A2/
da....

MAT 1014
Discrete Mathematics & Graph Theory

SHARADINDU ADHIKARI | 19BCE2105

Q1. Obtain the PDNF [canonical sum-of-products form] of $P \vee (\sim P \wedge \sim Q \wedge R)$.

Sol^n: We've:

$$P \vee (\sim P \wedge \sim Q \wedge R)$$

$$\Leftrightarrow [P \wedge (Q \vee \sim Q) \wedge (R \vee \sim R)] \vee [\sim P \wedge \sim Q \wedge R]$$

$$\Leftrightarrow [(P \wedge Q) \vee (P \wedge \sim Q) \vee (R \wedge \sim R)] \vee (\sim P \wedge \sim Q \wedge R)$$

$$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge R)$$

$$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge R)$$

is the required PDNF form.

—.—

Q2. Obtain the PCNF (canonical product-of-sums form) of $(\sim P \vee \sim Q) \rightarrow (\sim P \wedge R)$ using truth table.

Sol^n.:

We've:

$$(\sim P \vee \sim Q) \longrightarrow (\sim P \wedge R).$$

Taking negation,

$$\sim(\sim P \vee \sim Q) \vee (\sim P \wedge R)$$

$$\Leftrightarrow (P \wedge Q) \vee (\sim P \wedge R)$$

$$\Leftrightarrow (P \wedge Q \wedge (R \vee \sim R)) \vee (\sim P \wedge R \wedge (Q \vee \sim Q))$$

$$\Leftrightarrow (P \wedge Q \wedge R) \vee (P \wedge Q \wedge \sim R) \vee (\sim P \wedge Q \wedge R) \vee (\sim P \wedge \sim Q \wedge R)$$

$\therefore$ Remaining terms are:

$$(P \wedge \sim Q \wedge R) \vee (P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge \sim Q \wedge \sim R) \vee (\sim P \wedge Q \wedge \sim R)$$

By taking its negation, we get PCNF:

$$(\sim P \vee Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R)$$

.... contd...

19BCE2105

... contd.

- Truth Table :

P	Q	R	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$	$\sim P \wedge R$	$(\sim P \vee \sim Q) \rightarrow (\sim P \wedge R)$
F	F	F	T	T	T	F	F
F	F	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	T	T	T	F	T	T	T
T	F	F	F	T	T	F	F
T	F	T	F	T	T	F	F
T	T	F	F	F	F	F	T
T	T	T	F	F	F	F	T

∴ From the Truth table as well, the PCNF is:-

$$(\sim P \vee Q \vee \sim R) \wedge (\sim P \vee Q \vee R) \wedge (P \vee Q \vee R) \wedge (P \vee \sim Q \vee R)$$

Hence, it is verified both ways.

19BCE2105

Q3. Test the validity of the following argument by method of contradiction.

$$((\sim P \leftrightarrow Q) \wedge (Q \rightarrow R) \wedge \sim R) \rightarrow P.$$

Sol^n

	derivation	rule
1.	$\sim P$	assumed premise
2.	$Q \rightarrow R$	Rule P
3.	$\sim R$	Rule P
4.	$\sim Q$	$T\{2,3\}$, Modus Tollens
5.	$\sim P \leftrightarrow Q$	Rule P
6.	$(\sim P \rightarrow Q) \wedge (Q \rightarrow \sim P)$	$T\{5\}$
7.	$\sim P \rightarrow Q$	$T\{6\}$, $P \wedge Q \Rightarrow P$
8.	P	$T\{4,7\}$, $\sim Q, \sim P \rightarrow Q \Rightarrow P$
9.	$P \wedge \sim P$	$T\{1,8\}$
10.	F	$T\{9\}$

Hence, our assumption was wrong.

$\therefore$ $((\sim P \leftrightarrow Q) \wedge (Q \rightarrow R) \wedge \sim R) \rightarrow P$ is a true/valid statement.

19BCE2105

Q4. Establish the validity of the following arguments.

$\Longrightarrow$

D : Dominic goes to the racetrack.

H : Helen will be mad.

R : Ralph plays cards all night.

