Folk Theorem and, in particular, at the thresholds where cooperation is more beneficial than defection in the game of a Prisoners' Dilemma.

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SUMMARY

ACKNOWLEDGEMENTS

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Chapter 1

Analyses

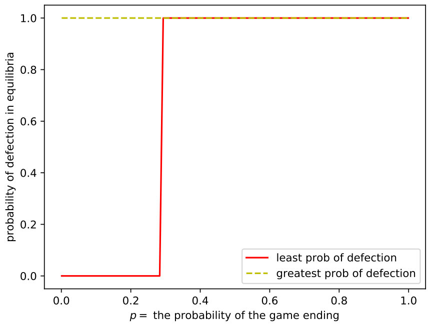
In this chapter, an analysis of the data collected via the methods described in the previous chapter (Chapter ??) is given. Firstly, a brief initial overview is provided, where descriptive statistics of the equilibria obtained and the overall characteristics of the strategies used is discussed. Following this, a critical analysis of the p-thresholds obtained is carried out. Here, the environmental effects, on the outcome of the game, discussed include: number and characteristics of opponents; noise; and degeneracy. Then a large-scale multivariate analysis is executed before considering the reliability of the collected data. Note, as of writing, the database currently has 825700 entries (rows) and a total number of 159 tournament sets.

1.1 Initial Analysis

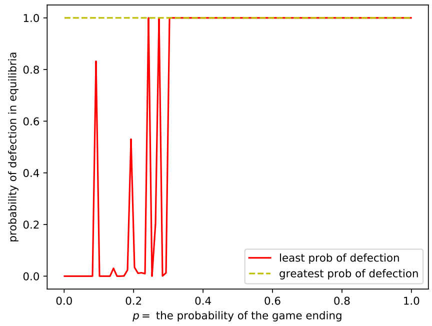
In this section, all the data (including those games which could be degenerate) are considered. Taking a brief look at the graphs produced for each set of tournaments, it can be seen that the main 'shapes' obtained are as seen in Figure 1.1.

This will be discussed further in the next section as follows: Figures 1.1a and 1.1d, and general properties of the p-threshold, will be described in Section 1.2; the stochasticity and accuracy visible in Figure 1.1b will be analysed in Sections ?? and 1.4.2, respectively; and finally Figure 1.1c, and degeneracy overall, will be considered in Section 1.2.4.

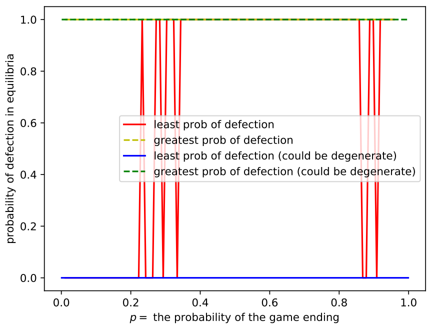
The summary statistics gained from running the *describe* method of a pandas database is given in Table 1.1. From this, it can be seen that the number of opponents the *Defector* played against ranged from one to seven, with an average



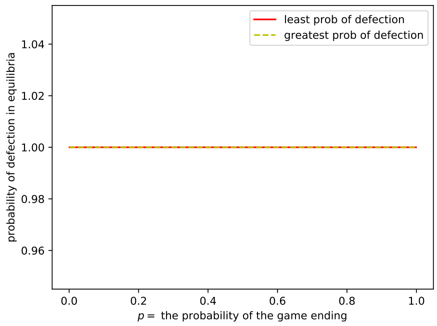
(a) An example of a graph with a clear p-threshold point of approximately 0.28. In this game there is no degeneracy and the opponent strategy playing in the tournament was *Inverse*, without noise.



(b) An example of a graph where the p-threshold is not as clear. Perhaps this is due to a small amount of noise and hence not enough repetitions or stochasticity of the players. In this case the threshold seems to lie in the range [0.1, 0.3]. There is no degeneracy in this game and opponent strategies in the tournament were: *Feld: 1.0, 0.5, 200*; *Cooperator*; *EvolvedLookerUp2_2_2*; *Tullock: 11*; and *ZD-GEN-2: 0.125, 0.5, 3*. Again, this tournament was run with no added noise.



