Folk Theorem and, in particular, at the thresholds where cooperation is more beneficial than defection in the game of a Prisoners' Dilemma.

Student: Sophie Shapcott

Supervisor: Dr Vincent Knight

Academic Year: 2019/20

School of Mathematics,
Cardiff University

A final year undergraduate project submitted in partial fulfiment of the requirements for MMORS (Masters in Mathematics, Operational Research and Statistics) taught programme.

SUMMARY

ACKNOWLEDGEMENTS

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Chapter 1

Literature Review

The folk theorems are a class of results which generally state that for repeated games, any feasible and individually rational payoff vector can be achieved as a subgame perfect equilibrium if the players are patient enough [27]. The origin of these theorems is unknown however written proof and research involving these ideas first appeared in the 1970s [15, 4, 33]. Since then, many generalisations and refinements of the ideas have been explored, for differing games, including: games with private monitoring [29, 21, 32], sequential games [19, 7, 36] and games of complete information [5, 1, 6], to name a few. Due to the identification of further equilibria (as compared with the stage game), which are key in predicting future behaviour, these theorems have been commented on as 'fundamental' in the theory of non-cooperative games [21, 25]. On the other hand, the majority of the strategies used in the proofs assume the identification of individual deviators [28], which may not be realistic in certain situations. Hence, an area of research on the so-called 'anti-folk theorems' was introduced [28, 39, 32]. Folk theorems appear to still be an active area of research today [22, 31, 34] with many differing applications. Therefore, in this chapter, a review of the literature on this topic is provided with papers ranging from the 'original' ideas in the 1970s to applications of the theorems in 2020.

1.1 First Papers

[15] is often cited as the earliest work on Folk-type theorems [1]. In [15], infinitely repeated games with discounting are considered. In particular, focus is on a class of strategies, now known as *grim trigger*, which are used to prove that, for any feasible and individually rational payoff vector, there exists a discount parameter such that a subgame perfect equilibrium with payoffs equal to that vector

exist. This is first shown under the constraints (in addition to those made on the strategy and payoff spaces) identical stage games, constant discount parameter and unique Nash equilibrium that is not Pareto optimal. However, a more general result is then given which removes these restrictive conditions. [15] uses the application of oligopoly throughout in addition to introducing the notation of 'temptation'. He motivates this through the explanation that 'threat' is no longer "credible" since players are unable to communicate in non-cooperative games. On the other hand, 'temptation' is said to be analogous to 'threat'.

In contrast to this, [4] presents folk theorems for infinite games without discounting, assuming payoffs take a "limiting average form". This choice is justified by stating that, as the discount rate approaches zero, the limit of the discounted sum behaves similarly to the limiting average payoff form. However, trigger strategies are still required in the proof. For simplicity, two player games are assumed throughout. Two folk theorems are proven in this paper with the latter one discussing subgame perfect equilibria similar to [15], whilst the former is a more generic version of the theorem. [4] discuss that, though the generic version of the theorem does exist, the subgame perfect equilibrium points allow for more 'believable' behaviour. They conclude by considering an example in which payoffs are discounted. A similar approach is taken in [33], with the statement of a folk theorem for infinite games with no discounting and the existence of subgame perfect equilibria. The only difference being the use of an "overtaking criterion" evaluation relation instead of a "limit of the mean criterion" evaluation relation as in [4].

1.2 Games with Complete Information

[1, 6] both study folk theorems of games assuming complete information. That is, all players have common knowledge of payoff functions, strategies etc. [1] focuses on the necessary and sufficient conditions required for a folk theorem proof. They state that feasibility and individual rationality of the payoff vector are necessary conditions and also approximately sufficient conditions for a payoff to be in the equilibrium. This is followed by a discussion on the full dimensionality constraint first introduced by [17] and often used in proving folk theorems. [1] highlight that the requirement of the convex hull, of the set of feasible payoffs, to be equal to the number of players is a sufficient condition. This is motivation for the main result of the paper; the introduction of a 'non equivalent utilities' condition, which is proved to be sufficient and almost necessary for a folk theorem. The condition, as indicated here, is weaker than the aforementioned full dimensionality condition.

It only requires “no pair of players have equivalent utility functions”.

