University of Basel

Bayesian probability

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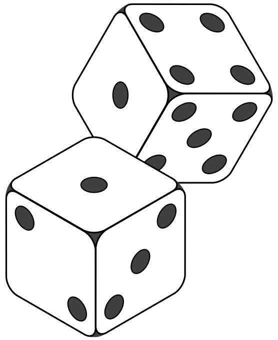
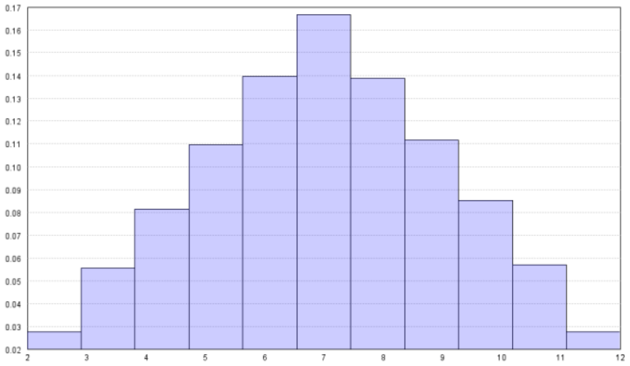
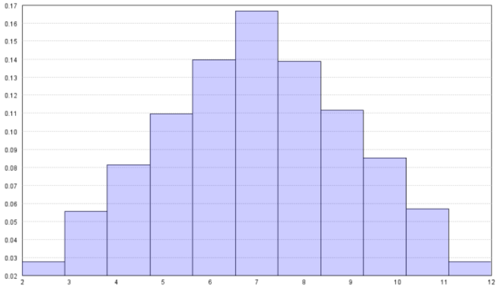
Different interpretations of probabilities

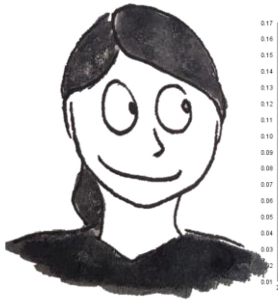
Long-term frequencies

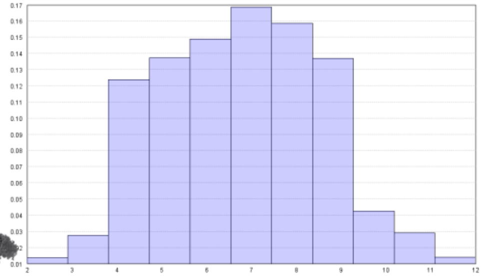




Degree of belief (Bayesian probability)





