Unit-3.

→ Mathematical Logics -

Logic provides rules by which we can determine whether a given argument is valid or not. In computer science, logics are used to verify the correctness of algorithm.

→ Proposition - (statement)

A proposition is a declarative sentence which is either true or false but not both.

Eg - a). Its raining outside - prop ✓

b). $x+y=7$ - not ✗

c). who are you? - not ✗

d). close the door - not ✗

c) The sun will come out tomorrow ✗

#. Statements which are exclamatory, interrogative in nature are not proposition.

Two values - "True" / "False" denoted by T and F.

→ Connectives -

The words or symbol that are used to make a sentence by two sentence are called logical connectives.

Some basic connectives -

1. Negation (NOT) $\sim$
2. Conjunction (AND) $\wedge$
3. Disjunction (OR) $\vee$
4. Conditional (If ~~and~~ then) $\rightarrow$
5. Biconditional (If and only if) $\leftrightarrow$

→ Negation -

If a proposition is denoted by p then its negation is denoted by $\sim p$.

If p is true, $\sim p$ is false and vice-versa.

eg - Delhi is a city - p

$\sim p$ - Delhi is not a city

Truth table

p	$\sim p$
T	F
F	T

→ Conjunction - If p and q are two proposition then conjunction of p and q is denoted by $p \wedge q$ and it is true only if both are true.

eg - Ram is honest = p

Rahim is honest = q

$p \wedge q$ = Ram and Rahim is honest

p	q	$p \wedge q$
T	T	T
F	F	F
T	F	F
F	T	F

→ Disjunction – If p and q are two prop., the disjunction of p and q is denoted by $p \vee q$ and it is false if both prop. are false

Eg – p = I will play cricket.
q = I will go to home

$p \vee q$ = I will play cricket or go to home

p	q	$p \vee q$
T	F	T
F	T	T
T	T	T
F	F	F

→ Conditional – (If.... then)

If p and q are prop. then compound statement 'If p then q' is denoted by $(p \to q)$ is called conditional prop.

$p \to q$

Hypothesis consequence

Eg: p: I am hungry q: I will eat
$p \to q$: If I am hungry then I will eat.

p	q	$p \to q$
T	F	F
F	T	T
T	T	T
F	F	T

→ <u>Biconditional</u> - (If and only if)

If p and q are two prop. then the compound prop. (p if and only if q) is called bicondition prop.

Denoted by $p \leftrightarrow q$.

p : He swims

q : water is warm.

$p \leftrightarrow q$: He swims if and only if water is warm.

p	q	$p \leftrightarrow q$
T	F	F
F	T	F
T	T	T
F	F	T

Q. using the statement -

n variables
rows $\rightarrow 2^n$

p : Mark is rich

q : Mark is happy.

~~Consid~~ Construct Truth table.

(i). Mark is neither happy nor rich $\sim q \wedge \sim p$

(ii). Mark is poor or he is both rich and happy.

$\sim p \vee (p \wedge \sim q)$

(i). $\sim q \wedge \sim p$

p	q	$\sim p$	$\sim q$	$\sim q \wedge \sim p$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(ii). $\sim p \vee (p \wedge \sim q)$

p	q	$\sim p$	$\sim q$	$(p \wedge \sim q)$	$\sim p \vee (p \wedge \sim q)$
T	T	F	F	F	F
T	F	F	T	T	T
F	T	T	F	F	T
F	F	T	T	F	T

→ **Tautology** – A proposition which is true for all possible truth values of its proposition variable is called tot tautology.

Contradiction – If it is false for all possible truth values then it is called contradiction.

Contingency – If it is neither tautology nor contradiction.

