A Comparative Survey of PSD Estimation Methods for EEG Signal Analysis

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***Abstract*-- Digital frequency domain analysis often involves spectral power determination. Although the Fourier Transform is often the primary tool, limitations exist with respect to power spectral density (PSD) measurements. Accurate PSD calculation requires interpolation and smoothing techniques, prior to most applications typically found in modern test equipment. Non-parametric methods provide a means to create representative periodograms, which provide a variety of flexibility with respect to pre-filtering, data windowing, and smoothing.**

Although more computationally demanding, parametric models employ predictive algorithms to represent the PSD, to arbitrary accuracy. A survey of both methods is presented, using real biometric data. Benefits and shortcomings are identified, and an attempt is made to qualitatively optimize each particular method. Results are presented in the form of numerical data, spectral estimates, and possible applications.

***Index Terms*— Periodogram, parametric and non-parametric PSD estimation.**

I. Introduction

AFTER Fourier presented his famous method for representing signals in the frequency domain, life scientists have employed his transform in order to determine cyclical events in nature. The PSD is a powerful method for the identification of repetitive and correlated events, and was first refined by Schuster in the late 1800s, in order to accurately quantify natural phenomena for lunar and sunspot-induced earthquake predictions [1], [2]. Known as the periodogram, the resulting PSD is formed by averaging many Fourier spectra in order to improve signal to noise, and preserve frequency resolution [3]. Since that time, signal processing has blossomed in the field of communications and biometric measurements.

Non-parametric estimation methods involve simple averaging techniques of multiple spectra, which is well within the capabilities of even modest computing equipment. In addition to the basic periodogram, methods by Bartlett and Welch offer more advanced forms involving pre-filtering, and data windowing.

Recent approaches involve developing parameterized models, whereby the PSD is represented by a high-order mathematical equation, instead of a simple running average. In this report, both methods are explored, in an attempt to understand proper application, and sensitivities to real-time data.

Biometric data was used for all tests, which was obtained from a one-minute electro-encephalogram (EEG) data file, sampled at 1 kHz. As an expedient for the calculations, MATLAB was used as a programming environment, and also for the design of the filters and windows.

II. Non-Parametric PSD Estimation

PSD estimation is a balance between smoothing and frequency resolution. Obtaining representative spectra is challenging, due to the large data sets (often continuous, real-time), and the computational complexity [4]. Methods which employ simple averaging techniques are generally known as non-parametric, and are non-predictive. True models, involving predictive and regression qualities are known as parametric methods, and seek to establish a behavioral description, and not simply an average or best-fit [3].

A periodogram is a description of the signal power as a function of frequency. For small signal sequences, this can be obtained by performing a DFT on the X(k) data series:

$$\frac{1}{M}\left|\mathrm{DFFT}(\mathrm{X}\omega)\right|^2 = \frac{1}{M}\left|\sum_{n=1}^{M-1} \mathrm{X}_{\omega}(k)e^{-j\omega n}\right|^2 \quad (1)$$

As a practical matter, the entire set may be quite long, or continuous, whereby a direct calculation is not possible. For these cases, smaller intervals are selected, usually with N equal to a power of 2 [3], and the DFT taken. The resulting PSD has larger frequency spacing, but the result of averaging many such spectra together returns an average, particularly if zero-padding is used in the DFT [3].

In general, PSD estimates using a limited number of samples tend to be quite poor [4]. In order to produce a smoother, well-behaved PSD estimate, many independent periodograms are averaged together, as proposed by Bartlett [3] . Bartlett's Method is a popular approach, which divides the long sequence into K non-overlapping sub-sequences, having length N. The DFT, $\widehat{\mathrm{Y}}_{\mathrm{k}}(\mathrm{f})$, is computed, squared, and added to the previous' segment, to produce a running average:

$$\hat{\mathrm{S}}_{\mathrm{cop}}(f) = \frac{1}{K}\sum_{k=1}^{K}\widehat{\mathrm{Y}}_{\mathrm{k}}(f) \quad (2)$$

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where multiple spectra, pertaining to smaller data slices, are averaged together.

