Estimation in Sensor Networks

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University of California, Santa Barbara

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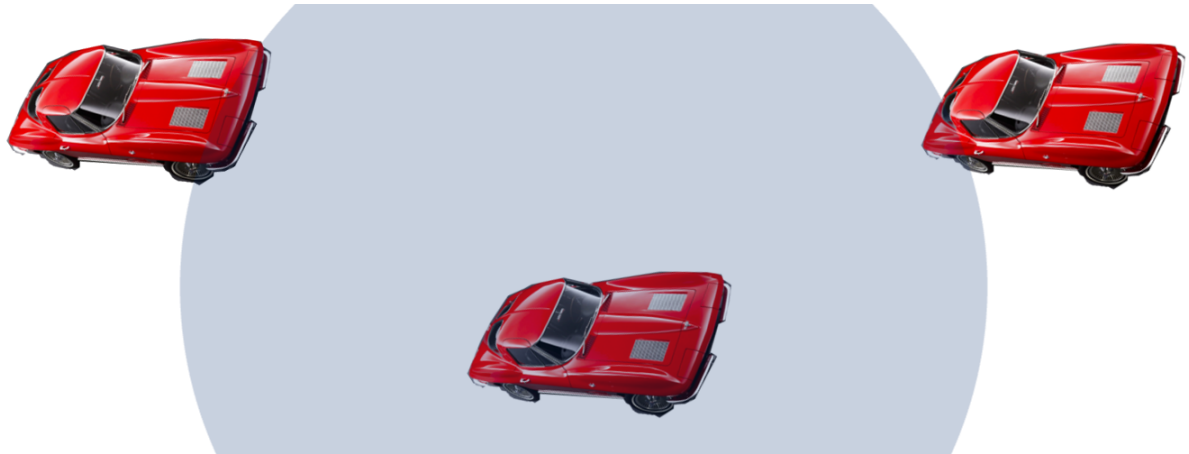
About me

I am a Bay Area native (Homestead High School). I got my Bachelor's in Electrical Engineering with a minor in Mathematics from UCLA and I am currently pursuing a PhD at UCSB focusing on novel ways of estimating the states of dynamical systems (our topic today).

When I was young I was interested in how things work, cars and planes, biology and ecosystems, the weather, cities, the economy, etc. I couldn't decide what to pursue, and so I guess I've ended up studying the common abstractions of these problems, the general rules which govern each of them.

Motivation

Consider a platoon of cars that can communicate with each within a 30 meters and want to maintain a constant velocity and spacing.



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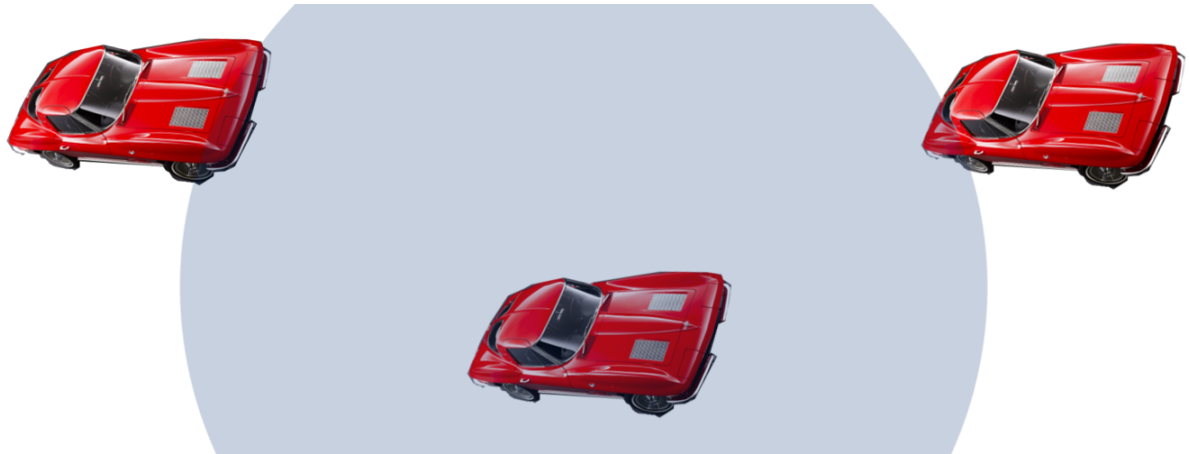