Similarly, [6] proves a folk theorem for complete information games with subgame perfect equilibria however, instead of exponential discounting (as in [1, 17]), he assumes discounting is present-biased. That is, the discount function implies a player is more willing to alter an event in the future than altering an event today. The folk theorem is proved in cases of where the player is time consistent (prefers to maximise their initial preferences) and time inconsistent (prefers to maximise current preferences) and the notion of ‘patience’ is taken to be the sum of all discount factors. Also considering time consistent and inconsistent players [27] discusses folk theorems with respect to time dependent discounting. In contrast to [6], the long-term characterisation of ‘patience’ is taken as discount factors at all stages uniformly converge to one. Motivation for the study of time dependent discounting is explained by empirical studies which seem to suggest that a player’s time is unstationary, rather than the stationarity assumed in most discounted game models.

1.3 Games with (Imperfect) Private Monitoring

In the late 1990s attention turned to games with imperfect monitoring. That is, a player’s actions can no longer be observed accurately, instead public or private signals are detected [11]. Games with (imperfect) public monitoring were the fist to be considered [29]. More recent studies in this area include [24, 9]. [24] states that repeated games with public monitoring give a suitable model for studying long term relationships. They state intuition suggests that cooperation becomes easier to sustain as action observability improves. However, [24] continues to show that this intuition is false, by considering a repeated public monitored game which satisfies “the limit perfect public equilibrium payoff set can achieve full efficiency asymptotically as public information becomes less sensitive to hidden actions.” They give an example which violates the sufficient condition given in Fudenberg et al. (1994) [FIND THIS PAPER] yet a folk theorem can be obtained. This paper mainly focuses on the Prisoner’s Dilemma but, similarly to [1], they state an example which satisfies the folk theorem without the full dimensionality condition.

[9] consider the notion of a robust equilibrium to incomplete information; that is, if the equilibrium yielded is near the original equilibrium for all perturbed games consisting of a small independent and identically distributed information shock.

Conversely to [15], they discuss the implication of grim trigger strategies not being robust when the aim is to sustain cooperation. A folk theorem is proved in robust equilibria for games of public monitoring, however [9] highlights that a much stronger condition is needed in comparison to Fudenberg et al. (1994) [FIND THIS PAPER] full dimensionality requirement.

As of 2004, repeated games with private monitoring was a relatively new area of study [29]. In this paper, conditional independence of signals is assumed in order to show a folk theorem for the IPD. Applications to duopolies are then described. Another study to consider a folk theorem for the IPD is [14]. Using a similar definition of robustness to [9], [14] proves that for a discounted Prisoner's Dilemma with private monitoring technologies, a folk theorem with robust equilibrium strategies can be obtained. In particular, they consider almost-perfect private monitoring and a limit folk theorem (for sequential equilibria) follows. In a similar paper, [13] introduce the idea of "belief free" equilibrium strategies; a property which implies the belief of an opponent's history is not required when obtaining a best response. They use these strategies in proving a folk theorem for the two-player Prisoner's Dilemma however, for general games, the set of belief free payoffs is not large enough to provide a folk theorem. Moreover, [13] highlight that for a larger number of players the calculation of the payoff set becomes significantly harder.

In contrast to this, [21] discusses a folk theorem using strategies that, although are not belief free, still make beliefs "irrelevant" at the start of each T-block of stage games. Their result is much more generic than [13] since it applies to N-player finite games, under the assumption of full dimensionality with private but almost-perfect monitoring. The T-block strategies mentioned above are modified in [37, 38] such that they "can support any vector in the belief free equilibrium payoff set". This modification results in belief free equilibrium strategies. [37, 38] generalise the results from the two-player Prisoner's Dilemma in [13] to the corresponding N-player game.

1.4 Games with Communication

Academics introduced games with communication to deal with the complications faced with imperfect monitoring. For example, games with public monitoring can obtain a folk theorem under weaker assumptions than those given in Fudenberg et al. (1994) [FIND THIS PAPER] if communication is introduced [23]. In this paper, communication is a message taken from the set of possible actions which the players give simultaneously after choosing an action and observing a signal.

[23] prove a folk theorem for symmetric games with four or more players without the assumption of the number of signals relevant to the number of actions.

Regarding private monitoring, [30] claims repeated games are "very difficult to analyse without communication". Thus examples of papers, proving folk theorems for these games with communication, include [16, 30, 26]. [16] proves a 'Nash threats' folk theorem, similar to [15], in the case of almost public information games without independent signals for two players. In this paper, communication is defined in the form of announcements where each set of announcements is the same for all players. They justify the decision to only consider two player games by highlighting that, although the results can be generalised, in certain cases it can be seen as advantageous to have additional players. Similarly, [30] proves a Nash threats folk theorem for private monitoring games with communication. He increases the number of environments where the folk theorem is applicable through developing further the idea of "delayed communication", as given in Compte [FIND THIS PAPER]. This paper, in addition, uses the assumptions of correlated private signals and each player's deviation from strategy is statistically identifiable from the other players' signals. [26] also considers a model of private monitoring and communication within games. He has the aim of increasing the number of applicable environments for Kandori and Matsushima (1998) [FIND THIS PAPER] frequent communication folk theorem. The paper states that Kandori and Matsushima (1998) [FIND THIS PAPER] assumed only private signals were publicised in this theorem but if other information was useful and communication was free and legal then players would also want to share their actions. This motivates the reasoning of the paper. However, assumptions of both full dimensionality, and the number of actions and signals, are still required.

Another paper which considers communication is [8], regarding self referential games with codes of conduct. These codes are descriptions of how the players and opponents should play. An application to computer algorithms is provided. Two folk theorems are proved: one assuming common knowledge of the codes of conduct, and the other where only certain players observe certain codes. The latter is the main result of the paper and is motivated by the fact that, often, individuals have good knowledge of those closest to them but not the whole community. [8] obtain the sufficient condition that, with public communication, if every player is observed by another two opponents, then a folk theorem is yielded.

1.5 Finite Horizon Games

Another area of interest is the existence of folk theorem-type results for games of finite repetition. [5] explores the case of finitely-repeated games of complete information and the associate subgame perfect equilibria. They state that, despite the existence of games, when repeated finitely, "produce no non cooperative equilibrium outcomes" besides those of the single game, there may be subgame perfect equilibria of finite-repeated games when the corresponding single game has multiple equilibria. Indeed, using their "three phase punishment", [5] prove that "any rational and feasible payoff vector can be obtained in the limit", assuming the feasible payoff region has dimension equal to the number of players and each player has two Nash equilibrium payoff values. In a similar manner, [3] considers an alternative version of the Prisoner's Dilemma in which an additional strategy is included. This gives a second pure-strategy equilibrium and a folk theorem result for a finitely repeated version of this game.

Although not strictly a finite game, [18] study a repeated game in which players may "strategically terminate" it. In particular, this involves the incorporation of a voting-step at the start of each repetition where a certain number of players decide whether to keep interacting. This is motivated by the increasing possibilities of ending business partnerships, due to more technology and knowledge. They prove a general folk theorem for any stage game (with the additional voting) which is satisfied "for all majority rules except the unanimous ending". Indeed, for the unanimous ending rule, they show that the theorem may not hold but sufficient conditions are provided for when it is satisfied.

1.6 Stochastic and Sequential Games

Other game types to have associated folk theorem results include stochastic [12] and overlapping generation [7, 19].

[12] explains that, often, the standard assumption of "a completely unchanging environment", within the theory of repeated games, is not reliable in applications. This reasoning is the motivation for studying the more generic stochastic games. These games may not have a pre-decided stage game, instead a 'state variable' is used to represent its environment which alters according to "initial conditions, player's actions, and the transition law". The paper discusses equilibrium payoffs in the case of very patient players without the need for the Markovian property. Specifically, perfect monitoring is assumed along with asymptotic state independence and either of payoff asymmetry or full dimensionality. [12] proves

two folk theorems: one with unobservable mixed strategies (in which case, fully dimensionality is required) and also, similar to [1], one with the slightly weaker condition of payoff asymmetry (here, mixed strategies have to be observed).

Both[7, 19] provide insight into folk theorems associated with overlapping generation games. These are similar to the repeated normal form games except that the set of players are considered to be finite. That is, each player is involved in a certain number of stage games before they are replaced by another, identical player. [19] proves a variety of folk theorems both with and without discounting and / or observable mixed strategies. He shows that the full dimensionality assumption is not required in these games since players are assumed to not end simultaneously. On the other hand, [7], although does provide a folk theorem, his main result is an anti-folk theorem, see 1.7. He studies games with imperfect public monitoring and states that, in such overlapping generational games, cooperation becomes impossible in contrast to repeated games. [7] constructs a mixed strategy equilibrium folk theorem however he goes on to show that these strategies are unstable to perturbations, resulting in an anti-folk theorem.

A similar study by [2] looks at dynastic repeated games. These differ from the overlapping generation games in two aspects: perfect observation of the past is assumed, and the payoffs obtain no dynastic component. Under the assumptions of full dimensionality and the existence of a payoff vector which strictly Pareto dominates the stage game equilibrium, [2] proves a folk theorem for private communication games with greater than three players in sequential equilibria.

