AI Th DA-2

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Q1. On an airport all passengers are checked carefully. Let T with $t \in \{0,1\}$ be the random variable indicating whether somebody is a terrorist $(t=1)$ or not $(t=0)$, and A with $a \in \{0,1\}$ be the variable indicating arrest. A terrorist shall be arrested with probability $P(A=1 \mid T=1) = 0.98$, a non-terrorist with probability $P(A=1 \mid T=0) = 0.001$. One in hundred thousand passengers is a terrorist, $P(T=1) = 0.00001$. What is the probability that an arrested person actually is a terrorist?

Soln: Given,

$$P(A=1 \mid T=1) = 0.98$$
$$P(A=1 \mid T=0) = 0.001$$
$$P(T=1) = 0.00001$$

From Bayesian theorem, we've:

$$P(T=1 \mid A=1) = \frac{P(A=1 \mid T=1) \cdot P(T=1)}{P(A=1)}$$

$$= \frac{P(A=1 \mid T=1) \cdot P(T=1)}{P(A=1 \mid T=1) \cdot P(T=1) + P(A=1 \mid T=0) \cdot P(T=0)}$$

$$= \frac{0.98 \times 0.00001}{(0.98 \times 0.00001) + (0.001 \times (1 - 0.00001))}$$

$$= \frac{980}{100097}$$

$$= 0.00979050321 1$$

$$\approx 0.01$$

Hence, the probability that an arrested person is actually a terrorist is 0.009 ($\approx$ 0.01).

It is an interesting result. Even though for any passenger, it can be decided with high reliability (98% and 99.9%) whether they are ~~not~~ a terrorist or not, if somebody gets arrested as a terrorist though, they are still most likely not a terrorist (with a probability of 99%)

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Q2. In an oral exam you've to solve exactly one problem, which might be one of 3 types: A, B, or C, which will come up with probabilities 30%, 20% and 50% respectively. During your preparation you have solved 9 out of 10 problems of type-A, 2 out of 10 problems of type-B, and 6 out of 10 problems of type-C.

(a) What is the probability that you will solve the problem of the exam?

(b) Given you have solved the problem, what is the probability that it was of type A?

Solⁿ:

(a) The probability to solve the problem of the exam is the probability of getting a problem of a certain type times the probability of solving such a problem, summed over all types. This is known as the total probability.

$$P(\text{solved}) = P(\text{solved} \mid A) \cdot P(A) +$$
$$P(\text{solved} \mid B) \cdot P(B) +$$
$$P(\text{solved} \mid C) \cdot P(C)$$

$$= \left(\frac{9}{10} \times \frac{30}{100}\right) + \left(\frac{2}{10} \times \frac{20}{100}\right) + \left(\frac{6}{10} \times \frac{50}{100}\right)$$

$$= \frac{27}{100} + \frac{4}{100} + \frac{30}{100}$$

$$= \frac{61}{100}$$

$$= 0.61$$

Hence my ~~the~~ probability of solving this problem of the exam is 61 %.

(b) For this part, I need Bayes Theorem.

$$P(A \mid \text{solved}) = \frac{P(\text{solved} \mid A) \cdot P(A)}{P(\text{solved})}$$

$$= \frac{\frac{9}{10} \times \frac{30}{100}}{\frac{61}{100}}$$

$$= \frac{27/100}{61/100}$$

$$= \frac{27}{61}$$

$$= 0.4426229508$$

Hence, the probability that it was of type - A, had I solved the problem, is 44.3~~3~~ %.

It is very interesting to ~~note~~ see, that, given I've solved the problem, the 'a posteriori' probability that the problem was of type-A is greater that its 'a priori' probability of 30 %, because problems of type-A are relatively easy to solve.

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