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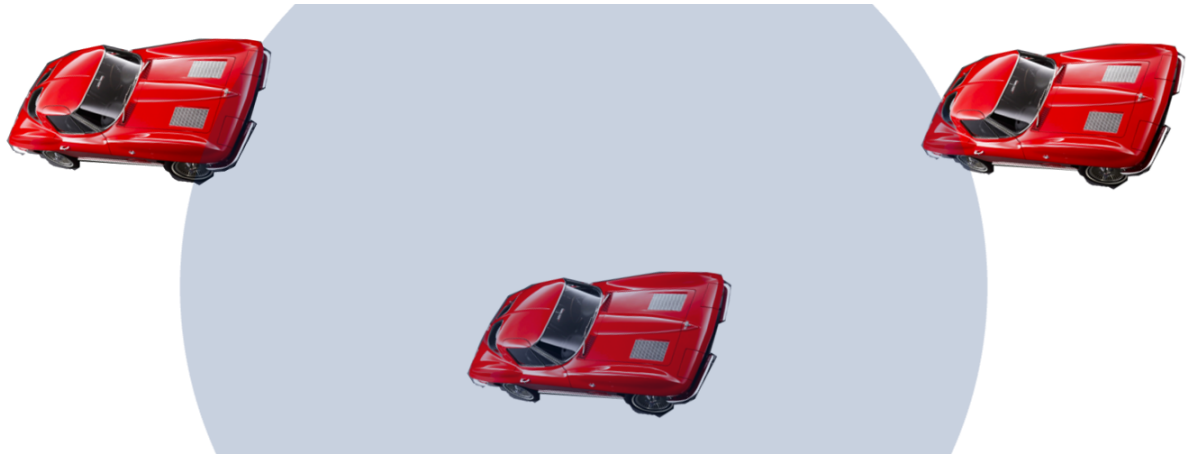

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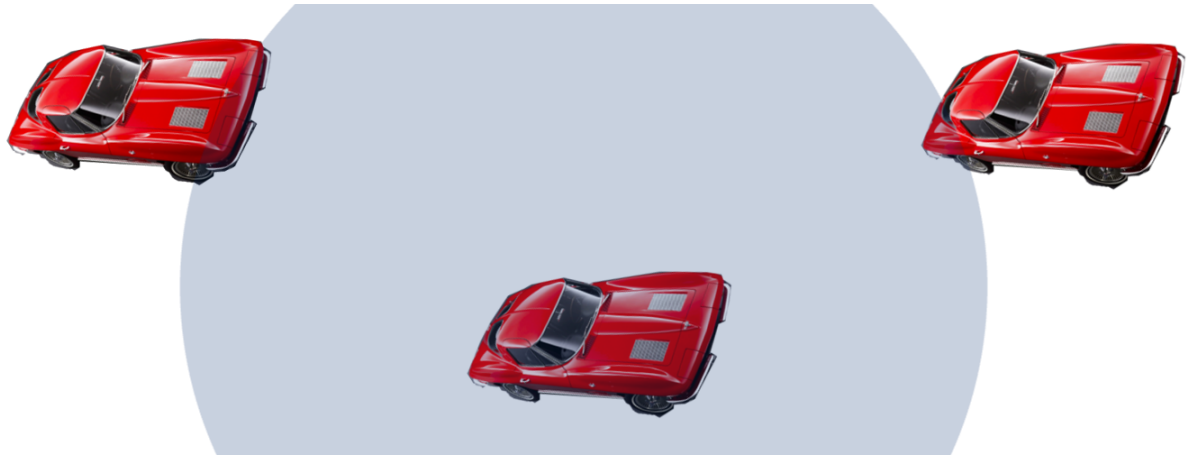

