

→ register number modulo 5

$$\Leftrightarrow 2105 \text{ modulo } 5 = 0 \quad \Leftrightarrow \text{Set-5}$$

Q1. find GCD (7469, 2464) using Euclidean algorithm.  
find  $x, y$ , MMI for the same using EEA.

Sol<sup>n</sup> : we've,  $c = dq + r$ ;  $0 \leq r < d$ .

here,  $c = 7469$  and  $d = 2464$  (at step-1).

Euclidean algorithm corresponds :

$$7469 = 2464 * 3 + \underline{77}$$

$$\Rightarrow 2464 = 77 * 32 + 0$$

$$\therefore \text{gcd}(7469, 2464) = 77$$

again, using the Extended Euclidean Algorithm, we've:

[assumption: at step-1,  $x=1$ ,  $s_1=0$ ,  $y=0$ ,  $t_1=1$ ,

$$\text{and: } s_2 = x - q \cdot s_1 \quad ; \quad t_2 = y - q \cdot t_1]$$

[[ table on the next page.  $s_2$  and  $t_2$ 's calculations here ]]

$$\hookrightarrow s_2 = 1 - 3 \cdot 0 = 1$$

$$t_2 = 0 - 3 \cdot 1 = -3$$

$$\text{again, } s_2 = 0 - 32 \cdot 1 = -32$$

$$\text{again, } t_2 = 1 - 32 \cdot (-3) = 97$$



EEA table:

<u>q</u>	<u>a</u>	<u>b</u>	<u>r</u>	<u>x</u>	<u>s<sub>1</sub></u>	<u>s<sub>2</sub></u>	<u>y</u>	<u>t<sub>1</sub></u>	<u>t<sub>2</sub></u>
3	7469	2464	77	1	0	1	0	1	-3
		↙	↙		↙	↙		↙	↙
32	2464	77	0	0	1	-32	1	-3	97
		↙	↙		↙	↙		↙	↙
x	77	0	x	1	-32		-3	97	

$$\therefore x = 1.$$

$$y = -3.$$

$$\text{again, } \gcd = 77$$

this also implies that,  $ax + by$  must be  $= 77$

let's check:

$$7469 \cdot (1) + 2464 \cdot (-3)$$

$$= 7469 - 7392$$

$$= 77.$$

checks out;

Also,  $\gcd(7469, 2464) = 77 \neq 1$

$\therefore$  MMI does not exist



Q2.(i) find the result:  $-5432 \bmod 38$ .

we've:  $-5432 \bmod 38$

$$= 38 - (5432 \bmod 38)$$

$$= 38 - 36$$

$$= 2$$

(ii) find the result:  $\phi(606)$ .

we've:  $\phi(606) = \phi(101 \times 6)$

$$= \phi(101) \cdot \phi(6)$$

$$= 100 \cdot \phi(3 \times 2)$$

$$= 100 \cdot \phi(3) \cdot \phi(2)$$

$$= 100 \cdot 2 \cdot 1$$

$$= 200$$

(7.7.0)



Q3.

Find  $18^{1001} \bmod 11$ .Sol<sup>n</sup>: we've:  $18^{1001} \bmod 11$ 

$$\Leftrightarrow 18^1 \bmod 11 = 7$$

$$\Leftrightarrow 18^2 \bmod 11 = 5$$

$$\Rightarrow 18^4 \bmod 11 = 25 \bmod 11 = 3$$

$$\Rightarrow 18^8 \bmod 11 = 9 \bmod 11 = 9$$

$$\Rightarrow 18^{16} \bmod 11 = 81 \bmod 11 = 4$$

$$\Rightarrow 18^{32} \bmod 11 = 16 \bmod 11 = 5$$

$$\Rightarrow 18^{64} \bmod 11 = 25 \bmod 11 = 3$$

$$\Leftrightarrow 18^{128} \bmod 11 = 9 \bmod 11 = 9$$

$$\Rightarrow 18^{256} \bmod 11 = 81 \bmod 11 = 4$$

$$\Rightarrow 18^{512} \bmod 11 = 5$$

now:

$$\Rightarrow 18^{1001} \bmod 11 = (18^{1000} \times 18^1) \bmod 11$$

$$= (18^{512} \times 18^{256} \times 18^{128} \times 18^{64} \times 18^{32} \times 18^8 \times 18^1) \bmod 11$$

(P.T.O.)



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$$\Leftrightarrow 18^{1001} \bmod 11 = (5 \times 4 \times 9 \times 3 \times 5 \times 9 \times 7) \bmod 11$$

$$= 170100 \bmod 11$$

$$= 7$$

$$\therefore 18^{1001} \bmod 11 = 7 \quad \underline{\text{ans.}}$$

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