C : Carmela will be mad.

V : Veronica will be notified.

premises :

(a) $D \rightarrow H$

(b) $R \rightarrow C$

(c) $(H \vee C) \rightarrow V$

(d) $\sim V$

Conclusion : $\sim D \wedge \sim R$

now,

	derivation	rule
1.	$(H \vee C) \rightarrow V$	P
2.	$\sim V$	P
3.	$\sim (H \vee C)$	T {1, 2}, Modus Tollens
4.	$\sim H \wedge \sim C$	T {3}, de Morgan
5.	$\sim H$	T {4}, simplification

... contd.

	derivation	rule
6.	$D \rightarrow H$	P
7.	$\sim D$	T{5,6}, simplification
8.	$\sim C$	T{4}, simplification
9.	$R \rightarrow C$	P
10.	$\sim R$	T{8,9} Modus Tollens
11.	$\sim D \wedge \sim R$	T{7,10}, conjunction

Hence, the given statement is valid.

Q5: Establish the validity of the given arguments [predicate calculus].

$\Rightarrow$

Let,

$J(x)$: x is a junior.

$S(x)$: x is a senior

$P(x)$: x is enrolled in physical education class.

contd....

premises : 1. No junior or senior is enrolled in a physical education class:-

$$(\forall x)\ [\ j(x) \vee s(x) \rightarrow \sim p(x)\ ]$$

2. Mary is enrolled in a physical education class :-

$$p(m)$$

Conclusion : Thus Mary is not a senior : $\sim s(m)$.

now,

derivation		rule
1.	$\forall x\ [j(x) \vee s(x) \rightarrow \sim p(x)]$	P
2.	$p(m)$	P
3.	$j(m) \vee s(m) \rightarrow \sim p(m)$	T, {1}, US
4.	$p(m) \rightarrow \sim[j(m) \vee s(m)]$	T{3}, $q \rightarrow t$, $\sim t \rightarrow \sim q$
5.	$p(m) \rightarrow [\sim j(m) \wedge \sim s(m)]$	T{4}
6.	$\sim j(m) \wedge \sim s(m)$	T, {2, 5}, modus ponens
7.	$\sim s(m)$	T, {6}, simplification

Hence, the given argument is valid.

Q6. Check the validity of the given arguments :-

$\Longrightarrow$

Let.

$L(x)$: x is a lion.

$F(x)$: x is fierce.

$C(x)$: x drinks coffee.

premises : 1. All lions are fierce : $\forall x \left(L(x) \rightarrow F(x) \right)$

2. Some lions don't drink coffee : $\exists x \left(L(x) \wedge \sim C(x) \right)$

Conclusion : So some fierce creatures do not drink coffee : $\exists x \left(F(x) \wedge \sim C(x) \right)$

solve :

	derivation	rule
1.	$\exists x \left(L(x) \wedge \sim C(x) \right)$	P
2.	$L(y) \wedge \sim C(y)$	T, {1}, ES
3.	$L(y)$	T {2}, simplification
4.	$\sim C(y)$	T {2}, simplification
5.	$\forall x \left[L(x) \rightarrow F(x) \right]$	P
6.	$L(y) \rightarrow F(y)$	T {5}, US
7.	$L(y)$	T {3,6}, modus ponens

contd.

19BCE2105

	derivation	rule
8.	$F(y)$	T{6}, simplification
9.	$F(y) \wedge \sim C(y)$	T{4,8}, conjunction
10.	$\exists x\,(F(x) \wedge \sim C(x))$	T{9}, EG

Hence, the given argument is valid.

Q7. Show that $(x)(\sim R(x) \rightarrow P(x))$ logically follows from $(x)(P(x) \vee Q(x))$ and $(x)((\sim P(x) \wedge Q(x)) \rightarrow R(x))$.

$\Longrightarrow$.

We've:

	derivation	rule
1.	$(x)(P(x) \vee Q(x))$	P
2.	$P(c) \vee Q(c)$	T{1}, US
3.	$(x)(\sim P(x) \wedge Q(x)) \rightarrow R(x)$	P
4.	$(\sim P(c) \wedge Q(c)) \rightarrow R(c)$	T{3}, US
5.	$\sim R(c) \rightarrow (P(c) \vee \sim Q(c))$	T{4}
6.	$\sim R(c)$	Additional premise (rule CP).