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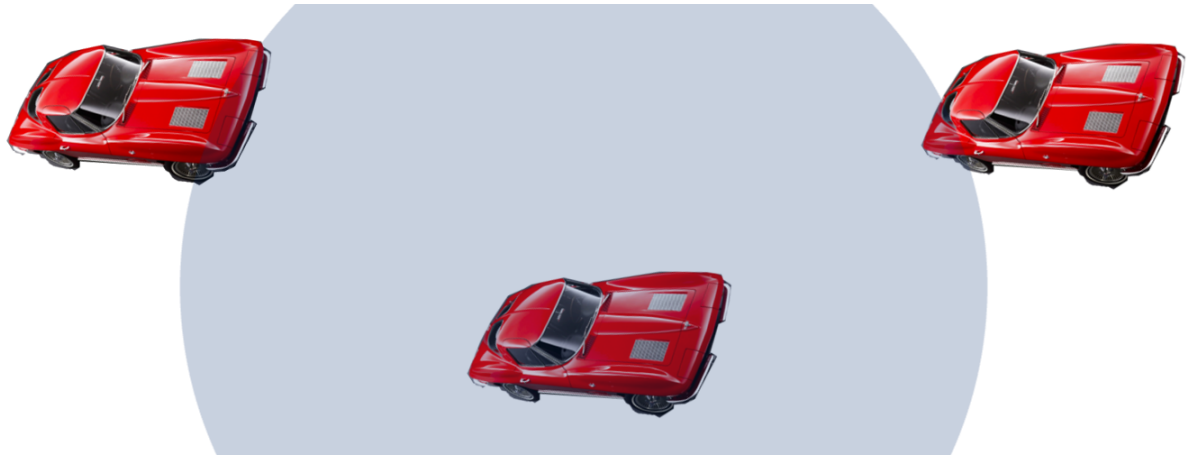

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- Should nodes agree (consensus) on a single estimate? If so, how?

Outline

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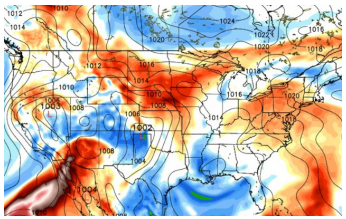

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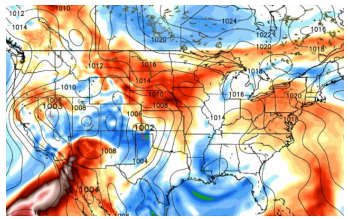

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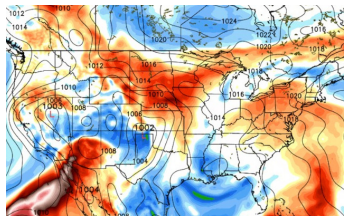

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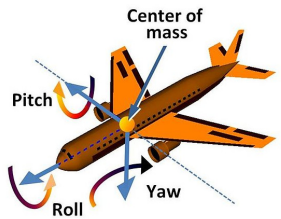

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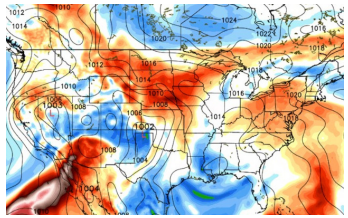

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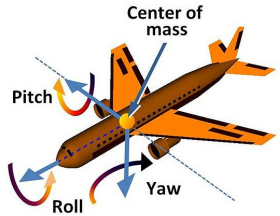

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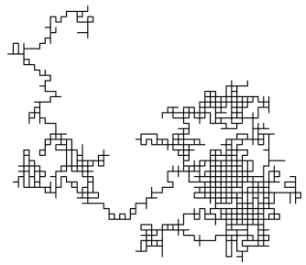

- Markov processes (e.g. random walk)

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Both prediction and filtering are examples of Estimation

Estimation

Our goal is to noisy sensor measurements and produce good estimates of the evolving quantity of interest. This has two parts:

- How do we merge data from many sensors (sensor fusion)?
- How do we use knowledge of the past to refine our present estimates?

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The famous Kalman Filter is an example of this method

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We also have a measurement of the state

$$y(t) = x(t) + n(t)$$

where $n(t)$ is drawn from a Gaussian distribution with mean 0 and variance 2.

