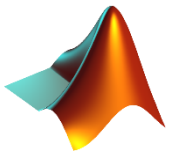
MAT 2002

MATLAB



Lab Assessment – 5

L29+L30

FALL SEMESTER 2020-21

by

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Question 1

Problem:

Solve the following difference equations using Z-transform:

(a) $y_{n+2} - 5y_{n+1} + 6y_n = 5^n,\ n \geq 0,\ y_0 = 1\ and\ y_1 = 1$

(b) $y(n+2) - y(n) = 2^n,\ n \geq 0,\ y_0 = 0\ and\ y_1 = 1$

Solutions:

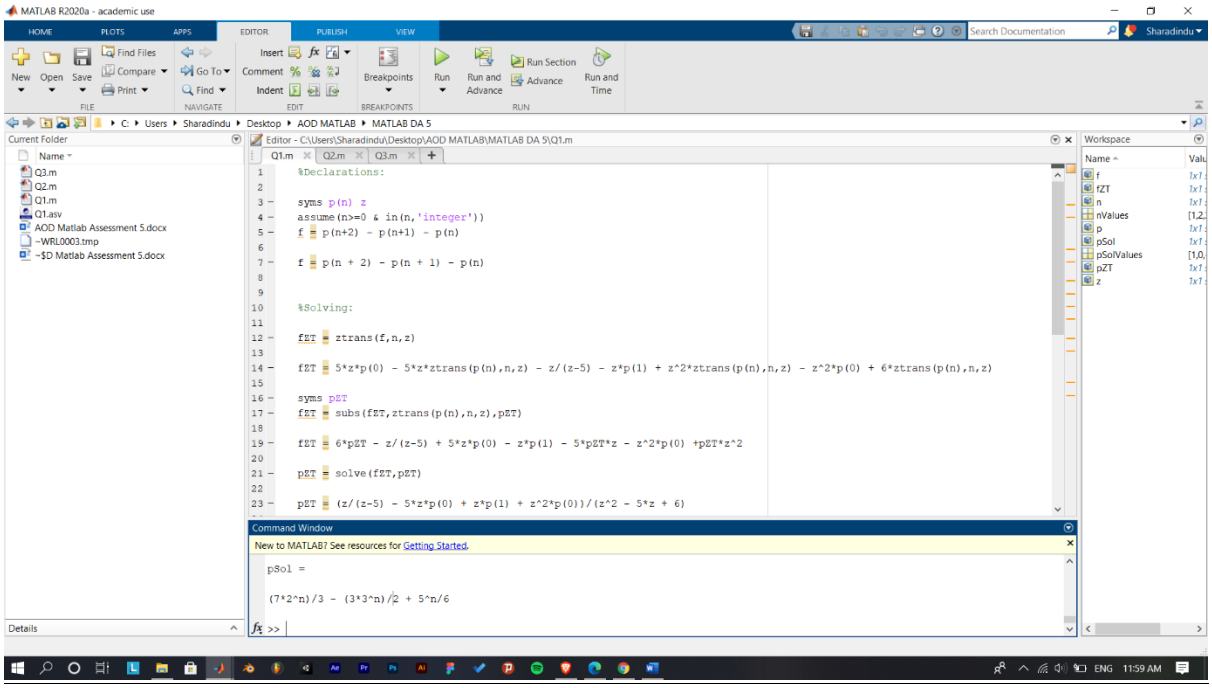
(a)

