Assessment - 3

Question-1 :

problem : Calculate Spearman's coefficient of correlation between marks assigned to students by three judges in a competition and determine which pair of judges has the nearest approach to common tastes.

1st Judge	98	79	83	57	88	95	86	62	76	67
2nd judge	75	68	56	72	58	50	89	94	62	52
3rd judge	61	67	52	55	89	78	75	49	65	59

Solution :

1st	98	79	83	57	88	95	86	62	76	67
2nd	75	68	56	72	58	50	89	94	62	52
3rd	61	67	52	55	89	78	75	49	65	59
R_1	1	6	5	18	3	2	4	9	7	8
R_2	3	5	8	4	7	10	2	1	6	9
R_3	6	4	9	8	1	2	3	10	5	7
d_{12}^2	4	1	9	36	16	64	4	64	1	1
d_{23}^2	9	1	1	16	36	64	1	81	1	4
d_{13}^2	25	4	16	4	4	0	1	1	4	1

$$\Sigma d_{12}^2 = 200$$

$$\Sigma d_{23}^2 = 214$$

$$\Sigma d_{13}^2 = 60$$

$$e_{12} = 1 - \frac{6 \sum d_{12}^2}{n(n^2-1)} = 1 - \frac{6 \times 200}{10(99)} = 1 - 1.2121$$

$$= -0.2121$$

$$e_{23} = 1 - \frac{6 \sum d_{23}^2}{n(n^2-1)} = 1 - \frac{6 \times 214}{10(99)} = 1 - 1.2969$$

$$= -0.2969$$

$$e_{13} = 1 - \frac{6 \sum d_{13}^2}{n(n^2-1)} = 1 - \frac{6 \times 60}{10(99)} = 1 - 0.3636$$

$$= 0.6464$$

$\therefore$ We take the decision presented by Judges 1 & 3 as: $e_{13} > e_{12} > e_{23}$

Code & Input in R-console :

```
> J1 = c(98, 79, 83, 57, 88, 95, 86, 62, 76, 67)
> J2 = c(78, 68, 56, 72, 58, 50, 89, 94, 62, 52)
> J3 = c(61, 67, 52, 55, 89, 78, 75, 49, 65, 59)
> cor.test (J1, J2, method = "spearman")
> cor.test (J2, J3, method = "spearman")
> cor.test (J1, J3, method = "spearman")
```

Question - 2 :

problem : Calculate the 2 regression equations and estimate the (i) productivity index of a worker whose test score is 92. ; (ii) test score of a worker whose productivity index is 75.

Solution :

X	60	62	65	70	72	48	53	73	65	82
Y	68	60	62	80	85	40	52	62	60	81
XY	4080	3720	4030	5600	6120	1920	2756	4526	3900	6642
X^2	3600	3844	4225	4900	5184	2304	2809	5329	4225	6724
Y^2	4624	3600	3844	6400	7225	1600	2704	3844	3600	6561

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$\Sigma x = 650$ $\quad \Sigma x^2 = 43144$

$\Sigma y = 650$ $\quad \Sigma y^2 = 44002$

$\Sigma xy = 43294$

$$b_{xy} = (r_{xy}) \cdot \left(\frac{\sigma_x}{\sigma_y}\right) = \frac{\text{cov}(x,y)}{\sigma_Y^2}$$

$$\text{cov}(x,y) = E(x^2) - \{E(x)\}^2$$

$$= \frac{\Sigma xy}{n} - \left(\frac{\Sigma x}{n}\right)\left(\frac{\Sigma y}{n}\right) = 38025$$

$$= \frac{43294}{10} - 65 \times 65$$

$$= 104$$

$$\sigma_y^2 = \frac{\Sigma y^2}{n} - \left(\frac{\Sigma y}{n}\right)^2$$

$$= \frac{44002}{10} - (65)^2$$

$$= 4400.2 - 4225$$

$$= 175$$

$$\therefore\ b_{xy} = \frac{104}{175} = 0.59$$

$$\sigma_x^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$$

$$= \frac{43144}{10} - (65)^2$$

$$= 89$$

$$\therefore\ b_{yx} = \frac{104}{89} = 1.16$$

$$(x - \bar{x}) = 0.59\,(y - \bar{y})$$

At $y = 75$.

$$x = 0.59\,(75 - 65) + 65$$

$$\boxed{x = \cancel{65.59}\ \ 70.958}$$

$$(y - \bar{y}) = 1.16\,(x - \bar{x})$$

At $x = 92$.

$$y = 1.16\,(92 - 65) + 65$$

$$\boxed{y = 96.529}$$

Code & Input in R console :

```
> x = c(60, 62, 65, 70, 72, 48, 53, 73, 65, 82)
> y = c(68, 60, 62, 80, 85, 40, 52, 62, 60, 81)
> g = lm(y~x)
> summary(g)
> K = lm(x~y)
> summary(k)
```

Question-3

problem : A research company summarised advertising expenditure and sales. Karl Pearson's coefficient : 0.6 . Derive the 2 regression equations. Estimate the sale if advertising expenditure is 15 crore.

Solution :

	Adv. Exp. (₹ in crore)	Sales (₹ in crore)
Mean →	20	200
SD →	18	170

Karl Pearson's coefficient : 0.6

$$b_{xy} = r \cdot \frac{\sigma_x}{\sigma_y} = 0.6 \times \frac{18}{170} = 0.06$$

$$b_{yx} = r \cdot \frac{\sigma_y}{\sigma_x} = 0.6 \times \frac{170}{18} = 5.6$$

$$(x - \bar{x}) = b_{xy}(y - \bar{y})$$

$$(x - 20) = 0.06(y - 200)$$

$$x = 0.06(y - 200)$$

$$\boxed{x = 0.06y + 8}$$

$$(y - \bar{y}) = b_{yx}(x - \bar{x})$$

$$(y - 200) = 5.6(x - 20)$$

$$\boxed{y = 5.6x + 88}$$

At $x = 15$,

$y = 172.$

Code & Input in R-console :

```
> X = c(~~60, 62, 65, 70, 72, 48, 53, 73, 65, 82~~)    c(20,18)
> Y = ~~c (68, 66, 62, 80, 85, 40, 52, 62, 60, 81)~~    c(200,170)
> yx = lm (y ~ x)
> summary (yx)
```

Question-4

Problem: Determine the relation using multiple regressions and interpret the result.

Solution:

x	12	14	15	16	18
y	32	35	45	50	65
z	34	44	43	45	36
xy	384	490	675	800	1170
yz	1088	1540	1935	2250	2340
xz	408	616	645	720	648
x^2	144	196	225	256	324
y^2	1024	1225	2025	2500	4225

$\Sigma xy = 3519$

$\Sigma yz = 9153$

$\Sigma xz = 3037$

$\Sigma x^2 = 1145$

$\Sigma y^2 = 10999$

$$202 = 5a_0 + 75a_1 + 227a_2$$

$$3037 = 75a_0 + 1145a_1 + 3519a_2$$

$$9153 = 227a_0 + 351a_1 + 10999a_2$$

$$\therefore a_0 = -18.16$$

$$a_1 = 7.928$$

$$a_3 = -1.329$$

Code & Input in R-console :-

```
> x = c(12, 14, 15, 16, 18)
> y = c(32, 35, 45, 50, 65)
> z = c(34, 44, 43, 45, 36)
> ~~reg~~ input_data = data.frame(x,y,z)
> input_data
> regmodel <- lm(z~x+y, data = input_data)
> regmodel
```

$$z = -18.163 + 7.298x - 1.329y$$