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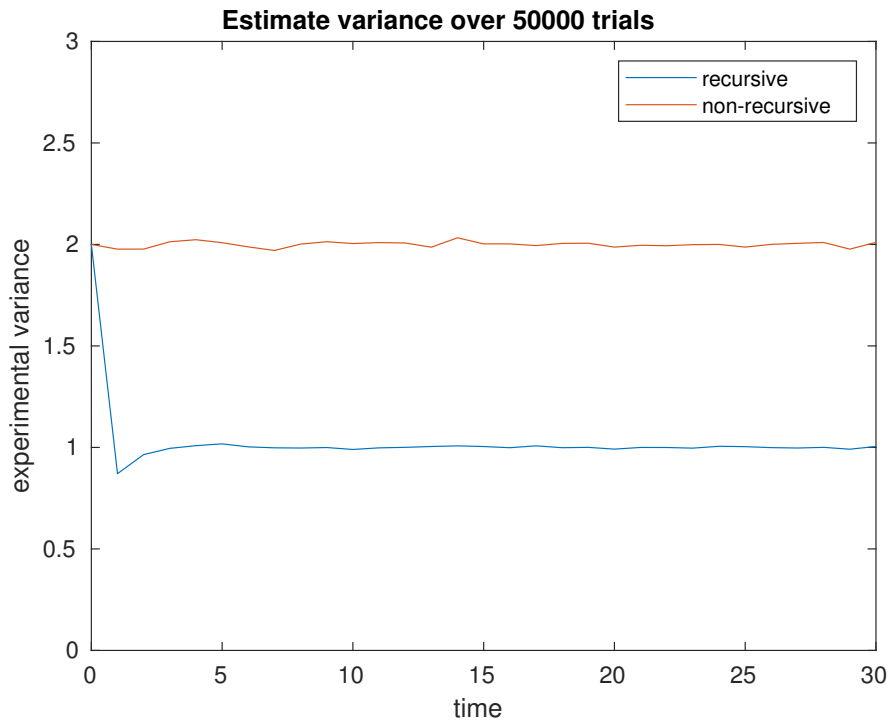
$$\hat{x}(1) = \frac{2}{2+3}\bar{x}(1) + \frac{3}{2+3}y = \frac{2}{5} \cdot 0 + \frac{3}{5} \cdot 0.5 = 0.3$$

Results

Without the recursive algorithm, just using $\hat{x} = y$, each estimate would have a variance of 2. The following compares the experimental variance of the recursive algorithm over many trials.



More measurements = better

One important fact to keep in mind:

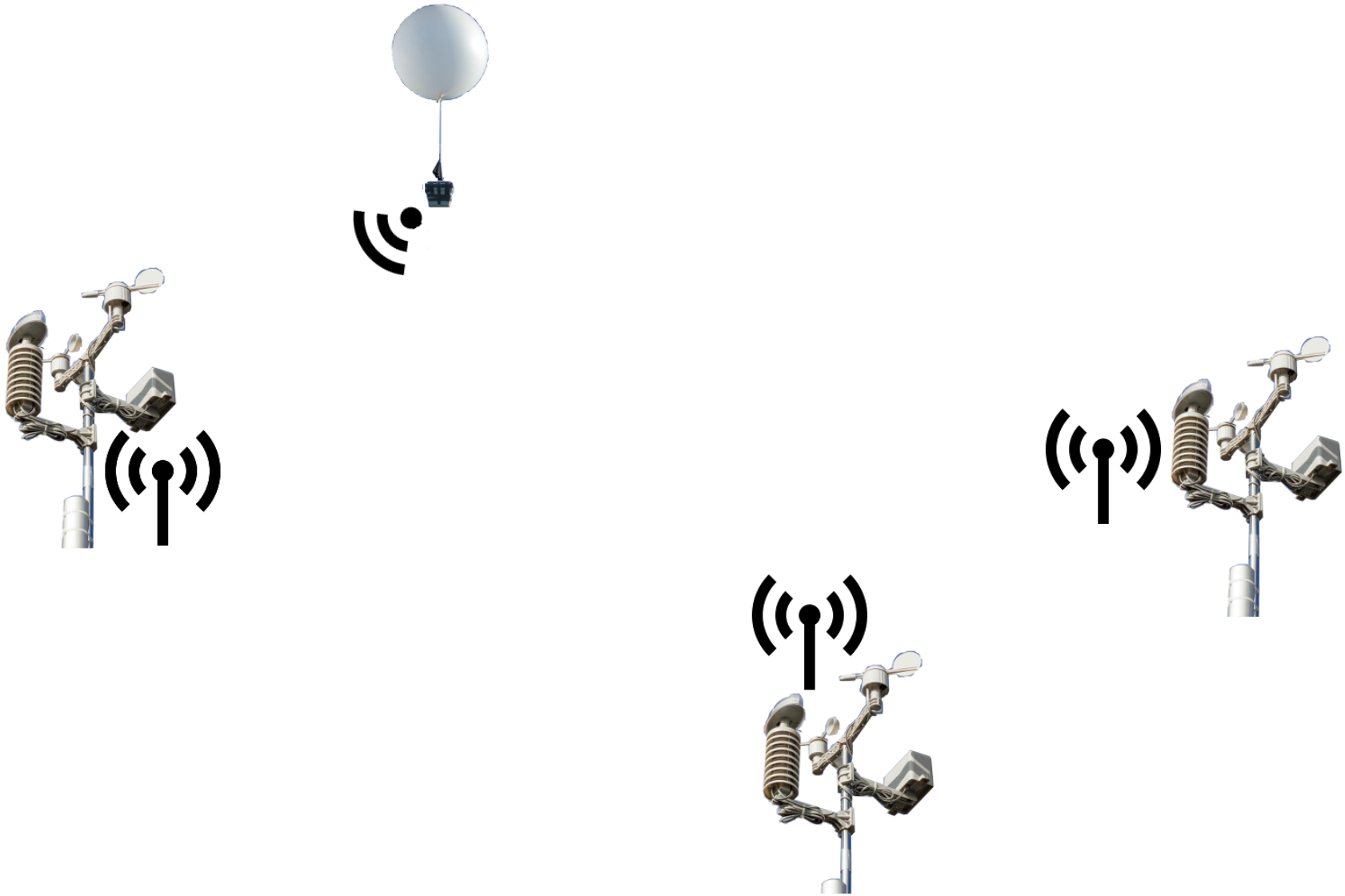
It is almost always better to have more measurements, this results in more accuracy in your estimates.

Sensor Networks

Let's consider cases where a dynamical process is monitored by disparate yet communicating sensors. Examples include monitoring of:

- weather (through balloons, stations, etc.)
- nuclear reactor cores
- multi-robot tasks
- large power grids
- inter-vehicle dynamics (platoons, collision avoidance)