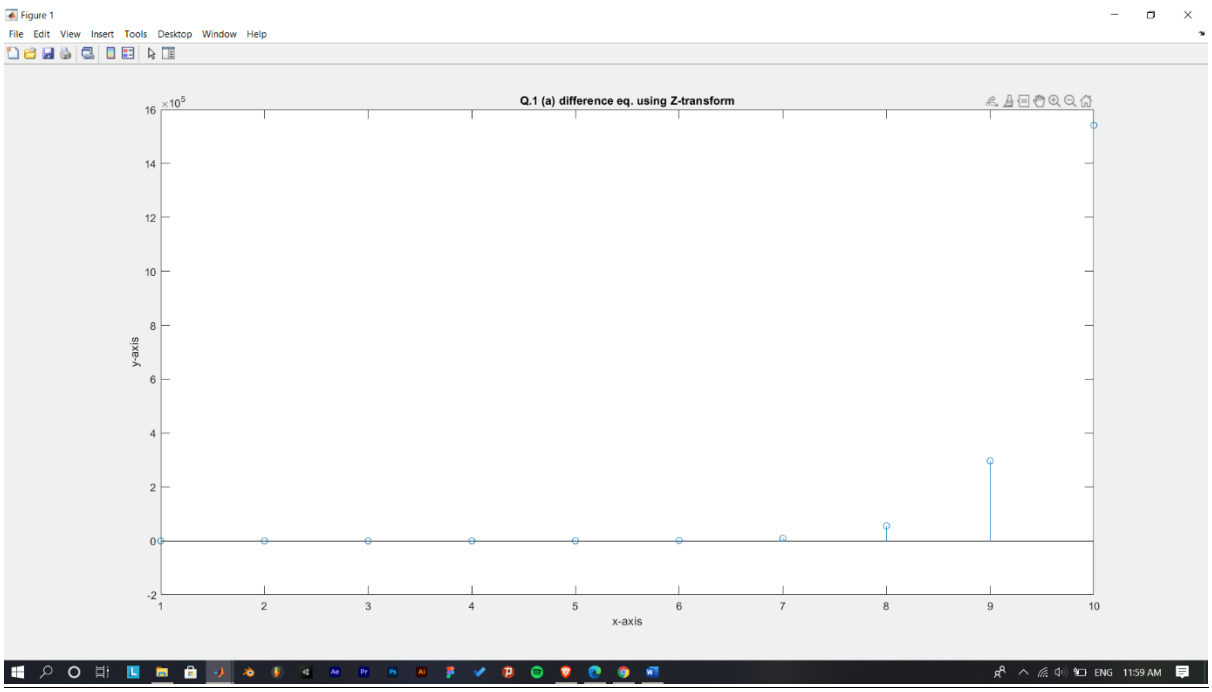
Code in MATLAB:

```
%Declarations:

syms p(n) z
assume(n>=0 & in(n,'integer'))
f = p(n+2) - p(n+1) - p(n)

f = p(n + 2) - p(n + 1) - p(n)


%Solving:

fZT = ztrans(f,n,z)

fZT = 5*z*p(0) - 5*z*ztrans(p(n),n,z) - z/(z-5) - z*p(1) +
z^2*ztrans(p(n),n,z) - z^2*p(0) + 6*ztrans(p(n),n,z)

syms pZT
fZT = subs(fZT,ztrans(p(n),n,z),pZT)

fZT = 6*pZT - z/(z-5) + 5*z*p(0) - z*p(1) - 5*pZT*z - z^2*p(0)
+pZT*z^2

pZT = solve(fZT,pZT)

pZT = (z/(z-5) - 5*z*p(0) + z*p(1) + z^2*p(0))/(z^2 - 5*z + 6)

pSol = iztrans(pZT,z,n);
pSol = simplify(pSol)

pSol = 3*2^n*p(0) - 2^n*p(1) - 2*3^n*p(0) + 3^n*p(1) + 2^n/3 - 3^n/2
+ 5^n/6


%Substitution:

pSol = subs(pSol,[p(0) p(1)],[1 1])

pSol = (7*2^n)/3 - (3*3^n)/2 + 5^n/6
```

```
%Plot:

nValues = 1:10;
pSolValues = subs(pSol,n,nValues);
pSolValues = double(pSolValues);
pSolValues = real(pSolValues);
stem(nValues,pSolValues)
title('Q.1 (a) difference eq. using Z-transform')
xlabel('x-axis')
ylabel('y-axis')
grid off
```

Screenshot & Graph:



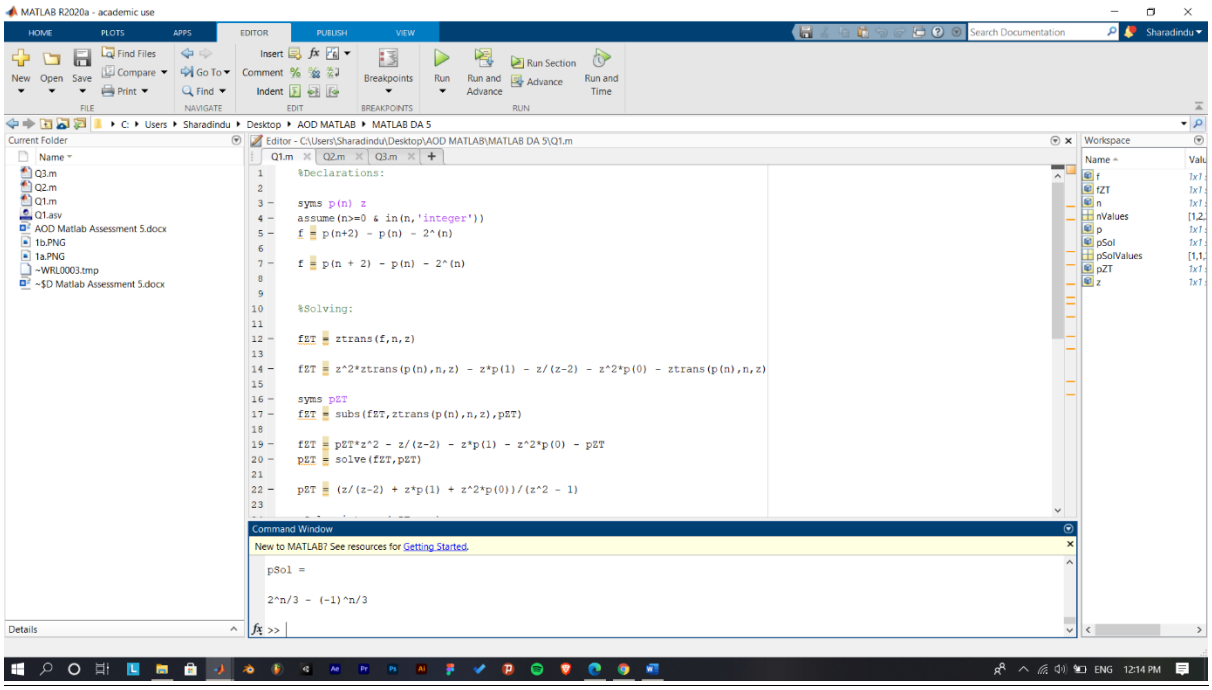


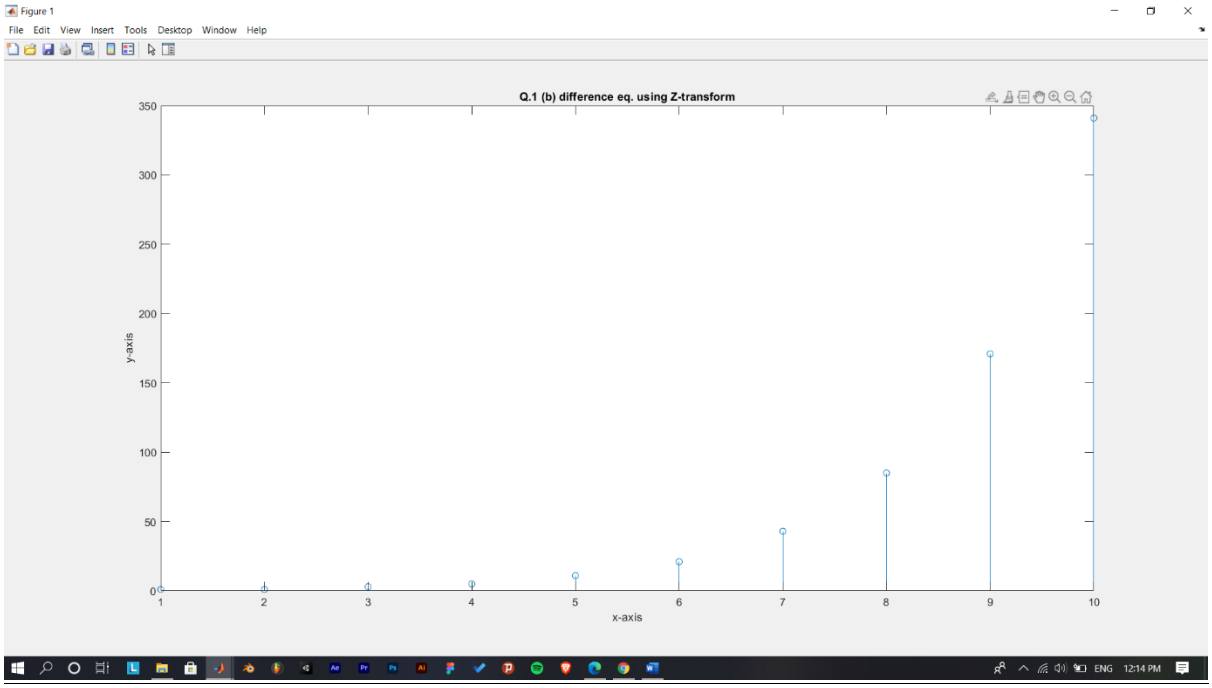
(b)

