PHYSIC 1701

DA - 1

by

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2.7 / Compton Effect

Q. 26. How much energy must a photon have if it is to have the momentum of a 10 MeV proton?

⇒ We've:

Kinetic energy of proton $= \frac{1}{2} mv^2$

$$\Rightarrow K = \frac{1}{2} mv^2$$

$$\Rightarrow v = \sqrt{\frac{2K}{m}}$$

$$= \sqrt{\frac{2 \times 10 \times 10^{6} \times 1.602 \times 10^{-19}}{1.67 \times 10^{-27}}} \; m/s$$

$$= 4.38 \times 10^{7} \; m/s$$

∴ Momentum of proton, $p = mv$

$$= (1.67 \times 10^{-27} \; kg) \times (4.38 \times 10^{7} \; m/s)$$

$$= 7.31 \times 10^{-20} \; kg \cdot m s^{-1}$$

∵ Momentum of photon = momentum of proton,

We've, Momentum of photon, $p = E/c$

$$\Rightarrow E = pc$$

$$= (7.31 \times 10^{-20}) \times (3 \times 10^{8}) \; J$$

$$= 2.193 \times 10^{-11} \; J$$

$$= 1.37 \times 10^{8} \; eV$$

∴ Reqd. energy of photon = 137 MeV

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Q. 28. A monochromatic X-ray beam whose wavelength is 55.8 pm is scattered through 46°. Find the wavelength of the scattered beam.

$\Rightarrow$

$\lambda = 55.8$ pm

$\theta = 46°$

We've : From Compton eqn,

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

$$\Rightarrow \lambda' = \lambda + \lambda_c \cdot (1 - \cos\theta) \quad , \text{ where } \lambda_c = \frac{h}{m_0 c}$$

$$= 55.8 + 2.43\,(1 - \cos 46°) \text{ pm}$$

Reqd. wavelength $= 56.54$ pm $\quad (\because \lambda_c = 2.43 \text{ pm})$

Q. 30 An X-ray photon whose initial frequency was 1.5×10^{19} Hz emerges from a collision with an electron with a frequency of 1.2×10^{19} Hz. How much kinetic energy was imparted to the electron?

$\Rightarrow$

Initial frequency, $\nu = 1.5 \times 10^{19}$ Hz

Frequency after collision, $\nu' = 1.2 \times 10^{19}$ Hz.

~~Frequency of~~

$\therefore$ Reqd. K.E. imparted to electron $= E - E'$

$$= h\,(1.5 \times 10^{19} - 1.2 \times 10^{19}) \text{ J}$$

$$= 6.626 \times 10^{-34} \times 0.3 \times 10^{19} \text{ J}$$

$$= 1.9878 \times 10^{-15} \text{ J}$$

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Q 32. Find the energy of an x-ray photon which can impart a maximum energy of 50 keV to an electron.

⇒ ∵ Max. energy is imparted, we've $\theta = 180^\circ$.

From Compton eqn, $\lambda' - \lambda = \lambda_c (1-\cos\theta)$, where, $\lambda_c = \frac{h}{m_0 c}$

$\Rightarrow \Delta\lambda = \lambda_c (1-\cos\theta)$,

$\Rightarrow \Delta\lambda = 2\lambda_c \quad [\because \cos 180^\circ = -1]$

$= 2 \times 0.0243 \text{ Å}$

$\Rightarrow \Delta\lambda = 0.0486 \text{ Å}$

$= 4.86 \times 10^{-12} m$

Now, we've, $\lambda' - \lambda = 0.0486 \text{ Å}$

$\Rightarrow \frac{hc}{E'} - \frac{hc}{E} = 0.0486 \text{ Å}$

$\Rightarrow hc \left[\frac{1}{E'} - \frac{1}{E}\right] = 0.0486 \text{ Å}$

$\Rightarrow hc \left[\frac{E - E'}{EE'}\right] = 0.0486 \text{ Å}$

$\Rightarrow \frac{50,000 \text{ eV} - E'}{50,000 \text{ eV} \cdot E'} = \frac{0.0486 \text{ Å}}{hc}$

