

## Definition of Turing m/c

$$M = \{ Q, \Sigma, \delta, q_0, F, B, T \}$$

where  $Q$  = finite set of internal states

$\Sigma$  = finite set of input symbols

$$\delta = Q \times T \rightarrow Q \times T \times \{L, R, N\}$$

$q_0$  = initial state  $q_0 \in Q$

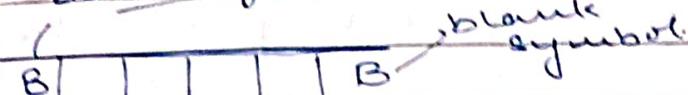
$F$  = final state  $F \subseteq Q$

$B$  =  $B \in T$ , special tape symbol

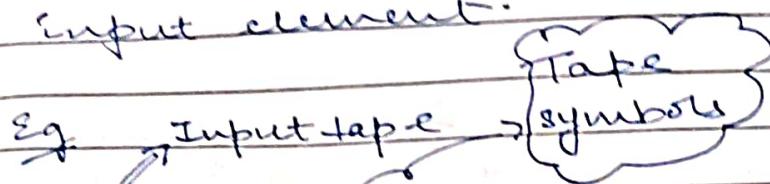
$T$  = finite set of | for Blank space  
tape symbols |

# Turing Machine

device to handle strings  
start with blank symbol



memory element divided into cells such that each cell can hold a single input element.



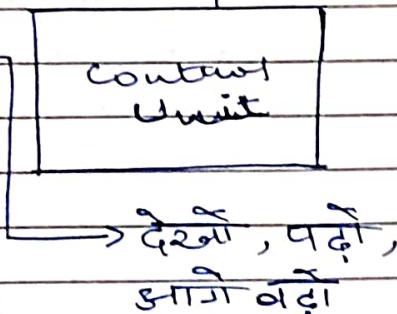
B | a | a | b | B

blank symbol ↑ can be any non-i/p symbol ( \$, # also allowed )  
read/write read.

BBBB --- ≡ B

↓ waiting no. of Bs  
≡ waiting one B  
in the beginning.

It can  
read  
asymbol  
or  
write  
↓  
Manipulate  
replace  
know



R/W head can

- move left (L)
- move right (R)
- halt at specific position (N)  
↓ no movement

Control unit  
associated with  
i/p tape through  
R/W head

Properties of R/W head

- read or write
- left/right/none

(\*) Can be designed in 3 ways

(1) Transition diagram

(2) Transition Table

(3) Instantaneous Description  
(String processing)

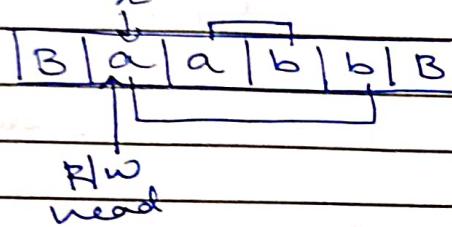
$$\delta = Q \times T \rightarrow Q \times T \times \{L, R, N\}$$

~~Part 1~~ Transition Table: of  $\{a^n b^n : n \geq 0\}$  ~~for all~~ question

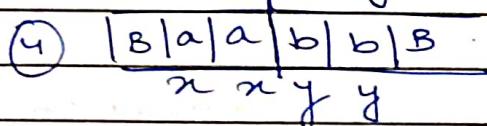
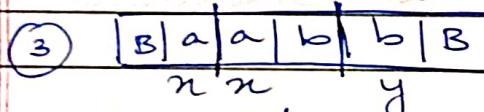
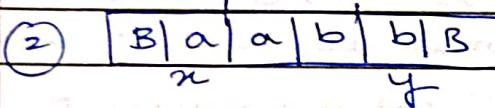
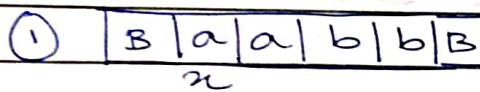
<u>S   T</u>	a	b	x	y	B
$\rightarrow q_0$	$q_1, x, R$	-	-	$q_4, y, N$	-
$q_1$	$q_1, a, R$	$q_1, b, R$	-	$q_2, y, L$	$q_2, y$
$q_2$	-	$q_3, y, L$	-	-	-
$q_3$	$q_3, a, L$	$q_3, b, L$	$q_0, x, R$	-	-
$q_4$	-	-	-	-	-

Eg: Design a Turing m/c for language  
 $L = \{a^n b^n : n \geq 0\}$  for  $\Sigma = \{a, b\}$

Step 1 make i/p tape



\* R/w head starts at blank processing starts at blank to just read vala symbol.



same  $a = x, b = y$ .

# Points to remember while designing a TM

- (1) In Turing m/c, if we need to remember any scanned symbol, then we CHANGE A STATE.
- (2) To minimize no. of states, change of states are generally done when either writing

a symbol, or, change in the movement of R/W head.

