

要求，麻烦您编程一下 Q2-Q5，测试包已经发给你，imm4（4 栋楼）-imm500000（500000 栋楼）是给练习一用的测试，res4（4 个顶点）-res500000（500000 个点）是给练习 2 用的。我简单的翻译了一下要点。

## The problem of the row of roofs

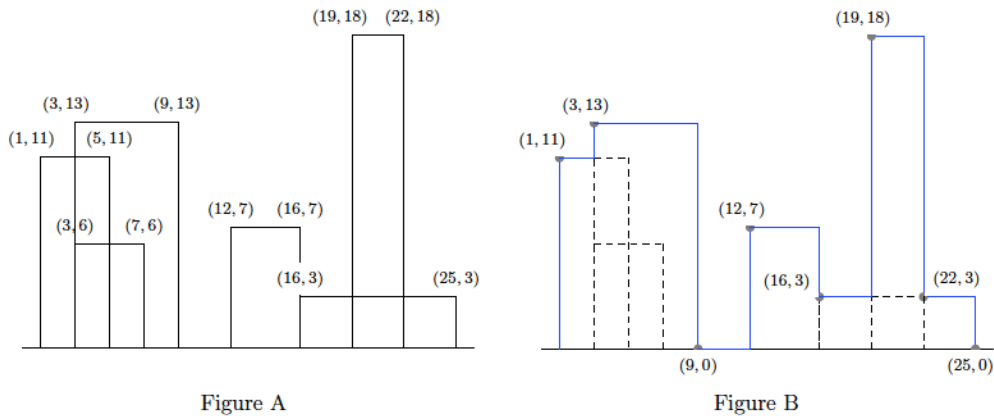
One of the classic problems for drawing images is the elimination of hidden lines. Here we are in a particular case of this problem. In the 2D case, the goal is to draw the line of roofs of buildings. For simplicity, it is assumed that all buildings correspond (by projection) to rectangles that all share the same base (the city is perfectly flat ...). A building is specified by a triplet  $(g; h; d)$ ,  $d > g > 0$ ;  $h > 0$ . which therefore represents the rectangle  $(g, 0); (g, h); (d, h); (d, 0)$ .

For example, for 6 buildings given by  $(3, 13, 9)$ ,  $(1, 11, 5)$ ,  $(19, 18, 22)$ ,  $(3, 6, 7)$ ,  $(16, 3, 25)$ ,  $(7, 16)$  -seeing Figure A, the row of roofs will be given in Figure B.

绘制图像的经典问题之一是消除隐藏线。在这里我们讨论一种特殊情况：在 2D 的情况下，目标是绘制线建筑屋顶。为了简单起见，假设所有建筑物（通过投影）都是举行（城市是完全平坦的）。一个建筑由 a 指定三重峰（左侧坐标  $g$ ；高度  $h$ ；右侧坐标  $d$ ）， $d > g > 0$ ,  $h > 0$ 。因此用四个点可以代表一个矩形： $(g, 0)$ ， $(g, h)$ ， $(d, h)$ ， $(d, 0)$ 。

例如， $(3, 13, 9)$ ， $(1, 11, 5)$ ， $(19, 18, 22)$ ， $(3, 6, 7)$ ， $(16, 3, 25)$ ， $(12, 7, 16)$  用于描述 6 个建筑物。

如图 A，屋顶线在图 B 中给出。



**Q 1.** If a row of roofs is a polyline, the converse is of course not true.

**Q 1.1.** Among the following polylines which are roof lines (with assumptions excluding any fancy taken here ... ie. all the walls are "vertical" and all the roofs are "horizontal" :-))?

- (2,0)(2,5)(4,4)(4, 7) (5, 7) (5, 0)
- (2,0)(1,4)(4,4)(4, 7) (5, 7) (5, 0)
- (2,0)(2,5)(4,5)(4,7) (5, 7) (5,0)
- (2,0) (2, 5) (4, 5) (4, 7)(6, 7) (5,0)
- (2,0)(2,5)(4,5) (4,8) (4,7) (5,7) (5,0)

**Q 1.2.** What are the conditions for a polyline to be a row of roofs?  
These conditions can be used to choose a more compact internal representation of a row of roofs.

For example :

(1,0)(1,11)(3,11)(9,13)(9,0)(12,0)(12,7)(16,7) (16,3) (19,3)  
(19,18)(22, 3) (25,3) (25,0)

can be represented (see figure B) by:

(22,3) (19,18) (22,3) (25,0)

**Q 1.3.** What line of roofs is represented by (1,1) (5,13) (9,20) (12,27) (16,3)(19,18)(22,3)(25,0).

The first representation proposed has the advantage of being close to the format svg, which will be used for the output.

The second is more compact and may be simpler to handle for some algorithms.

The transformation from one to the other is done very simply (How?).

You can choose the one you like best - or any other representation!

请参看 xhtml 文件，双击显示 svg 图片文件。从观察可以看出，这种屋顶线是按照顺序，从左而右连续链接，能够形成长方形的大楼。如果只给左上角的点，按照屋顶线的算法，是不能画成大楼的。

### First approaches

So the problem is

Entrance : n, a positive integer: the number of buildings

a list of n triplets of integers (g,h,d),  $d > g > 0$ ;  $h > 0$ . representing the buildings.

Output: a svg file representing the corresponding roof row

Note: You will first be able to exit the roof line as a list of pairs of integers as mentioned above. The only way to get it in svg format is to let you view line !