$\Rightarrow \frac{5 \times 10^4 \text{ eV} - E'}{5 \times 10^4 \text{ eV} \cdot E'} = \frac{4.86 \times 10^{-12} m}{6.626 \times 10^{-34} \times 3 \times 10^8}$

$= 2.44 \times 10^{13}$ —— (1)

Isolating E' in eqn (1),

we get : $E' = 41.8 \text{ eV}$

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Q. 34. (a) Find the change in wavelength of 80 pm γ-rays that are scattered $120°$ by a target.

(b) Find the angle between the directions of the recoil electron and the incident photon.

(c) Find the energy of the recoil electron.

$\Rightarrow$ (a) From the Compton eq^n,

$$\Delta\lambda = \lambda' - \lambda = \lambda_c (1-\cos\theta)$$

$$\Rightarrow \lambda' - 80\,pm = 2.43\,pm\,(1-\cos 120°)$$

$$\Rightarrow \lambda' = (3.645 + 80)\,pm$$

$$= 83.645\,pm$$

(b) From momentum conservation of Compton Scattering, we know:

$$P_e \sin\phi = h\nu' \sin\theta \quad \text{——— (1)}$$

$$P_e \cos\phi = h\nu - h\nu'\cos\theta \quad \text{——— (2)}$$

Dividing (1) by (2),

$$\Rightarrow \boxed{\tan\phi = \frac{\nu' \sin\theta}{\nu - \nu'\cos\theta}}$$

$$= \frac{\lambda \sin\theta}{\lambda' - \lambda\cos\theta}$$

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$\tan\phi = (80 \times \sin 120^\circ) / (83.645 - 80 \cdot \cos 120^\circ)$

$= \dfrac{80 \times \sqrt{3}/2}{83.645 + 40}$

$= \dfrac{80\sqrt{3}}{2(123.645)}$

$= \dfrac{138.5}{247.29}$

$= 0.56$

$\therefore \phi = \tan^{-1}(0.56) \approx 29.24^\circ$

(c) Recoil energy of electron

$= hc\left[\dfrac{1}{\lambda} - \dfrac{1}{\lambda'}\right]$

$= \dfrac{hc\,[3.645]}{80 \times 83.645}$

$= \dfrac{6.626 \times 10^{-34} \times 3 \times 10^{8} \times 3.645}{80 \times 83.645}$ J

$= 0.0108 \times 10^{-26}$ J

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Q. 36 In a Compton eff exp., in which the incident x-rays have a wavelength of 10 pm, the scattered x-rays at a certain angle have a wavelength of 10.5 pm. Find the momentum of the corresponding recoil electron.

$\Rightarrow$

Welve, K.E. of electrons $= \frac{hc}{\lambda} - \frac{hc}{\lambda'}$

$$= \frac{hc\,(0.5)}{10 \times 10.5} \text{ J}$$

$$= \frac{hc \times 1/2}{105} \text{ J}$$

$\because KE = \frac{p^2}{2m}$ $$= \frac{19.869 \times 10^{-26}}{210} \text{ J}$$

$$= 0.094 \times 10^{-26} \text{ J}$$

$$= 9.4 \times 10^{-28} \text{ J}$$

So, $p^2 = 2 \times m_e \times 9.4 \times 10^{-28}$

$$= 2 \times 9.1 \times 10^{-31} \times 9.4 \times 10^{-28}$$

$$p = \sqrt{1710.8 \times 10^{-60}} \text{ kg m/s}$$

$$= 41.36 \times 10^{-30} \text{ kg ms}^{-1}$$

For direction, we can see the angle of scattering,

$$\lambda' - \lambda = \lambda_c (1 - \cos\theta)$$

$$\Rightarrow \cos\theta = \frac{\lambda_c - \lambda' + \lambda}{\lambda_c} \quad \text{------} \quad (1)$$

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From ①, $\cos\theta = \frac{2.43 - 10.5 + 10.0}{2.43}$

$= 0.79$

$\Rightarrow \theta = \cos^{-1}(0.79) = 37.8^{\circ}$

So, the electron moves at an angle of 37.8° after the scattering.