Bartlett's method improves signal to noise by factor K, but results in a loss of resolution of K/N, where N is the original size of the data set [3]. It is possible to show that, for large numbers of K, the estimate improves, and the variance tends to zero [3]. For simple estimates, this method achieves good results with minimal computation.

A. *Windowing*

To create the Bartlett sub-sequence, the data is effectively multiplied by a rectangular function, or time-windowed. From signal theory, multiplication in the time domain is equivalent to convolution in the frequency domain [5]. In Bartlett's example, the rectangular window results in a sinc(f) frequency spectrum, which is then squared to obtain the PSD. The resulting window frequency spectrum is sinc(f) convolved with itself, producing a triangular window, shown in Figure 1 [5]:

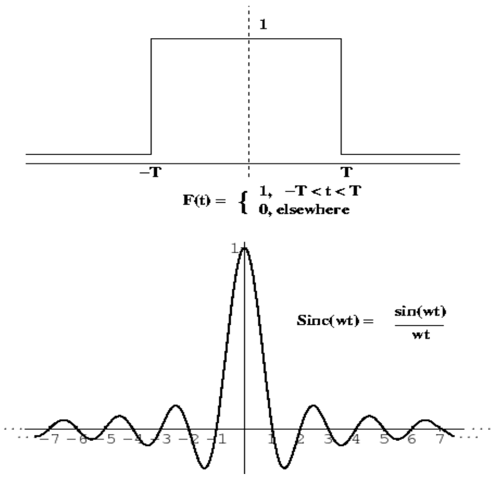

Fig. 1. A rectangular window used to select sub-sequence data for Bartlett's method has a sinc frequency response.

This window, when finally convolved with the signal spectrum, distorts the original by smearing the narrow spectral components, possibly eliminating them, altogether [4]. This effect is remedied by extending the observation period (N) [3]. Improvement is possible by careful selection of windows, which all have the basic tradeoff between main lobe width, and sidelobe supression. Several windows are demonstrated in Figure 2.

B. *Welch's Method*

As was noted with windowing, energy from sidelobes leaks into the desired spectrum, creating fluctuations in the periodogram. Similar to Bartlett's Method, Welch computes periodograms of length N, and averages the spectra. However, two additional parameters permit the use of overlapping segments, and freedom to use any window [6]. The result is control over the amount of smoothing, as well as improving resolution by reducing energy leakage from excessive sidelobes. For example, a rectangular window is only able to

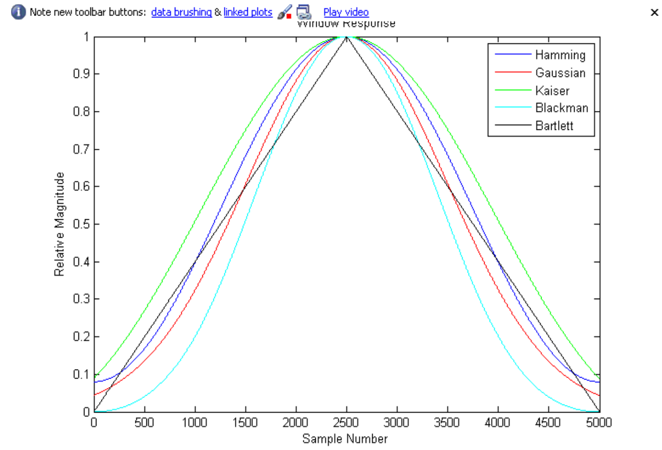

Fig. 2. Various window functions displaying different bandwidths. These were used in the simulation.

supress out-of-band signals by ~ 25 dB [4]. Numerically, Welch's method is similar to Bartlett's, but with an additional index, and scaling factor:

$$S(\omega) = \sum_{n=0}^{L-1} x(n)w(n)e^{-2\pi\left(\frac{\omega}{\omega_s}\right)n} \tag{3}$$

$$\hat{I}_{xx}(\omega) = \frac{1}{LU}|S(\omega)|^2 \tag{4}$$

$$U = \frac{1}{L}\sum_{n=0}^{L-1}|w(n)|^2 \tag{5}$$

Where *S(ω)* is the windowed DFT, I_{xx} is the spectral estimate, and U is the normalizing factor, based on the particular windowing function, w(n) [3,6].