### 第一个练习：

(练习一是给长方形，生成屋顶线)

输入：长方体的左顶点坐标，高度，右侧顶点坐标。

输出：生成屋顶线图片（生成屋顶线的一串坐标点就可以了，生成图片太简单了，我会做。）

For example for the example above, one might have something like:

```
<svg xmlns="http://www.w3.org/2000/svg" width="300"
height="200" viewBox="-10 -150 200 150">
<polyline points="1,0 1,11 3,11 3,13 9,13 9,0 12,0 12,7 16,7 16,3
19,3 19,18 22,18 22,3 25,3 25,0"
stroke="blue" stroke-width="1" fill="none" transform="scale(5,-
5)"/></svg>
```

Note: The y-axis is directed by default with the y-crescents down. The scale (3, -3) allows to have the y croissants upward on one hand and on the other hand to change the scale. To verify the accuracy of the

the solution, one could also represent the buildings and the row of roofs:

```
<svg xmlns="http://www.w3.org/2000/svg" version="1.1"
width="300" height="300" viewBox="-10 -150
200 150">
<rect x ="3" y ="0" width ="6" height="13" transform=" scale(5,-5) "
/>
<rect x ="1" y ="0" width ="4" height="11" transform=" scale(5,-5) "
/>
<rect x ="19" y ="0" width ="3" height="18" transform=" scale(5,-5) "
"/>
<rect x ="3" y ="0" width ="4" height="6" transform=" scale(5,-5) "
/>
<rect x ="16" y ="0" width ="9" height="3" transform=" scale(5,-5) "
/>
<rect x ="12" y ="0" width ="4" height="7" transform=" scale(5,-5) "
/>
<polyline points=
"1,0 1,11 3,11 3,13 9,13 9,0 12 ,0 12,7 16,7 16,3 19,3 19,18 22,18
22,3 25,3 25,0 "
stroke="green" stroke-width="1" fill="none" transform=" scale(5,-5)
" /></svg>
```

这是画图的代码， 详细内容参见 xhtml 文件。

Q 2. A first approach can be to construct a table of integers (or booleans) representing the pixels of the visualization window and for each building to indicate all the "pixels of the building" at 1. Then it just "automaticly" scan this table to calculate the row of roofs.

What would be the complexity of such an approach? What disadvantage do you think she has?

Q 3. A second approach is to build incrementally the line by adding the buildings one by one.

Propose an algorithm for inserting a building into a row of roofs.

What will be its complexity in

function of  $n$ , the number of points defining the row of roofs? Deduce an algorithm to construct the line and analyze its complexity.

请编程实现：

Q2. 把整数表自动转化成为屋顶线坐标。不用时间复杂度。

Q3. 写个算法把图标中的建筑物一个一个加进去。不用分析时间复杂度。

## 练习二

### (练习二是给坐标，生成坐标)

We now want to solve the problem with a "divide to conquer" approach, which is quite close to that of the sorting: the list of buildings is divided into two lists of equal size (to 1 near), one generates recursively for the rows of roofs are merged.

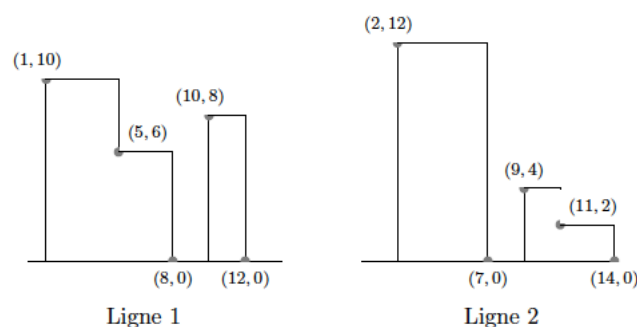
#### Q 4. Merge lines

We will therefore have to merge two rows of roofs into the corresponding row of roofs.

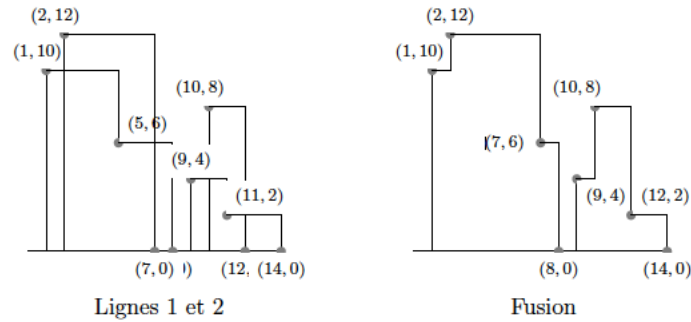
For example, the first line given by  $(1,10)$ ,  $(5,6)$ ,  $(8,0)$ ,  $(10,8)$ ,  $(12,0)$  and the second by  $(2,12)$

$(7,0)$ ,  $(9,4)$ ,  $(11,2)$ ,  $(14,0)$ , as in the figures below:

只给左上角顶点时（如图 ligne 1, ligne 2），



如果出现楼宇重叠的现象（如图 ligne 1 et 2）时，就这样画图（如图 Fusion）：



只给左上角的点，也可以画出整个屋顶线图，请写出算法。

（这个算法就是，给左上角的点，然后你就向右一直画，出现下一个点时，停止向右画，在横线都画完以后，你就把竖线都画上，就连好了，不需要画图，只需要生成有顺序的点坐标就可以了）

Propose an algorithm in  $O(n)$  to merge two lists of roofs,  $n$  being the maximum of the two lengths roof lines.

Q 5. Deduce and implement an algorithm in  $O(n \log n)$  for the construction of the row of roofs.

Q 6. Test, experiment. To enable you to test, data sets are available to you

Q4 写一个复杂度为  $O(n)$  的算法

Q5 写一个复杂度为  $O(n \log n)$  的算法（我觉得二分法，分类来做，每次除以 2，就能出现  $\log$ ）

Q6 测试一下你的算法。

测试包已经发给你，imm4（4 栋楼）-imm500000（500000 栋楼）是给练习一用的测试，res4（4 个顶点）-res500000（500000 个点）是给练习 2 用的。