Q. Is the following compound statement is tautology.

(a). $(p \to (q \to r)) \to ((p \to q) \to (p \to r))$

(b) $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

p	q	r	$q \to r$	$p \to (q \to r)$	$p \to q$	$p \to r$	$(p \to q) \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	F	T	T	T	F	T	T
T	F	F	T	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$(p\to(q\to r))\to(p\to q)\to(p\to r)$

~~T~~	T
~~F~~	T
~~T~~	T
~~F~~	T
~~T~~	T
~~F~~	T
~~T~~	T
~~F~~	T

Tautology.

(b). $(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$

p	q	r	$\sim p$	$\sim q$	$\sim p \leftrightarrow \sim q$	$q \leftrightarrow r$	$(\sim p \leftrightarrow \sim q) \leftrightarrow (q \leftrightarrow r)$
T	T	T	F	F	T	T	T
T	T	F	F	F	T	F	F
T	F	T	F	T	F	F	T
T	F	F	F	T	F	T	F
F	T	T	T	F	F	T	F
F	T	F	T	F	F	F	T
F	F	T	T	T	T	F	F
F	F	F	T	T	T	T	T

contingency.

→ Equivalence of Proposition :

Two compound proposition $A(p_1, p_2, \dots p_n)$ and $B(p_1, p_2, \dots p_n)$ are labelled equivalent or logically equivalent, if they have identical truth values. We denote $A \Leftrightarrow B$ or $A \equiv B$

Q. P.T. $p \to q \equiv \sim p \vee q$

sol^n

p	q	$\sim p$	$p \to q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\because$ All the truth values of $p \to q$ and $\sim p \vee q$ are same therefore. $p \to q \equiv \sim p \vee q$

H.P.

→ ~~Durability~~ :

Duality law - The dual of prop. can be obtained by replace $\wedge$ to $\vee$ and $\vee$ to $\wedge$ and T to F and F to T.

(1). $(p \wedge q) \vee r$ $\qquad$ $(p \vee q) \wedge r$

(2). $(p \wedge T) \wedge F$ $\qquad$ $(p \vee F) \vee T$

→ Duality Theorem : If $A \equiv B$ then $A^* \equiv B^*$. (where A^* and B^* are dual of A and B).

Q. $p \vee q \equiv q \vee p$

Dual $p \wedge q \equiv q \wedge p$.

→ **Algebra of propositions** –

Name of Law	Primary	Dual
1. Commutative	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
2. Associative	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
3. Distributive	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
4. Idempotive	$p \vee p \equiv p$	$p \wedge p \equiv p$
5. Identity	$p \vee F \equiv p$	$p \wedge T \equiv p$
6. Dominant	$p \vee T \equiv T$	$p \wedge F \equiv F$
7. Compliment	$p \vee \sim p \equiv T$	$p \wedge \sim p \equiv F$
8. Absorption	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
9. Demorgan's law	$\sim (p \vee q) \equiv \sim p \wedge \sim q$	$\sim (p \wedge q) \equiv \sim p \vee \sim q$

Q. Without using truth table prove –

$\cancel{\sim p \wedge}$ $(\sim p \vee q) \wedge (p \wedge (p \wedge q)) \equiv p \wedge q$

$= (\sim p \vee q) \wedge ((p \wedge p) \wedge q)$ Associative law

$= (\sim p \vee q) \wedge (p \wedge q)$ Idempotive

$= (p \wedge q) \wedge (\sim p \vee q)$ Commutative

$= ((p \wedge q) \wedge \sim p) \vee ((p \wedge q) \wedge q)$ Distributive

$= (\sim p \wedge (p \wedge q)) \vee (p \wedge (q \wedge q))$ Communicative in I
Associative in II.

