UNIT-1 Numbers.

In decimal number system, there are ten symbols namely 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 called digits. A number is denoted by group of these digits called as numerals.

Face Value:- Face value of a digit in a numeral is value of the digit itself. for eg. 321, face value of 2 is 2 and face value of 3 is 3.

Place Value:- Place Value of a digit in a numeral is value of the digit multiplied by 10^n where n starts from 0.

for eg. 321.

- Place value of 1 = $1 \times 10^0 = 1 \times 1 = 1$
- Place Value of 2 = $2 \times 10^1 = 2 \times 10 = 20$
- Place Value of 3 = $3 \times 10^2 = 3 \times 100 = 300$

Types of Numbers

- **Natural numbers** $n > 0$ where n is counting number [1, 2, 3, ---]
- **Whole numbers** $n \geq 0$ where n is counting number. [0, 1, 2, 3, - -]
  - 0 is the only whole number which is not a natural number.
  - Every natural number is a whole number.
- **Integers:-** $n \geq 0$ & $n \leq 0$ where n is counting number.
  ---, -3, -2, -1, 0, 1, 2, 3, - - are integers.

* Positive Integers: $n>0$, [1, 2, 3, --]
* Negative Integers $n<0$; [-1, -2, -3, --]
* Non-Positive Integers :→ $n \leq 0$; [0, -1, -2, -3, --]
* Non-Negative Integers: $n \geq 0$, [0, 1, 2, 3, --]

0 is neither positive nor negative integer.

* Even Numbers, $n/2 = 0$ where n is counting number.
  [0, 2, 4]
* odd Number: $\frac{n}{2} \neq 0$ where n is counting number
  [1, 3, 5, --]
* Prime Numbers Numbers which is divisible by themselves only apart from 1.

Ex → 191 is prime number or not?
Solⁿ → $14^2 > 191$
Prime Numbers Less than 14 are 2, 3, 5, 7, 11 & 13.
191 is not divisible by any above prime number.
191 is a prime number

* 187 is a prime number or not.
  $14^2 > 187$.
  187 is divisible by 11.
  187 is a prime number.

Composite Number:- Non prime numbers > 1
for eg. 4, 6, 8, 9 etc.

* 1 is neither prime nor composite number.
* 2 is the only even prime number.

Primes Numb... their ...

Coprimes Numbers :- Two natural numbers are coprimes if their HCF is 1. for eg. (2,3) (4,5) are coprimes.

Divisibility by 2 : A number is divisible by 2 if its unit digit is 0, 2, 4, 6 or 8.

Eg. 64578 is divisible by 2 or not.
Yes it is divisible by 2

eg. 64575 is divisible by 2
No, it is not divisible by 2.

Divisibility by 3 : A number is divisible by 3 if sum of its digits is completely divisible by 3.

Eg. 64578 is divisible by 3 or not?
Soln: 6+4+5+7+8 = 30
It is divisible by 3.

Divisibility by 4 ⇒ A number is divisible by 4 if number formed using its last two digits is completely divisible by 4.

eg. 64578 is divisible by 4 or not.
Not because last two digit it is not divisible by 4.

Divisibility by 5 :- A number is divisible by 5 if its unit is unit digit 0 or 5.

Eg. 64578 is not divisible by 5 because unit digit is not divisible 0 or 5.
64576 is not divisible by 0 or 5.

Divisibility by 6 : A number is divisible by 6 if the number is divisible by both 2 & 3.

64578 is divisible by 2 because it unit digit is 0
by 2.

is divisible by 3

6+4+5+7+8 = 30

30 is divisible by 3.

So 64578 is divisible by 6.

eg. 64576

* Divisibility by 8 :→ A number is divisible by 8 if number formed using its last three digit is completely divisible by 8.

eg. 64578 is divisible by 8.

64578 is not divisible by 8.

Last three digit 578 is not divisible by 8.

* 64576 is divisible by 8.

* Divisibility by 9 :→ A number is divisible by 9 if ~~no~~ sum of its digit is divisible by 9.

eg. 64579 is divisible by 9 or not.

6+4+5+7+9 = 31.

64579 is not divisible by 9 because the sum of all digit is 31 and 31 is not divisible by 9.

* Divisibility by 10 : A number is divisible by 10 if its unit digit is 0.

eg. 64575 its unit digit is 5 so it is not divisible by 10.

