UNIVERSITY OF PETROLEUM & ENERGY STUDIES, DEHRADUN

Program	**B. Tech (All SoCS Branches)**	**Semester**	**I**
Course	**Mathematics-I**	**Course Code**	**MATH 1002**
Session	**July-Dec 2019**	**Topic**	**Mathematical Logic**

1. If p be "He is rich" and q be "He is happy". Write each statement in symbolic form using p and q. Note that "He is poor" and "He is unhappy" are equivalent to $\sim p$ and $\sim q$, respectively.
   (i) If he is rich, then he is unhappy.
   (ii) He is neither rich nor happy.
   (iii) It is necessary to be poor in order to be happy.
   (iv) To be poor is to be unhappy.
2. If p, q and r are three statements, construct the Truth Table of the following propositions
   $(i)\ p \vee \sim q \wedge r$ $(ii)\ r \wedge \sim (p \Rightarrow q)$ $(iii)\ r \Leftrightarrow (p \wedge q)$.
3. If p, q and r are three statements, then check whether the following statements are tautology or not.
   $(i)\ (p \wedge q) \Rightarrow (p \Rightarrow q)$ $(ii) \sim (p \vee q) \vee [\sim p \wedge q] \vee p$
   $(iii)\ (p \Rightarrow q) \Leftrightarrow (\sim q \Rightarrow \sim p)$.
4. Determine whether the following propositions are a tautology, contingency or contradiction :
   $(i)\ p \leftrightarrow (p \wedge q)$ $(ii)\ p \rightarrow (q \vee p)$ $(iii)\ (\sim p \wedge q) \wedge p$.
5. Show that the following propositions are equivalent or not?
   $(i)\ p \leftrightarrow q \cong (p \wedge q) \vee (\sim p \wedge \sim q)$ $(ii)\ [(p \rightarrow q) \rightarrow r] \cong (p \wedge \sim q) \rightarrow r$.
6. Determine the principal disjunctive normal form (PDNF) and principal conjunctive normal form (PCNF) of the following propositions
   $(i)\ p \leftrightarrow q$ $(ii)\ (q \wedge p) \vee (\sim q \wedge r)$.
7. If $D=\{1,2,3,....9\}$. Determine the truth value of each of the following statements.

(i) $(\forall\, x \in D),\ x+4<15,$

(ii) $(\exists\, x \in D),\ x+4=10,$

(iii) $(\forall\, x \in D),\ x+4\le 10,$

(iv) $(\exists\, x \in D),\ x+4>15.$

8. Write the negation of the following statements

(i) All natural numbers are less than 10.

(ii) For all real numbers x, if $x>4$ then $x^2>16$. .

9. A certain country is inhabited by people who either always tell truth or always lie, and also they will respond to questions only with 'yes' or 'no'. A tourist comes to a fork in the road, where one branch leads to capital and other does not. There is no sign indicating which branch to take, but there is an inhabitant Mr. Z standing at the fork. What a single question should tourist ask him to determine which branch to take?

p: Mr. Z always tells truth,

q: Left hand road goes to capital.

10. Determine the validity of the following arguments

i. Either I will pass the examination, or, I will not graduate.
If I do not graduate, then I will go to Canada.
I failed.

Thus, I will go to Canada.

ii. If the market is free, then there is no inflation.
If there is no inflation then there are price controls.
Since there are price controls.

Therefore, the market is free.