$= (F \wedge q) \vee (p \wedge q)$

$= F \vee (p \wedge q)$

$= p \wedge q$ Identity

→ Equivalence Invading Conditional. –

i). $p \to q \equiv \sim p \vee q$

ii). $p \to q \equiv \sim q \to \sim p$

iii). $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

iv). $(p \to q) \vee (p \to r) \equiv \cancel{(p \vee q) \to r}\ p \to (q \vee r)$

v). $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$

vi). $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

Q. Prove -

$$p \to (q \to p) \equiv \sim p \to (p \to q)$$

LHS $p \to (q \to p) = p \to (\sim q \vee p)$

$= \sim p \vee (\sim q \vee p)$

$= (\sim q \vee p) \vee \sim p$

$= \sim q \vee (p \vee \sim p)$

$= \sim q \vee T$

$= T$

RHS. $\sim p \to (p \to q) = \sim p \to (\sim p \vee q)$

$= \sim(\sim p) \vee (\sim p \vee q)$

$= p \vee (\sim p \vee q)$

$= (p \vee \sim p) \vee q$

$= T \vee q = T$

→ Equivalence invading Biconditional -

1. $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
2. $p \leftrightarrow q \equiv \sim q \leftrightarrow \sim p$

Q. Without using truth table, prove -

$$\sim(p \leftrightarrow q) \equiv (p \vee q) \wedge \sim(p \wedge q) \equiv (p \wedge \sim q) \vee (\sim p \wedge q)$$

LHS $= \sim(p \leftrightarrow q) = \sim((p \to q) \wedge (q \to p))$

$= \sim((\sim p \vee q) \wedge (\sim q \vee p))$ $[\because p \to q \equiv \sim p \vee q]$

$= \sim((\sim p \vee q) \wedge \sim q) \vee ((\sim p \vee q) \wedge p)$ Distributive

$= \sim[(\sim q \wedge (\sim p \wedge q)) \vee (p \wedge (\sim p \vee q))]$ Commutative

$= \sim[\{(\sim q \wedge \sim p) \vee (\sim q \wedge q)\} \vee ((p \wedge \sim q) \vee (p \wedge q))]$

Distributive

$= \sim[(\sim(q \vee p) \vee (F)) \vee (F \vee (p \wedge q))]$

$= \sim[\sim(q \vee p) \vee (p \wedge q)]$ Identity law

$= \boxed{p \vee q \wedge \sim(p \wedge q)}$ Demorgan's law.

$= (p \vee q) \wedge (\sim p \vee \sim q)$ Demorgans

$= ((p \vee q) \wedge \sim p) \vee ((p \vee q) \wedge \sim q)$ Distributive

$= (\sim p \wedge (p \vee q)) \vee (\sim q \wedge (p \vee q))$

$= ((\sim p \wedge p) \vee (\sim p \wedge q)) \vee ((\sim q \wedge p) \vee (\sim q \wedge q))$

Distributive

$= (F \vee (\sim p \wedge q)) \vee [(p \wedge \sim q) \vee F]$ ~~commu~~ compliment

$= (\sim p \wedge q) \vee (p \wedge \sim q)$ Identity law

$= (p \wedge \sim q) \vee (\sim p \wedge q)$ commutative law.

Q. Prove the following equivalence by proving the equivalence of ~~their~~ the duals.

(i). $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

(ii). $(p \wedge (p \leftrightarrow q)) \rightarrow q \equiv T$

Sol^n (i). $(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

$\sim(p \vee q) \vee r \equiv (\sim p \vee r) \wedge (\sim q \vee r)$ — ① $\left[\because p \rightarrow q \equiv \sim p \vee q\right]$

Dual of — ① is :

$\sim(p \wedge q) \wedge r \equiv (\sim p \wedge r) \vee (\sim q \wedge r)$

LHS $= \sim(p \wedge q) \wedge r$

$= (\sim p \vee \sim q) \wedge r$ Demorgans.

$= r \wedge (\sim p \vee \sim q)$ commutative

$= (r \wedge \sim p) \vee (r \wedge \sim q)$ Distributive.

$= (\sim p \wedge r) \vee (\sim q \wedge r)$ = RHS.

Since dual are equivalent ⇒ primals are also equivalent. (By duality th.).