The provision for adjustment of the two parameters is an attractive element, and adaptable to many analysis applications.

C. *EEG PSD Estimation*

Welch's Method

In order to evaluate each of the various estimation methods, actual biometric data was used to gain qualitative insight into the behaviors and limitations. The data set was obtained from 60 seconds of electroencephalogram (EEG) measurements, whereby a subject was asked to perform a series of tasks while driving [7]. The sample rate was fixed at 1 kHz, and includes artifacts as well as noise. Figure 3 depicts the raw data.

As an expedient for the calculations, MATLAB was used as a programming environment, easily facilitating all of the necessary formulas and computations.

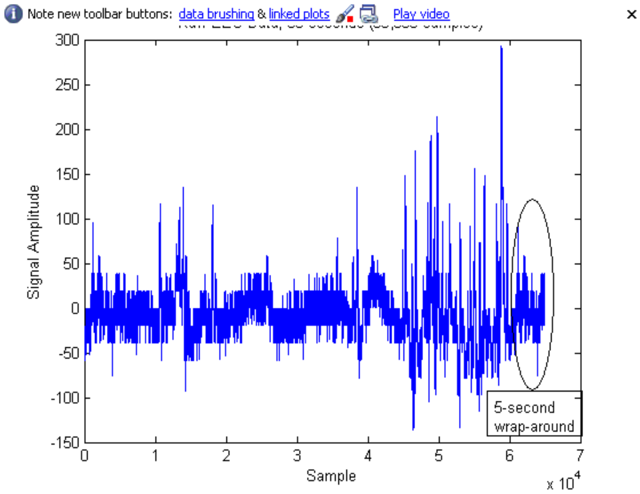

Fig. 3. 60 seconds of Raw EEG data, sampled at 1 kHz.

Pre-Filtering

As a preliminary step to the PSD estimation, the entire data set was pre-filtered, using a 50 Hz Butterworth lowpass filter (LPF). Several orders of filter were tried, resulting in narrow transition bands. Although the selection was subjective, other literature suggests using lower-order filters to minimize phase distortion, caused by steep filter skirts, as well as possible quantization effects [8]. Ultimately, a 4-pole filter was designed, and is shown in Figure 4. The raw data was convolved with the filter, removing signal energy above the frequency bands of interest.

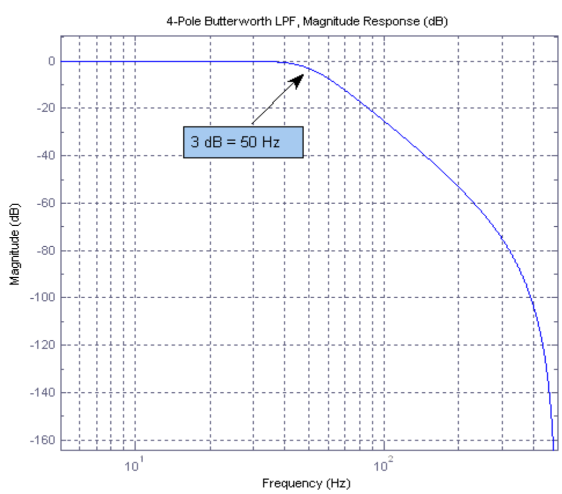

Fig. 4. Magnitude response of designed 4-pole Butterworth low-pass filter, for prefiltering EEG data.