Sensor Networks: Central Estimation



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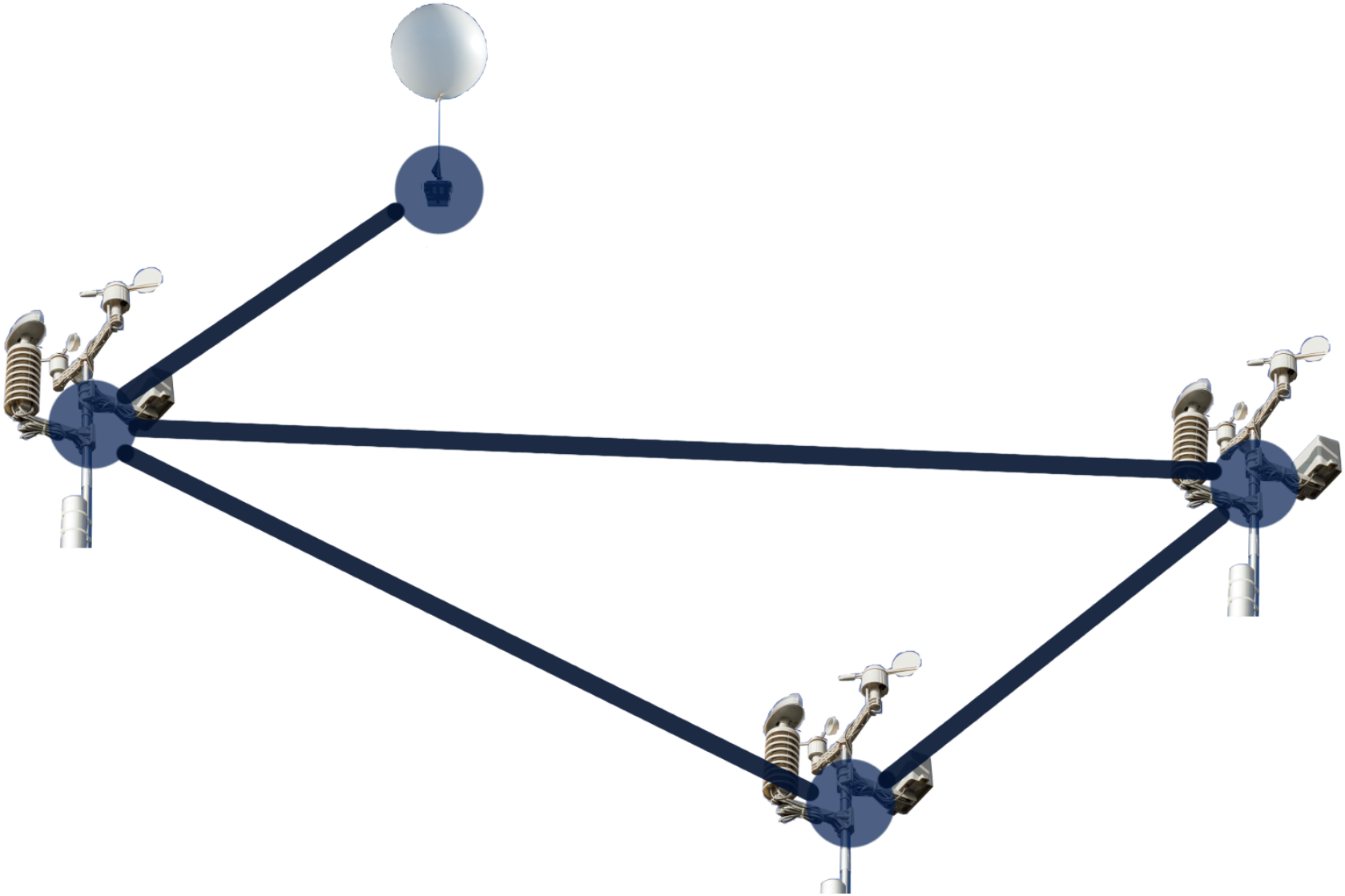