64570 its unit digit is 0 so it is divisible by 10.

* Divisibility by 11: A number is divisible by 11 if difference between odd places and even places is 0

1 2 3 4 5
64575 is

4+7 =

6+5

$\overset{1}{6}\overset{2}{4}\overset{3}{5}\overset{4}{7}\overset{5}{5}$ is divisible by 11 or not.

$4+7 = 11$

$6+5+5 = 16$

$16-11 = 5$

difference is not 0 so it is not divisible by 11.

eg. $\overset{1}{6}\overset{2}{4}\overset{3}{0}\overset{4}{7}\overset{5}{5}$

$4+7 = 11$

$6+5 = 11$

$11-11 = 0$

difference is 0 so it is divisible by 11.

Tips On Division

- If a number n is divisible by two coprimes numbers a, b then n is divisible by ab.
- $(a-b)$ always divides (a^n-b^n) if n is a natural number.
- $(a+b)$ always divides (a^n-b^n) if n is a even number.
- $(a+b)$ always divides (a^n+b^n) if n is odd number.
- $(a+b)$ always

Division Algorithm

When a number is divided by another number then

Dividend = (Divisor × Quotient) + Reminder.

Series

following are formulaes for basic number series.

- $(1+2+3+ \cdots +n) = \frac{n(n+1)}{2}$
- $(1^2+2^2+3^2+ \cdots +n^2) = \frac{n(n+1)(2n+1)}{6}$
- $(1^3+2^3+3^3+ \cdots +n^3) = \frac{n^2(n+1)^2}{4}$

Basic formulaes

These are the basic formulae:

$$(a+b)^2 = a^2+b^2+2ab$$

$$(a-b)^2 = a^2+b^2-2ab$$

$$(a+b)^2-(a-b)^2 = 4ab$$

$$(a+b)^2+(a-b)^2 = 2(a^2+b^2)$$

$$(a^2-b^2) = (a+b)(a-b)$$

$$(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca)$$

$$(a^3+b^3) = (a+b)(a^2-ab+b^2)$$

$$(a^3-b^3) = (a-b)(a^2+ab+b^2)$$

$$(a^3+b^3+c^3-3abc) = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

Highest Power of a number in factorial

What is factorial?

Factorial of a number n is given by

$$n! = n(n-1)(n-2) \times (n-3) \cdots \times 2 \times 1.$$

for eg $4! = 4 \times 3 \times 2 \times 1 = 24$.

Greatest Integer function

If x is an integer, then the greatest integer function of x is the greatest integer that is less than or equal to x. It is represented by $[x]$.

Eg Greatest integer less than equal to $2.1 \Rightarrow [2.1] = 2$

$[1.9] = 1$, $[-2.1] = -3$

Calculating Highest power of a number in a factorial

If p is prime number, then the highest power of p in a factorial n is given by.

$$\left[\frac{n}{p}\right] + \left[\frac{n}{p^2}\right] + \left[\frac{n}{p^3}\right] + \cdots$$

Highest Power of prime number in factorial

Calculation of the maximum power of a prime number in a factorial simply involves using the formula described above.

Q1 → Highest power of 3 in 15!

$$\frac{15!}{3} = \frac{5}{3} = 1$$

$$5 + 1 = 6.$$

Highest power of 3 is 6.

Q→ Maximum power of 2 in 48!

$\frac{48!}{2} = \frac{24}{2} = \frac{12}{2} = \frac{6}{2} = \frac{3}{2} = 1$

$24+12+6+3+1$

$\Rightarrow 46$

Maximum power of 2 is 46.

Q→ Maximum power of 3 in 50!

$\frac{50!}{3} = \frac{16}{3} = \frac{5}{3} = 1$

$16+5+1 = 22$

The maximum power of 3 is 22.

Highest power of composite number in a factorial

To calculate the highest power of composite number in a factorial, we first express the composite number as a product of primes. Among these prime factors, the highest power of the largest prime factor will be equal to the highest power of that composite number.

Q→ Maximum power of 15 in 24!

$\frac{24!}{15} = \frac{24!}{3\times5} = \frac{24}{3} = \frac{8}{3} = 2$

$8+2 = 10$

$\frac{24!}{5} = 4$

Maximum power of 15 is 4.