Code in MATLAB:

```
%Declarations:

syms p(n) z
assume(n>=0 & in(n,'integer'))
f = p(n+2) - p(n) - 2^(n)

f = p(n + 2) - p(n) - 2^(n)


%Solving:

fZT = ztrans(f,n,z)

fZT = z^2*ztrans(p(n),n,z) - z*p(1) - z/(z-2) - z^2*p(0) -
ztrans(p(n),n,z)

syms pZT
fZT = subs(fZT,ztrans(p(n),n,z),pZT)

fZT = pZT*z^2 - z/(z-2) - z*p(1) - z^2*p(0) - pZT
pZT = solve(fZT,pZT)

pZT = (z/(z-2) + z*p(1) + z^2*p(0))/(z^2 - 1)

pSol = iztrans(pZT,z,n);
pSol = simplify(pSol)

pSol = p(0)/2 + p(1)/2 + ((-1)^n*p(0))/2 - ((-1)^n*p(1))/2 + (-
1)^n/6 + 2^n/3 - 1/2


%Substitution:

pSol = subs(pSol,[p(0) p(1)],[0 1])

pSol = 2^n/3 - (-1)^n/3


%Plot:

nValues = 1:10;
pSolValues = subs(pSol,n,nValues);
pSolValues = double(pSolValues);
pSolValues = real(pSolValues);
stem(nValues,pSolValues)
title('Q.1 (b) difference eq. using Z-transform')
xlabel('x-axis')
ylabel('y-axis')
grid off
```

Screenshot & Graph:





Question 2

Problem:

Formulate the difference equation for Fibonacci numbers and hence solve by Z-transforms.

Solution:

The difference equation for Fibonacci series is:

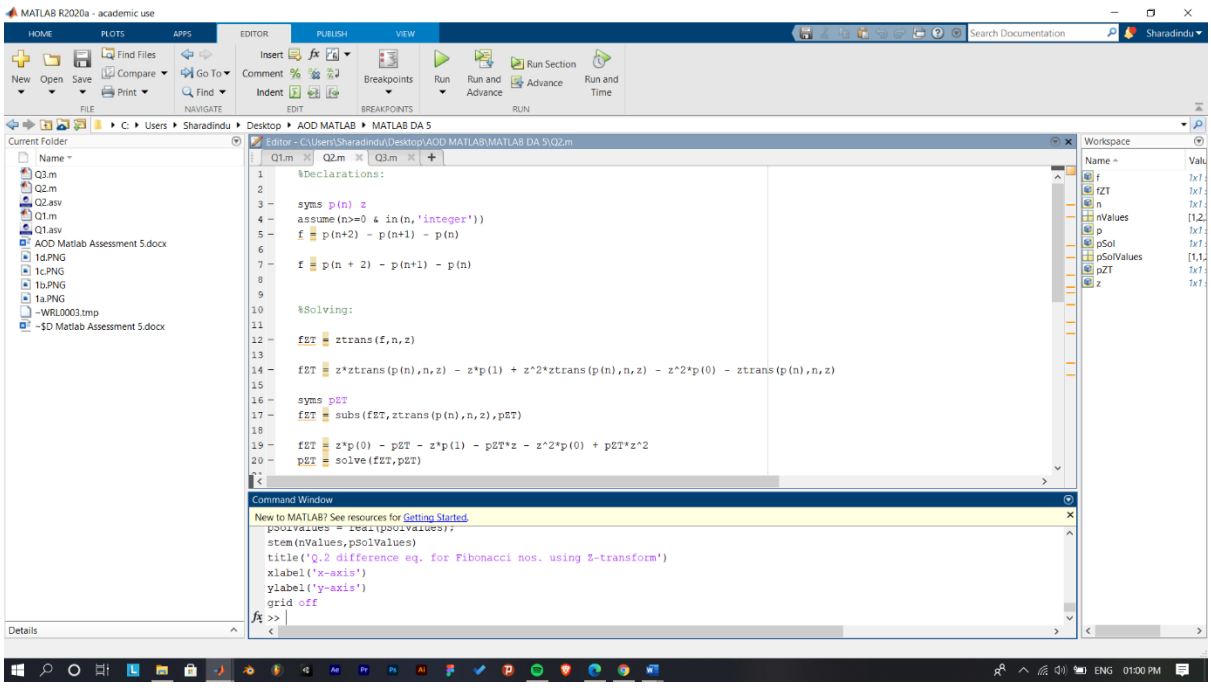
$$u[n+2] = u[n+1] + u[n] \quad with \quad u[0] = 0, \; u[1] = 1$$