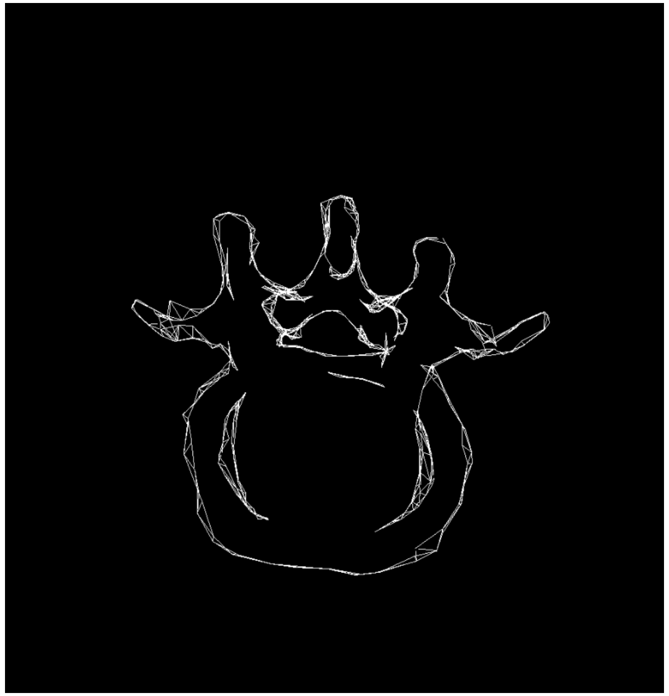


Bayesian probability in shape and image analysis

Sources of uncertainty in image and shape analysis

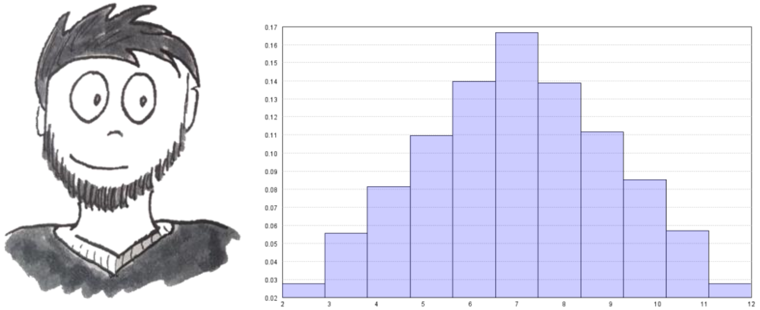
- Measurement noise
- Calibration of acquisition device
- Limited measurement accuracy
- Missing data

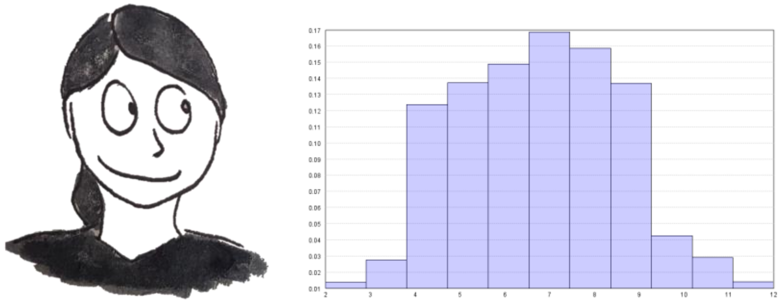
Repeating a measurement does not give us much more information.



Bayesian probability

Bayesian probabilities rely on a *subjective* perspective:

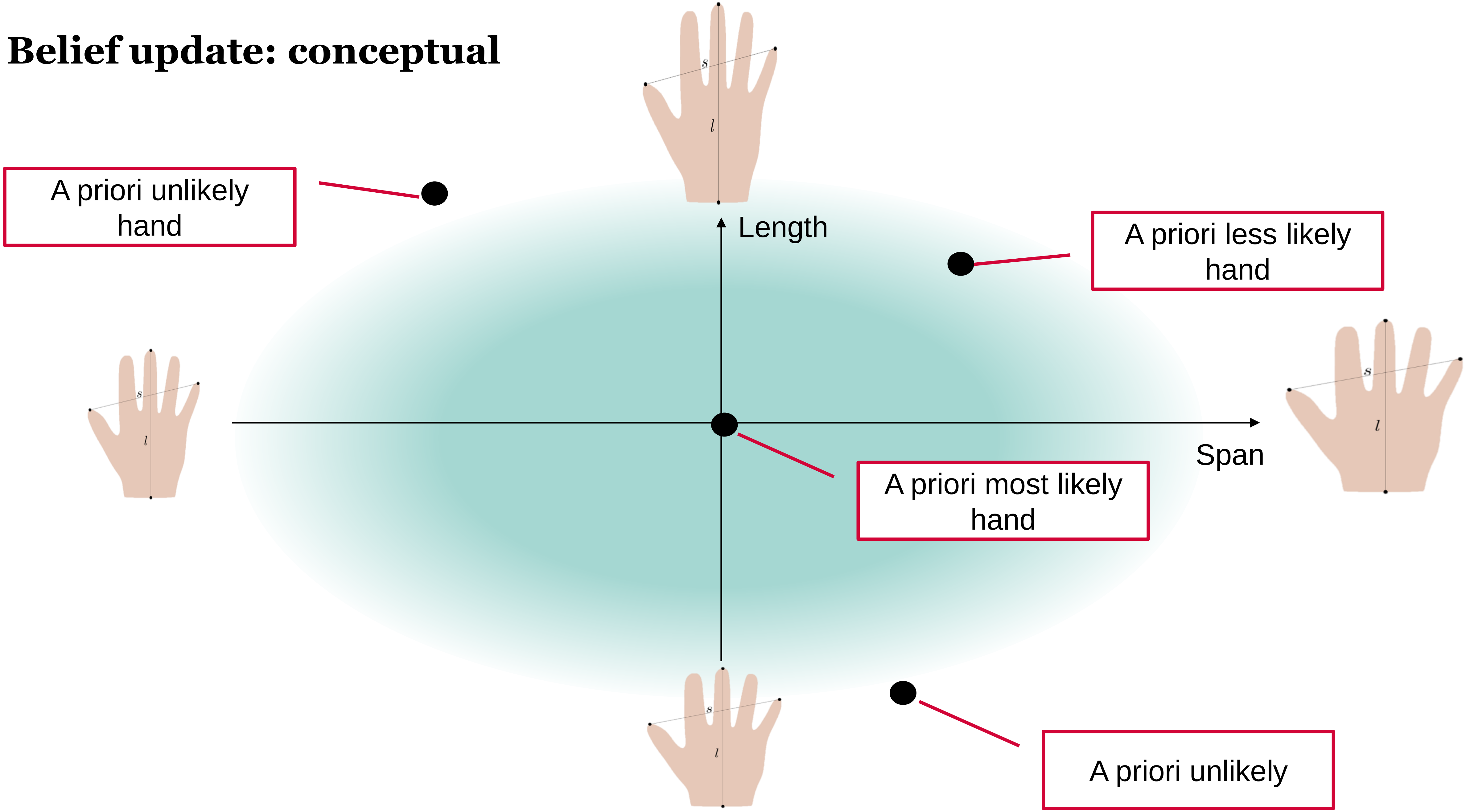




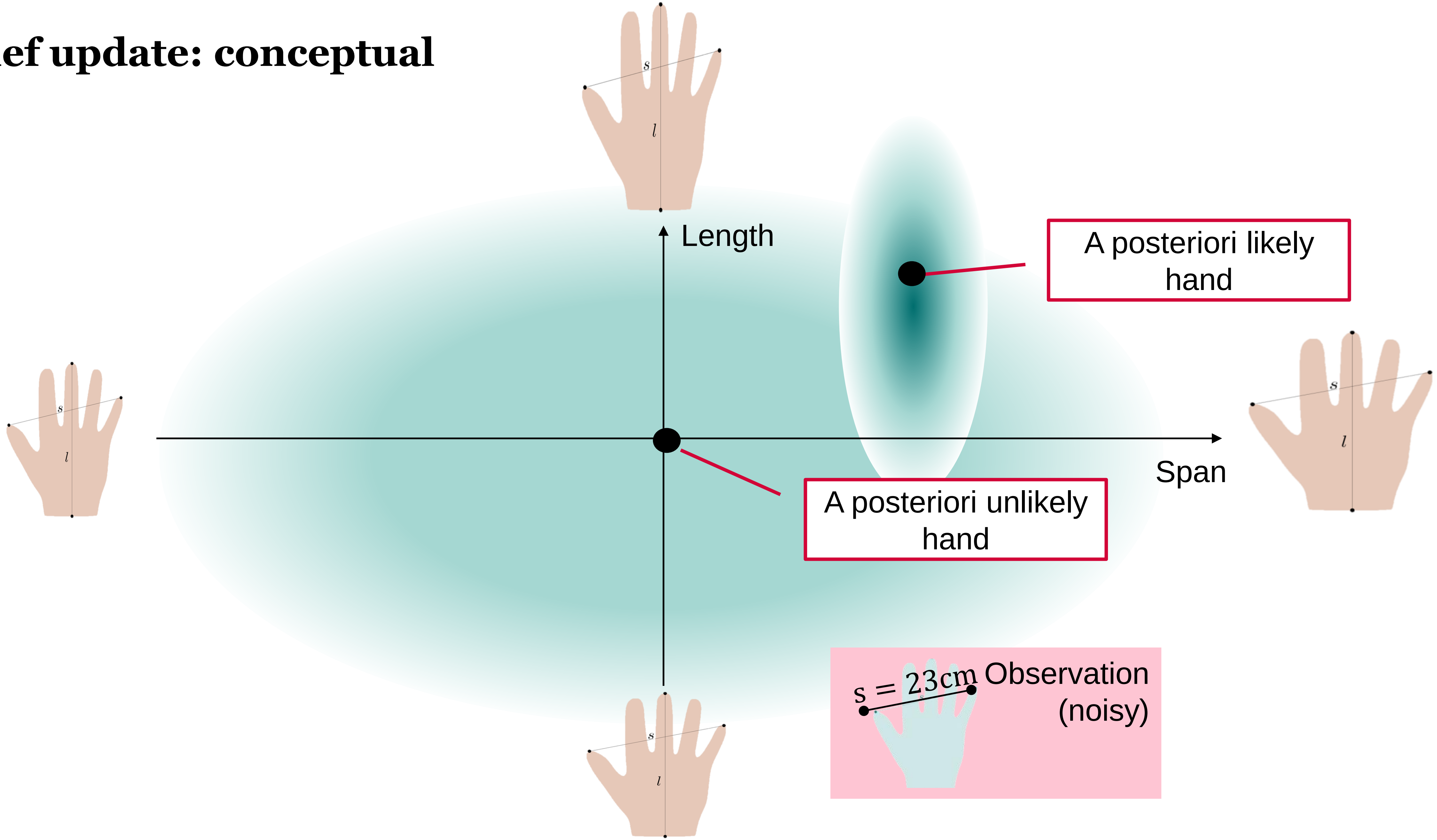
Subjective is not arbitrary!

