Introductory Control Theory Syllabus

Sharad Shankar

June 2020

Introduction

Control can be exciting course for students. A wide array of fields and engineering application involve control or tracking somehow, and notions of performance, stability, robustness, and safety learned in this course are foundational to engineering practice. However, the introductory controls course is often taught as a historical edifice, caught up in details of 100 year old theory and 50 year old practice. This guarantees that students learn neither fundamental concepts nor useful skills all that effectively. The following revision of the 10 week undergraduate Introductory Controls syllabus aims to give students a solid foundation and insight into the theory of control, with the understanding that learning modern control practice is a multi-semester effort. The syllabus is annotated in blue throughout to help explain course design decisions.

Important points include that a basis in Differential Equations is emphasized throughout the course, even if State Space design is only significantly covered towards the end. There is also no midterm or final exam, instead replaced by three quizzes and a final project, which may be more appropriate for the content of the course. Quizzes are strategically placed to help ensure understanding of concepts which will be built on later or provide feedback to the instructor on the state of the class. Week 4 is designed assumed that students have already taken a signals course; if not this material can be stretched into week 5 at the expense of some week 5/6 material like optimal control. It is worth noting that all topics needed for a second semester of controls is covered, however procedural knowledge like Routh-Hurwitz, lead-lag, and hand drawing of visualization plots is either left out or mentioned only briefly. A potential revision of this course could omit discussion of visualization entirely, however it is still a very useful tool in understanding material from further controls courses.

1 Week 1: Motivation and Basic Concepts

Learning objectives: students can identify control problems and controls in the world, students can write systems in block diagram form and explain the value/purpose of feedback

1.1 Control Examples

Instructor should introduce examples to illustrate three purposes of control,

- *tracking* - for example temperature control, or cruise control
- *performance* - for example vehicle suspension or an industrial robotic arm
- *stability* for example traction control, or a nuclear reactor

This is best done as a set of video modules or perhaps interactive applications where students can see the controller equations and change the values of control parameters (or remove the controller altogether) to see the effects. As a segue to Block diagrams, a note that all these controllers use *feedback* of the system state to achieve the design objectives above.

1.2 Block Diagrams

Instructor should use the previous or new examples to show how block diagrams are used to break apart engineering problems and visualize feedback. Since Block Diagrams are the highest level conceptualization in this course, this is the best time for the instructor to provide an overview of the course using block diagrams as an organizer. A discussion section which challenges students to formulate word problems into block diagrams could be useful

2 Week 2: Modeling and Dynamical Systems

Learning Objectives: students understand the general principle of modeling, students are reminded of differential equations and can explain their use a modeling tool

2.1 Models Conceptually

Given a block diagram, a model describes the internal mechanism of any given block, examples abound. Examples which illustrate the *descriptive* rather than predictive power of models are useful in the context of this class. Distinctions between black and white box models are perhaps best saved for later, or for courses which cover System ID.

2.2 Dynamical Systems as Differential Equations

Some simple examples using Newtonian Physics can presented such as pendulums and carts, but also talking about chemical processes and population/ecosystem/epidemic dynamics for example will help show the scope of these methods

3 Week 3: Solutions and Stability of Dynamical Systems

Learning Objectives: students understand how differential equations describe dynamics and can find solutions to linear ODEs using complex exponentials.

3.1 Qualitative Analysis of ODE's

Given an ODE of the form $\dot{x} = f(x)$ trajectory solutions will be shown and the fixed points will be pointed out and shown to be stable or unstable (for example in a pendulum).

3.2 Linear ODEs and Their Solutions

The special class of linear ODEs is introduced with unique fixed point at the origin, local linearization of a nonlinear ODE is introduced. Recall of complex exponential forcing term method of solution.

3.3 The Imaginary Plane - Poles and Zeros

More detail on how a complex exponential represents solutions of a differential equation as a Real decaying/growing portion and an Imaginary oscillatory portion.

3.4 Week 3 Quiz

Key Concepts: role of feedback, purpose of models, stability
Key Skills: drawing block diagrams, solving linear ODEs, correspondence of imaginary plane to signals, identifying stable/unstable systems and fixed points

Quizzes are best offered as multi-opportunity or including a collaborative stage

4 Week 4: Responses and Filtering of Linear Systems

Learning Objectives: students understand the connection between differential equations and the Laplace/Fourier domains based on the Imaginary plane, students can find step, impulse, and frequency responses

4.1 Transfer Functions

Replacing the complex exponential equivalently with the transform variable s to find solutions, the meaning/effect of poles and zeros and why LHP poles means stability, and using the derived input-output relation to combine differential equation blocks in series with correspondence.

4.2 Frequency Response

Isolating the response of the system to periodic signals (the imaginary axis), basic notion of filtering, resonance.

4.3 Step and Impulse Responses

Meaning and importance of step and impulse signals in tracking and response to disturbance, transform domain representations, using transform domain to find solutions. Show how all of this can be implemented/plotted in Matlab/Simulink.

