ECE147A
Homework 8 Solutions

April 5, 2017

1 Problem Statement

In this assignment, we were given four distinct specifications and the following plant:

$$P(s) = \frac{10}{s+5}$$

We want to relate these specifications to loop shaping goals so that we can come up with a design strategy.

The first specification was that the system be Type I, or in other words we want zero steady state error to step responses. We can achieve this by placing an integrator term, $\frac{1}{s}$ in our controller.

The second specification is on the overshoot, we must stay below 25% overshoot, which using the following second order approximation gives us a phase margin requirement of:

$$\Phi_M \approx 100 \cdot \zeta = 100 \frac{-\ln(0.25)}{\sqrt{\pi^2 + \ln^2(0.25)}} = 41^\circ$$

Our rule of thumb for a good phase margin is $> 45^\circ$, so we shouldn't actually run up against this requirement if we are designing correctly!

The third specification is on disturbance attenuation, which we want to *improve* by a factor of 20 from our initial disturbance gain at frequencies at or below 1 rad/s. With no feedback control, our plant has a DC gain of 2 (6dB), and our gain up to 1 rad/s will be less than or equal to this value (in this case the disturbance adds to our reference before going throught the plant). Therefore once we have completed our design, the transfer function $\frac{Y}{D}$ should have a gain of:

$$20\log_{10}(\frac{2}{20}) = 20\log_{10}(10^{-1}) = -20\text{dB}$$

in decibels. This corresponds to a high gain at low frequences in our loop gain, $C(s)P(s)$. Recall that in feedback, $\dfrac{Y}{D} = \dfrac{P(s)}{1+KC(s)P(s)}$.

The fourth and final specification is on noise attenuation, we would like high frequency ($\geq 10^3$ rad/s) noise to be attenuated by a factor of 1000, or -60dB in decibels. This corresponds to a low gain at high frequencies in our loop gain. Recall that in feedback, $\dfrac{Y}{N} = -\dfrac{C(s)P(s)}{1+KC(s)P(s)}$. We may ignore the sign of this transfer function, since we are only interested in its magnitude.

2 Attempt at PI Control

From our first specification, we know that we require an integrator in our controller. We also know that this will have the effect of better disturbance and noise attenuation. We need to find out if we can meet these specifications using only Integral or PI control. With only Integral control, the best disturbance attenuation achieved with PM$\geq 45°$ was 12dB:

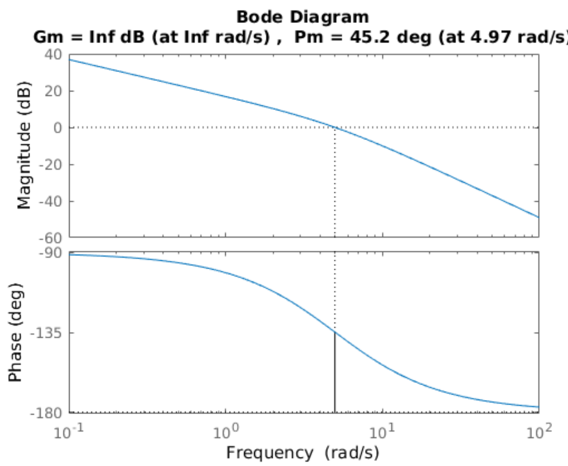
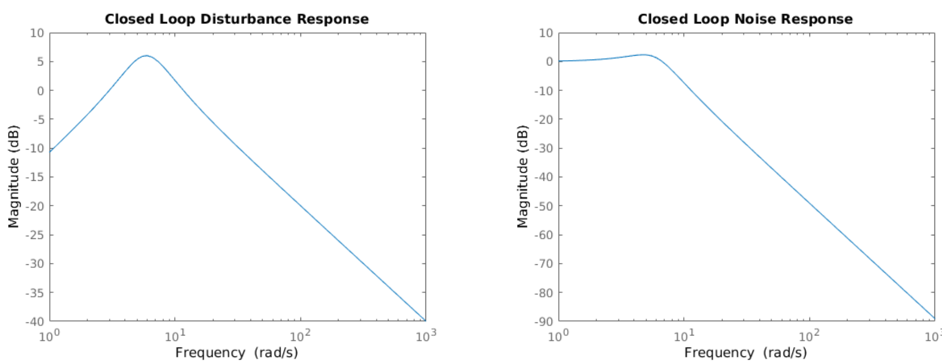