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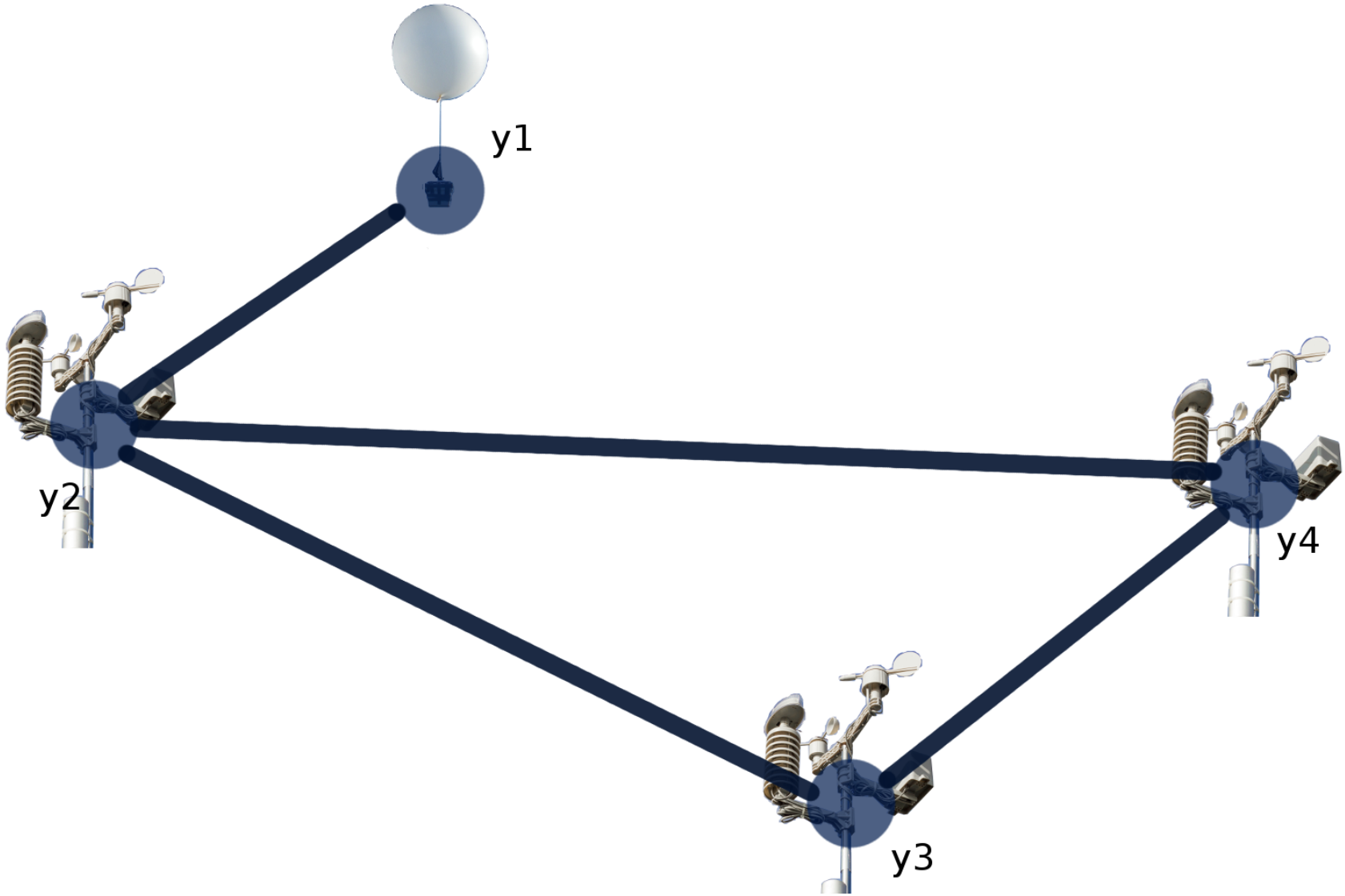

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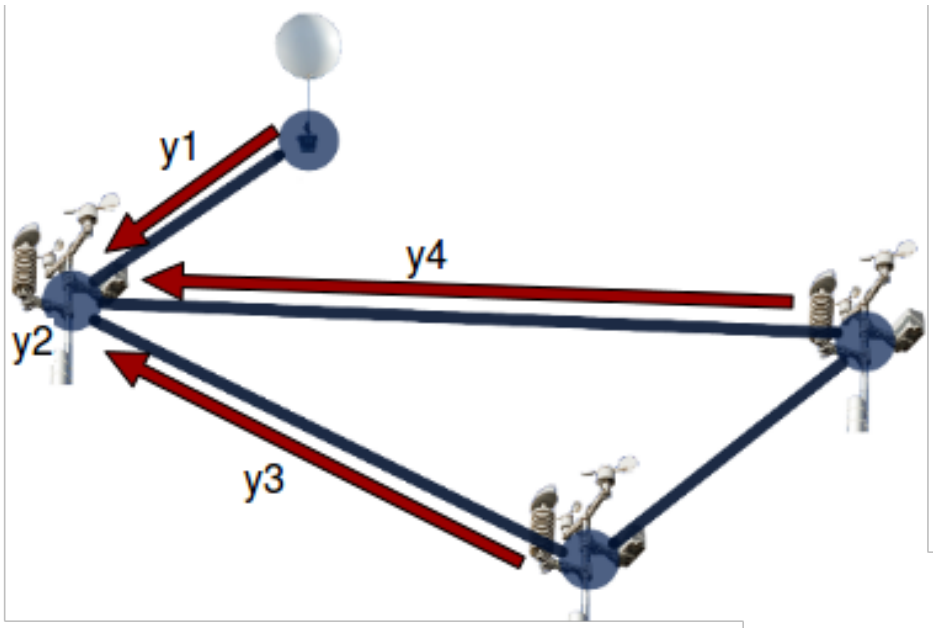