Q→ Maximum power of 6 which can divide 120!

$\frac{120!}{6} = \frac{120!}{3\times2} = \frac{120}{3} = \frac{40}{3} = \frac{13}{3} = \frac{4}{3} = 1$

$40+13+4+1 = 58$

$\frac{120}{2} = \frac{60}{2} = \frac{30}{2} = \frac{15}{2} = \frac{7}{2} = \frac{3}{2} = 1$

$60+30+15+7+3+1 = 116.$

Maximum

Find

Q→

S

Maximum power of 6 which divide 120! is 58.

Q→ find the maximum power of 15 in 75!

Sol^n ⇒ $\frac{75!}{15} = \frac{75}{3\times5}$

$\frac{75}{3} = \frac{25}{3} = \frac{8}{3} = 2$

$25+8+2 = 35$

$\frac{75!}{\cancel{3}5} = \frac{15}{5} = 3$

18

The maximum power of 15 divide 75! is 18.

Note :→ If p is a prime number, the highest power of p^a present in a factorial a n is given by

$\left[\frac{\text{Highest of p in n!}}{a} \right]$

Q→ Maximum power of 9 in 32!

$\frac{32!}{3^2} = \frac{32}{3} = \frac{10}{3} = \frac{3}{3} = 1$

$\cancel{8}\ 10+3+1 = 14$

$\frac{14}{2} = 7$.

Q→ find the maximum power of 8 in 100!

$\frac{100!}{2^3} = \frac{100}{2} = \frac{50}{2} = \frac{25}{2} = \frac{12}{2} = \frac{6}{2} = \frac{3}{2} = 1$

$50+25+12+6+3+1 = \frac{97}{3}\ 32$

32 is the maximum power of 8 in 100!.

Q→ What is the highest power of 12 that divides 54!

$\frac{54!}{2^2 \times 3} = \quad \frac{54}{3} = \frac{18}{3} = \frac{6}{3} = 2$

$18+6+2 = 26$

$\frac{54}{2} = \frac{27}{2} = \frac{13}{2} = \frac{6}{2} = \frac{3}{2} = 1$

$27+13+6+3+1 = \frac{50}{2} = 25$.

25 is the highest power of 12 divides 54!

Q→ find the maximum power of 40 in 120!

$\frac{120!}{2^3 \times 5} = \quad \frac{120}{2} = \frac{60}{2} = \frac{30}{2} = \frac{15}{2} = \frac{7}{2} = 3$

$= \cancel{120}\ 60+30+15+7+3$

$= 115$.

$\frac{120}{5}\ \frac{24}{5}\ 4$

$= 24+4 = 28$.

28 is the maximum power of 40 in 120!

HCF and LCM

54

HCF and LCM of two Numbers

~~LCM~~ If a and b be two numbers then the HCF & LCM of two numbers is.

LCM × HCF (a, b) = Product of two numbers (a, b)

$$LCM(a,b) = \frac{a \times b}{HCF(a,b)}$$

$$HCF(a,b) = \frac{a \times b}{LCM(a,b)}$$

Q1 → Two numbers are in the ratio 3:5 and their LCM is 1500. find the HCF of the numbers.

(a) 100 (b) 110 (c) 120 (d) 130.

Let $3x$ & $5x$

$LCM = 15x$

$\cancel{1500} = \cancel{15}x$
100

$x = 100$

Q2 → find the smallest number that leaves a remainder 4 on division by 5, 5 on division by 6, 6 on division by 7, 7 on division by 8 and 8 on division by 9?

(a) 2519 (b) 2520 (c) 2619 (d) 2700.

LCM of 4, 5, 6, 7, 8, 9

$N = 5a - 1$
$N = 6b - 1$
$N = 7c - 1$
$N = 8d - 1$
$N = 9e - 1$

$= 2520 - 1$
$= 2519$ Ans

Q3 → How many pairs of positive integers x, y exist su
HCF of $x, y = 35$ and sum of $x + y = 1085$?
(a) 10 (b) 15 (c) 20 (d) 25

Q4 → Find the greatest number that will divide 72, 96 and 120
Leaving the same remainder in each case.

Q5 → If the HCF of two no. is 12 and their LCM is 360,
find the numbers.

Q6 → What is the largest three-digit number that is exactly
divisible by the HCF of 24 and 36.

