-p) f_{BP}(x,\, y \,|\, \lambda_1,\, \lambda_2,\, \lambda_3) & x \neq y \end{cases}$$

where p is an inflation parameter, f_{BP} is the probability function of a bivariate Poisson distribution and f_D is the probability function of a discrete distribution with parameter vector θ. Variables x, y are goals scored. Parameter λ_3 is the correlation between the marginal distributions. Parameter λ_1 is porportional to the expected goals scored by team j against team i and is defined as

$$\log \lambda_1 = \log \mu + \beta_a \log a_j + \beta_d \log d_i$$

where μ is the average goals scored in a home or away game of the contest, variables a_j, d_i are attack and defense ratings of teams j, i and parameters β_a, β_d are weights to attack and defense ratings. Likewise for λ_2.

We now introduce the objective function. Its parameters are

(1) The exponential decay constant k used to compute the weight of pre-game vs. in-game ratings.

(2) The constant c used to ensure convergence of Algorithm 2.

(3) The parameters $\beta_a, \beta_d, \lambda_3$ and p from the explanation of DIBP.

(4) The linearized version of the default rating r_i of each confederation (ratings are linearized by taking their log).

The objective function minimizes the squared error of actual game results (win, loss or tie) and expected game results, which are computed with the probability function f_{DIBP}.

Verification