Q. 38. A photon of energy E is scattered by a particle of rest energy E_0. Find the max. K.E. of the recoiling particle in terms of E and E_0.

$\Rightarrow$

Energy of photon, $= E$

Energy of particle $= E_0$

For max. KE, $\theta = 180^{\circ}$

$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}[1 - \cos 180^{\circ}]$

$= \frac{2h}{m_0 c}$

$\Rightarrow \frac{\lambda'}{hc} - \frac{\lambda}{hc} = \frac{2}{m_0 c^2}$

$\Rightarrow \frac{1}{E'} - \frac{1}{E} = \frac{2}{m_0 c^2}$

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$$\Rightarrow \frac{E - E'}{EE'} = \frac{2}{E_0} \qquad \left(\because E_0 = mc^2\right)$$

$$\Rightarrow E_0 E - E' E_0 = 2EE'$$

$$\Rightarrow E' \left[2E + E_0\right] = E_0 E$$

$$\Rightarrow \boxed{E' = \frac{E_0 E}{(2E + E_0)}}$$

Hence, proved

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3.1 <u>de Broglie Waves</u>

Q. 2 Find the de Broglie wavelength of an electron whose speed is (a) 1.0×10^8 m/s, and (b) 2.0×10^8 m/s.

$\Rightarrow$ $\because$ In both (a) and (b), speed of electron is comparable to the speed of light, we've:

(a) $$\lambda = \frac{h}{p} = \frac{h}{\dfrac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$= \frac{6.626 \times 10^{-34}}{\dfrac{9.1 \times 10^{-31} \times 1.0 \times 10^8}{\sqrt{1-\frac{(1.0 \times 10^8)^2}{(3 \times 10^8)^2}}}}$$

$$\lambda = 6.86 \times 10^{-12} \text{ m}$$

(b) Similarly, $$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}}{\dfrac{9.1 \times 10^{-31} \times 2 \times 10^8}{\sqrt{1-\left(\frac{2 \times 10^8}{3 \times 10^8}\right)^2}}}$$

$$\lambda = 2.7 \times 10^{-12} \text{ m}$$

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Q. 4. Find the de Broglie wavelength of the 40 keV electrons used in a certain electron microscope.

→ Energy of electron, $E = 40 \text{ keV}$

$= 40 \times 10^{3} \times 1.602 \times 10^{-19}$ J

Also, $\lambda = \frac{h}{p}$ and $p = E \cdot c.$ and $E = \frac{p^2}{2m}$.

$\therefore \lambda = \frac{h}{\sqrt{2mE}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times E}}$ m

$\therefore$ Reqd. wavelength, $\lambda = 6.135 \times 10^{-12}$ m

Q. 6. Find the de Broglie wavelength of a 1 MeV proton. Is a relativistic calculation needed?

→ We've, $E = \frac{hc}{\lambda} \Rightarrow \boxed{\lambda = \frac{hc}{E}}$ —— (1)

where, $E = 1 \times 10^{6}$ eV

Now, $1 \text{ eV} = 1.6 \times 10^{-19}$ J

and, $10^{6} \text{ eV} = 1.6 \times 10^{-13}$ J

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Now, using eqn (1), we have:-

$$\lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-13}} \, m = \frac{19.869 \times 10^{-26}}{1.6 \times 10^{-13}} \, m$$

$$\approx 12.4 \times 10^{-13} \, m$$

Reqd. debroglie wavelength, $\lambda \approx 1.24$ pm.