Figure 1: Loop Gain, $|C(s)P(s)|_{dB}$, with Integral Control



(a) $|\frac{Y}{D}|_{dB}$

(b) $|\frac{Y}{N}|_{dB}$

Figure 2: Optimal Disturbance and Noise Attenuation with Integral Control

Only the noise attenuation requirement was fulfilled. In order to see if our specifications are met, we can look at the closed loop plots. However we can also see both of these on our loop gain plot. Convince yourself with some quick math, that the high frequency gain will be the same for both open and closed loop (with unity feedback). Low frequency gain in our loop gain on the other hand is inversely proportional to our closed loop disturbance gain at those frequencies. Raising the open loop gain 26 dB will *approximately* lower $|\frac{Y}{D}|_{dB}$ by 26dB (assuming unity feedback)

With PI control we can no longer meet our noise attenuation requirement (convince yourself of this!). It is implied that we will need a strictly proper, higher order controller in order to achieve both the desired roll-off at high frequencies and increased gain at low frequencies.

3 Integral + Lead-Lag Controller

There are many valid ways of approaching this problem, however the most helpful perspective might be this: Our Integral control provides us with good noise attenuation, then use a Lag compensator to improve gain at low frequencies (to improve disturbance attenuation), while the Lead compensator should maintain the phase margin we had with only integral control.

The reason this is useful, is that it gives us some intuition of where to place our poles and zeros and how to choose our gain. We know that we could fulfill our noise attenuation requirement using just an integral term with adequate phase margin, so as long as our lead-lag network has sufficiently small zeros, then our noise attenuation should not be affected much.

$$\text{Lead-Lag Network: } \frac{s+\frac{1}{T_1}}{s+\alpha\frac{1}{T_1}} \cdot \frac{s+\frac{1}{T_2}}{s+\beta\frac{1}{T_2}} \quad \text{s.t. } \alpha > 1, \quad \beta < 1$$

The amount of disturbance attenuation we achieve is determined by K and how close the lag controller's pole is to the origin, and the range of frequencies for this disturbance attenuation is determined by the zero. So intuitively we would like to place our lag zero somewhere greater than 1 rad/s (ie $T_2 < 1$). Then the larger K is and the smaller β is, the more disturbance attenuation we will get.

Maintaining our phase margin is a little trickier, since there will be some interplay of the lead and lag controllers. For this reason, the design strategies in the book and slides use $\beta = \frac{1}{\alpha}$. We won't go over this process in detail here, but in the end it will take some tuning regardless of what method you use. The following is a controller informed by the process laid out in Chapter 11.5, as well as the Lecture 18 slides.

$$\text{Integrator + Lead-Lag Network: } 50\frac{1}{s} \cdot \frac{s+2}{s+20} \cdot \frac{s+2}{s+0.2}$$

