Assignment- 4

\# 19BCE2105

Question - 1 :

- **problem** : 8 coins are thrown simultaneously. Find the chance of obtaining — (i) at least 6 heads ; (ii) no heads , and (iii) all heads.

- **Manual Calculation** : Here, $n = 8$, $p = \frac{1}{2}$, $q = 1 - p = \frac{1}{2}$.

(i) $P(X \geq 6) = P(X=6) + P(X=7) + P(X=8)$

$$= {}^8C_6 \left(\frac{1}{2}\right)^6 \left(\frac{1}{2}\right)^2 + {}^8C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^1 + {}^8C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^0$$

$$= 28 \times \frac{1}{2^8} + 8 \times \frac{1}{2^8} + 1 \times \frac{1}{2^8}$$

$$= \frac{37}{256} = 0.1445$$

(ii) $P(X=0) = {}^8C_0 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^0$

$$= \frac{1}{2^8}$$

$$= \frac{1}{256} = 0.0039$$

(iii) $P(X=8) = {}^8C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^0$

$$= \frac{1}{2^8}$$

$$= 0.0039$$

- **Code in R-console** :

```
> 1- pbinom (5, 8, 0.5)          // P(X>=6)
[1] 0.1445312

> dbinom (0, 8, 0.5)             // P(X=0)
[1] 0.00390625

> dbinom (8,8, 0.5)              // P(X=8)
[1] 0.00390625
```

19BCE2105

Question - 2 :

- **problem :** A multi-choice test questions consists of 8 questions with 3 answers to each question (of which only one is correct). A student answers each question by rolling a balanced dice and checking the first answer if he gets 1 or 2, the second answer if he gets 3 or 4 and the three answers if he gets 5 or 6. To get a distinction, the student must secure at least 75% correct answers. If there is no negative marking, what is the probability that the student secures a distinction ?

- **Manual Calculation :** Here, $p = \frac{1}{3}$, $q = 1 - p = 2/3$, $n = 8$

Acc. to questⁿ, in order to get at least 75%, the student must answer at least 6 or more questions.

$$\therefore P(x \geq 6) = P(x=6) + P(x=7) + P(x=8)$$

$$= {}^8C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^2 + {}^8C_7 \left(\frac{1}{3}\right)^7 \left(\frac{2}{3}\right)^1 + {}^8C_8 \left(\frac{1}{3}\right)^8 \left(\frac{2}{3}\right)^0$$

$$= 28 \times \frac{1}{729} \times \frac{4}{9} + 8 \times \frac{1}{2187} \times \frac{2}{3} + 1 \times \frac{1}{6561}$$

$$= 0.01707 + 0.002438 + 0.00015$$

$$= 0.01965$$

- **Code in R-console :**

```
> 1 - pbinom (5, 8, 1/3)        // P(x>=6)
[1] 0.01966164
```

19BCE2105

Question - 3 :

- **problem :** In a town 10 accidents took place in a span of 50 days. Assuming that the number of accidents per day follows the Poisson distribution, find the probability that there will be 3 or more accidents in a day.

- **Manual calculation :** Here, $\lambda = \frac{10}{50} = \frac{1}{5}$

$$p(x \geq 3) = 1 - [p(x=0) + p(x=1) + p(x=2)]$$

$$= 1 - \left[e^{-1/5}\left(\frac{(\frac{1}{5})^0}{0!} + \frac{(\frac{1}{5})^1}{1!} + \frac{(\frac{1}{5})^2}{2!}\right)\right]$$

$$= 1 - [2.7183]^{-1/5}\left(1 + \frac{1}{5} + \frac{1}{50}\right)$$

$$= 1 - (2.7183)^{-1/5} \times \frac{61}{50}$$

$$= 1 - 0.81873 \times \frac{61}{50}$$

$$= 0.0011494$$

- **Code in R- console :**

```
> 1 - ppois(2, lambda = 1/5)          // p(x≥3).
[1] 0.001148481
```

Question-4 :

- **problem :** 1000 light bulbs with a mean life of 120 days are installed in a new factory ; their length of life is normally distributed with standard deviation 20 days. (i) how many bulbs will expire in less than 90 days ? (ii) if it is decided to replace all the bulbs together, what intervals should be allowed between replacements if not more than 10% should expire before replacement ?

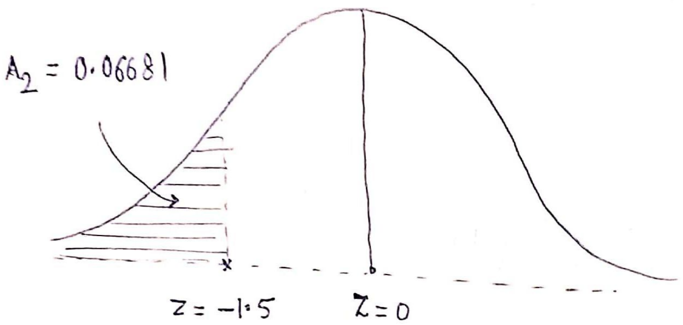
19 BCE 2105

Manual calculation :

Here, $\mu = 120$, $\sigma = 20$

(i) $x = 90$

$\therefore Z = \frac{x-\mu}{\sigma} = \frac{90-120}{20} = -1.5$



$\therefore$ No. of bulbs that will expire in less than 90 days

$= A_2 \times 1000$

$= 0.06681 \times 1000$

$= 66.8$ bulbs

≈ 67 bulbs

(ii) $P(X \leq X_1) = 0.1$

$P\left(Z \leq \frac{X_1 - 120}{20}\right) = 0.1$

or, $\frac{X_1 - 120}{20} = -1.3$ [from table]

$\therefore X_1 = (-1.3 \times 20) + 120$

$= (120 - 26)$ days

$= 94$ days

19BCE2105

Code in R-console :-

```
> pnorm(90, mean = 120, sd = 20) * 1000
[1] 66.8072
```

— END —