... contd.

derivation		rule
7.	$P(c) \vee \sim Q(c)$	T{5, 6}, modus ponens
8.	$(P(c) \vee Q(c)) \wedge (P(c) \vee \sim Q(c))$	T{2, 7}, conjunction
9.	$P(c) \vee [Q(c) \wedge \sim Q(c)]$	T{8}
10.	$P(c) \vee F$	T{9}
11.	$P(c)$	T{10}
12.	$\sim R(c) \longrightarrow P(c)$	rule CP
13.	$\forall x \, (\sim R(x) \rightarrow P(x))$	T{12}, UG

Hence, the given argument logically follows from the other two.

Q8. Let $P(x): |x| > 3$ and $Q(x): x > 3$.

Is the statement $(x)\,(P(x) \longrightarrow Q(x))$ true? Also, find the converse, inverse and contrapositive of $(x)\,(P(x) \longrightarrow Q(x))$ and verify the trueness of those statements.

$\Rightarrow$

contd.

contd. . . .

$\Rightarrow$ We've: $P(x) : |x| > 3$

and. $Q(x) : x > 3$

$\therefore (x)\,(P(x) \rightarrow Q(x)) \Rightarrow$ false

- Converse: $(x)\,(Q(x) \rightarrow P(x)) \Rightarrow$ True
- Inverse: $(x)\,(\sim P(x) \rightarrow \sim Q(x)) \Rightarrow$ True
- Contrapositive: $(x)\,(\sim Q(x) \rightarrow \sim P(x)) \Rightarrow$ False

---

Q9. Let $P(x,y)$, $Q(x,y)$ and $R(x,y)$ represents 3 statements. What is the negation of the given statement?

$\rightarrow$

Given statement: $(x)\ \exists(y)\ [P(x,y) \wedge Q(x,y)) \rightarrow R(x,y)]$.

$\therefore$ Negation: $(\exists x)\cdot(y)\ [P(x,y) \wedge Q(x,y) \wedge \sim R(x,y)]$.

19BCE2105

Q10. Using rule CP, derive $P \to \sim S$ from $P \to (Q \vee R)$, $Q \to \sim P$, $S \to \sim R$.

Sol^n: we've:

	derivation	rule
1.	P	Assumed premise
2.	$Q \to \sim P$	P
3.	$\sim Q$	T {1, 2}, modus tollens
4.	$P \to (Q \vee R)$	P
5.	$Q \vee R$	T, {1, 4}, modus ponens
6.	R	T, {3, 6}, disjunctive syllogism
7.	$S \to \sim R$	P
8.	$R \to \sim S$	T, {7}
9.	$\sim S$	T, {6, 8}, modus ponens
10.	$P \to \sim S$	CP.

Hence, the given statement is derived using rule CP from the other three.

— . —

Q11. Check the validity of the given argument.

⇒ Let,

p : The band could play rock music
q : refreshments were delivered on time
r : New Year's party was cancelled.
s : Alicia was angry
t : Refunds had to be made.

premises :-

1. $(\sim p \vee \sim q) \rightarrow (r \wedge s)$
2. $r \rightarrow t$
3. $\sim t$

Conclusion :- p

we've,

	derivation	rule
1.	$r \rightarrow t$	P
2.	$\sim t$	P
3.	$\sim r$	T {1, 2}, modus tollens
4.	$\sim r \vee \sim s$	T {3}, addition

.... contd.

derivation		rule
5.	$\sim(r \wedge s)$	T{4}, de Morgan
6.	$(\sim p \vee \sim q) \rightarrow (r \wedge s)$	P
7.	$\sim(\sim p \vee \sim q)$	T{5,6}, modus tollens
8.	$p \wedge q$	T{7}, de Morgan
9.	p	T{8}, $P \wedge Q \Rightarrow P$

Hence, the given argument is valid.

—·—

END