Also, $m_p \cdot c^2 = 1.6 \times 10^{-27} \times (3 \times 10^{8})^2$ J

$= 14.4 \times 10^{-11}$ J

$\because$ 14.4×10^{-11} J $>>$ 1.6×10^{-13} J

$\Rightarrow$ $(m_p c^2) >> (E)$, [where, $m_p c^2$ = rest mass energy]

Hence, No relativistic calculation is needed.

Q.8 Find the K.E. of an electron whose de Broglie wavelength is the same as that of a 100 keV x-ray.

$\Rightarrow$ Given, Energy of X-ray, = 100 keV

$E = (100 \times 10^3)$ eV

$= (10^5 \times 1.602 \times 10^{-19})$ J

Also, $E = \frac{hc}{\lambda}$

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$$\Rightarrow \lambda = \frac{hc}{E} = \frac{6.626\times10^{-34}\times3\times10^{8}}{1.602\times10^{5}\times10^{-19}} \, m$$

$$\therefore \lambda = 1.240\times10^{-11} \, m$$

Now, this wavelength, λ is same for electron also.

$$\therefore K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{\lambda^2 \cdot 2m}$$

$$K = \frac{(6.626\times10^{-34})^2}{(1.240\times10^{-11})^2 \cdot 2\cdot(9.1\times10^{-31})} \, J$$

$$\Rightarrow K = 1.56\times10^{-15} \, J$$

$$K = 9.782 \, keV$$

$\therefore$ Reqd. kinetic energy of electron = 9.782 keV.

Q. 10. Show that the de Broglie wavelength of a particle of mass m & kinetic energy, K is given by :-

$$\lambda = \frac{hc}{\sqrt{K(K+2mc^2)}}$$

We've, Energy of particle, $E = \sqrt{p^2c^2 + (mc^2)^2}$

Also, $E = K + mc^2$

Comparing these eq^{ns}, we've :

$$(K + mc^2)^2 = \left(\sqrt{p^2c^2 + (mc^2)^2}\right)^2$$

$$\Rightarrow \quad K + m^2c^2 + 2mc^2 \cdot K = p^2c^2 + m^2c^4$$

$$\Rightarrow \quad \frac{\sqrt{K(K + 2mc^2)}}{c} = p \quad \text{———} (1).$$

Also, $\lambda = \frac{h}{p}$

$$\therefore \lambda = \frac{hc}{\sqrt{K(K + 2mc^2)}} \quad \left[\text{from } (1)\right]$$

(a) Derive a relativistically correct formula that gives the de Broglie wavelength of a charge particle in terms of potential difference, through which it has been accelerated.

(b) What is the non-relativistic approximation of this formula, valid for $qV \ll mc^2$?

We've, debroglie wavelength, $\lambda = \frac{h}{p}$

$$\Rightarrow \lambda = \frac{hc}{pc}$$

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$$\Rightarrow \lambda = \frac{hc}{\sqrt{E^2 - m^2c^4}}$$

$$= \frac{hc}{\sqrt{(K^2 + mc^2)^2 - (mc^2)^2}}$$

$$= \frac{hc}{\sqrt{K^2 + \cancel{(mc^2)^2} + 2Kmc^2 - \cancel{(mc^2)^2}}}$$

$$= \frac{hc}{\sqrt{K^2 + 2Kmc^2}}$$

w.k.t.,

$$E^2 = p^2c^2 + m^2c^4$$

$$\Rightarrow pc = \sqrt{E^2 - m^2c^4}$$

where, E = kinetic energy of particle + rest mass energy

$$\Rightarrow E = K + mc^2$$

Now, According to question, $K = qV$.

$$\therefore \lambda = \frac{hc}{\sqrt{q^2V^2 + 2qVmc^2}}$$

$$= \frac{hc}{\sqrt{2mqVc^2\left[1 + \frac{qV}{2mc^2}\right]}}$$

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(b) $$\lambda = \frac{h}{\sqrt{2mqV\left(1+\frac{qV}{2mc^2}\right)}}$$

For, non-relativistic approximation, $qV << mc^2$

then, λ reduces to $\frac{h}{\sqrt{2mqV}}$

Q. 30 Discuss the prohibition of $E = 0$ for a particle trapped in a box L wide in terms of the uncertainty principle. How does the minimum momentum of such a particle compare with the momentum uncertainty required by the uncertainty principle if $\Delta x = L$?