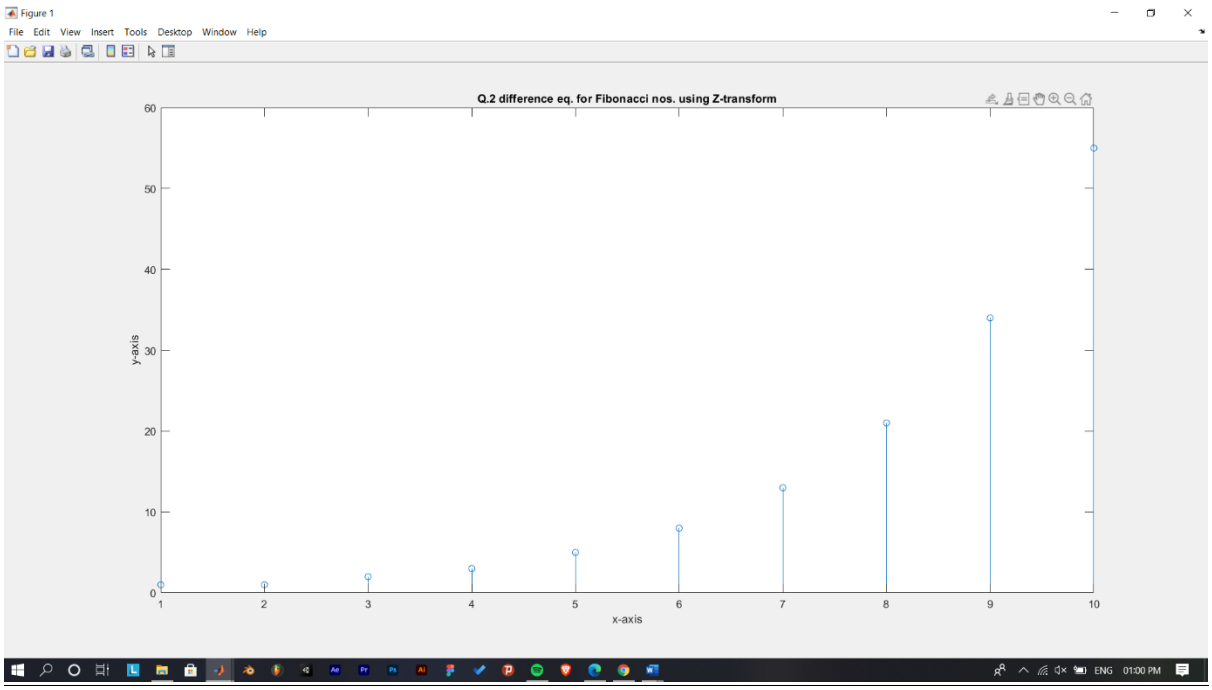
Code in MATLAB:

```
%Declarations:

syms p(n) z
assume(n>=0 & in(n,'integer'))
f = p(n+2) - p(n+1) - p(n)

f = p(n + 2) - p(n+1) - p(n)


%Solving:

fZT = ztrans(f,n,z)

fZT = z*ztrans(p(n),n,z) - z*p(1) + z^2*ztrans(p(n),n,z) - z^2*p(0)
- ztrans(p(n),n,z)

syms pZT
fZT = subs(fZT,ztrans(p(n),n,z),pZT)

fZT = z*p(0) - pZT - z*p(1) - pZT*z - z^2*p(0) + pZT*z^2
pZT = solve(fZT,pZT)

pZT = - (z*p(1) - z*p(0) + z^2*p(0)) / (- z^2 + z + 1)

pSol = iztrans(pZT,z,n);
pSol = simplify(pSol)

pSol = 2*(-1)^(n/2)*cos(n*(pi/2 + asinh(1/2)*1i))*p(1) + (2^(2 -
n)*5^(1/2)*(5^(1/2) + 1)^(n - 1)*(p(0)/2 - p(1)))/5 - (2*2^(1 -
n)*5^(1/2)*(1 - 5^(1/2))^(n - 1)*(p(0)/2 - p(1)))/5


%Substitution:

pSol = subs(pSol,[p(0) p(1)],[0 1])

pSol = 2*(-1)^(n/2)*cos(n*(pi/2 + asinh(1/2)*1i)) - (2^(2 -
n)*5^(1/2)*(5^(1/2) + 1)^(n - 1))/5 + (2*2^(1 - n)*5^(1/2)*(1 -
5^(1/2))^(n - 1))/5
```

```
%Plot:

nValues = 1:10;
pSolValues = subs(pSol,n,nValues);
pSolValues = double(pSolValues);
pSolValues = real(pSolValues);
stem(nValues,pSolValues)
title('Q.2 difference eq. for Fibonacci nos. using Z-transform')
xlabel('x-axis')
ylabel('y-axis')
grid off
```