(ii). $\sim(p \wedge (p \leftrightarrow q)) \vee q \equiv T$

$\sim(p \wedge ((p \rightarrow q) \wedge (q \rightarrow p))) \vee q \equiv T$

$\sim(p \wedge ((\sim p \vee q) \wedge (\sim q \vee p))) \vee q \equiv T$ — ②

Dual of ②

$$\sim(p \vee ((\sim p \wedge q) \vee (\sim q \wedge p))) \wedge q \equiv F$$

$\underline{LHS} = \sim(p \vee ((\sim p \wedge q) \vee (\sim q \wedge p))) \wedge q$

$= \sim p \wedge \sim((\sim p \wedge q) \vee (\sim q \wedge p)) \wedge q$

Demorgan's

$= \sim p \wedge [\sim(\sim p \wedge q) \wedge \sim(\sim q \wedge p)] \wedge q$

Demorgan's

$= \sim p \wedge [(p \vee \sim q) \wedge (q \vee \sim p)] \wedge q$ Demorgans

$= \sim p \wedge [[(p \vee \sim q) \wedge q] \vee [(p \vee \sim q) \wedge \sim p]] \wedge q$

$\sim q \vee p$ Distributive

~~$= \sim p \wedge [p \vee (\sim q \wedge q)] \vee [\sim q \vee ($~~

$= \sim p \wedge ([(p \wedge q) \vee (\sim q \wedge q)] \vee [(p \wedge \sim p) \vee (\sim q \wedge \sim p)]) \wedge q$

Distributive

$= \sim p \wedge [((p \wedge q) \vee F) \vee$

→ Tautological Implication :-

A compound prop. $A(p_1, p_2, \ldots p_n)$ is said to be tautologically imply $B(p_1, p_2, \ldots p_n)$ iff

$A \rightarrow B$ is tautology

and we denote $A \Rightarrow B$

Eg: $p \Rightarrow p \vee q$ i.e.

$p \rightarrow (p \vee q)$ is a tautology.

p	q	$p \vee q$	$p \rightarrow (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

or

$p \rightarrow (p \vee q) \equiv T$

$\sim p \vee (p \vee q)$

$(\sim p \vee p) \vee q$

$T \vee q \equiv T$

Q. By truth table prove -

$$p \rightarrow ((p \rightarrow r)) \Rightarrow (p \rightarrow q) \rightarrow (p \rightarrow r)$$

Sol^n i.e. $[p \rightarrow ((p \rightarrow r))] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$

is a tautology.

p	q	r	$p \rightarrow r$	$p \rightarrow q$	$p \rightarrow (p \rightarrow r)$ (a)	$(p \rightarrow q) \rightarrow (p \rightarrow r)$ (b)	$a \rightarrow b$
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Given statement is a tautology.

$$p \rightarrow ((p \rightarrow r)) \Rightarrow (p \rightarrow q) \rightarrow (p \rightarrow r)$$

Q. Without Truth table prove -

$$(p \vee q) \wedge (p \to r) \wedge (q \to r) \Rightarrow r$$

$\{p \to q = \sim p \vee q\}$

i.e. prove $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r \equiv T$

LHS. $= [(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

~~$[(p \vee q) \wedge (\sim p \vee r) \wedge (\sim q \vee r)]$~~

$= [(p \vee q) \wedge [(p \vee q) \to r]] \to r$ $\quad [\because (p \to r) \wedge (q \to r) = (p \vee q) \to r]$

$= [(p \vee q) \wedge (\sim (p \vee q) \vee r)] \to r$

$= [((p \vee q) \wedge \sim (p \vee q)) \vee ((p \vee q) \wedge r)] \to r$ Distributive

$= [F \vee ((p \vee q) \wedge r)] \to r$ compliment law.

$= [(p \vee q) \wedge r)] \to r$ Identity law

$= [(p \wedge r) \vee (q \wedge r)] \to r$ Distributive

$= \sim ((p \wedge r) \vee (q \wedge r)) \vee r$ $\quad p \to q = \sim p \vee q$

$= [\sim (p \wedge r) \wedge \sim (q \wedge r)] \vee r$ Demorgans.

