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TOPIC: Phase and group velocity of EM waves

AIM: To understand the nature of EM waves travelling in a medium with the help of phase & group velocities.

THEORY & FORMULAE:

Any real signal consists of travelling-waves of many different frequencies, which travel together as a group, at a speed that will always be less than or equal to the speed of light in vacuum.

To gain some insight into what may happen when a real signal travels through a dispersive medium, we consider adding 2 waves of equal amplitude.

When 2 travelling waves with unit amplitude $f_1(z,t) = \cos(K_1 z - \omega_1 t)$ and $f_2(z,t) = \cos(K_2 z - \omega_2 t)$ are added, we get:-

$$f_1(z,t) + f_2(z,t) = \cos(K_1 z - \omega_1 t) + \cos(K_2 z - \omega_2 t)$$

$$= 2\cos\left(\frac{\Delta K}{2} z - \frac{\Delta\omega}{2} t\right) \cdot \cos(\bar{K} \cdot z - \bar{\omega} t)$$

where, $\frac{\Delta K}{2} = \frac{K_1 - K_2}{2}$, $\frac{\Delta\omega}{2} = \frac{\omega_1 - \omega_2}{2}$,

$\bar{K} = \frac{K_1 + K_2}{2}$, and $\bar{\omega} = \frac{\omega_1 + \omega_2}{2}$

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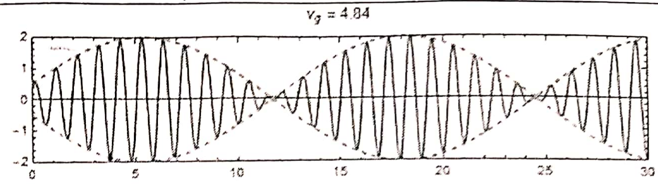
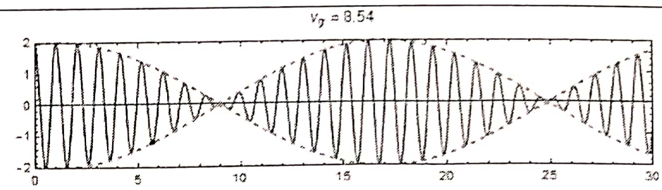
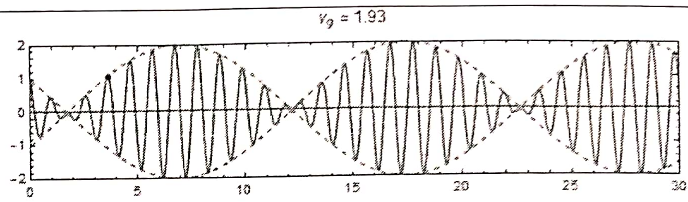
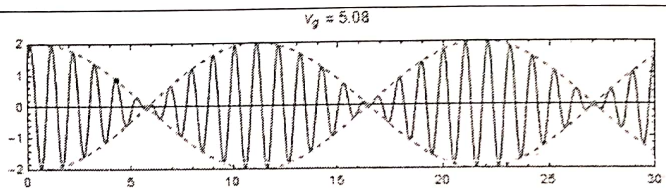
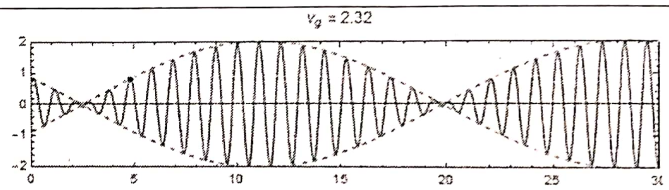
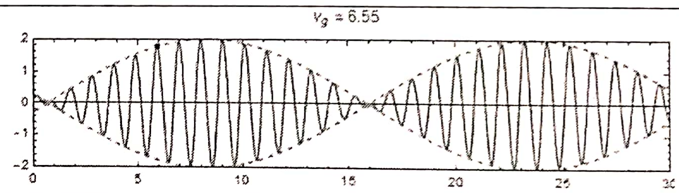
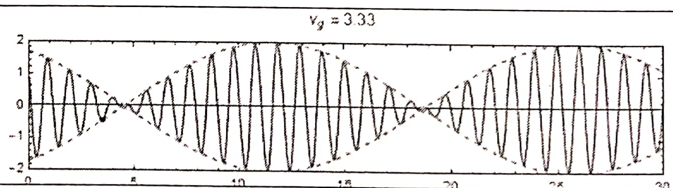
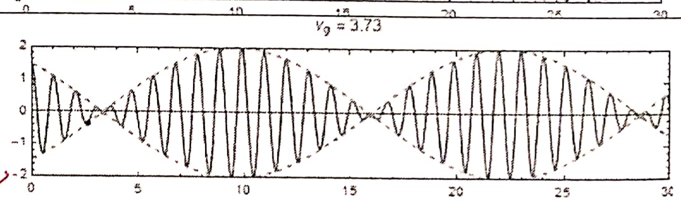
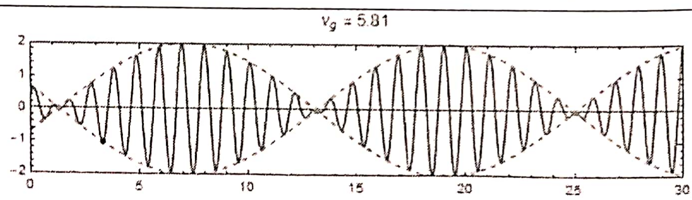
S.No	Δω	Δk	t	Wave pattern of the resultant waves	V_g
1.	1.19	0.246	3.74		4.84
2.	1.69	0.198	5.7		8.54
3.	0.58	0.3	3.58		1.93
4.	1.5	0.295	4.28		5.08
5.	0.42	0.181	4.8		2.32
6.	1.35	0.206	5.92		6.55
7.	0.74	0.222	3.48		3.33
8.	0.93	0.249	2.58		3.73
9.	1.54	0.265	3.28		5.81