Screenshot & Graph:





Question 3

Problem:

Solve the following difference equations using method of undetermined coefficients:

(a) $y_{n+2} - 9y_n = 0$

(b) $y_{n+2} - 6y_{n+1} + 8y_n = 0,\ y_0 = 1\ and\ y_1 = 0$

Solutions:

(a)

Code in MATLAB Editor:

```
clc
syms n k1 k2 L
F = input('Input the coefficients [a,b,c]: ');
a=F(1);b=F(2);c=F(3);
ch_eqn=a*L^2+b*L+c; %Characteristic equation
L=solve(ch_eqn);
L1=L(1);L2=L(2);
D=b^2-4*a*c;
if(D>0) % Roots are real and different
y1=L1^n;
y2=L2^n;
elseif (D==0)% Roots are real and equal
y1=L1^n;
y2=n*L1^n;
else % Roots are complex
rho=abs(L1); t=angle(L1);
y1 = (rho^n)*cos(n*t);
y2 = (rho^n)*sin(n*t);
end
yn = k1*y1+k2*y2;
check=input('If initial conditions are known, then enter 1 else
enter 0: ');
if (check == 1)
IC=input('Enter the initial conditions [y(0),y(1)]');
eq1=(subs(yn,n,0)-IC(1));
eq2=(subs(yn,n,1)-IC(2));
[k1,k2]=solve(eq1,eq2);
yn=simplify(subs(yn));
m=0:20;
y=subs(yn,n,m);
stem(y)
title('Q.3 (a) Difference equation using undetermined
coefficients');
xlabel('n'); ylabel('y(n)');
end
disp('The Solution of the given Homogeneous equation is y_n= ');
disp(collect(collect(yn,y1),y2))
```