B	a	a	b	b	B
1°	4°	3°	2°		

1° pe start karw

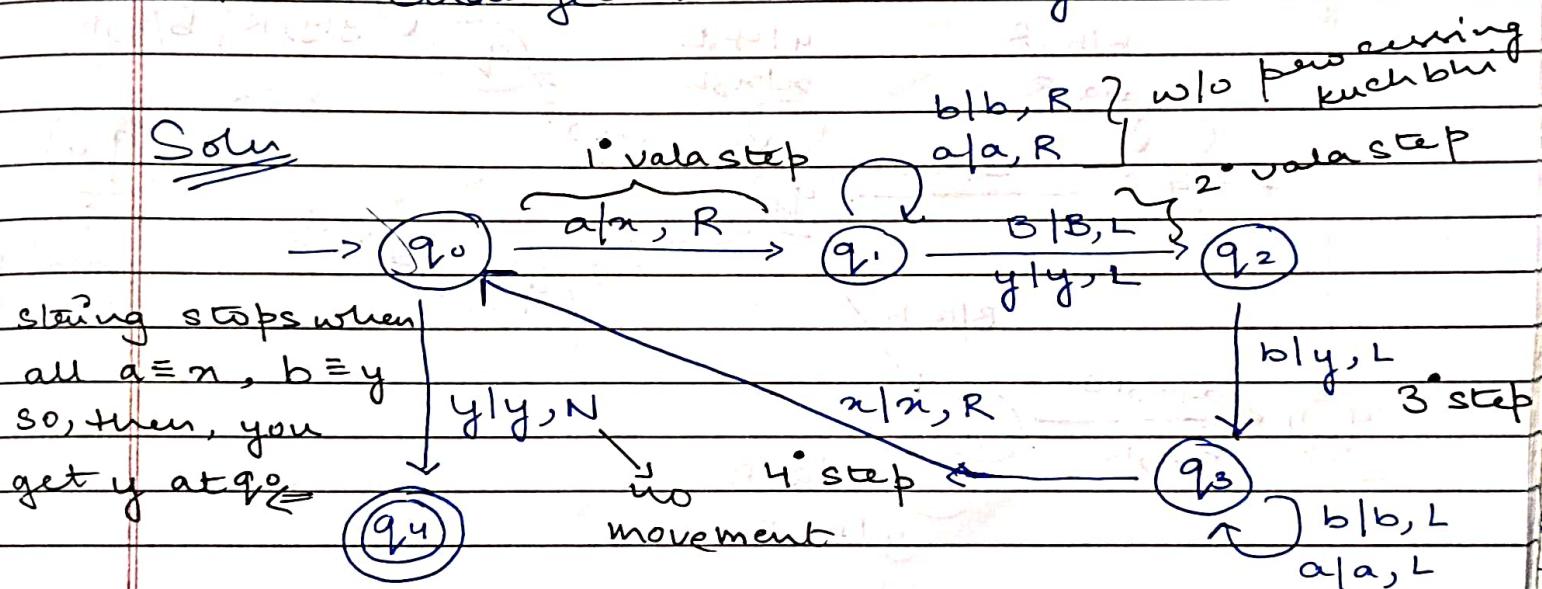
a ko dekho, change a to n

fir 2° tak bhaag ke chale jao, w/o processing  
beech ka kuch bhi

when u reach 2°, as if  $\overrightarrow{q_0 q_1}$  se takrate  
vaapas giye gaye  $\Rightarrow$  you see b at 3°  
change this b to y.

Then, 3° ke b se bhaag ke reach  
4°, ~~because~~ because uske parne ka a has  
been changed to n already

Solu



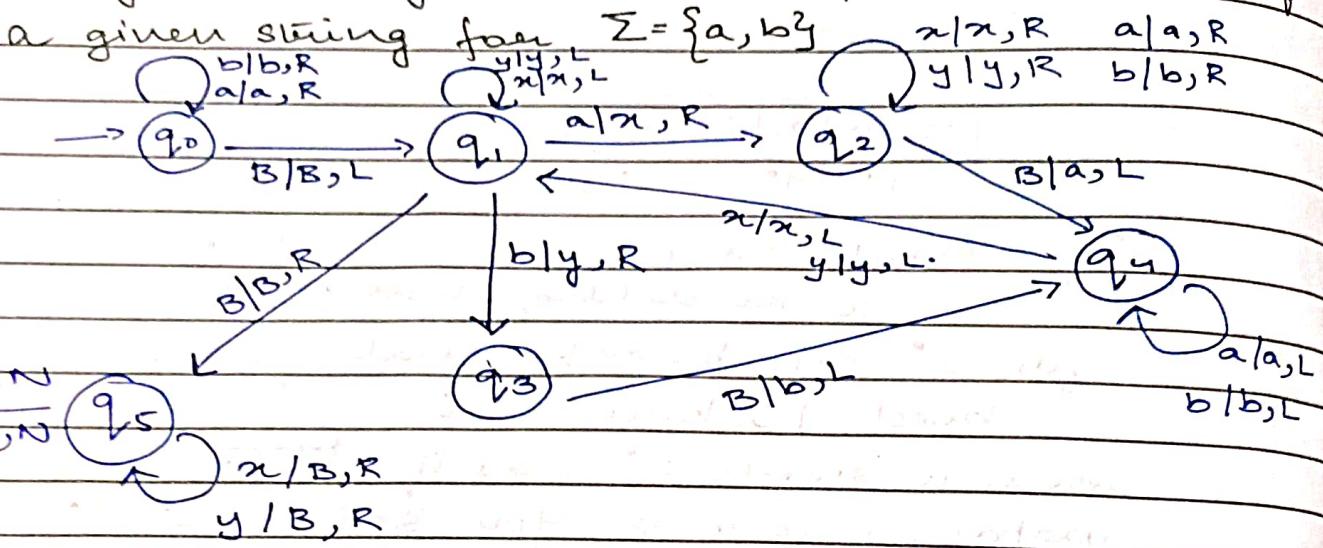
$a/x, R$  means change a to n; move right  
 $b/y, L$  means blank to blank rene do, then move left