(c) An example of a graph where the p-threshold is not clear at all due to some (or all) of the games, which were played by the particular tournament set, being degenerate. In this case, the opponent strategies playing in the tournament were: *Random: 0.5*; *Grumpy: Nice, 10, -10*; *Fortress3*; and *Negation*. This tournament had no added noise.



(d) An example of a graph where the least probability of defection was constant throughout the varying ending probabilities. In this particular tournament there was no ending probability in which cooperation was beneficial; with the least probability of defection equalling the greatest probability of defection at one. Here, the opponents were: *AntiCycler*; *e*; and *Stalker: (D,)*.

Figure 1.1: Example graphs obtained from the experiment.

Table 1.1: A table of the summary statistics produced from the data of the experiment.

	experiment_number	number_of_players	tournament_player_set	num_of_equilibria
mean	107364.481228	5.435509	97.105850	1.913727
std	46880.538807	1.726832	42.619612	2.022014
min	0.000000	2.000000	0.000000	1.000000
25%	72231.000000	4.000000	65.000000	1.000000
50%	114641.000000	6.000000	104.000000	1.000000
75%	147396.000000	7.000000	133.000000	3.000000
max	175399.000000	8.000000	159.000000	39.000000

of four opponents. Also, as expected, the mean probability of the game ending encountered was 0.5. Observe that, overall, there were $175,399$ distinct tournaments played (*experiment_number*) with a total of 159 distinct sets of strategies (*tournament_player_set*). Looking now at the statistics for Nash equilibria, it can be seen that a total of $823,823$ equilibrium points were calculated in this experiment, with an average of $1.914 \approx 2$ equilibria per game. However, observe, at least one game obtained 39 equilibria which will be explored into later on in this section. Considering the probabilities of defection within these equilibria, notice that both the greatest and the least probabilities of defection ranged from zero to one inclusive with a 50th percentile of zero. But, looking at the average values, the least probability has a mean of 0.342 and only just above this, the greatest probability has a mean of 0.460.

Next, further descriptive statistics are calculated for the strategies. This is to obtain a more in-depth view on the types of strategies randomly chosen to play and their characteristics. Executing *value_counts* method on the column of strategy names, it is observed that the player which appeared the most times (9 times) is *ZD-GEN-2: 0.125, 0.5, 3*; followed closely by *Tideman and Chieruzzi* with 7 sets of tournaments. On the other hand 38 out of the 200 strategies playing in this experiment appeared only once. Running the *value_counts* method again, but this time on the memory depths of the strategies found the majority of strategies to have an infinite memory depth. On the other hand, strategies having no memory or a depth equal to one were also significant. Considering the stochasticity of players alongside how many appearances each strategy made yielded the following chart in Figure 1.2.

It is clear that there is a clear bias towards deterministic strategies in this exper-