$= (\sim (p \wedge r) \vee r) \wedge (\sim (q \wedge r) \vee r)$ Distributive

$= ((\sim p \vee \sim r) \vee r) \wedge ((\sim q \vee \sim r) \vee r)$ Demorgans.

$= (\sim p \vee (\sim r \vee r)) \wedge (\sim q \vee (\sim r \vee r))$ Associative

$= (\sim p \vee T) \wedge (\sim q \vee T)$ Compliment law.

$= T \wedge T \equiv T$ Dominant law.

Q. Prove the following by duals -

If, $A \equiv B$.
then, $A^* \equiv B^*$

$$(p \to (p \leftrightarrow q)) \to q \equiv T$$

Sol^n $\{p \to [(p \to q) \wedge (q \to p)]\} \to q \equiv T$

$$\sim (p \vee ((\sim p \vee q) \wedge (\sim q \vee p))) \to q \equiv T$$

→ Theory of Inference -

Inference theory is concerned with the inferring conclusion from certain hypothesis called premises. By applying rules of inference -

⊛ Argument - A sequence of statements is called argument. It has premises and conclusion.

We write

$$\left.\begin{array}{l} p_1 \\ p_2 \\ \vdots \\ p_n \end{array}\right\} \text{premises}$$

$$\overline{\quad c \quad} \leftarrow \text{conclusion.}$$

Q. If it rains heavily then travelling will be difficult.

If students arrive on time, then travelling will not be difficult.

They arrived on time. Therefore, it didn't rain.

Solⁿ. p: It rain heavily

q: Travelling is difficult

r: Students arrive on time

$$\begin{array}{l} p \to q \\ r \to \sim q \\ r \\ \hline \sim p \end{array}$$

(cont.) - -

(*). An argument is called a valid argument if conclusion can be drawn by the set premises by foll. rules of inference.

(1) Truth table techniques - If

$$\begin{array}{c} H_1 \\ H_2 \\ \vdots \\ H_n \\ \hline C \end{array}$$

then argument is valid

iff $H_1 \wedge H_2 \wedge H_3 \ldots H_n \Rightarrow C$

i.e. $H_1 \wedge H_2 \wedge H_3 \ldots H_3 \to C$ is tautology

Sol^n continued..

$$((p \to q) \wedge (r \to \sim q) \wedge r) \to \sim p$$ is tautology.

p	q	r	$\sim p$	$\sim q$	$p \to q$ (a)	$r \to \sim q$ (b)	$a \wedge b \wedge r$ (c)	$c \to \sim p$
T	T	T	F	F	T	F	F	T
T	T	F	F	F	T	T	F	T
T	F	T	F	T	F	T	F	T
T	F	F	F	T	F	T	F	T
F	T	T	T	F	T	F	F	T
F	T	F	T	F	T	T	F	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	F	T

This is a tautology, so given argument is valid.

(2). Rules of Inference -

(a). Rule P - A premises may be introduced at any step of derivation.

(b). Rule T - A formula S which is tautologically equivalent to a given premises may be used ~~as~~ at any step in derivation.

Statement	Name of Rule
(i). $p \rightarrow p \vee q$ $q \rightarrow p \vee q$	Addition
(ii). $p \wedge q \rightarrow p$ $p \wedge q \rightarrow q$	Simplification
(iii). $(p) \wedge (q) \rightarrow p \wedge q$	Conjunction
(iv). $p \wedge (p \rightarrow q) \rightarrow q$	Modus ponens
(v). $\sim q \wedge (p \rightarrow q) \rightarrow \sim p$	Modus tollens
(vi). $(p \rightarrow q) \wedge (q \rightarrow r) \rightarrow p \rightarrow r$	Hypothetical syllogism
(vii). $(p \vee q) \wedge \sim p \rightarrow q$	Disjunctive syllogism.

