ASSIGNMENT 1
MAT2002
MATRICES

July 29, 2020

Solve the following questions:

1. Verify Cayley-Hamilton theorem for the given matrix and hence find A^{-1}.
$$A = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{bmatrix}.$$

2. Find the eigenvalues and eigenvectors of
$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 2 \end{bmatrix}$$
and verify the following
(i) sum of the eigenvalues= trace of A.
(ii) product of eigenvalues = Det(A).

3. Find the eigenvalues and eigenvectors of
$$A = \begin{bmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{bmatrix}$$
and verify the following: (1) A^{-1} exists or not. (2) eigenvectors are mutually orthogonal or not.

4. Reduce the quadratic form $2x_1x_2 + 2x_1x_3 - 2x_2x_3$ to the canonical form by an Orthogonal transformation.

5. Reduce the quadratic form $Q = -6x^2 + 6xy - 6y^2$ to the canonical form by an Orthogonal transformation.What kind of conic section is described by the equation if $Q = -18$.

6. Determine the eigenvalues of matrix given below(using properties) and verify whether matrix is diagonalizable or not
$$A = \begin{bmatrix} 6 & 2 & 3 \\ 0 & 6 & 6 \\ 0 & 0 & 6 \end{bmatrix}$$
.

7. Find the Fourier series for the function $f(x) = |x|$, in the interval $x \in (-\pi, \pi)$.

8. Find the Fourier series for the function $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 < x \le \pi. \end{cases}$ and hence deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$.

9. Obtain Fourier series of cosine terms for the function $f(x) = \begin{cases} x, & 0 < x \leq \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \leq x < \pi. \end{cases}$

10. Find the Fourier series of y up to second harmonic from the following table:

x	0	1	2	3	4	5
y	4	8	17	7	6	2

11. Obtain the first three coefficients in the Fourier cosine series for y, where y is given in the following table:

x	0	30	60	90	120	150	180
y	0	5224	8097	7850	5499	2626	0