The result is a fast oscillating wave that travels with a phase velocity,

$$V_p = \frac{\bar{\omega}}{\bar{k}}$$ and the

amplitude of this wave is being modulated in space and time by

$$2 \cdot \cos\left(\frac{\Delta K}{2} z - \frac{\Delta \omega}{2} t\right).$$

This modulated wave travels at the group velocity, given by :

$$V_g = \frac{\Delta\omega/2}{\Delta K/2} = \frac{\Delta\omega}{\Delta K}$$

1. Are the wave patterns for various values of $\Delta\omega$ and ΔK same? If not, why?

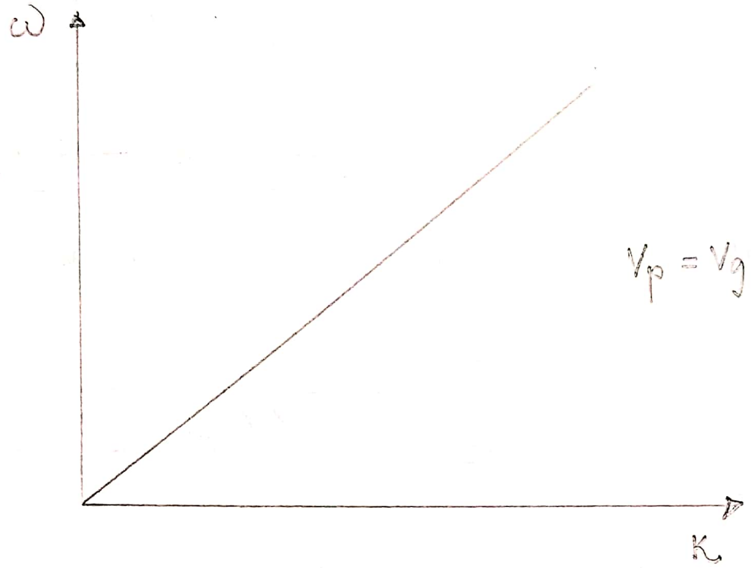
⇒ No, the wave patterns for various values of $\Delta\omega$ and ΔK are not the same as amplitude of the wave is modulated and follows the equation :