Q. $p \to q$

$r \to \sim q$

r

$\sim p$

Proof: - We will prove by rule of inference.

S.N.	Statement	Reason
1.	$r \to \sim q$	Rule P
2.	$p \to q$	Rule P
3.	$\sim q \to \sim p$	rule T, contrapositive in 2.
4.	$r \to \sim p$	rule T; hypothetical syllogism 1 and 3.
5.	r	Rule P.
6.	$\sim p$	Rule T, 5 and 4 modus ponens

Q. Show that $(t \wedge s)$ can be derived from the premises

$p \to q$, $q \to \sim r$, r, $p \vee (t \wedge s)$

Sol:

S.N.	Statement	Reason
1.	$p \to q$	Rule P ~~(hypothetical syllogism)~~
2.	$q \to \sim r$	Rule P
3.	$p \to \sim r$	Rule T, (hypothetical syllogism) 1, 2.
4.	$\sim(\sim r) \to \sim p$ $r \to \sim p$	Rule T, equivalent of 3.
5.	r	rule P.
6.	$\sim p$	Rule T, Modus ponens
7.	$p \vee (t \wedge s)$	Rule P.
8.	$t \wedge s$	Rule T Disjunctive syllogism. 6, 7.

→ Rule- CP (Conditional Preposition Rule)

$$(p \wedge r) \rightarrow s \equiv p \rightarrow (r \rightarrow s)$$

If the conclusion is of the form $r \rightarrow s$ then we will take r as additional premises and using r and given premises derive r.

Q. ~~Derive~~ Derive $p \rightarrow (q \rightarrow s)$ using CP rule from the premises $p \rightarrow (q \rightarrow r)$ & $q \rightarrow (r \rightarrow s)$

Sol^n. Let p be an additional proposition then ⇒

$$\begin{array}{c} p \\ p \rightarrow (q \rightarrow r) \\ q \rightarrow (r \rightarrow s) \\ \hline (q \rightarrow s) \end{array}$$

S.N.	Statement	Reason.
1.	p.	Rule P, additional premises
2	$p \rightarrow (q \rightarrow r)$	Rule P
3.	$q \rightarrow r$	Rule T, modus ponens
4	$\sim q \vee r$	Rule T, equivalence in 3
5.	$q \rightarrow (r \rightarrow s)$	Rule P
6.	~~$\sim q \vee (p \rightarrow r)$~~ $\sim q \vee (r \rightarrow s)$	Rule T, equivalence in 5

7. ~~$\sim q \vee (r \vee t$~~

$\sim q \vee (r \wedge (r \to s))$ — rule T, distributive in 4 and 5.

8. $\sim q \vee s$ — Rule T, Modus ponens in 7

9. $q \to s$ — Rule T, equivalence in 8.

10. $p \to (q \to s)$ — by C.P. rule.

→ <u>Inconsistent Premises</u> - A set of premises $P_1, P_2, \ldots P_n$ is called inconsistent iff $P_1 \wedge P_2 \wedge P_3 \wedge \ldots P_n \Rightarrow R \wedge \sim R$ ← false

for some formula R.

otherwise it is <u>consistent</u>.

Q Prove that premises $p \to q$, $q \to r$, $s \to \sim r$ and $q \wedge s$. are inconsistent.

<u>Sol^n</u> If we derive contradiction (False) then the given premises are inconsistent.

S.N.	Statement	Reason
1.	$p \to q$	Rule P
2	$q \to r$	Rule P
3	$p \to r$	Rule T, Hyp. syll.

4. $s \to \sim r$ Rule P

5. $r \to \sim s$ Rule T, equivalence in 4.

~~6. $p \to \sim s$ Rule T, Hyp. syll. in 3 & 5~~

6. $q \to \sim s$ Rule T, Hyp. syll. in 2 & 5

7. $\sim q \vee \sim s$ Rule T, equivalence in 6.