Number of factors

Prime factorisation :→ We all know that every composite number can be written as a product of some prime numbers. for eg 90 as $2 \times 3^2 \times 5$. This process is called prime factorisation and it is the very first step to solve any questions related to factors.

Important formula

Number of factors

If we have to find the number of factors of any number say N, then we should follow below steps.

Step 1 : Prime factorize $N = p^a \times q^b \times r^c \times \ldots$

Step 2 : The number of factors of $N = (a+1)(b+1)(c+1)\ldots$

Sum of factors

To find the sum of all the factors

$$\frac{(p^0+p^1+\ldots+p^a)(q^0+q^1+\ldots+q^b)(r^0+r^1+\ldots+r^c)}{(p^a-1)(q^b-1)(r^c-1)}$$

Q1→ Consider the number 120. find the following for n

1. Sum of factors
2. Number of factors
3. Product of factors ⇒ $N^{\frac{\text{No. of factors}}{2}}$

$120 = 2^3 \times 3^1 \times 5^1$. By applying the formulae

- Sum of factors = $\frac{[(2^0+2^1+2^2+2^3)(3^0+3^1)(5^0+5^1)]}{[(2-1)(3-1)(5-1)]}$

$= 45$

- No. of factor $= (3+1)(1+1)(1+1)$

$= 4 \times 2 \times 2 = 16$.

- Product of factors $= (120)^{16/2} = (120)^8$.

Q→ find the following for the number 84.

1. Number of odd factors.
2. Number of even factors.

Sol^n→ By the prime factorisation of 84

$$84 = 2^2 \times 3^1 \times 7^1$$

Total number of factors $= (2+1)(1+1)(1+1)$
$= 3 \times 2 \times 2$
$= 12$

odd factors $= (1+1)(1+1)$
$= 2 \times 2 = 4$

No. of even factors = Total no. of factors − odd factor.
$= 12 - 4$
$= 8$.

Imp

Q→ Let $N = 3^{15} \times 7^{43}$. How many factors of N^2 are less than N but not divide completely?

The

be

Unit 1
Logarithm

The Logarithm is an exponent or power to which a base must be raised to obtain a given number. Mathematically, Logarithms are expressed as, m is the Logarithm of n to the base b if $b^m = n$, which can also be written as $m = \log_b n$.

$\boxed{b^y = a \Rightarrow \log_b a = y}$ ⇒ where a & b = two positive real no., y = real no. a = argument, b = base

for eg $4^3 = 64$ ⇒ hence 3 is the Logarithm of 64 to base 4, or $3 = \log_4 64$.

Similarly, we know $10^3 = 1000$, Logarithms with base 10 are usually known as common or Briggsian Logarithms and are simply expressed as log n. In this article, we will discuss What is Logarithms formulas, basic Logarithm formulas, change of base rule, Logarithms rule and formulas.

Logarithms Rules

There are 7 Logarithm rules which are useful in expanding Logarithm, contracting Logarithms, and solving Logarithmic equations. The seven rules of Logarithms are discussed below:

1. Product Rules

$$\log_b(P \cdot Q) = \log_b P + \log_b Q.$$

The Logarithm of the product is the total of the logarithm of the factors.

2. Quotient Rule:

$$\log_b\left(\frac{P}{Q}\right) = \frac{\log_b P}{\log_b Q} = \log_b P - \log_b Q.$$

The Logarithm of the ratio of two numbers is the difference between the logarithm of the numerator and denominator.

3. Power Rule:-

$$\log_b(p^q) = q \times \log_b p$$

The above Property of the product rule states that the logarithm of a positive number p to the power q is equivalent to the product of q and log of p.

4. Zero Rule:

$$\log_b(1) = 0$$

The Logarithm of 1 such that b greater than 0 but $b \neq 1$, equals zero.

5. The Logarithm of a Base to a Power Rule

$$\log_b b^y = y$$

The logarithm of a base to a power rule states that the logarithm of b with a rational exponent is equal to the exponent times its logarithm.

6. A Base to a Logarithm Power Rule

$$b^{\log y} = y$$

The above rule states that raising the logarithm is equal to the number.

7. Identity Rule:-

$$\log_y y = 1$$

The argument of the logarithm is similar to the base. As the base is equal to the argument, y can be greater than 0 but cannot be equals to 0.

