Discrete-time Control Contraction Metrics (DCCM) for Quasistatic Planar Pushing using Smoothed Dynamics

MIT 2.152[J] Nonlinear Control Project Report

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***Abstract*—**

I. Introduction

Planning and control through contact is an important task in Robotics. Robots interacting with the environment need to make and break contact in order to complete tasks but this has proved challenging due to the non-smooth nature of contact dynamics. This has traditionally been done using hybrid planners and controllers with guards and resets where different dynamics are considered based on enumerated contact modes. An alternative approach has been proposed in [1] where contact dynamics are smoothed to create more informative gradients in global planning of contact-rich manipulation tasks. In this paper we apply these same smoothing techniques to explore the controllers that this enables.

Another way in which control through contact is difficult is that these systems are highly underactuated. The state of the unactuated objects cannot be controlled directly.

This work builds on [2] which demonstrates the effectiveness of CCMs on cannonical underactuated systems such as Cart-Pole.

as well as [3] that extends work in [4] to discrete time.

II. Preliminaries and Problem Formulation

A. Quasistatic Assumptions

We will assume that our system is quasistatic, meaning at each time step velocities and accelerations of the system are 0. This corresponds to having a high amount of damping and is a reasonable assumption in the 2D planar pushing setup where we restrict pushing velocities to be low and there is a large amount of friction between the object and table surface. As a result, the state of our system only consists of positions.

B. Analytically Smoothed Contact Dynamics

In this work we use the analytically smoothed contact dynamics and corresponding simulator developed by [1]. Contact dynamics are formulated as an unconstrained convex program where the contact and friction contraints are moved into the objective function using a log barrier function. The effect of this is that there is a log barrier penalty for violating the contact constraints. Constraints can exert force even if they are not active and this translates to producing a force at a distance.

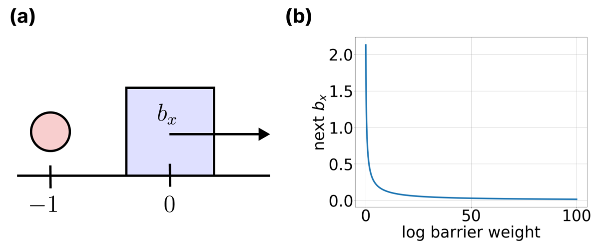

Fig. 1. Force at a distance effect of smoothed contact dynamics. (**a**) A system consisting of an actuated point finger at $x = -1$ and unactuated box at $x = 0$, (**b**) The next b_x after rolling out one step of the analytically smoothed dynamics with different log barrier weights.

We plot the force at a distance effect of the smoothed contact dynamics in Figure 1. We see that for a high weight, which corresponds to a small force at a distance, the next b_x is close to 0, but as the log barrier weight decreases, the box is pushed further to the right.

C. Planar Pushing System

The state of

$$x = \begin{bmatrix} b_x \\ b_y \\ b_\theta \\ s_x \\ s_y \end{bmatrix} \tag{1}$$

control input u are absolute position commands for the sphere

$$u = \begin{bmatrix} u_x \\ u_y \end{bmatrix} \tag{2}$$

The system evolves in discrete time and is control affine and nonlinear.

The differential dynamics are defined as

$$\delta_{x_k} = A(x_k)\delta_{x_k} + B(x_k)\delta_{u_k} \tag{3}$$

We can define the state feedback control law

$$\delta_{u_k} = K(x_k)\delta_{x_k} \tag{4}$$

Generalized infinitesimal squared distance in the positive definite metric M is denoted V_k

$$V_k = \delta_{x_k}^\top M_k \delta_{x_k} \tag{5}$$

And by substituting the differential dynamics and control law, we can see that the generalized infinitesimal squared distance at the next time step is

$$\begin{aligned} V_{k+1} &= \delta_{x_{k+1}}^\top M_{k+1} \delta_{x_{k+1}} \\ &= \delta_{x_k}^\top (A_k + B_k K_k)^\top M_{k+1} (A_k + B_k K_k) \delta_{x_k} \end{aligned} \tag{6}$$

The contraction condition can then be expressed as

$$V_{k+1} - V_k \leq -\beta V_k < 0 \tag{7}$$

which simplifies to

$$(A_k + B_k K_k)^\top M_{k+1} (A_k + B_k K_k) - (1-\beta) M_k < 0 \tag{8}$$

[3] showed that equation 8 can be transformed via Schur's complement (among other transformations) into

$$\begin{bmatrix} W_{k+1} & A_k + B_k L_k \\ (A_k + B_k L_k)^\top & (1-\beta) W_k \end{bmatrix} > 0 \tag{9}$$

where $W := M^{-1}$ and $L := KW$

III. Methods

A. Contraction Metric and Controller Synthesis

1) Sum of Squares (SOS) Programming:

2) Enforcing Contraction on Samples:

3) Sampling Strategy:

B. Online Geodesic and Controller Computation

References

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