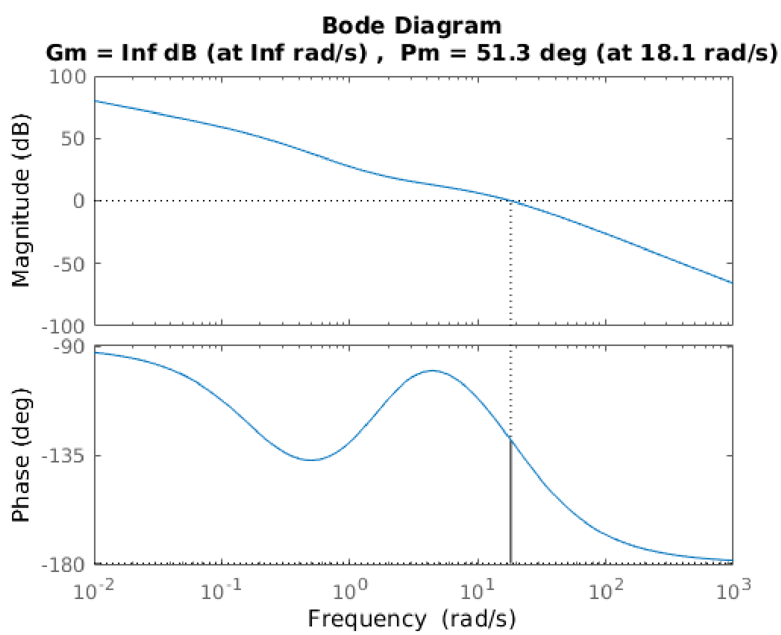
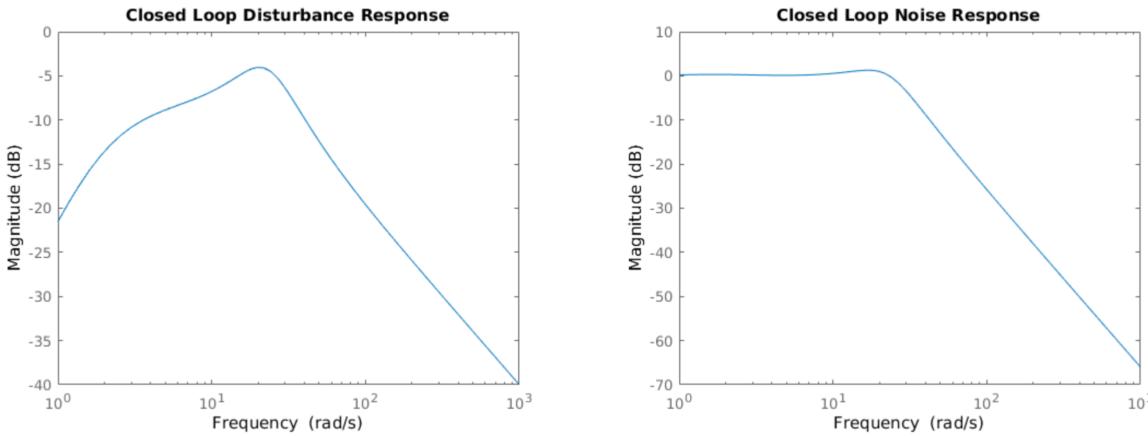


Figure 3: Loop Gain, $|C(s)P(s)|_{dB}$, with Integral + Lead-Lag Network



(a) $|\frac{Y}{D}|_{dB}$ (b) $|\frac{Y}{N}|_{dB}$

Figure 4: Requirements Satisfied by Integrator + Lead-Lag Network

Finally, we should run some simulations to test our design. First our entire system:

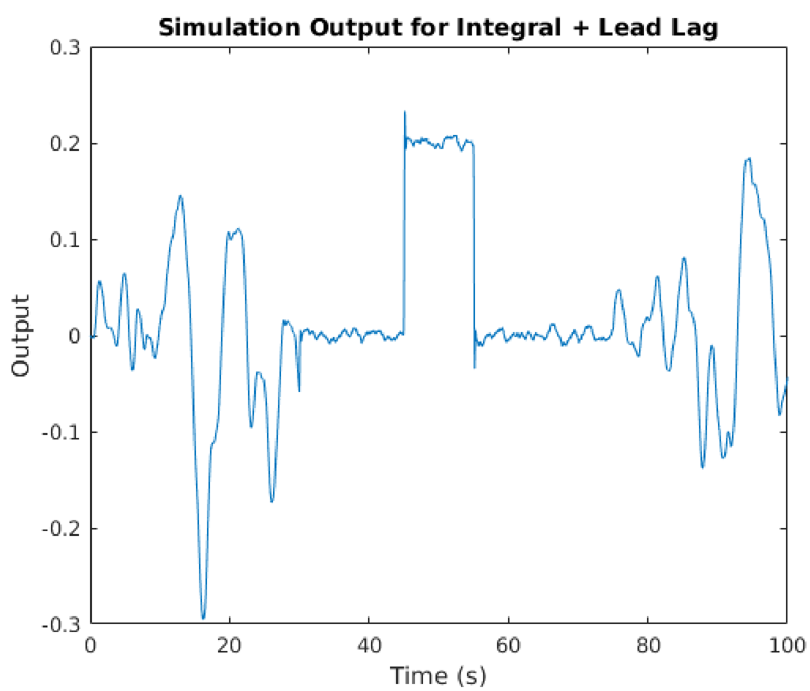


Figure 5: Output with Integral + Lead-Lag Network

We measure an overshoot of 16.6% in our simulation. To measure our noise attenuation, we turn off the disturbance and reference. Then by taking the average magnitude of the noise after the controller is turned on we determine that the actual noise attenuation is by a factor of 650 (we expected 1000). This could be partially because the noise does has some frequency content below 1000 rad/s (160 Hz).

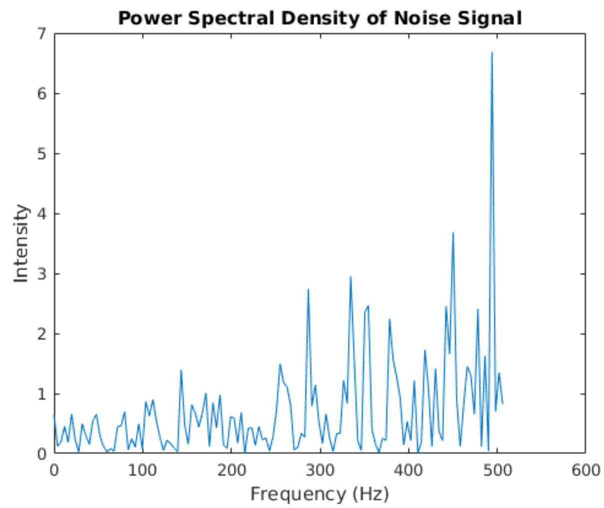


Figure 6: Spectrum of Noise

Although this may not account completely for lower attenuation than we expected, it certainly will have some effect. However looking at our plot with the disturbance turned off, we see that improving our disturbance attenuation would be more helpful.

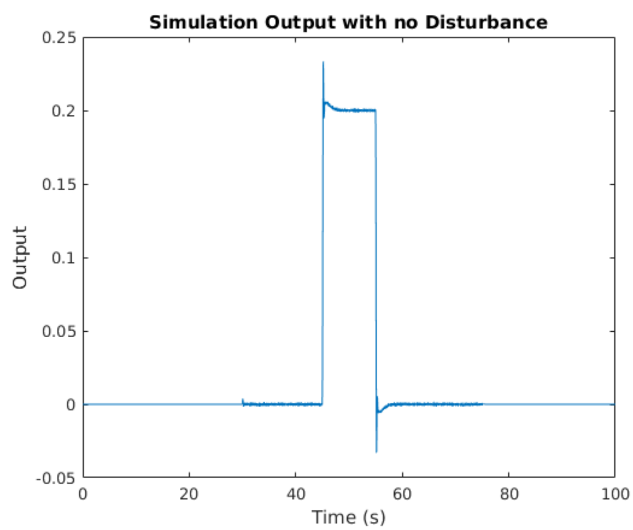


Figure 7: Output without Disturbance

To find the disturbance attenuation, we then turned off the noise and measured the output before and after the controller is turned on. We found that the actual improvement in disturbance attenuation was by a factor of 23, which is about what we expected. You may see such random disturbances refered to as process noise to distinguish them from the measurement noise were considering in this assignment.

Hopefully this assignment helped you to apply the insights you developed throughout the course to a design problem!