5 Week 5: Effect of Feedback

Learning Objectives: students can introduce feedback into both differential equations and transfer functions, and understand the possible effects of feedback

5.1 Effect on Response/Stability

Show how negative feedback is written in block diagrams, is represented in differential equations and transfer functions and therefore affects the system response/stability. Simply examples of unstable ODEs/correseponding TF are stabilized using simple feedback, some discussion of necessary magnitude of feedback. Discuss effect on signals originating in different parts of the block diagram (i.e. reference, process and observation disturbances)

5.2 Robustness

Introduce sensitivity for a given parameter and show how feedback can reduce the sensitivity to parameter changes. Show how feedback can also reduce the effect of disturbances both through the transfer function and simulation.

6 Week 6: Model-Based vs. Model-Free Control

Learning Objectives: students can explain the benefits/drawbacks of the major control design/tuning methods

6.1 PID Control/Tuning

Teach each component and ODE/TF representation, as well as how it would affect the feedback response including overshoot and convergence time. Learn idea behind iterative tuning method, noting that it does not require knowledge of the model, only the observed response. Make note of need to filter integral and derivative terms in order to avoid windup and twitchy behavior respectfully. Provide scripts which help students perform the iterative tuning themselves.

6.2 Optimal Control

Show how to formulate an optimal control problem in terms of the response and inputs, explain the basic idea behind optimization as hill climbing until flat regions are reached. Contrast this method which relies on knowledge of the system model with the iterative model-free approach in PID tuning. Provide tools so they can try this out in Matlab without fully understanding numerical optimization.

6.3 Week 6 Quiz

Key Concepts: Laplace domain, frequency response, control design, robustness.
Key Skills: computing responses in transform domain, control design formulation

7 Week 7: Visualizing Control

Learning Objectives: students understand generally how visualizations are produced, what they represent, and how they are related.

7.1 Root-Location Plots

Show how the effect of a given control parameter (usually proportional gain) on the system poles can be generally plotted. Justify any plotting rules by showing how poles change in the transfer function. Now is a reasonable time to introduce Routh-Hurwitz stability criteria to help reinforce the current as well as previous lessons.

7.2 Bode Plots and Nyquist Plots

Switch to Frequency Response context, show how we can find the response to any point on the imaginary axis by measuring the contribution of each pole/zero. Show how this can produce either separate magnitude/phase plots or a single Nyquist plot. Use Root-Location plot to show how stability conditions on these plots can be derived.

8 Week 8: Controller Design/Validation with Visualizations

Learning Objectives: student are able to read stability and performance, and can validate/choose controller parameters based on visualizations

8.1 Relation to Performance, Robustness

Show how performance can generally be read off of each plot assuming dominant poles, zeros (this follows in a straightforward way from understanding of the complex plane). Show how robustness relates to Phase Margin in Bode and Nyquist plots and why root-location on its own is misleading for robustness.

8.2 Loop Shaping

Explain the desirable qualities of a Nyquist/Bode plot and their justification for a nominally stable plant:

- High gain at low frequencies - leads to better reference tracking and process disturbance attenuation according to Feedback TF and low-frequency assumtion on these signals
- Small slope at crossover - relates to phase margin because magnitude slope and phase drop are related
- Low gain at high frequncies - helps attenuate observation noise according to Feedback TF and high frequency assumption on this signal

Show how this can be performed in Matlab and used to validate the controls they designed previously.

8.3 Control Project Selection

Two project tracks should be offered:

- A skills-oriented track where students choose from a small (2-4 options) set of pre-specified projects that either use existing simulations or hardware. Students should be able to complete the design using only tools from the course and discussed in the project description.

- A research-oriented track where students are asked to examine either an industrial or academic control method/problem of their choice and report on it. Students should be asked to focus on some main points: the main challenge or difficulty that the problem poses, the reasoning behind the proposed method, and the results achieved.

9 Weeks 9 and 10: State Spaces and Discrete Time

Learning Objectives: students recognize how to extend their knowledge to multi-dimensional and discrete time systems

9.1 Multi-Input, Multi-Output, Multi-State Systems

Noting that only scalar examples have been examined so far, larger richer systems are introduced with examples. Basics of how to find solutions should be discussed with Matlab implementation.

9.2 Difference Equations

Introduce difference equations, noting interpretation as sampled continuous time systems. Stress that digital controller implentation requires understanding of differences in discrete-time systems. Give examples of digital controllers and their performance. This serves as the bridge to the next quarter's material.

9.3 Week 9 Quiz

Key Concepts: visualization, stability, performance, discrete time
Key Skills: loose sketching of visualizations, deriving properties from first principles, solving larger systems, difference equations

9.4 Control Project Completion

In late week 9, a project check in should occur so that advice/assistance can be provided. Students should be expected to give a brief explanation of their problem/approach to the TA/Professor and can discuss any difficulties they have had with implementation.