Input  $B \underline{aabbbB}$

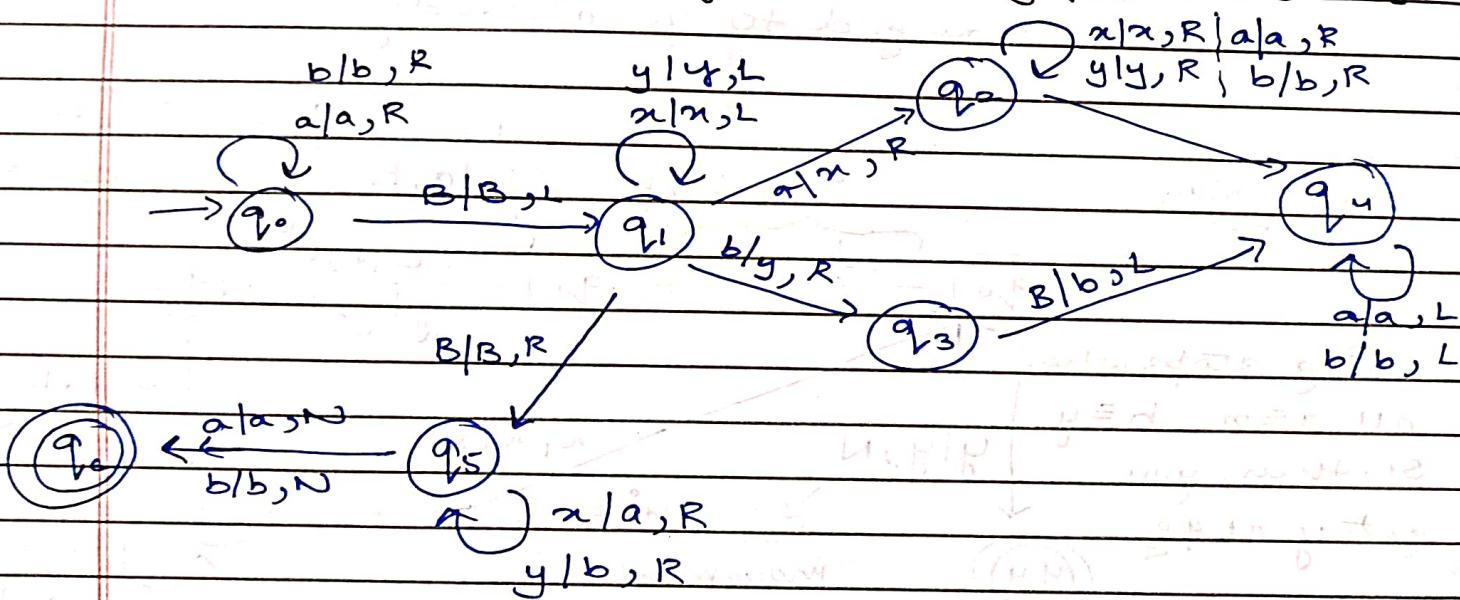
find  $w^R$  of  $w$  for  $\Sigma = \{a, b\}$

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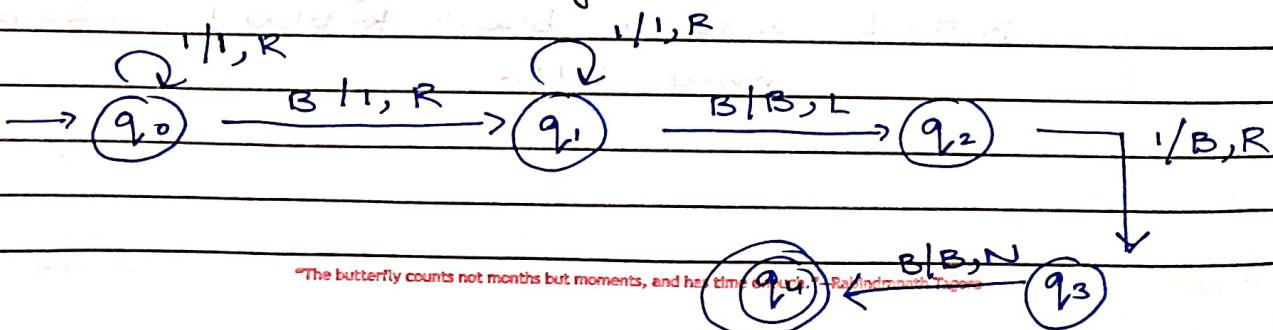
Eg<sup>2</sup> Design a Turing m/c that computes reverse of a given string for  $\Sigma = \{a, b\}$



Q<sup>3</sup> Design a Turing machine that computes  $w \rightarrow ww^R$  of a given string for  $\Sigma = \{a, b\}$



Q<sup>4</sup> Design a Turing machine that computes the addition of 2 unary numbers.