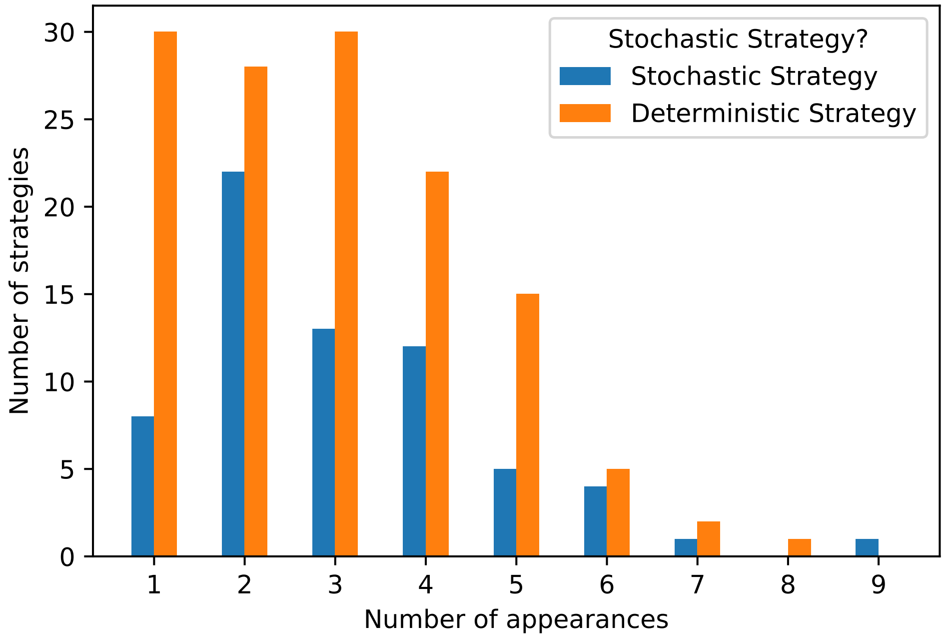


Figure 1.2: A plot to show the ratio of stochastic to deterministic strategies randomly chosen throughout the experiment.

iment. However, this is to be expected as running the following code:

```
len(axl.filtered_strategies(filterset={"stochastic": True})),
len(axl.filtered_strategies(filterset={"stochastic": False}))

(86, 156)
```

it can be seen that over half the strategies coded into the Axelrod library are classed as deterministic. Looking at Figure 1.2 again, observe, the majority of deterministic strategies were executed either once or three times. On the other hand, a large proportion of the stochastic strategies were played twice.

Further, the number of Nash equilibria obtained for each game was analysed and their distributions with respect to the number of opponents of the *Defector* plotted. Executing the *value_counts* method on the 'num_of_equilibria' column gave the conclusion that the majority of games (131773) yielded one Nash equilibria with 28793 games obtaining 3 equilibria. Also, the maximum number of equilibria yielded, 39, was for one game, which, doing a search on the database, was found to be a six player game with noise=0.1. The opponents of the *Defector* were: *Inverse Punisher*; *Prober*; *PSO Gambler 2_2_2 Noise 05*; *Handshake*; and *More Tideman and Chieruzzi*.

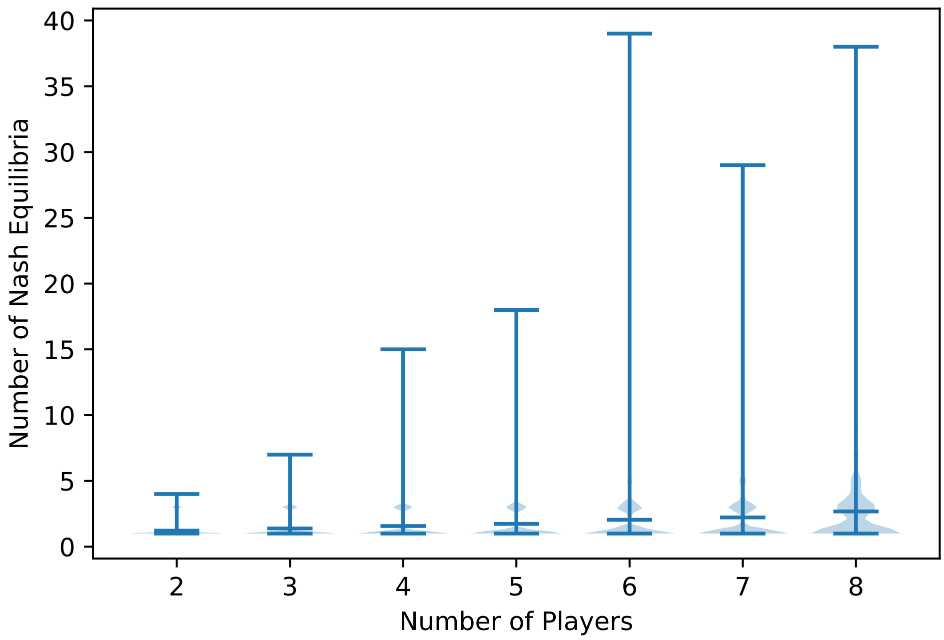


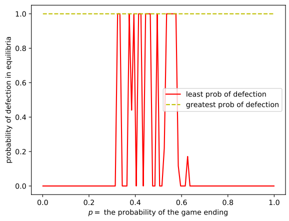
Figure 1.3: A violinplot showing the distribution of the number of equilibria obtained for varying number of players.