→ For a trapped particle, the uncertainty in position is very less. So, uncertainty of momentum should be finite.

$$\Delta x = L$$

$$\Rightarrow \Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\Rightarrow \Delta p \geq \frac{\hbar}{2L}$$

$$\Rightarrow \Delta p \approx \frac{\hbar}{2L}$$

Now, $p_n = \dfrac{h}{\lambda_n}$, $L = \dfrac{n\lambda_n}{2}$

$$\Rightarrow \lambda_n = \frac{2L}{n}$$

$$\Rightarrow p_1 = \frac{h}{2L}$$

$$\therefore p_1 > \Delta p$$

⇒ The value of p_1 is greater than minimum value of Δp.

Q.32 Compare the uncertainties in the velocities of an electron and a proton confined in a 1.00 nm box.

⟹ W.K.T. From de Broglie's uncertainty principle,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$
$$\geq \hbar/2$$

Here, the confined space, $\Delta x = 1.00$ nm

$$\therefore \Delta p \geq \frac{\hbar}{2 \cdot \Delta x}$$

$$\geq \frac{h}{4\pi \cdot \Delta x} = \frac{6.626 \times 10^{-34}}{4\pi \times 1 \times 10^{-9}}$$

$$\Rightarrow \Delta p = 5.27 \times 10^{-26} \ kg \cdot ms^{-1}$$

Now, the uncertainties in their velocities are :—

For electron,
$$\Delta V_{electron} \geq \frac{\hbar}{2 \cdot m_{electron} \Delta x}$$

$$= 5.79 \times 10^{4} \ ms^{-1}$$

For proton,
$$\Delta V_{proton} \geq \frac{\hbar}{2 \cdot m_{proton} \cdot \Delta x}$$

$$= 3.15 \times 10^{1} \ ms^{-1}$$

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Q. 34 (a) How much time is needed to measure the K.E. of an electron whose speed is $10\ m s^{-1}$ with an uncertainty of no more than 0.100 % ? How far will the electron have travelled in this period of time? (b) Make the same calculations for a 1.00 g insect whose speed is the same. What do these sets of figure indicate ?

⟹ (a) speed of e^- = 10 m/s

uncertainty = 0.100%.

From the uncertainty principle, we've :-

$$\Delta E \cdot \Delta t \geq \hbar/2$$

$$\Rightarrow \Delta E \cdot \Delta t \geq h/4\pi$$

Now,

$$\frac{\Delta E}{E} = 0.100\ \%$$

$$\Rightarrow \frac{h}{4\pi \cdot E \cdot \Delta t} = 0.100\ \% = 10^{-3}$$

So,

$$\Delta t = \frac{h}{4\pi \cdot E \cdot 10^{-3}}$$

$$= \frac{h}{4\pi \cdot \frac{1}{2} m_e v^2 \cdot 10^{-3}}$$

$$\approx 1.16 \times 10^{-3}\ s$$

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Hence, the electron will travel = $(1.16 \times 10^{-3}\, s \times 10\, m/s)$

$= 1.16 \times 10^{-2}\, m$

(b) Mass of insect = M_i = 1g = 10^{-3} kg.

speed of insect = $10\, ms^{-1}$

From the uncertainty principle, $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

$\therefore \frac{\Delta E}{E} = 0.100\% = 10^{-3}$

$\Rightarrow \frac{\hbar}{2 \cdot E \cdot \Delta t} = 10^{-3}$

$\Rightarrow \Delta t = \frac{\hbar}{M_i V^2 \times 10^{-3}}$

$= 1.06 \times 10^{-30}\, s$

$\therefore$ The insect will travel a distance of

$(10\, m/s \times 1.06 \times 10^{-30} s)$

$= 1.06 \times 10^{-29}\, m.$

Hence, the results indicate that the time to measure the K.E. of an object with good precision is easier for massive case than light mass case.