The filtered data stream was first subjected to non-parametric methods, including the simple periodogram, Bartlett's Method, and Welch's Method. The 60-second set was sub-divided into 5-second slices, which were then used to analyze the models. After the window and overlap parameters were optimized, the MATLAB routine was then allowed to run, sliding the PSD window by 1-second increments through the data set. Since Welch's Method requires an extra 5 seconds of data, the raw data file was appended with the first 5 seconds, permitting the PSD window to “run-out” past the 60-second mark. The remaining 5 seconds were disregarded.

D. *Non-parametric Analysis*

For each slice, a simple periodogram was performed on the entire 5-second interval, which resulted in a rather noisy PSD estimate. Subtle features within the frequency band of interest (1 – 100 Hz) are difficult to differentiate. Bartlett and Welch Methods were performed using N = 1000-point sub-sequences (1-second). Although smaller values of N result in more averaged spectra, frequency resolution is decreased beyond 1 Hz, possibly interfering with subsequent power band measurements.

Smoothing is seen with the averaging techniques, as well as some structure within the desired frequency band. Figure 5 compares the three approaches, where the Welch Method employed a simple rectangular window, and 40% overlap.

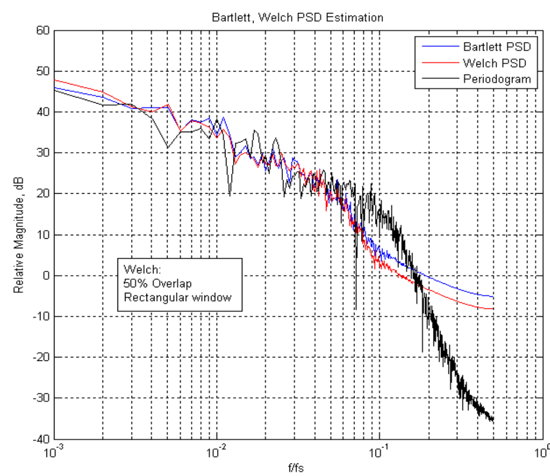

Fig. 5. Non-parametric PSD comparison, on single 5000-sample data slice, representing periodogram, Bartlett's method, and Welch's method (rectangular window, 40% overlap). Good agreement, and SNR improvement is demonstrated. Periodogram is slightly offset, due to graphical effects, not numerical.

Many trials were run, with changes made to both the window type and amount of overlap. Subjectively, it was determined that a 50% overlap and Kaiser windowing produced the best result with the Welch Method. The tradeoffs in dynamic range tracking are evident in Figure 6, particularly with respect to the endpoints at the higher frequencies. This is likely a result of the energy leak associated with Bartlett’s default square window, and highlights the benefit of Welch’s flexibility with respect to window function selection. In-band structure is accurately reproduced by all of the models.

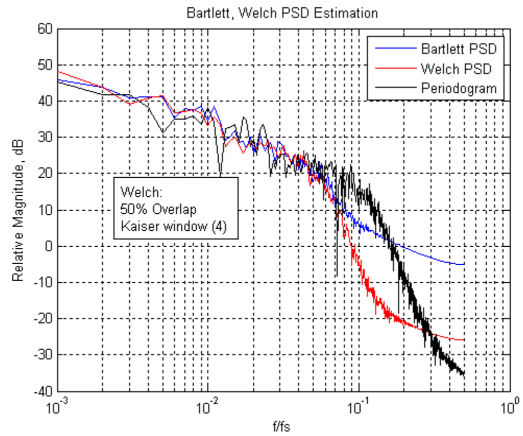

Fig. 6. Non-parametric comparison, employing 50% overlap and Kaiser windowing for Welch. Again, slight high-frequency shift in periodogram is due to graphic alignment, and not a numeric artifact. Good agreement is demonstrated.

III. PARAMETRIC PSD ESTIMATION

Much greater time was needed to evaluate and understand the parametric models, due to the complexity and amount of additional computation time. As before, the three popular parametric models, Yule-Walker, Burg, and Least-squares was applied to each 5-second data slice, and adjusted for best qualitative fit. The order of the model proved to be somewhat sensitive, with an exact value difficult to ascertain. Previous reports indicate rather high values for n, ranging from 100-1000 [9]. Although these values were tried, extreme computation times and instabilities were observed.