Figure 1.3 shows the distributions of the number of Nash equilibria as per the number of players. Observe, all of the distributions observed on this plot have a clear modal value of one, that is, irrespective of the number of players, the majority of games yielded only one equilibria. As can be seen from the plot, the variance in the number of equilibria increases with the number of players, apart from when there were 6 players (5 opponents of the defector), where the spread is maximum. This could be due to the 39 equilibria gained for one game as discussed in the paragraph above. Looking now at the mean of the distributions, observe that these are also slightly increasing as the number of players increase.

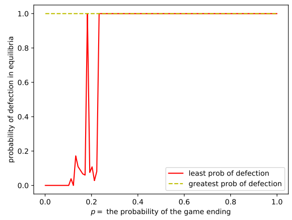
1.2 Analysis of the p-Threshold

In order to analyse the p-thresholds of the tournaments, two csv files were created [1] containing the minimum, mean, median and maximum probabilities for each set of tournaments. This was in order to gain as much information as possible from tournaments which gave graphs as shown in Figure 1.4. That is, tournaments in which the number of repetitions was not sufficient to omit the 'noise' which could affect the results.

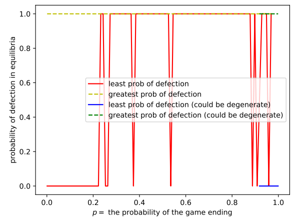
[1] Please see Appendix ?? for the code used to obtain these files.



(a) This was an eight player tournament with no added noise. The opponent strategies were: *Tester*; *Gradual Killer: (D, D, D, D, D, C, C)*; *Win-Shift Lose-Stay: D*; *Evolved FSM 16*; *Retaliate: 0.1, 20*; *GEN-2: 0.125, 0.5, 3*; and *Joss: 0.9*.



(b) This was a five player tournament with no added noise. The opponent strategies were: *WmAdams*; *Joss: 0.9*; *Limited Yamachi*; and *ZD- WSLS: 0.05*.



(c) This was a two player tournament with no added noise. The *Defector*'s opponent was *Stochastic Fortress3*.

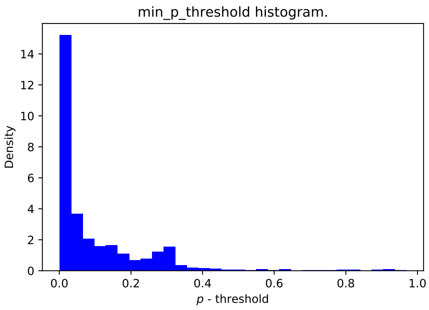
Figure 1.4: Examples of plots where the p-threshold isn't as clear.

For clarity, here is a restatement of the definition of a *p-threshold*: The probability of the tournament ending for which the least probability of defection in Nash equilibria is non-zero.

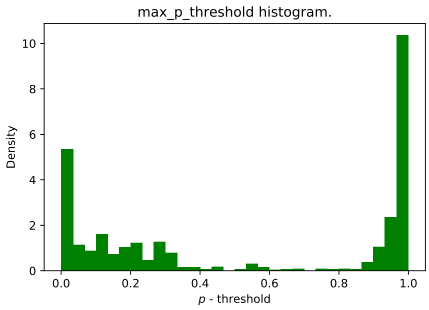
The first csv file created ignored whether a game could be degenerate and, by a small alteration, the second file contained only the non-degenerate games. This was in order to help identify whether degeneracy has any effect on the p-threshold. Within these files, other than the varying thresholds, the information about the number of players, tournament strategy set and level of noise was retained.

Firstly, an exploration into the overall p-thresholds is given.

Observe, in Figure 1.5a, the majority of minimum thresholds were less than 0.4, with a clear modal value of 0. That is, in a significant proportion of the tournaments, there was no probability of the game ending for which the probability of defection was zero. Now considering Figure 1.5b, it can be seen that the modal value for the maximum threshold is 1. Also, in comparison with the minimum threshold, there is a larger spread in the density. Yet, there is still a peak around zero as with the minimum threshold data. Looking at the mean and median threshold data, Figure 1.6, observe that the distributions obtained are similar with, again, a modal value of zero, indicating there is always a chance of defection irrespective of the game ending probability. However, in these histograms

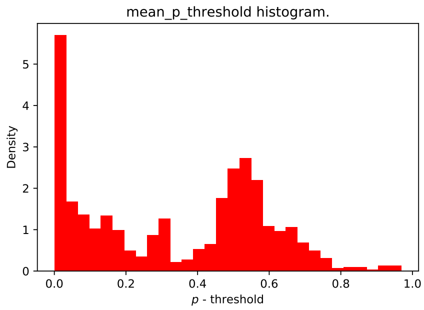


(a) A plot to show the minimum p-thresholds.