Logarithm
Below
help

the

Logarithm formulas

Below are some of the different Logarithm formulas which help to solve the logarithm equations.

Basic Logarithm formula

Some of the different Basic logarithm formulas which help to solve the logarithm equations.

Are given below:

- $\log_b(m \times n) = \log_b m + \log_b n \Rightarrow$ for eg $\log_3(4y) = \log_3(4) + \log_3(y)$
- $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n \Rightarrow \log_3\left(\frac{5}{y}\right) = \log_3(5) - \log_3(y)$
- $\log_b(x^y) = y \times \log_b x \Rightarrow \log_b(4^3) = 3\log_b 4$
- $\log_b \sqrt[m]{n} = \log_b n^{1/m}$
- $m\log_b(x) + n\log_b(y) = \log_b(x^m y^n)$

Addition and Subtraction

- $\log_b(m+n) = \log_b m + \log_b\left(1 + \frac{n}{m}\right)$
- $\log_b(m-n) = \log_b m + \log_b\left(1 - \frac{n}{m}\right)$

Change of Base formula

In the change of base formula, we will convert the Logarithm from a given base 'n' to base 'd'.

$$\log_n m = \frac{\log_d m}{\log_d n}.$$

* **Change of Base Rule**

$\log_b(m^n) = n\log_b m$

for eg. $\log_b(\cancel{4}3) = \log_a 3 / \log_a b$

* **Base Switch Rule**

$$\log_b(a) = \frac{1}{\log_a b}$$

Ex. $\log_b 7 = \dfrac{1}{\log_7 b}$.

* **Derivative of Log**

$f(x) = \log_b(x)$, then the derivative of $f(x)$ is given by;

$$f'(x) = \frac{1}{x\log b}$$

for eg :→ $f(x) = \log_8(x)$

then $f'(x) = \dfrac{1}{x\log 8}$

* **Integral of Log**

$$\int \log_b(x)dx = x\left\{\log_b(x) - \frac{1}{\log b}\right\} + C$$

Eg :→ $\int \log_8(x)\,dx = x \cdot \left\{\log_8(x) - \dfrac{1}{\log 8}\right\} + C$

Other Properties

Some related properties of Logarithmic functions are.

- $\log_b b = 1$
- $\log_b 1 = 0$
- $\log_b 0 =$ undefined.
- $\log_a a^N = N$
- $a^{\log_a b} = b$
- $a^{\log_b c} = c^{\log_b a}$
- $\log_{a^b} M = \dfrac{1}{b}\log_a M$
- $\log_b M = \dfrac{\log_a M}{\log_a b}$
- $\log_a b = \dfrac{1}{\log_b a}$
- $\log_a b \cdot \log_b c = \log_a c$

Basic logarithm formulas

- $\log_b(mn) = \log_b m + \log_b n$
- $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$
- $\log_b(x^y) = y\log_b x$
- $\log_b m\sqrt{n} = \log_b \frac{n}{m}$
- $\log_b(x) + n\log_b(y) = \log_b(x^m y^n)$
- $\log_b(m+n) = \log_b m + \log_b(1+nm)$
- $\log_b(m-n) = \log_b m + \log_b(1-nm)$

Remember

$\log_a(b+c) \neq \log_a b + \log_a c$

Solved Eg

Q1 → Solve $\log_2(32) = ?$

Solⁿ → Since $\log_2(2^5) = 5$. Ans

Q2 → What is the value of $\log_{10}(100)$?

Solⁿ → $\log_{10}(10^2) = 2$.

Q3 → Use of the property of Logarithms solve for the value of x for $\log_3 x = \log_3 5 + \log_3 4$.

Solⁿ → $\log_3(x) = \log_x(5\times4)$

$\log_3(x) = \log_x(20) \Rightarrow \boxed{x = 20}$

Q4 → Solve for x in $\log_2 x = 4$

Solⁿ → $2^4 = x$

$16 = x$ Ans

Characteristic of logarithm

The characteristic of the logarithm of a number greater than one is one less than the number of digits in it.

Eg ⇒ characteristic of 98765 = 4 so total digits in the given number is 5.

We know that $\log 100 = \log 10^2 = 2\log 10 = 2.00$

As the characteristic of 100 is 2, the total digits in 100 are 3.