Other studies of biometric data suggested values from 10-60 were sufficient for similar-sized data sets, and proved to be a better guide for this study [10]. Adhering to this range, many trials were performed using the various windows and values for n. Very low values produced poor results, suggesting the model is unable to accurately predict the nuances of the PSD, as demonstrated in Figure 7.

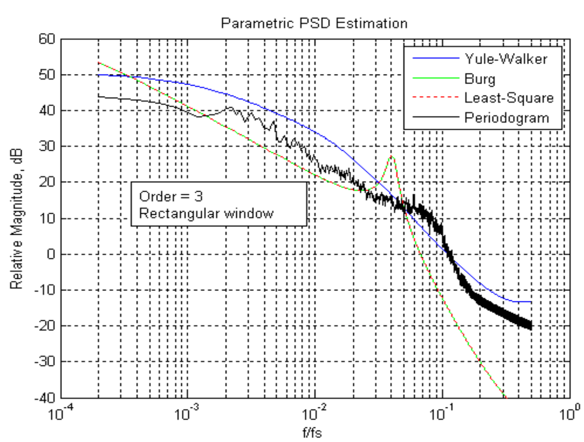

Fig. 7. Parametric model comparison of full minute data set, displaying inadequate PSD prediction for low order (n=3). For n =15, the three models begin to converge, with only slight variance at the 50 Hz LPF corner frequency

For extreme values, the three models show excellent agreement, with only miniscule differences. Peaking at 50 Hz is evident in all cases, as this represents the most distinguishable feature of the PSD. Figure 8 compares the periodogram and parametric cases for n=29, Hamming window.

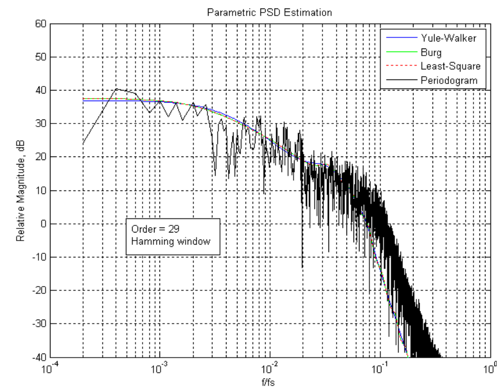

Fig. 8. Parametric model comparison for n=29, and Hamming window. Excellent agreement with periodogram is shown, as well as model-to-model consistency.

After analysis was performed on a slice-by-slice basis, some overall parameters were selected for all models, and the simulation allowed to run to completion. The entire 60-second data set was processed non-parametrically, using a 50% overlap and Hamming window for Welch. Hamming was chosen as it most closely mimicked the Gaussian window, but without the computational overhead. The resulting PSD estimate demonstrates good smoothing, while retaining some of the structure evident in the averaged periodogram. Frequency resolution was retained at 1 Hz, sufficient for the subsequent power band measurements. The resulting PSD is shown in Figure 9, demonstrating solid agreement with the periodogram, and consistency between all three methods.

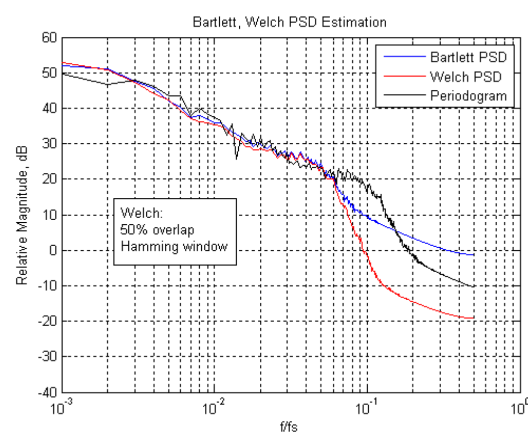

Fig. 9. Fully-processed data set, utilizing non-parametric estimation (50% overlap, Hamming window). SNR improvement and model consistency are excellent. Periodogram high-frequency shift is an artifact.