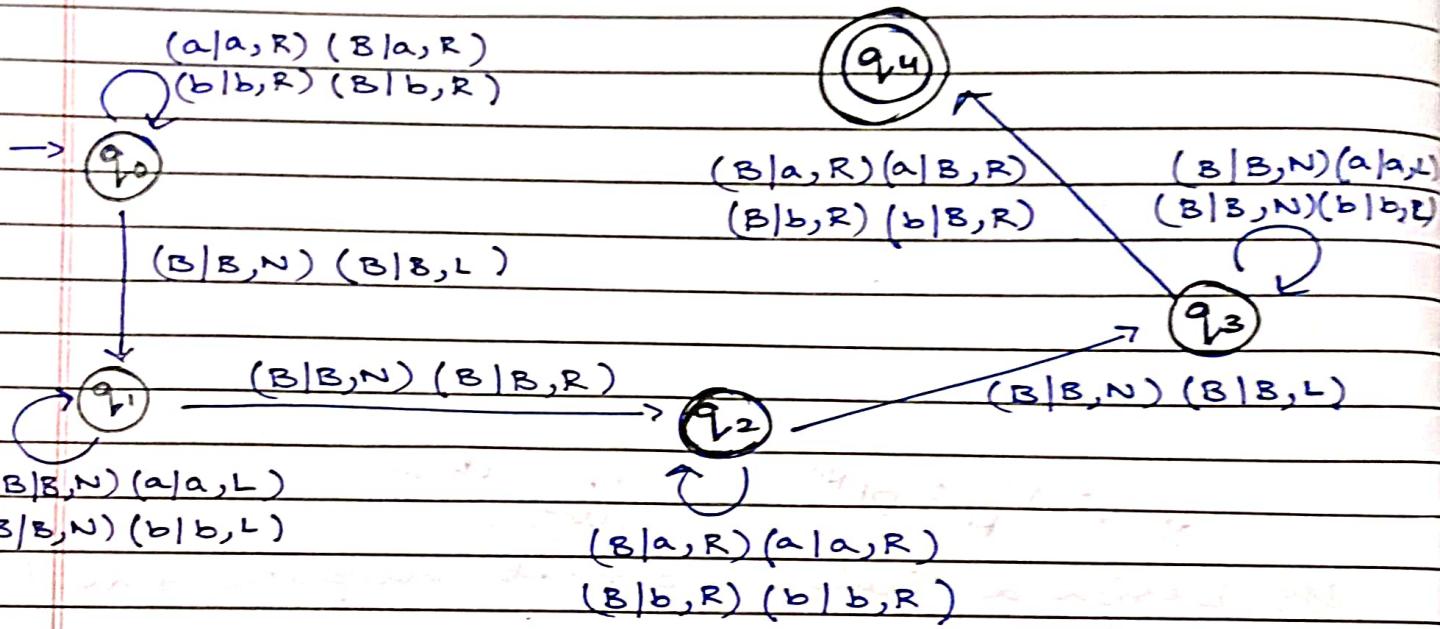
Ex7

Design a Turing m/c that computes  $W \rightarrow WWW$   
 for  $\Sigma = \{a, b\}$ .

Tape 1

$$\boxed{B | a | b | b | B | B}$$

Tape 2

$$\boxed{B | B | B | B | B | B}$$


# Try  $m * n$  and  $n^2$  using Two-tape Turing m/c

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# use 2 tape  
T.M as trick to  
design Turing  
m/c

## Extensions of Turing m/c

Eg8 Design a Two tape Turing m/c that computes  
 $W \rightarrow WW^R$  for  $\Sigma = \{a, b\}$

concept:  $| B | a | b | b | B | B$  Tape 1

$| B | B | B | B | B | B$  Tape 2

Step 1 Tape 1 mei a dekho,

Tape 2 mei B ko a banao

Tape 1 mei b dekho

Tape 2 mei B ko b banao

Step 2 Tape 1 mei B aagaya ; stay

Tape 2 mei B aaya ; left none

Step 3 Tape 2 mei a ya b ko B banao

Tape 1 mei B ko a ya b banao

for tape 1

for tape 2

(a/a, R) (B/a, R)

(b/b, R) (B/b, R)

→  $q_0$  (B/B, N) (B/B, L)

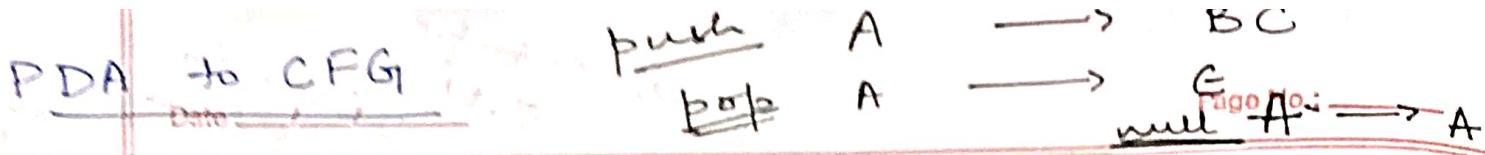
(B/b, R) (b/B, L)

(B/a, R) (a/B, L)

$q_1$

(B/B, N) (B/B, N)

$q_2$



①  $S \longrightarrow [q_0 z_0 q_0]$   $q_0 \in Q$

②  $\delta(q_i, a, A) = \{(q_j, BC)\}$   $\Rightarrow$  push  
 Push operation  $[q_i A q_k] \xrightarrow{a} [q_j B q_l] [q_m C q_n]$   
 $q_j, q_l, q_m, q_n \in Q$

No operation  $[q_i A q_k] \xrightarrow{a} [q_i A q_k]$

$\delta(q_i, a, A) = \{(q_j, B)\}$

$[q_i A q_k] \xrightarrow{a} [q_j B q_k]$

Pop operation

$\delta(q_i, a, A) \xrightarrow{a} \{(q_j, \epsilon)\}$

$[q_i A q_j] \xrightarrow{a} [q_i \epsilon q_k]$

So;

Push  $\delta(q_i, a, A) = \{(q_j, BC)\}$

$[q_i A q_k] \xrightarrow{a} [q_j B q_l] [q_m C q_n]$

No  $\delta(q_i, a, A) = \{(q_j, B)\}$

$[q_i A q_k] \xrightarrow{a} [q_j B q_k]$

Pop  $\delta(q_i, a, A) = \{(q_j, \epsilon)\}$   $[q_i A q_j] \xrightarrow{a} [q_i \epsilon q_k]$