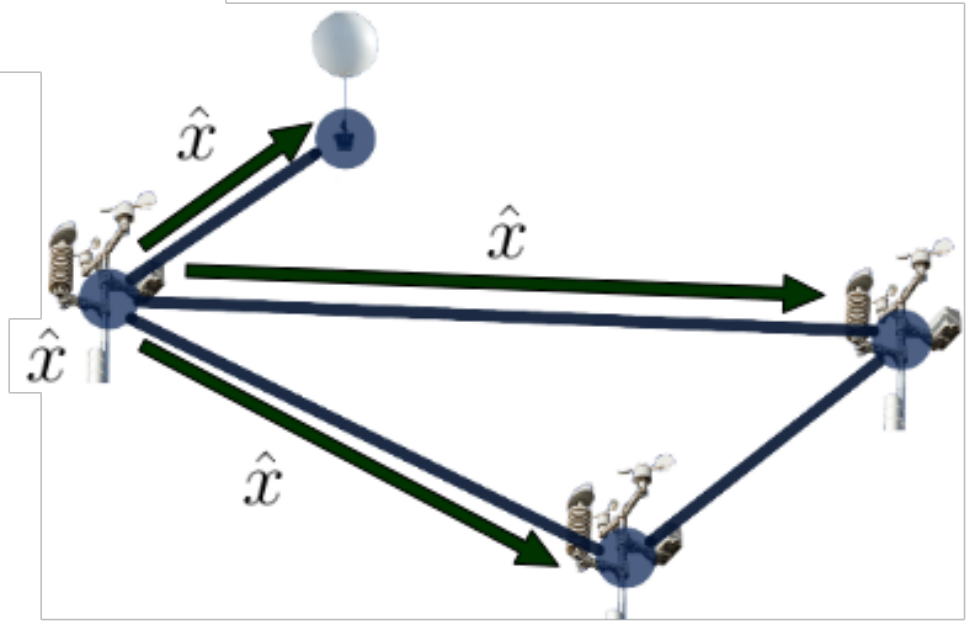

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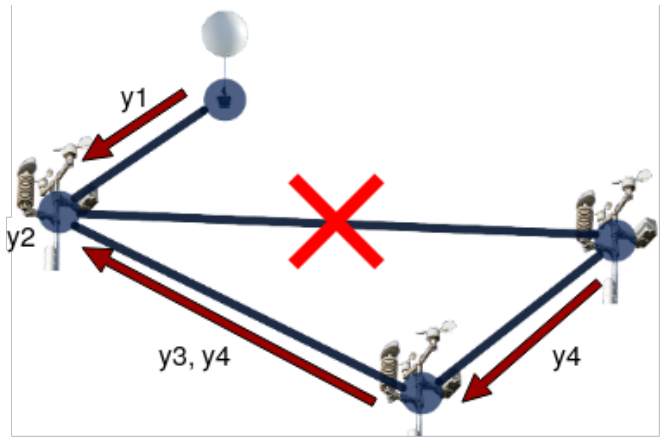
Sensor Networks, cont.

Computing estimates based on *all* measurements in the network may be inconvenient for various reasons:

- a central node is infeasible due to distances
- communication cost/delay for computation is unacceptable



So a decentralized scheme may sacrifice performance for speed

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With either approach we hope to approximate some properties of the centralized scheme, especially accuracy and consensus.

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- consensus? $\approx$ estimates propagate through the network

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The answer is that we considered only simple process, measurement, and noise models. Many of the applications we talked about have much more complicated behavior not captured by these models.

Furthermore, the communication graph structure may change in time! How do we adjust our algorithm?

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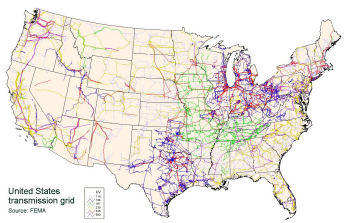

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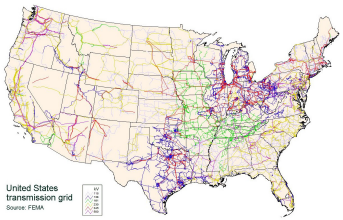

- traffic control