Parametric models were also allowed to run the entire data set, after selecting a Kaiser window, and an n=50 for order. As was depicted in the slice-analysis, excellent agreement is evident, as well as model consistency. Although lacking the fine detail of the simple averaging methods, the predictions appear to be solid, and extendable past the edges, in the event extrapolation is required. Although the computation time took tens of seconds, this is well within the acceptable bounds of modest computing equipment. Figure 10 compares the three models with the averaged periodogram

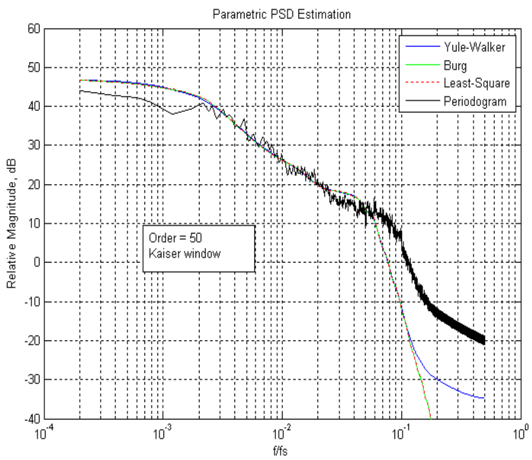

Fig. 10. Fully-processed data set, utilizing parametric modeling (n = 50, Kaiser window). SNR improvement and model consistency are excellent. Fine-frequency fluctuations are not present, as with the non-parametric PSDs.

IV. EEG Frequency Band Measurements

Since the ultimate goal of the study was to identify changes in the spectrum resulting from brain activity, each of the three EEG frequency bands, given in Table 1, were used to extract the total integrated power from each data slice, and store in a vector to analyze over the 60-second subject test. Model parameters were identical to those depicted for the entire runs, in the previous section.

Table 1. Frequency bands for EEG Alpha, Beta, and Gamma wave activity.

Alpha Band	8 Hz	12 Hz
Beta Band	16 Hz	24 Hz
Gamma Band	31 Hz	43 Hz

In addition, this data was saved to a file on disk, such that it might be further processed using a spreadsheet or other numerical package. Output was available for each particular method, whereby comparisons can be performed between non-parametric and parametric estimations.
Figure 11 shows the result of Alpha activity vs time, obtained using non-parametric estimation.

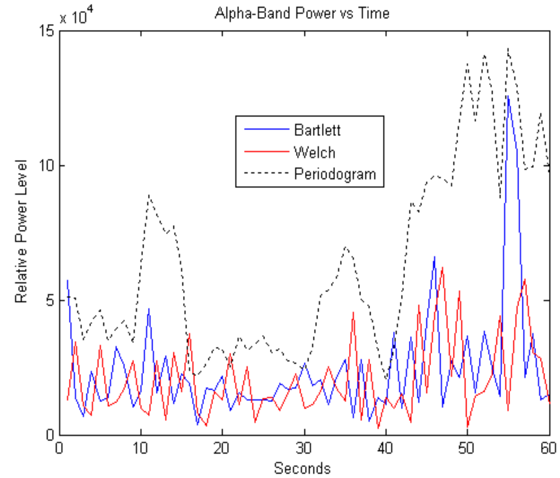

Fig. 11. Alpha frequency-band power measurement vs time for EEG subject test, using non-parametric PSD estimation. Each 5000-sample data slice was processed, integrated and stored in a vector. Moderate agreement is demonstrated, with large improvement over simple periodogram.

Reasonable agreement is apparent between the averaged spectra, as well as improvement in frequency resolution over the averaged periodogram. Due to overlap, Welch appears to better identify persistent background frequencies, while Bartlett responds to instantaneous events (evidenced by the smoothing at 55 seconds). Both methods demonstrate finer detail in the structure with respect to the simple periodogram, despite having processed the identical data set.