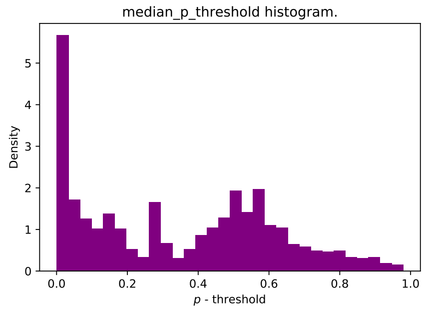


(b) A plot to show the maximum p-thresholds.

Figure 1.5: Plots to show the p-thresholds for all 1001 sets of tournaments.



(a) A plot to show the mean p-thresholds.



(b) A plot to show the median p-thresholds.

Figure 1.6: Plots to show the p-thresholds for all 1001 sets of tournaments.

there is also a clear peak around a threshold of 0.5. This suggests that some of the tournaments may have several p-thresholds, perhaps due to stochasticity or degeneracy. Indeed, obtaining the minimum and maximum p-thresholds for those tournaments where the mean threshold was within the range $[0.5, 0.6]$, it can be seen that, from Figure 1.7for a significant proportion, their minimum threshold was around zero and their maximum around one. Moreover, looking closer at three of the tournaments which satisfied the above, there is a clear stochasticity within the corresponding graphs, Figure **??**.

Before continuing onto the general overview of the thresholds for those tournaments which led to definite non-degenerate games, a brief point is made regarding those tournaments which, for all probabilities of the game ending, had no positive probability of defection. Indeed, out of the 1754 total tournaments 753 of them had the above property of no threshold. That is, the plots obtained for these tournaments were (approximately) constant at zero for all probabilities of the game ending. Further investigation into these tournaments will be executed

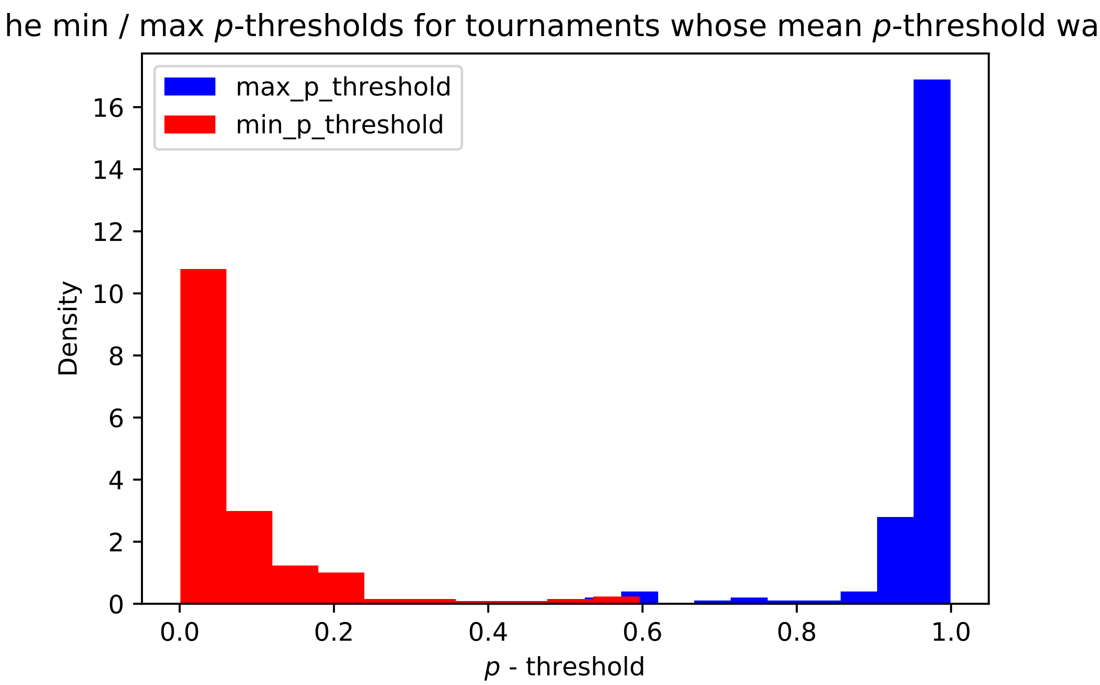
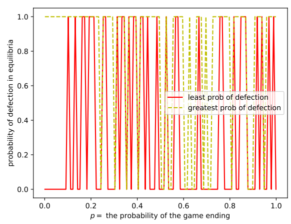
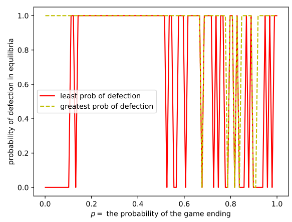