- Belief are updated when new data becomes available
- Belief update follow rules of probability theory.

Belief update: conceptual



Belief update: conceptual



Bayes rule

Rule for updating our belief in X, after we have observed data Y

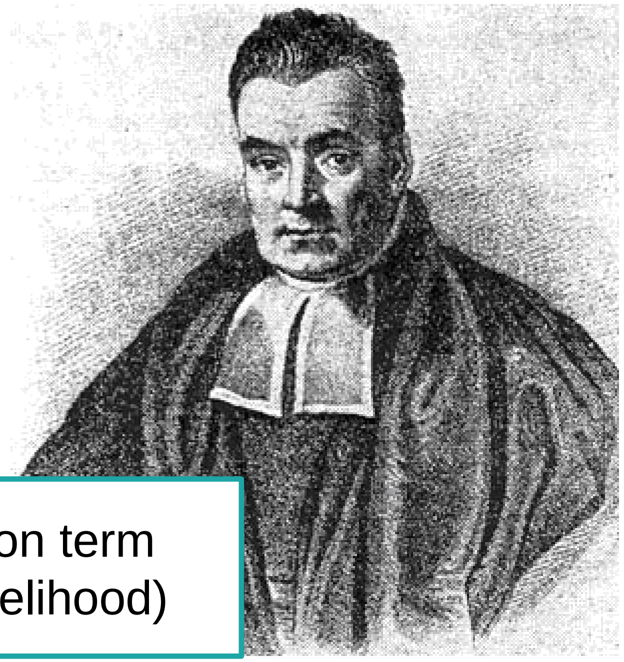
Likelihood

Prior

$$P(X|Y) = \frac{P(Y|X)P(X)}{P(Y)} = \frac{P(Y|X)P(X)}{\int_X P(X)P(Y|X)}$$

Posterior

Normalization term
(marginal likelihood)



Example: Belief update for the hand model

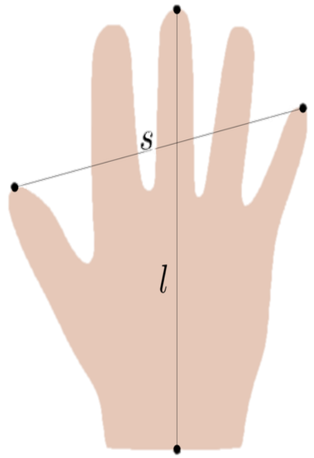
Model

1. *Length of an average hand is approximately 24 centimeter*
   ca 2/3 of the hands are within a range of ± 2 *cm*

$$p(L) = N(24, 2)$$

2. *On average, length and span are approximately the same*
   The span varies more than the length. 2/3 of the hands are within a 4 cm range