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Q. 36. (a) Find the magnitude of the momentum of a particle in a box in its nth state.

(b) The minimum change in particle's momentum that a measurement can cause corresponds to a change of ± 1 in the quantum number n. If $\Delta x = L$, show that $\Delta p \cdot \Delta x \geq \hbar/2$

$\Rightarrow$

(a) we know, $\boxed{\lambda = \frac{2L}{n}}$ — (1) $n = 1, 2, 3$

$$\frac{h}{p} = \frac{2L}{n} \qquad (\lambda = h/p)$$

$$\boxed{p = \pm \frac{nh}{2L}}$$

(b) From uncertainty principle, $\Delta p \cdot \Delta x \geq \frac{\hbar}{2}$

$$\Rightarrow \Delta p \cdot L \geq \frac{\hbar}{2}$$

$$\Rightarrow \boxed{\Delta p \geq \frac{\hbar}{2L}} \text{ — (ii)}$$

In both (a) and (b), when $n = 1$,

Δp from (1) $= \frac{h}{2L}$ — (3)

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we've : ③ > ②

$\therefore$ Minimum value is ②

Hence, $\Delta p \cdot \Delta x \geq \hbar/2$. Hence, proved

Q. 38 An unstable elementary particle called the eta meson has a rest mass of 549 MeV/c^2 and a mean life of 7×10^{-19} s. What is the uncertainty in its rest mass?

$\Rightarrow$ We've from de broglie eq^n,

$$\boxed{\Delta E \cdot \Delta t \geq \frac{h}{4\pi}} \quad \text{①}$$

So, in ① $\Delta E \geq \dfrac{h}{4\pi \cdot \Delta t}$

where,
$E = 549$ MeV
$\Delta t = 7 \times 10^{-19}$ s

Now, dividing on both sides with 'E' to get the uncertainty, we've :

$$\frac{\Delta E}{E} \geq \frac{h}{4\pi \cdot E \cdot \Delta t}$$

$$\Rightarrow \frac{\Delta E}{E} \geq \frac{\hbar}{2E\Delta t}$$

$$\Rightarrow \frac{\Delta E}{E} \geq \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 549 \times 10^{6} \times 1.6 \times 10^{-19} \times 7 \times 10^{-19}}$$

$$\geq 8.56 \times 10^{-7}$$

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$$\therefore \quad \frac{\Delta M}{M} = \frac{\Delta E}{E} = 8.56 \times 10^{-7}$$

Q. 40

(a) Verify that the uncertainty principle can be expressed in the form $\Delta L \cdot \Delta\theta \geq \frac{\hbar}{2}$, where ΔL is the uncertainty in the angular momentum of a particle and $\Delta\theta$ is the uncertainty in its angular position.

(b) At what uncertainty in L will the angular position of a particle become completely indeterminate?

(a) W.K.T.,

Angular momentum, $L = mvr$

$$\Rightarrow L = pr.$$

$$\Delta L = \Delta p \cdot r$$

$$\Rightarrow \Delta p = \frac{\Delta L}{r}.$$

Let's consider a particle of mass m moving in a circle of radius r at speed v, for which $L = mvr$.

$$\therefore \quad x = r\theta.$$

$$\Rightarrow \Delta x = r\,\Delta\theta$$

Now, W.K.T., $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$

$$\Rightarrow r \cdot \Delta\theta \cdot \frac{\Delta L}{r} \geq \frac{\hbar}{2}$$

$\Rightarrow \quad \Delta\theta \cdot \Delta L \geq \frac{\hbar}{2}$ —— ①

(b) Again, $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$.

From ①, we've, $\Delta\theta \cdot \Delta L \geq \frac{\hbar}{2}$

when uncertainty of L is 0, uncertainty of θ is ∞ and thus position becomes indeterminate.