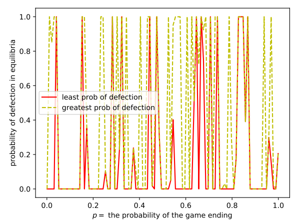
Figure 1.7: A plot to show the minimum and maximum p-thresholds for those tournaments which had an mean threshold within the range $[0.5, 0.6]$.



(a) This plot was obtained from the tournament in which the five opponents of the *Defector* were: *RichardHufford*; *Soft Joss: 0.9*; *Feld: 1.0, 0.5, 200*; *UsuallyDefects*; and *Handshake*. The tournament also included an additional 0.2 of noise.

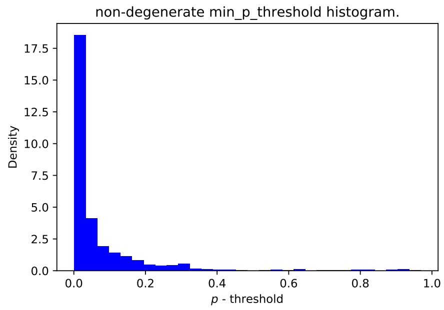


(b) This plot represents the, 0.1 noise, tournament with the three players: *Worse and Worse 2*; *Raider*; and, *Defector*.

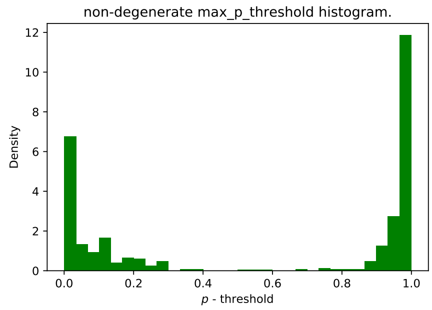


(c) This plot was produced from the tournament in which the *Defector* was playing *ZD-Extort-2: 0.1111111111111111, 0.5* and *Adaptive Tit For Tat: 0.5*. There was the addition of 0.5 noise within the tournament.

Figure 1.8: Example plots of the tournaments where the mean p-threshold was within the range $[0.5, 0.6]$.

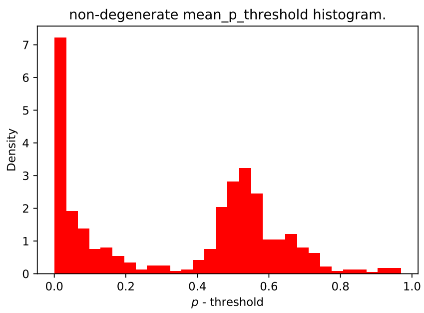


(a) A plot to show the minimum p-thresholds.

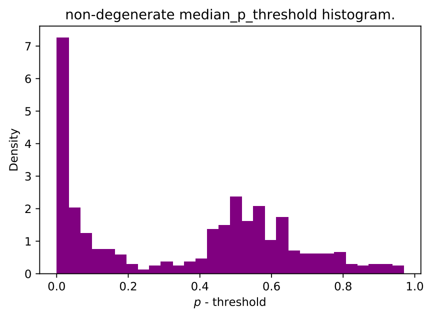


(b) A plot to show the maximum p-thresholds.