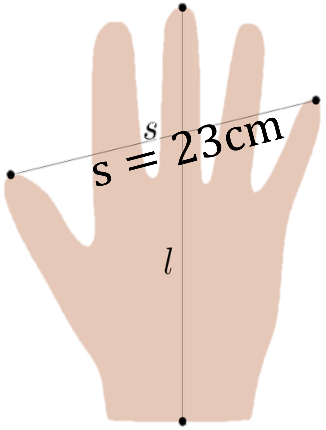
$$p(S|L = l) = N(l, 4)$$



Example: Belief update for the hand model

Belief update

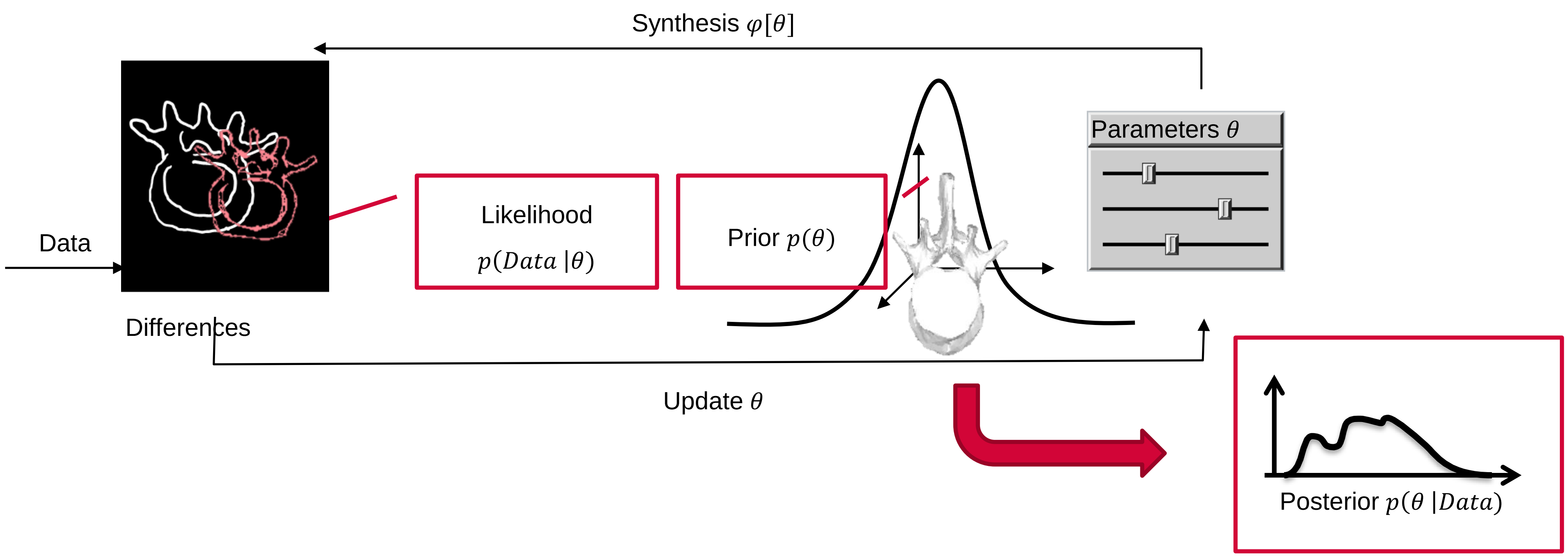
$$p(L|S=23) = \frac{p(S=23|L)p(L)}{p(S=23)} = \frac{p(S=23|L)p(L)}{\int p(S=23|L)p(L)dL}$$



All quantities are known

- *Everything can be computed (in principle)*

Bayesian probabilities in our Project



To specify:

- Prior distribution over parameters θ
- Likelihood function $p(Data\,|\theta)$
  - Contains synthesis function: How parameters map to contours

To compute:

- Posterior