Similarly, Gamma activity was examined, using parametric estimation. Here, better overall estimation comes at the expense of fine-frequency fluctuations. Again, reasonable agreement with the periodogram is apparent, as well as the excellent consistency between the models, as demonstrated in Figure 12.

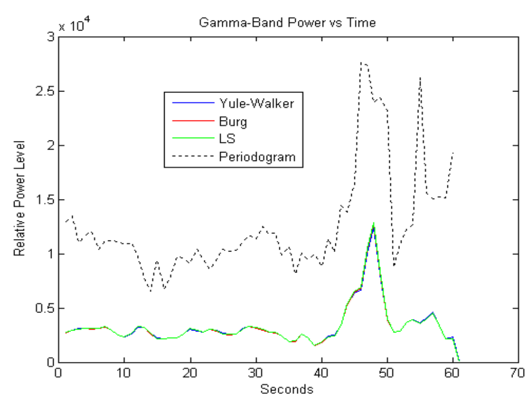

Fig. 12. Gamma frequency-band power measurement vs time for EEG subject test, using parametric PSD modeling. Each 5000-sample data slice was processed, integrated and stored in a vector. Excellent agreement and model consistency is apparent.

Finally, Beta activity was analyzed using a comparison of both non-parametric and parametric methods, in an effort to determine consistency. Figure 13 shows reasonable consistency between the models, as well as the periodogram. However, near the middle of the subject test, the periodogram suggests activity not displayed by either estimation method, perhaps due to the fact that the periodogram is inaccurately portraying the data, or there exists a numerical remnant. In either case, each method strongly suggests its strength and weakness, when viewed in the final analysis.

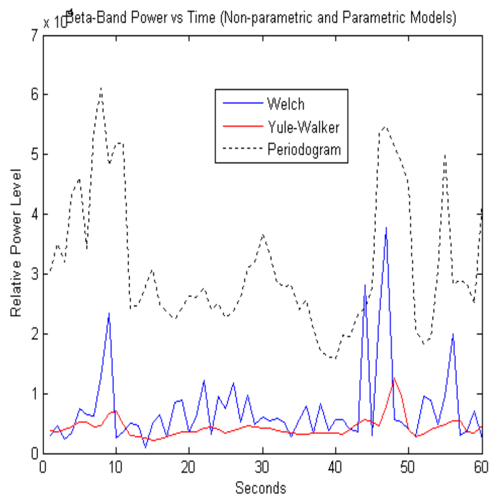

Fig. 13. Beta frequency-band power measurement vs time for EEG subject test, using non-parametric and parametric PSD modeling. Each 5000-sample data slice was processed, integrated and stored in a vector. Moderate agreement and model consistency is apparent.

V. CONCLUSION

PSD estimation is a prevalent, but difficult task in DSP. Hindered by excessive record lengths, presence of noise, as well as tradeoffs involving frequency resolution and reduction of variance, proper selection and implementation is largely application-specific. Non-parametric methods are useful for smoothing data, and are computationally efficient, especially when coupled with the Fast-Fourier Transform. Although they tend to preserve fine-frequency fluctuations, they are also subject to corruption by noise. In addition, the entire data record must ultimately be stored as a vector, which adds memory requirements to a measurement system. Many combinations of windows and overlapping were explored, each with their own attributes. As a starting point, Welch's Method, employing 50% overlap and a Hamming window produced good general results, and is recommended for generic applications.

Parametric model estimation is a powerful tool for PSD estimation, particularly for band-data extraction. Since the result is a continuous model, little memory is required to represent the PSD. The cost is clearly computational overhead, but not excessive by modern computer metrics. Additionally, these models require additional tweaking and intervention. Undoubtedly, numerical "goodness of fit" algorithms exist for these cases, but gaining a qualitative understanding of the behavior appeared to be adequate for the EEG analysis.

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