Figure 1.9: Plots to show the p-thresholds for all tournaments which led to non-degenerate games.



(a) A plot to show the mean p-thresholds.



(b) A plot to show the median p-thresholds.

Figure 1.10: Plots to show the p-thresholds for all tournaments which led to non-degenerate games.

later.

Regarding degeneracy, out of all 1754 tournaments, 372 were highlighted as potentially leading to degenerate games. Furthermore, out of those tournaments which had a p-threshold, 253 of them were identified as possibly degenerate. These are omitted from the following plots in order to focus solely on non-degenerate games.

Consider now, the overall p-threshold for those tournaments which lead to a non-degenerate game. Comparing Figures 1.9a, 1.9b, 1.10a and 1.10b with Figures 1.5a, 1.5b, 1.6a and 1.6b, respectively, it can be seen that the majority of thresholds remain the same with only a very small number changing. This could be due to the number of games that were removed since obtaining the length of both csv files suggested that only four full tournaments were removed. The effects of degeneracy will be investigated in more depth in Section 1.2.4.

`folk_thm/main_analysis/player_mean_thresh_violinplot.pdf`

Figure 1.11: A violinplot showing the distribution of mean p-thresholds for each number of opponents.

1.2.1 Effects of the Number of Players

In this section, the p-thresholds will be analysed with respect to the number of opponents the *Defector* played against. Note, in this section, only the mean p-thresholds of games which are non-degenerate will be discussed.

From Figure 1.11, it can be seen that the distributions of the p-thresholds with respect to the number of players are approximately the same. The means of the data falls around 0.7 with the main mode for each number of players being close to 1. Observe, though, that for the larger group of players (6, 7, 8), the distributions become bimodal with the second mode around 0.5. This is most significant in the eight player tournaments however, note that, as of writing, the experiment was still running for eight player games and hence less data is available currently for this situation.

1.2.2 Effects of Stochastic Players

In order to analyse the effect of stochastic strategies on the p-threshold, tournaments in which these strategies featured are repeated but omitting the stochastic players to identify any alterations in the results.

1.2.3 Effects of Noise

Here, an analysis on the effects of noise on the p-threshold is provided. The addition of noise to a tournament indicates that, with a certain probability, the action of a particular strategy is altered [1]. That is, an action of *coop* changes to *defect* and vice versa.

Figure 1.12 shows the mean p-thresholds for all non-degenerate tournaments with the addition of varying probabilities of noise. Firstly, observe that for those tournaments with the addition of noise at least 0.6 (Figures 1.12g, 1.12g, 1.12g, 1.12g, 1.12g) the majority of thresholds are equal to one. This is due to the definition of the parameter noise in the tournaments as described above, That is, there is a greater chance that the *Defector's* strategy will change to cooperation. Hence, only tournaments with noise≤ 0.5 will be considered.

1.2.4 Effects of Degeneracy

1.3 Multivariate Data Analysis

1.4 Reliability of Data

1.4.1 Comparison of Databases

1.4.2 Accuracy of Thresholds

(a) A plot to show the mean p-thresholds for all tournaments with no added noise.

(b) A plot to show the mean p-thresholds for all tournaments with noise = 0.1.

(c) A plot to show the mean p-thresholds for all tournaments with noise = 0.2.

(d) A plot to show the mean p-thresholds for all tournaments with noise = 0.3.

(e) A plot to show the mean p-thresholds for all tournaments with noise = 0.4.

(f) A plot to show the mean p-thresholds for all tournaments with noise = 0.5.

(g) A plot to show the mean p-thresholds for all tournaments with noise = 0.6.

(h) A plot to show the mean p-thresholds for all tournaments with noise = 0.7.

folk_thm/main_analysis/noise_mean_thresh_violinplot.pdf

Figure 1.13: A violinplot showing the distribution of mean p-thresholds for each level of noise.

References

[1] Nikoleta E. Glynatsi and Vincent A. Knight. *A meta analysis of tournaments and an evaluation of performance in the Iterated Prisoner's Dilemma*. 2020. arXiv: `2001.05911 [cs.GT]`.