$\log 10 = \log 10^1 = 1 \log 10 = 1$ so the digits are 2 in 10.

~~Sol~~ Eg - How many digits are contained in the number 2^{100}.

Sol ⇒ $\log 2^{100} = 100 \times \log 2 = 100 \times .3010 = 30.10$

No. of digits in 2^{100} are ~~30~~ $30 + 1 = 31$.

To determine the characteristic of the logarithm of a decimal fraction: (Numbers between 0 to 1)

Look at this example:

find the total zeroes after the decimal point of the expansion 2^{6}.

We know that $2^{6} = \frac{1}{64} = .015625$.

$\log \frac{1}{64} = -1.806$

how many zero and the fi

log

?

8

How many zeroes are there between the decimal point and the first significant digit in $\left(\frac{1}{2}\right)^{1000}$.

Sol^n ⇒ $\log\left(\frac{1}{2}\right)^{1000} = 1000 \times \log\left(\frac{1}{2}\right) = 1000 \times -.30102$
$= -301.02$.

But in log. ⇒ $-301.02 = -301 + (-.002)$
$= -302 + (1 - .02) = \bar{302}.98$

So, Number of zeroes are $302 - 1 = 301$

Q→ $\frac{1}{1+\log_a bc} + \frac{1}{1+\log_b ca} + \frac{1}{1+\log_c ab} =$

(a) 0 (b) 3 (c) 2 (d) 1.

Ans d.

Sol^n ⇒ $\frac{1}{\log_a a+\log_a bc} + \frac{1}{\log_b b+\log_b ca} + \frac{1}{\log_c c+\log_c ab}$

$\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} =$

$= \log_{abc} a + \log_{abc} b + \log_{abc} c$

$= \log_{abc} abc = 1$

Q→ The value of $(yz)^{\log y - \log z} \times (zx)^{\log z - \log x} \times (xy)^{\log x - \log y}$

(a) 2 (b) 1 (c) 0 (d) 3.

Ans b.

Sol^n ⇒ Assume $K = (yz)^{\log y - \log z} \times (zx)^{\log y - \log x} \times (xy)^{\log x - \log y}$

Taking log on both sides

$\log K = \log\left[(yz)^{\log y - \log z} \times (zx)^{\log y - \log z} \times (xy)^{\log x - \log y}\right]$

$= \log (yz)^{\log y - \log z} + \log (zx)^{\log y - \log z} + \log (xy)$

$= (\log y - \log z) \log (yz) + (\log z - \log x) \log (zx) + (\log_a = \log_a^2$

$\log K = (\log y - \log z)(\log y + \log z) + (\log z - \log x)(\log z + \log x)$, If

$(\log x - \log y)(\log x + \log y) = 0$

$\log K = 0$ $\quad \log K = 0 \Rightarrow K = 1$ Ans

Q→ $\log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$ then $\left(\frac{x}{y} + \frac{y}{x}\right)$ is

(a) 5 (b) 9 (c) 7 (d) 0

Sol^n→ $\log\left(\frac{x+y}{3}\right) = \frac{1}{2}(\log x + \log y)$

$2 \log\left(\frac{x+y}{3}\right) = \log (xy)$

$\cancel{\log}\left(\frac{x+y}{3}\right)^2 = \cancel{\log}(xy)$

$(x+y)^2 = 9xy$

$x^2 + y^2 + 2xy = 9xy$

$x^2 + y^2 = 7xy$

$\frac{x^2}{xy} + \frac{y^2}{xy} = \frac{7xy}{xy}$

$\boxed{\frac{x}{y} + \frac{y}{x} = 7}$

Q→ The value of $7 \log_a \frac{16}{15} + 5 \log_a \frac{25}{24} + 3 \log_a \frac{81}{80}$ is

(a) $\log_a 5$ (b) $\log_a 3$ (c) $\log_a 2$ (d) $\log_a 0$

Sol^n→ $7 \log_a\left(\frac{2^4}{3 \times 5}\right) + 5 \log_a\left(\frac{5^2}{3 \times 2^3}\right) + 3 \log_a\left(\frac{3^4}{5 \times 2^4}\right)$