Considering now sequential games, in which players do not choose their action simultaneously, [36] introduces a concept of effective minimax values before proving a corresponding folk theorem. In his model, players pick actions in groups and thus the effective minimax value is defined to be the lowest equilibrium payoff a player will receive even if none of their opponents have equivalent utilities. [36] claims his folk theorem can be applied to other game models as it is a “uniform characterisation”.

1.7 Anti-Folk Theorems

A common theme in the proofs of folk theorems is the use of strategies which “identify and punish” deviators [28]. However, as soon as the game contains incomplete / imperfect information, deviators cannot necessarily be identified. This yields a much smaller equilibrium set and these results are termed “Anti-Folk Theorems”. This description of anti-folk theorems is adapted from [28], who state the original term was given in Kaneko (1982) and Dubey & Kaneko (1984) [FIND PAPERS]. [28] prove an anti-folk theorem using the "long-run average criterion" in contrast to the discounted criterion.

Considering similar models to [7, 19], [39] obtain an anti-folk theorem for a limited observability overlapping generations model. They show that cooperation cannot be sustained when new players can only observe recent history. This is in contrast to the folk theorems obtained under the assumption of common knowledge of all past actions. Though their results are restrictive in certain cases, they justify the work by stating it is suitable for modelling "high turnover" rates.

Another game model where an anti-folk theorem has resulted is a repeated game with private monitoring [32]. In this paper, the following three assumptions are made: infinite and connected private monitoring (that is, infinitely many connected signals); finite past; and independent and identically distributed shocks affect the payoffs. Under these assumptions [32] shows the violation of the folk theorem. That is, the equilibria of the repeated game consist only of the equilibria of the single game.

1.8 Evolutionary Stability

Recently, there has been research into the results of the folk theorem with respect to the evolutionary game theoretic paradigm. Indeed, [25] state that the folk theorem is often used to characterise evolutionary stable strategies since exact solutions using theoretical results from evolutionary game theory are hard to obtain. However, they show the assumption that the folk theorem yields all Nash equilibria to be misleading. This is achieved by defining "type-k equilibria" which are a refinement of the Nash equilibria. [25] go on to prove that the set of type-k equilibria is contained within the set of repeated game Nash equilibria using "reactive strategies".

In contrast to [37, 38, 13] who discuss folk theorems using belief-free strategies, [20] discusses the instability in an evolutionary sense. He shows that the belief-free equilibria are not robust to small perturbations in games with private monitoring and, in certain cases, this is extreme. Similar to [25], he states that Nash equilibria are used to predict evolutionary behaviour since they are thought of as stable. However, he goes on to show that only the choice of repeated stage game Nash equilibria satisfy evolutionary stability.

1.9 Recent Applications

In recent years, studies have been applying results of the folk theorem in various areas for creating algorithms, for example. This section briefly discusses a few of these.

[10] uses the folk theorem in the creation of a model which aims to suppress the effects of distributed denial of service attacks. They claim that all networks suffer from attacks to infrastructure and services. Thus, they use the programmability of software defined network environments to perform a game theoretic analysis. [10] create an algorithm for reward and punishment based on the Nash folk theorem. Similarly, [34] make use of the cooperative equilibrium solution from [15]s folk theorem to create an algorithm suggested to optimise a multi-period production planning based real-time scheduling method for a job shop. Moreover, [35] uses the result of the folk theorem in the IPD to study the cooperation rates of varying agent strategies in a multi-agent system.

Another application of the folk theorem is an algorithm used to obtain equilibria of a discounted repeated game [31]. [31] introduce a new algorithm, entitled "Communicate & Agree", which can find equilibria in incomplete information, but perfect monitoring, games. Using the folk theorem in the algorithm enables the payoffs obtained to be potentially higher than those achieved by repeating the Nash equilibria of the single game. However, [31] go on to highlight that the algorithm is not guaranteed to always find equilibria. They say it is dependent on: the discount factor, sampling density and whether it is a zero-sum game or not. Finally, [22] discuss the potential of using game theoretic ideas in quantum optimal transport. In particular, they define the Quantum Prisoner's Dilemma and explore the possibility of a quantum folk theorem in relation to the corresponding repeated game.

1.10 Conclusion

In this section, an overview into research regarding the folk theorem has been provided. The research history of the folk theorem spans from the 1970s until now, with many different models being considered. Examples include: games with complete information, games with imperfect private monitoring and finite-horizon games. However, there has also been studies into situations where the folk theorem does not hold, or the equilibrium strategies used in proving the theorems are unstable and / or not robust.

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Appendix A

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Appendix B

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