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PDA to CFG

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Eg' Design an equivalent ~~pushdown automaton~~ context free grammar for given ~~the~~ PDA

$$\delta(q_0, 1, z_0) = \{(q_0, xz_0)\} \quad (1)$$

$$\delta(q_0, 1, x) = \{(q_0, xx)\} \quad (2)$$

$$\delta(q_0, 0, x) = \{(q_1, x)\} \quad (3)$$

$$\delta(q_0, \epsilon, z_0) = \{(q_0, \epsilon)\} \quad (4)$$

$$\delta(q_1, 1, x) = \{(q_1, \epsilon)\} \quad (5)$$

$$\delta(q_0, 0, z_0) = \{(q_0, z_0)\} \quad (6)$$

~~Soln'~~ first find  $Q = \{q_0, q_1\}$  finite set of internal states

Step 1  $S \rightarrow [q_0 z_0 q_1]$

That is  $S \rightarrow [q_0 z_0 q_0] \mid [q_0 z_0 q_1]$

Step 2

for transition

$$\delta(q_0, 1, z_0) = \{(q_0, xz_0)\}$$

$$q_k = q_0 \Rightarrow [q_0 z_0 q_0] \rightarrow 1 [q_0 x q_0] [q_0 z_0 q_0] /$$

$$q_k = q_0 \Rightarrow [q_0 z_0 q_0] \rightarrow 1 [q_0 x q_1] [q_1 z_0 q_0] /$$

$$q_k = q_1 \text{ and } [q_0 z_0 q_1] \rightarrow 1 [q_0 x q_1] [q_1 z_0 q_1] /$$

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push  $q_i A q_k \rightarrow a [q_j B q_e] [q_e C q_k]$ 