$= 7 (4 \log 2 - \log_a 3 - \log_a 5] + 5 [2 \log 5 - \log_a 3 - 3 \log_a 2]$

$+ 3 [4 \log_a 3 - \log_a 5 - 4 \log_a 2]$

$= 28 \log 2 - 7 \log_a 3 - 7 \log_a 5 + 10 \log 5 - 5 \log 3 - 15 \log 2 + 12 \log 3 - 3 \log_a 5 - 12 \log_a 2$

$= \log_a 2$ Ans

→ If $a = \log_{105} 7$, $b = \log_7 5$ then $\log_{35} 105 =$

(a) ab (b) $(b+1)a$ (c) $\frac{1}{ab}$ (d) $\frac{1}{a(b+1)}$

Sol^n →

$$ab = \log_{105} 7 \cdot \log_7 5$$

$$= \log_{105} 5$$

$$\log_{35} 105 = \frac{1}{\log_{105} 35} = \frac{1}{\log_{105} 7 + \log_{105} 5}$$

$$= \frac{1}{ab + a}$$

$$= \frac{1}{a(b+1)}$$ Ans

Q→ If $a = \log_4 5$ and $b = \log_5 6$ then $\log_2 3 = ?$

(a) $1-2ab$ (b) $1+2ab$ (c) $2ab-1$ (d) $\frac{a-b}{a+b}$

$$ab = \log_4 5 \cdot \log_5 6 = \log_4 6 = \frac{1}{2}\log_2 6$$

$$\log_2 6 = \log_2 3 \times 2 = \log_2 3 + \log_2 2$$

$$= \frac{1}{2}(\log_2 3 + \log_2 2)$$

$$= \frac{1}{2}(\log_2 3 + 1)$$

$$ab = \frac{1}{2}(\log_2 3 + 1)$$

$$2ab = \log_2 3 + 1$$

$$\boxed{\log_2 3 = 2ab - 1}$$

Important concepts of Remainder theorem and unit digit

Basic Remainder formula :

Dividend = Divisor × Quotient + Remainder

If remainder = 0, then it the number is perfectly divisible by divisor.

Note :- If we have a negative remainder (which was assumed for the convenience during solving of the problem) the divisor is to be added to get the real remainder.

Cyclicity ⇒

Cyclicity is an important concept ~~when~~ which can be used to solve questions on remainder theorem and unit digit.

Numbers	Cyclicity
0, 1, 5, 6	1
2, 3, 7, 8	4
4, 9	2

Euler's Remainder theorem

for coprime numbers M & N, Remainder $[M^{E(N)}/N] = 1$,
where E(N) is Euler number of N.

If $N = a^n \times b^s \times c^m \times d^v \times f^r$ such that a, b, c, d, f are prime numbers.

Euler's number of $N = N(1 - \frac{1}{a}) \times (1 - \frac{1}{b})(1 - \frac{1}{c})(1 - \frac{1}{d})(1 - \frac{1}{f})$

Remainder theorem and unit digit : Some important Points to Remember

- (a^n+b^n) is divisible by $(a+b)$, when n is odd.
- (a^n-b^n) is divisible by $(a+b)$, when n is even.
- (a^n-b^n) is always divisible by $(a-b)$, for every n.
- Remainder $\left(\frac{ab}{c}\right)$ = Rem. (Rem. $\frac{a}{c}$) × Rem$\left(\frac{b}{c}\right)$/c)

Practice questions on Remainder thm and unit digit

Q1→ What is the unit digit of $1!+2!+3!+--+88!+89!$?

(a) 5 (b) 3 (c) 1 (d) 8.

$1! = 1,\ 2! = 2,\ 3! = 6,\ 4! = 24,\ 5! = 120,\ 6! = 620, ..$

Unit digit $= 1+2+6+4+0+0+\cdot$

$= 13.$

Q2→ If 3 divided the integer n, the remainder is 2. then, what will be the remainder when 7n is divided by 3.

(a) 3 (b) 2 (c) 6 (d) 4.

Solⁿ→ b.

Remainder $\left(\frac{7n}{3}\right) =$

$\frac{n}{3} =$

$n = 3a+2$

$7n = 7 \times n$

$= 7(3a+2)$

$= 21a + 14$

$= 2$

$R = 2.$

Q3→ What is the remainder when 1294×1298 is divisible by 16.

(a) 14 (b) 11 (c) 12 (d) 10.