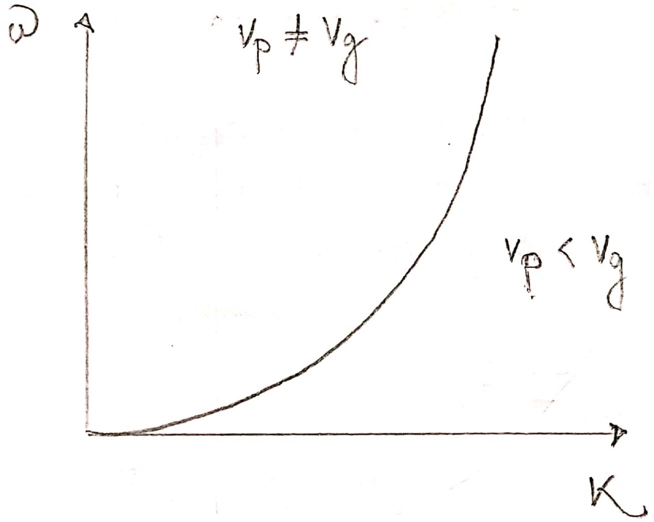
$$2 \cdot \cos\left(\frac{\Delta K}{2} z - \frac{\Delta \omega}{2} t\right)$$

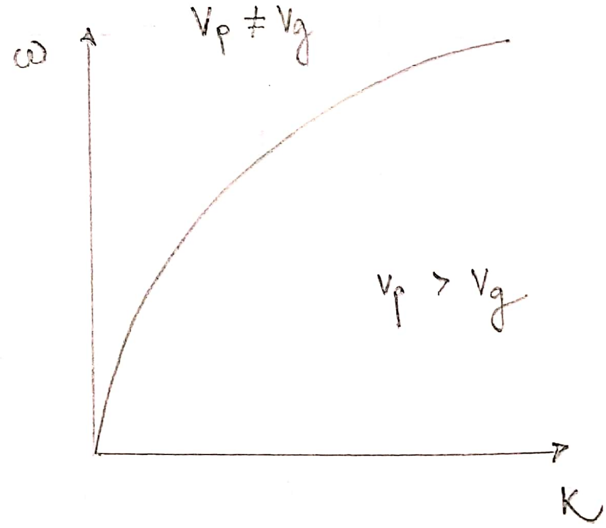
As time goes on, the wave moves towards right, changing the pattern. The individual peak travels with the group speed. By varying ΔK and $\Delta\omega$, wave pattern changes as distribution between consecutive particle changes.

2. Draw a typical dispersion relation curve [ω-k curve] for $V_p = V_g$ and $V_p \neq V_g$ case.

⟹







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3. When do we see V_p and V_g being the same?

⟹ When there is no dispersion medium, the phase velocity and group velocity is the same.
If the derivative term in Rayleigh formula is zero, $V_p = V_g$.
Phase velocities of the components of the envelope cause the wave packets to spread out.

4. Comment on the phase velocity (V_p) of the waves for increased values of $\Delta\omega$ and ΔK.

⟹ Phase velocity depends on refractive index of medium, i.e.,

$$V_p = \frac{c}{\mu}, \qquad V_p = \frac{\omega}{K},$$

$$V_g = V_p + K\frac{\partial V_p}{\partial K},$$

So, on increasing the values of $\Delta\omega$ and ΔK, only the amplitude changes; there is no change in phase velocity (V_p).

$\frac{7.0}{10}$

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$\omega_1 = 1$; $K_1 = 3$ $\quad \omega = \frac{\omega_1 + \omega_2}{2}$

$\omega_2 = 2$ $\quad K_2 = 4$

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