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~~Step<sup>3</sup>~~

for transition 2

$$\delta(q_0, 1, \alpha) = \{ q_0, q_0 q_1 \}$$

$q_i \in A$        $q_j \in B$        $q_e \in C$

$$[q_0 q_0] \rightarrow 1 [q_0 q_0] [q_0 q_0] /$$

$q_k = q_0$        $q_e = q_0$

$$1 [q_0 q_1] [q_1 q_0] /$$

$q_e = 1$

$$[q_0 q_1] \rightarrow 1 [q_0 q_0] [q_0 q_1] /$$

$q_k = q_1$        $q_e = q_0$

$$1 [q_0 q_1] [q_1 q_1] /$$

$q_e = q_1$

\* To push mei

$$q_k = q_0 \rightarrow q_e = q_0 \mid q_e = q_1$$

$$q_k = q_1 \rightarrow q_e = q_0 \mid q_e = q_1$$

~~Step<sup>4</sup>~~

for transition 3

No operation

$$\delta(q_0, 0, \alpha) = \{ (q_1, \alpha) \}$$

$q_i \in A$        $q_j \in B$

no ka formula :  $[q_i A q_k] \rightarrow a [q_j B q_k]$ 

$$[q_0 q_0] \rightarrow 0 [q_1 q_0]$$

$$[q_0 q_1] \rightarrow 0 [q_1 q_1]$$

~~steps~~

for transition 4

pop operation

$$\delta(q_0, \epsilon, z_0) = \{ (q_0, \epsilon) \}$$

$q_i \quad a \in A \quad q_j \in$

pop ka formula

~~$$[q_i A q_i] \rightarrow a$$~~

$$\Rightarrow [q_0 z_0 q_0] \rightarrow \epsilon$$

~~steps~~

for transition 5

Pop

$$\delta(q_1, l, x) = \{ (q_1, \epsilon) \}$$

$q_i \quad a \in A \quad q_j$

$$[q_i A q_i] \rightarrow a$$

$$[q_1 x q_1] \rightarrow l$$

~~steps~~ for transition 6

No movement

$$\delta(q_0, o, z_0) = \{ (q_0, z_0) \}$$

$q_i \quad a \in A \quad q_j \in B$

$$[q_i A q_k] \rightarrow a [q_j B q_k]$$

$$\Rightarrow [q_0 z_0 q_0] \rightarrow o [q_0 z_0 q_0]$$

$$[q_0 z_0 q_1] \rightarrow o [q_0 z_0 q_1]$$

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## Turing m/c

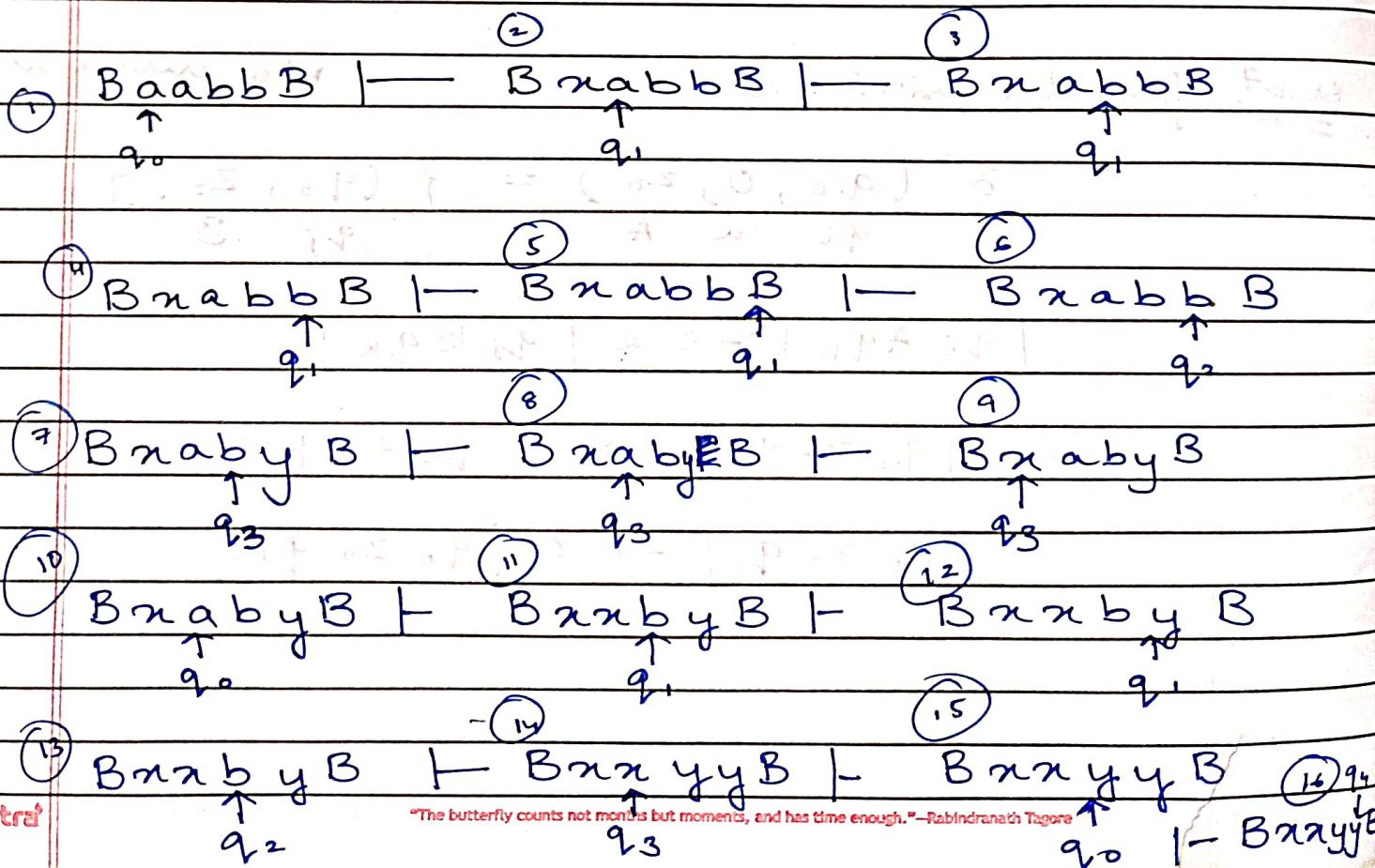
## String processing

## Transition Table

$g \mid r$	$a$	$b$	$n$	$y$	$B$
$q_0$	$q_1, n, R$			$q_4, y, N$	
$q_1$	$q_1, a, R$	$q_1, b, R$		$q_2, y, L$	$q_2, B_L$
$q_2$		$q_3, y, L$			
$q_3$	$q_3, a, L$	$q_3, b, L$	$q_0, n, R$		
$q_4$					

## String processing

$$w = aabb$$



PCP  $\rightarrow$  NP complete problem

Date \_\_\_\_\_ Non polynomial type . Page No.: \_\_\_\_\_ Unit 5

## Post Correspondence Problem.

Eg' For the given two lists

$$A = \{a, abaaa, ab\}$$

$B = \{aaa, ab, b\}$ , does this PCP have a solution? If yes, give the solution.

Aus.

$x_2$	$x_1$	$x_3$
abaaa	a   a / ab	
abaaa	a aa   b	
y <sub>2</sub>	y <sub>1</sub>	y <sub>3</sub>

The soln of the PCP is (2, 1, 1, 3)

$x_2$	$x_1$	$x_3$
a b   a aa	a   a a / ab	
a b   a a a	a   a a a / a b	
y <sub>2</sub>	y <sub>1</sub>	y <sub>3</sub>

$x_2$  add karna toh  $y_2$  bhi karna hai

Eg 2

$$A = \{abbb, abb, a\}$$

$$B = \{ab, bb, a\}$$

This PCP does not have a solution.

(3) is not a valid solution.

① ~~for~~ general purpose computer

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Universal Turing m/c

K=1

L

K=2

R

K=3

N

② ~~for~~  $(q_i, a_i; q_j, a_j, m_f)$

③ ~~for~~ State  $q_i$ ,  $i$ th symbol  $a_i$  ;

State changes to  $q_j$  ; symbol changes to  $a_j$   
 $m_f$  is movement of R/W Head.

④ ~~for~~ In the form of 0 and 1 where 1 is used as separator symbols and 0 is the representation of states & tape symbols.

e.g.  $q_0 \quad 0$   
 $q_1 \quad 00$   
 $q_2 \quad 000$   
 $q_3 \quad 0000$

$1 \ 0^{i+1} 1 0^i 1 0^j 1 0^k \rightarrow$   $\underbrace{\hspace{10em}}$   
movement of R/W Head.  
K=1 L  
2 R  
3 N

17/10/24 Modified Post Correspondence Problem

MPCP

$$A = \{x_1, x_2, \dots, x_n\}$$

$$B = \{w_1, w_2, \dots, w_k\}$$

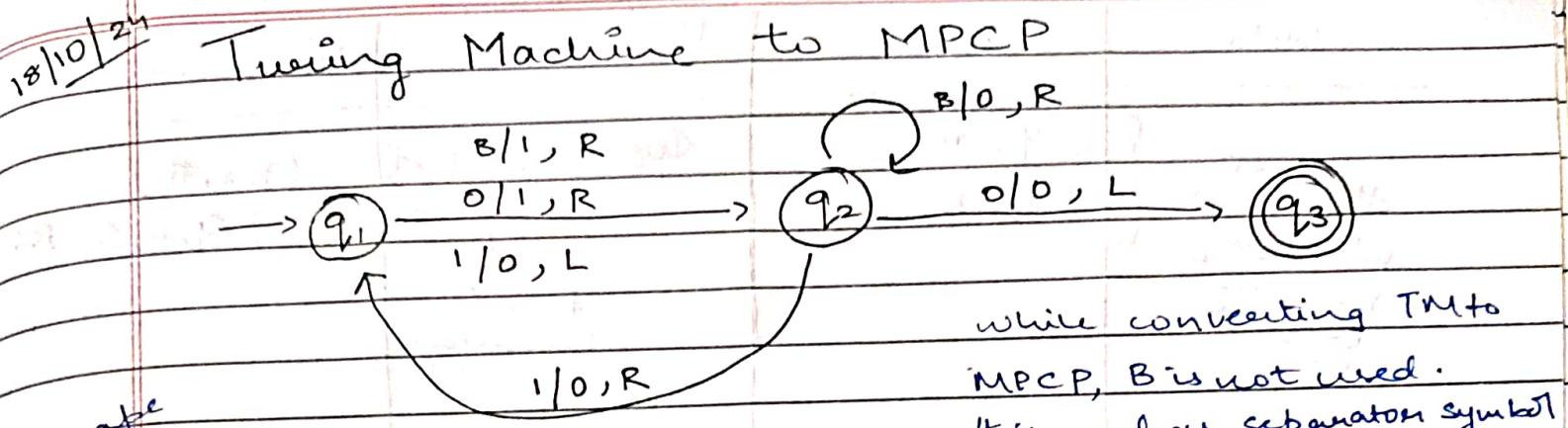
\* decidability using  
Turing m/c

$\downarrow$   
MPCP

$\downarrow$   
PCP

Always exists a solution  $(x_i, w_j)$   
starting with

"The butterfly counts not months but moments, and has time enough." - Rabindranath Tagore



while converting TM to MPCP, B is not used.  
# is used as separator symbol

Convert given TM to MPCP, and find out the solution using  $w = 01$

Soln	Rule	A	B	Transition
Pre-defined rules	initial state 1.	#	# $q_1$ 01 #	
	tape symbols 2.	0 { Tape } 1 { Symbols } #	0 1 #	
	All non final states 3.	$q_1$ 0	1 $q_2$	$\delta(q_1, 0) = \{q_2, 1, R\}$
	$0 q_1 1$ $1 q_1 1$	$q_2 0 0$ $q_2 1 0$	$\delta(q_1, 1) = \{q_2, 0, L\}$	
	$0 q_1 #$ $1 q_1 #$	$0 1 q_2 #$ $1 1 q_2 #$	$\delta(q_1, B) = \{(q_2, 1, R^2)\}$	
# Jabs left move tab para left tab right (01) hi wait 2 cases will will	$0 q_2 0$ $1 q_2 0$	$q_3 0 0$ $q_3 1 0$	$\delta(q_2, 0) = \{q_3, 0, L\}$	
	$q_2 1$	$0 q_1$	$\delta(q_2, 1) = \{q_1, 0, R\}$	

Rule	A	B	Transition
# aaya # Parve jan Parve dikhaai dikhaai yari	{ 0q₂ # 1q₂ #}	0/0q₂ # 1/q₂ # # koo banaya.	$\delta(q₂, \#)$ $= \{q₂, 0, R\}$

(final states) 4.	0q₃ 0 1q₃ 1 0q₃ 1 1q₃ 0 1q₃ 0q₃ q₃ 0 q₃ 1	q₃ q₃ q₃ q₃ q₃ q₃ q₃ q₃	all possibilities dikhaao w/ new no transition or final state
-------------------	--	--	--

5.	q₃ #	#
----	------	---

last mei  
hamer  
yahi dikhaao  
ende

(final states) 4.

0q <sub>3</sub> 0	q <sub>3</sub>
1q <sub>3</sub> 1	q <sub>3</sub>
0q <sub>3</sub> 1	q <sub>3</sub>
1q <sub>3</sub> 0	q <sub>3</sub>
1q <sub>3</sub>	q <sub>3</sub>
0q <sub>3</sub>	q <sub>3</sub>
q <sub>3</sub> 0	q <sub>3</sub>
q <sub>3</sub> 1	q <sub>3</sub>

all possibilities  
dikhauswer  
no transition  
or final state

5.

q<sub>3</sub>#

#

last we  
haven't  
yahi dikha  
era

: it is MPCP, it will always start  
with the first state.

A # q<sub>1</sub>0 | # q<sub>2</sub>1 | q<sub>1</sub> # q<sub>2</sub>1 | 0 q<sub>1</sub> # 1 0 q<sub>2</sub> # 1 0 q<sub>2</sub> # 0

B # q<sub>1</sub>0 | q<sub>2</sub>1 # | q<sub>1</sub> # q<sub>2</sub>1 | 0 q<sub>1</sub> # 1 0 q<sub>2</sub> # 1 0 q<sub>2</sub> # 0

Blank can be used anywhere. { end marker used only in the end } \* : separator  
~~date~~ \_\_\_\_\_ \$ : end marker Page No. \_\_\_\_\_

# # ~~excess~~

Conversion.

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Modified PCP to PCP

# add two stars

MPCP → PCP

A	B			B
$x_i$	$w_i$	0 min 1 copying		$q_i$
1	111	range begins A		$*1*1*1*$
$m=3$	10111	range ends Yi		$1*1*1*1*$
10	0 3 ] )	0 *1*		$1*0*$
	123 of MPCP	1 1 *		0 *
		2 1*0*1*1*1*		
		3 1*0*		
		4 \$		* \$

sc 123 of PCP becomes fixed (4th).

ke liye  
next element

ke baad star lagado

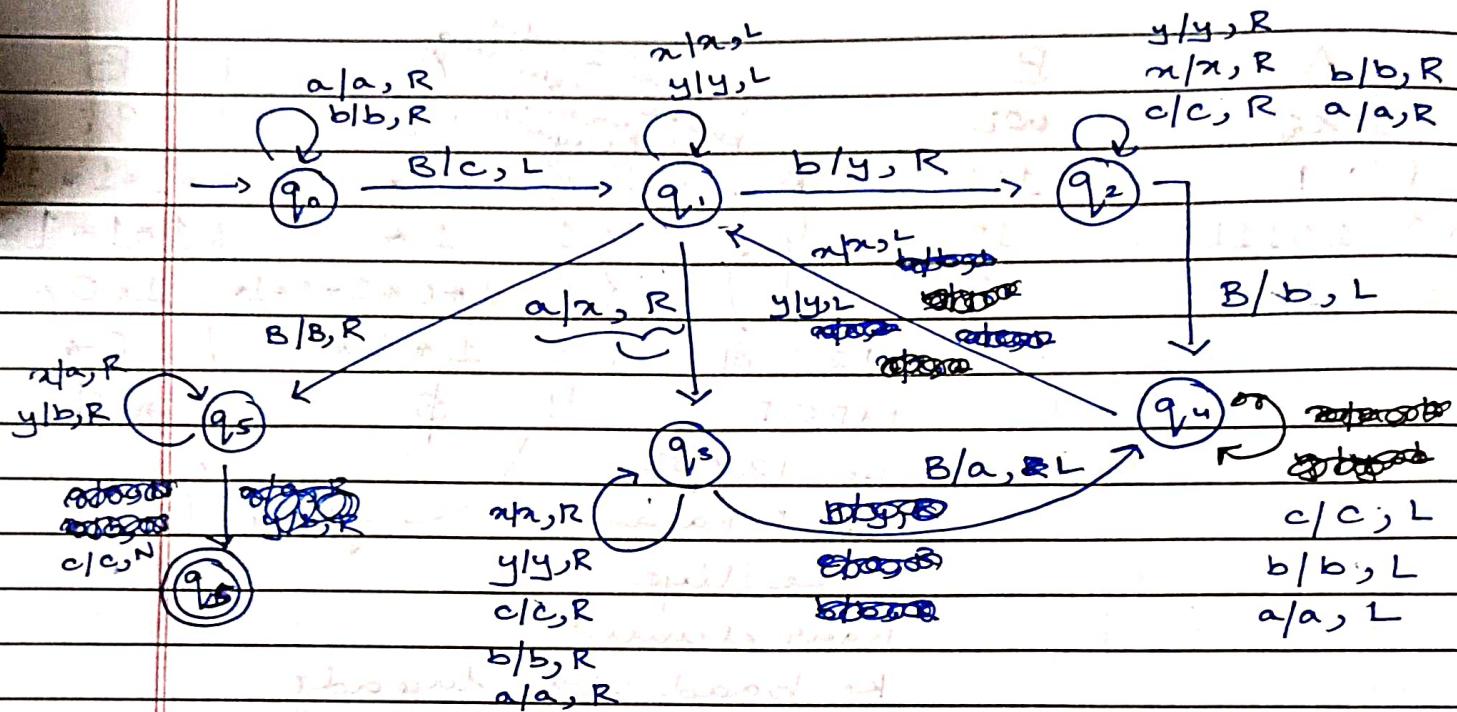
X ————— X

Expt

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## Turing m/c

 $BababBB \rightarrow Bababcbaab$  $BababBBB \rightarrow$ Eg:  $w \rightarrow wcw^R$ 

$$\begin{array}{ccccccccc} & & c & & b & a & & & \\ & & y & & y & b & a & & \\ & & x & & x & a & & & \\ & & B & & & & & & \end{array}$$
 $BababBB \rightarrow Bababcbaab$  $Baabb BBBB B B$ 

$y$	$c$	$b$	$b$
$y$	$\therefore$	$b$	$b$
$x$	$\therefore$	$a$	$a$

 $Baabbcbbaa B$

Date 1/1

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## Transition Tab (a).

$q_1 r$	a	b	c	n	y	B
$q_0$	$q_0, a, R$	$q_0, b, R$	-	-	-	$q_1, c, L$
$q_1$	$q_3, x, R$	$q_2, y, R$	-	$q_1, n, L$	$q_1, y, L$	$q_5, B, R$
$q_2$	$q_2, a, R$	$q_2, b, R$	$q_2, c, R$	$q_2, n, R$	$q_2, y, R$	$q_4, b, L$
$q_3$	$q_3, a, R$	$q_3, b, R$	$q_3, c, R$	$q_3, n, R$	$q_3, y, R$	$q_4, a, L$
$q_4$	$q_4, a, L$	$q_4, b, L$	$q_4, c, L$	$q_1, n, L$	$q_1, y, L$	-
$q_5$	<del><math>q_5, x, R</math></del>	-	-	$q_6, c, N$	$q_5, a, R$	$q_5, b, R$
$q_6$	-	-	-	-	-	-