8. $\sim(q \wedge s)$ Rule T, Demorgan's

9. $q \wedge s$ ~~Rule~~ P.

10. $(q \wedge s) \wedge \sim(q \wedge s)$ conjunction in 8, 9

11. F equivalence in 10

→ Indirect method - To prove the validity of

$$\begin{array}{c} H_1 \\ H_2 \\ \vdots \\ H_n \\ \hline C \end{array}$$

we can add $\sim C$ as additional proposition and try to prove contradiction i.e. False

Q. use indirect method to show that

$$(r \to \sim q,\ r \vee s,\ s \to \sim q,\ p \to q) \Rightarrow \sim p.$$

sol^n for indirect method take $\sim(\sim p) \equiv p$ as additional proposition and we prove that this given set of premise is inconsistent.

S.N.	Statement	Reason
1.	$p \to q$	Rule P.
2.	p	Rule P
3.	q	Rule T., modus ponens in 1, 2
4.	$s \to \sim q$	Rule P.
5.	$\sim s$	Rule T, modus tollens.
6.	$r \vee s$	Rule P.
7.	r	Disjunctive syll. 5, 6
8.	$r \to \sim q$	Rule P
9.	$\sim q$	modus ponens.
10.	$q \wedge \sim q$	conjunctive

→ Mathematical Induction -

principle of mathematical Induction -

Let $P(n)$ be a statement, satisfying the following condition.

(1). $P(n_0)$ is true for some $n_0 \in Z$

(2) If $P(k)$ is true for all $k \geq Z$, then $P(k+1)$ is true.

Then $p(n)$ is true for n.

Q. Prove that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots \frac{1}{\sqrt{n}} > \sqrt{n}$

for $n \geq 2$

Let $S_n = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots \frac{1}{\sqrt{n}}$

if $n = 2$. $S_2 = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} = 1 + \frac{\sqrt{2}}{2} = 1.704 > \sqrt{2}$

1.414

Let the result be true for all integer less than or equal to k.

$S_k = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots \frac{1}{\sqrt{k}} > \sqrt{k}$

now, $S_{k+1} = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \cdots \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}}$ ~~$\sqrt{k} + \frac{1}{\sqrt{k+1}}$~~

$> \sqrt{k} + \frac{1}{\sqrt{k+1}}$

will become small because $k+1 > k$.

$> \frac{\sqrt{k(k+1)} + 1}{\sqrt{k+1}} > \frac{\sqrt{k(k)} + 1}{\sqrt{k+1}}$

$> \frac{\sqrt{k^2} + 1}{\sqrt{k+1}}$

$> \frac{k+1}{\sqrt{k+1}}$

$\geq \sqrt{k+1}$

Hence, By Induction method, $S_n > \sqrt{n}$ $\forall n$.

Q. Use mathematical induction method to prove $3^n + 7^n - 2$ is divisible by 8, $\forall n \geq 1$

Sol^n Let $S_n = 3^n + 7^n - 2$

if $n = 1$ $S_1 = 3 + 7 - 2 = 8$ is div. by 8.

Let S_n be true for integers less than equal to k.

$S_k = 3^k + 7^k - 2$ is divisible by 8.

for $\underline{k+1}$.

$$S_{k+1} = 3^{k+1} + 7^{k+1} - 2$$

$$= 3.3^k + 7.7^k - 2$$

~~3+5~~

$$= 3.3^k + 3.7^k - 6 + 4.7^k + 4$$

$$= \underbrace{3\left[3^k + 7^k - 2\right]}_{S_k} + 4(7^k + 1)$$

S_k. div. by 8.

7^k, 1 → odd + odd → even

(4 x even) → div. by 8.

Hence S_{k+1} is div. by 8. ~~by~~

Therefore, by induction method S_n is div. by 8 $\forall n \geq 1$