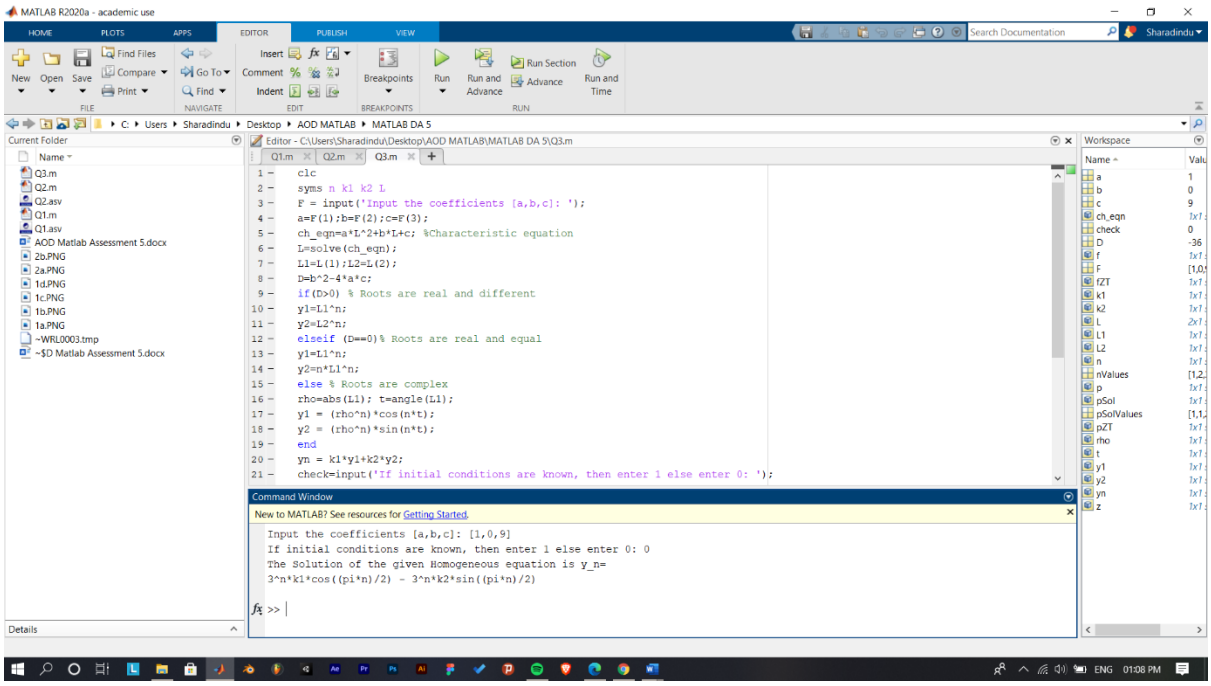
Input in Command Window:

```
Input the coefficients [a,b,c]: [1,0,9]
If initial conditions are known, then enter 1 else enter 0: 0
```

Output in Command Window:

```
The Solution of the given Homogeneous equation is y_n=
3^n*k1*cos((pi*n)/2) - 3^n*k2*sin((pi*n)/2)
```

Screenshot:



(b)

Code in MATLAB Editor:

```
clc
syms n k1 k2 L
F = input('Input the coefficients [a,b,c]: ');
a=F(1);b=F(2);c=F(3);
ch_eqn=a*L^2+b*L+c; %Characteristic equation
L=solve(ch_eqn);
L1=L(1);L2=L(2);
D=b^2-4*a*c;
if(D>0) % Roots are real and different
y1=L1^n;
y2=L2^n;
elseif (D==0)% Roots are real and equal
y1=L1^n;
y2=n*L1^n;
else % Roots are complex
rho=abs(L1); t=angle(L1);
y1 = (rho^n)*cos(n*t);
y2 = (rho^n)*sin(n*t);
end
yn = k1*y1+k2*y2;
check=input('If initial conditions are known, then enter 1 else
enter 0: ');
if (check == 1)
IC=input('Enter the initial conditions [y(0),y(1)]: ');
eq1=(subs(yn,n,0)-IC(1));
eq2=(subs(yn,n,1)-IC(2));
[k1,k2]=solve(eq1,eq2);
yn=simplify(subs(yn));
m=0:20;
y=subs(yn,n,m);
stem(y)
title('Q.3 (b) Difference equation using undetermined
coefficients');
xlabel('n'); ylabel('y(n)');
end
disp('The Solution of the given Homogeneous equation is y_n= ');
disp(collect(collect(yn,y1),y2))
```

Input in Command Window:

```
Input the coefficients [a,b,c]: [1,-6,8]
If initial conditions are known, then enter 1 else enter 0: 1
Enter the initial conditions [y(0),y(1)]: [1,0]
```

Output in Command Window:

```
The Solution of the given Homogeneous equation is y_n=
2*2^n - 2^(2*n)
```

Screenshot & Graph: