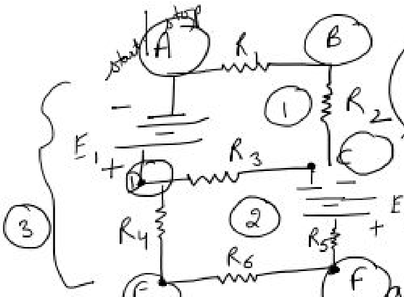
Network Analysis & Synthesis

Any arrangement of various electrical energy source along with different circuit elements is called an Electrical n/w



Loop :- (Mesh)

→ Closed path which originates from a particular node, terminates at same node, travelling along various other nodes without travelling through any node twice. OR

set of branches forming a closed path in a n/w in such a way that if one branch is removed then remaining branches don't form a closed path.

Network Element → R

Any individual circuit element with two terminals which can be connected to other circuit element is called a n/w element.

Active Elements

Elements which supply power or Energy (eg. Voltage source, current source)

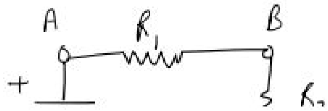
imp (supplies infinite power for infinite time)

Passive Elements

→ Element which either store energy or dissipate energy in form of heat. (eg. resistance, inductor, capacitor)

imp (supply or absorb finite amount of power for a finite time).

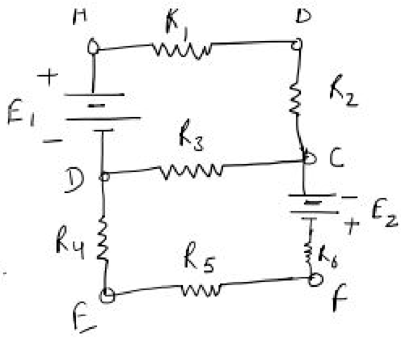
Branch :-



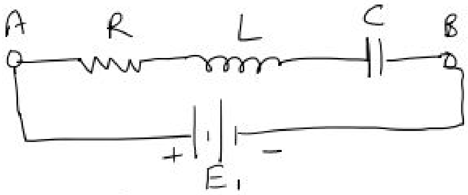
Branch : AB ED

Branch :-

A Branch may consist more than one Element.



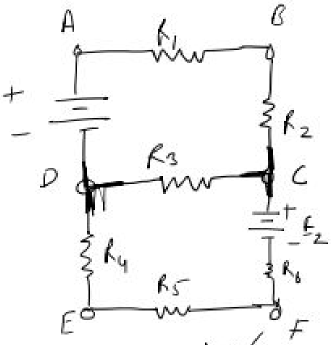
Branch: AB ED
BC
CF
CD
FE
DA



→ AB

Junction Points :-

Points where 3 or more branches meet.



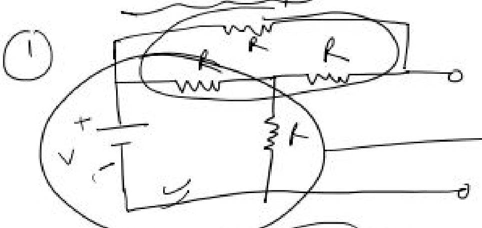
* No. of Jun pt. C & D

Node :- where 2 or more branches/~~elements~~ are joined together.

* Points A, B, C, D, E, F ⇒ Nodes

Circuit & N/w :-

①

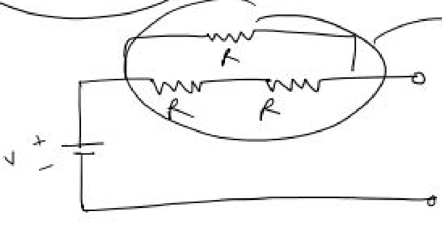


→ Network & Circuit {with source}

∵ it has source

Closed Loop {with source}

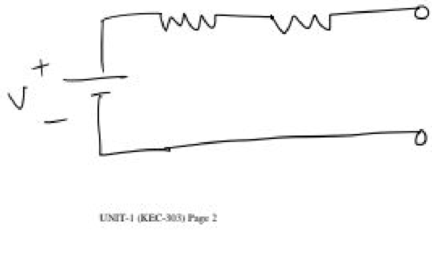
②



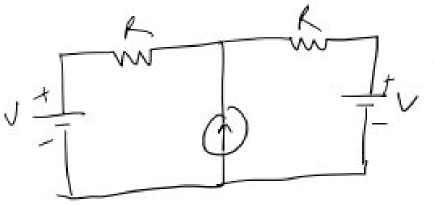
Closed path {but no source}

→ Network only {No closed path → No current}

③



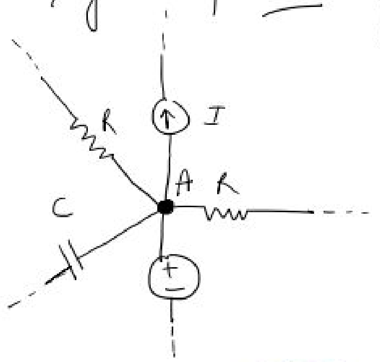
→ Network only {no closed Loop}

(4) 

→ Network & Circuit. {closed path with source}

Imp * Every circuit is a network but all n/w may not be circuit. ⟶ True

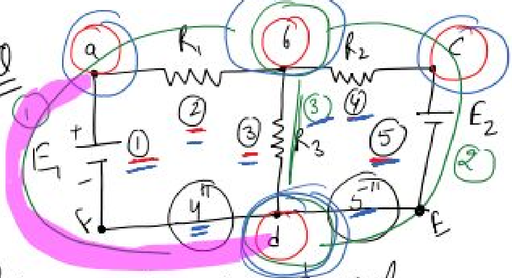
* Degree of node:- → Number of branches connected to a node is called degree of node.



~~Degree~~ = 5

Degree of Node = Branch Connected

* Q



find no. of nodes & no. of branches?

(A) $n=4$, $b=5$ {a, b, c, d; 1, 2, 3, 4, 5}

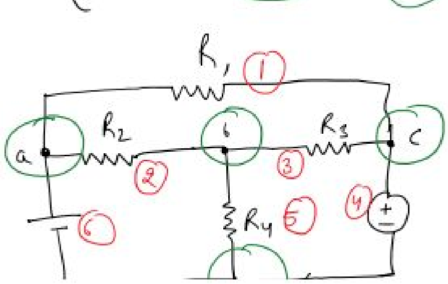
(B) $n=3$, $b=4$ {a, b, d}{1, c, d}

(C) $n=2$, $b=3$ {b, d}{1, 2, 3}

(D) All of Above

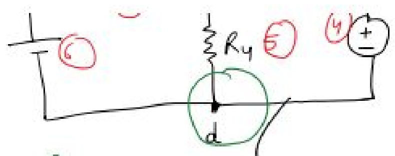
Imp {$b=n+1$} → {branch = node + 1} → Not necessary → {single loop}

* Q

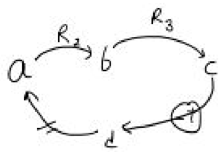


$\boxed{n = 4}$

$\boxed{b = 6}$



$\boxed{b = 6}$

* **Path**:- → 

→ closed path or/Loop/Mesh

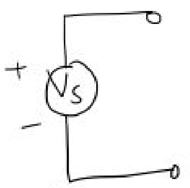
↳ a, c, b, d → path a, b, d → path a, b, c, d → path

Energy Sources

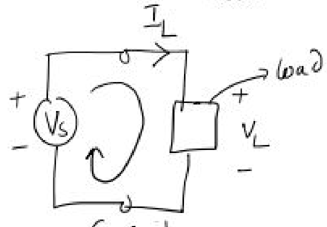
- Voltage Source
  - → Ideal Source
  - → practical source
- Current Source
  - → Ideal Source
  - → Practical Source

① Voltage Source:-

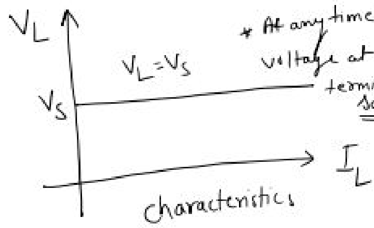
↳ (Ideal) → energy source which gives constant voltage across its terminals irrespective of current drawn through its terminal.



Symbol



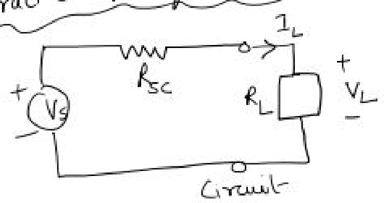
Circuit



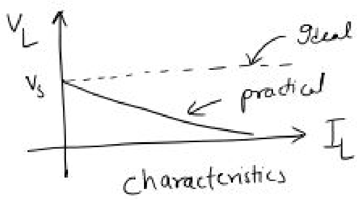
characteristics

* At any time value of voltage at load terminal remain same.

Practical Voltage Source:-



Circuit



characteristics

* Every Voltage source has small internal resistance shown in series with voltage source R_{SC}.

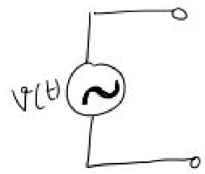
* If there is no load

$I_L = 0$, $V = V_S$

$V_L = -R_{sc} I_L + V_s$ or $V_L = V_s - I_L R_{sc}$ = $\begin{cases} V = IR \\ V_{R_{sc}} = I_L R_{sc} \end{cases}$

Voltage source

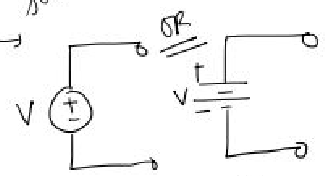
→ Time Variant source



AC Source * Voltage is varying with time.

* Represented by small $v(t)$

→ Time invariant source



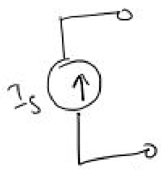
DC Source

* Voltage is not varying with time.

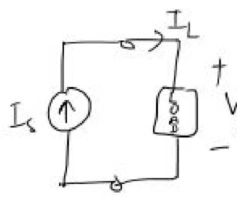
* Represent by Capital letter → V

(2) Current Source: Ideal:-

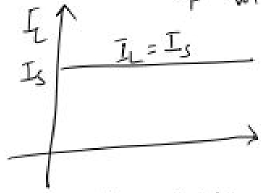
* source which gives constant current at its terminal irrespective of voltage appearing across its terminals.



Symbol



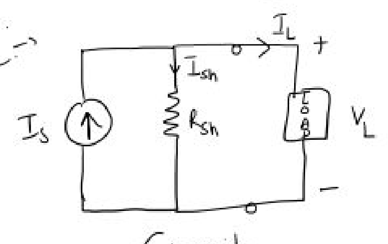
Circuit



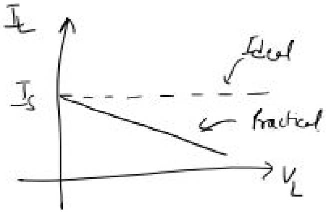
characteristics

* At any time, Value of current through load I_L is same

Practical :-



Circuit



* Every current source has high internal resistance shown in ||al with current source & represent by R_{sh}.

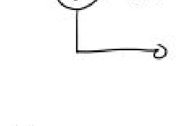
by * $I_L + I_{sh} = I_s$

As $I_{sh} \uparrow$ $I_L \downarrow$

$I_L < I_s$

* Current Source!

→ Time Variant ↓ (AC Source)



$i(t) = A \sin \omega t$

→ Time Invariant (DC Source)

by * for ideal current source $R_{sh} = \infty$

Electrical Circuit OR Networks

(A) (B) (C) (D)

Electrical Circuit / Networks

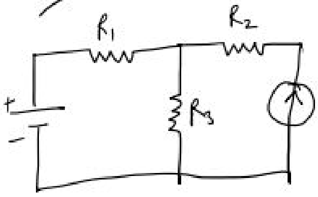
(A)
- Active n/ω
- Passive n/ω

(B)
- Unilateral n/ω
- Bilateral n/ω

(C)
- Linear n/ω
- Non-linear n/ω

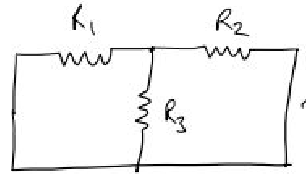
(D)
- Lumped n/ω
- Distributed n/ω

(1) Active N/ω :-



* A circuit having one source of energy
  - voltage source
  - Current source

Passive N/ω :-



→ A circuit with no energy source (Passive n/ω or passive circuit)

(B) Unilateral N/ω :- A circuit whose characteristics / behaviour is dependent on Direction of current through various element. eg. Diode $\xrightarrow{\text{allow}}$ flow of current only in one direction

Unilateral Element or transistor

Bilateral N/ω :-
→ eg. Resistor o—R—o

→ A circuit whose characteristics & behaviour is same irrespective of direction of current through various element. (eg. R, L, C) -

(C) Linear & Non-linear n/ω :-

(C) Linear & Non-linear n/ω :-

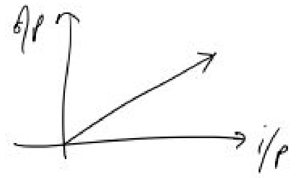
→ A circuit/n/ω whose parameter like, resistance, inductance, capacitance are $\xrightarrow[\text{always}]{}$ constant (irrespective of change in time, temp., voltage etc.)

* Ohm's Law $\xrightarrow[\text{applied}]{\text{can be}}$ to such n/ω.

* Using Law of superposition, mathematical eqⁿ of such n/ω can be obtained.

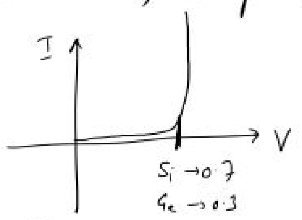
Imp * Response of various n/ω element $\xrightarrow{\text{is}}$ linear
→ wrt to excitation applied to them.

Response =



Non-linear n/ω :- → A circuit whose parameter changes with change in time, temp., voltage etc.

→ ckt having
* diode
→ current doesn't vary linearly with voltage applied to it.



* Ohm's Law $\xrightarrow[\text{be}]{\text{may/does not}}$ applied to such n/ω.

(4) Lumped / Distributed n/ω :-

→ A n/ω in which all the n/ω elements are physically seperable. e.g. Most of electric n/ω
→ lumped
having element R, L, C etc.

→ N/ω in which ckt element (R, L, C) etc can't be physical seperable.

→ eg Transmission lines

... physical seperable.

↳ eg. Transmission lines
↳ R, L, C of Tx-line are distributed over complete length & can't be seperate element.

*Q Which is an Ideal Source?

a) Resistor

b) Capacitor

c) ✓ Voltage Source

d) ✓ Op-amp. (operational-amplifier)

Imp *Q Which is non-linear element?

a) Fuse

b) Transistor

c) Thermistor

d) ✓ All of above

Imp * Transistor / op-amp $\xrightarrow{\text{are}}$ special case of active element.

* Linear Element :→ which obey ohm's law. (R, L, C)

* Non-linear Element :→ which doesn't obey ohm's law (Diode)

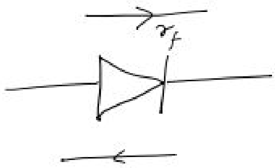
* Imp points :-
It should not be considered that non-linear devices are undesirable elements.

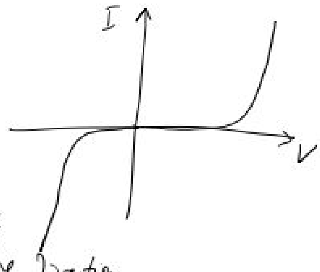
eg. ① Zener Diode :→ Non linear element → Used in voltage regulation.

② Fuse :- → Non-linear element → used for protecting from over current.

③ Thermistor :- → non-linear element → used for all thermostat application

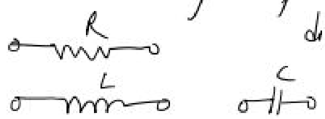
Unilateral Element :-
→ Diode
Transistor.
→ Element allows flow of current in one Direction.





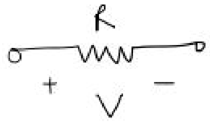
* Element whose parametric value is not constant with reversal of current.

Bilateral Element :- → Element allow flow of current in both direction
→ e.g. R, L, C



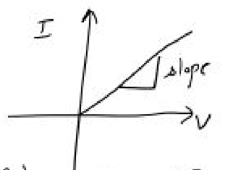
* Elements whose parametric value does not change (constant) with reversal of current.

*



$i = \frac{V}{R}$

$i \propto V$



$i = G.V$

or $G = \frac{1}{R}$
conductance

$R \rightarrow$ ohm (Ω)

$G \rightarrow$ mho ($\mho$)

$\text{slope} = \frac{dI}{dV} = \frac{1}{R}$

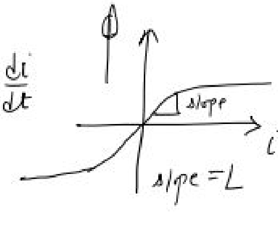
$\text{slope} = \boxed{G}$

*

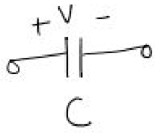
flux $\phi \propto i$

$\phi = L i$
self Inductance (Henry (H))

$V = L\frac{di}{dt}$



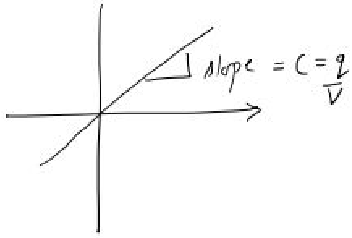
slope = L

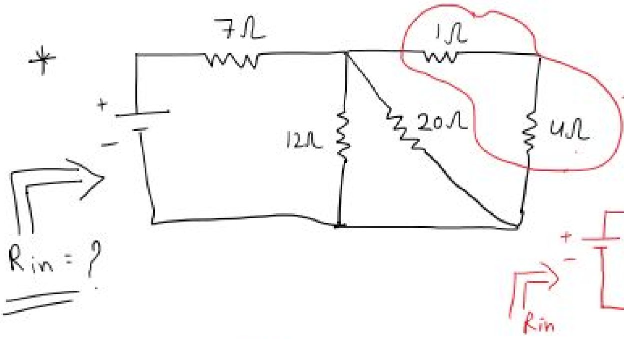
* 

$q \propto V$

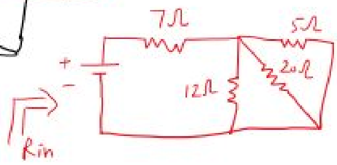
$q = CV$

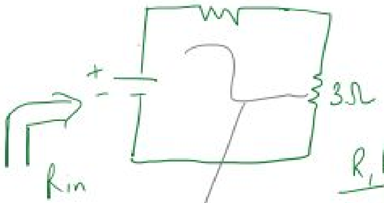
$i = \frac{dq}{dt} = C\frac{dv}{dt}$



* 

$1 + 4 = 5\Omega$





$10 \parallel 20 \parallel 5$

$\cancel{\frac{5 \times 20 \times 12}{5 + 20 + 12} = \frac{1200}{37} = 32.43}$

$\frac{R_1 R_2}{R_1 + R_2}$

* $\frac{20 \times 5}{20 + 5} = \frac{100}{25} = 4$

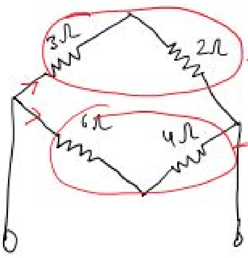
* $12 \parallel 4 = \frac{12 \times 4}{12 + 4} = \frac{48}{16} = 3\Omega$

Series

* $7 + 3 = 10\Omega$

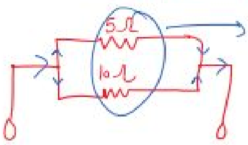
$R_{in} = 10\Omega$

* Q



Series $3 + 2 = 5\Omega$

Series $6 + 4 = 10\Omega$



||al

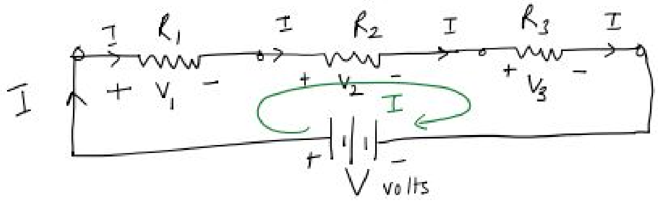
$\frac{R_1 R_2}{R_1 + R_2} = \frac{5 \times 10}{5 + 10} = \frac{50}{15}$

$\boxed{R = 3.33\Omega}$

Analysis of Series Circuit:-

Analysis of Series Circuit:-

① Resistors in Series:-



* Current same & Voltage Division

$$V = V_1 + V_2 + V_3$$

$$V_1 = IR_1 \quad , \quad V_2 = IR_2 \quad , \quad V_3 = IR_3$$

$$V = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3$$

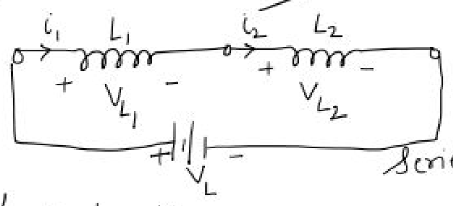
$$= I(R_1 + R_2 + R_3)$$

or $\boxed{V = IR_{eq}}$ where $R_{eq} = R_1 + R_2 + R_3$

$R_{eq} > R_1$
$R_{eq} > R_2$
$\vdots$
$R_{eq} > R_n$

Total Eq^n resistance for n resistance in Series of series ckt is $\longrightarrow$ $R_{eq} = R_1 + R_2 + R_3 + \text{- - - -} R_n = \sum_{i=1}^{n} R_i$

② Inductors in Series:-



$L_1 \xrightarrow{\text{current flow}} i_1$

$L_2 \xrightarrow{\text{current flow}} i_2$

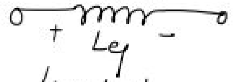
Series $\rightarrow$ $i = i_1 = i_2$ $\rightarrow$ same current

$$V_{L_1} = L_1 \frac{di_1}{dt}$$

$$V_{L_2} = L_2 \frac{di_2}{dt}$$

$$V_L = V_{L_1} + V_{L_2}$$

$$L_{eq}\frac{di}{dt} = L_1 \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$



$L_{eq} = L_1 + L_2$

$$L_{eq} \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} \qquad \{\because i = i_1 = i_2\}$$

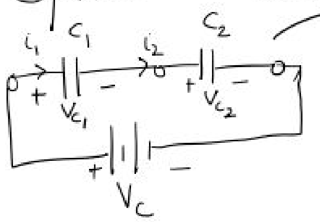
$$L_{eq} \frac{di}{dt} = (L_1 + L_2) \frac{di}{dt}$$

Int. both side

$\boxed{L_{eq} = L_1 + L_2}$ $\rightarrow$ $\therefore$ Total Eq^n inductance of series Circuit is sum of inductance

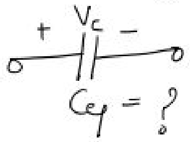
$$\boxed{L_{eq} = L_1 + L_2}$$ → ∴ ... Circuit is sum of inductance connected in series.

③ Capacitors in Series :-



∵ Series → same current is flowing.

$$i = i_1 = i_2$$



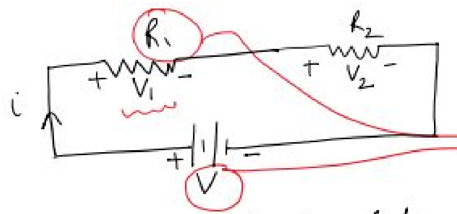
$$V_{C_1} = \frac{1}{C_1}\int_{-\infty}^{t} i_1\,dt \quad , \quad V_{C_2} = \frac{1}{C_2}\int_{-\infty}^{t} i_2\,dt$$

$$V_C = V_{C_1} + V_{C_2}$$

$$\frac{1}{C_{eq}}\int_{-\infty}^{t} i\,dt = \frac{1}{C_1}\int_{-\infty}^{t} i\,dt + \frac{1}{C_2}\int_{-\infty}^{t} i\,dt$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}} \quad \text{or} \quad C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

or $$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots \frac{1}{C_n}$$ → Reciprocal of total Eq^n Capacitor of series connection is sum of reciprocal of individual Capacit., connected in series.

* 

$$V_1 = iR_1 \qquad V_2 = iR_2$$

$$\left\{R = \frac{1}{G}\right\}$$

Voltage divider Rule in series of R → $V_1 = V \cdot \dfrac{R_1}{R_1 + R_2}$ → total Resistance.

② $V_1 = V \cdot \dfrac{\frac{1}{G_1}}{\frac{1}{G_1} + \frac{1}{G_2}}$ → $V_1 = V \cdot \dfrac{G_2}{G_1 + G_2}$ ④

|||ly $V_2 = V \cdot \dfrac{R_2}{R_1 + R_2}$ ① → $V_2 = V \cdot \dfrac{G_1}{G_1 + G_2}$ ③

So

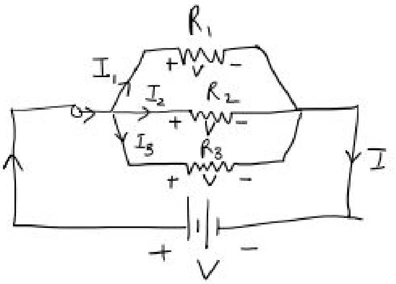
eq $\frac{2}{1}$ $$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$ OR eq $\frac{4}{3}$ → $$\frac{V_1}{V_2} = \frac{G_2}{G_1}$$

R → Resistance ... Conductance

$R \rightarrow$ Resistance
$G \rightarrow$ Conductance
$R = \frac{1}{G}$

Analysis of Parallel Ckt :-

① Resistors in ||al :-



$\underline{I = I_1 + I_2 + I_3}$

$V = I_1 R_1$
$I_1 = \frac{V}{R_1}$, $V = I_2 R_2$
$I_2 = \frac{V}{R_2}$, $V = I_3 R_3$
$I_3 = \frac{V}{R_3}$

$I = I_1 + I_2 + I_3$ { In ||al current will divide }

$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ → Eq^n Resistance.

we know $R = \frac{1}{G}$ or $G = \frac{1}{R}$

$\boxed{G_{eq} = G_1 + G_2 + G_3}$ → G = conductance.

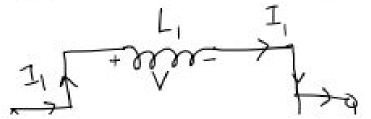
* Imp
for $n = 2$ R_1 & R_2 → are in ||al

$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$

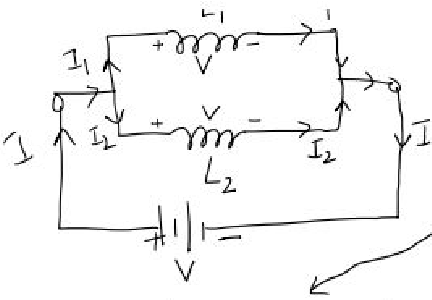
$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$

{ $R_{eq} < R_1$
$R_{eq} < R_2$
⋮
$R_{eq} < R_n$ }

② Inductors in ||al :-



{ in ||al current → divide
voltage → same }



{ in ||al current → divide
voltage → same }

$$I = I_1 + I_2$$

$$I_1 = \frac{1}{L_1}\int_{-\infty}^{t} V_{L_1}\,dt \quad \Bigg\} \quad I_2 = \frac{1}{L_2}\int_{-\infty}^{t} V_{L_2}\,dt$$

So, $$\frac{1}{L_{eq}}\int_{-\infty}^{t} V_L\,dt = \frac{1}{L_1}\int_{-\infty}^{t} V_{L_1}\,dt + \frac{1}{L_2}\int_{-\infty}^{t} V_{L_2}\,dt$$

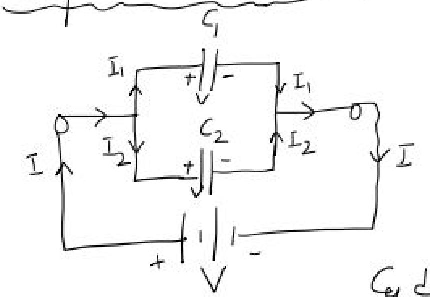
differentiate both side

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$$

{ $V_L = V_{L_1} = V_{L_2} = V$
∵ voltage across ||al ckt → same }

OR $$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} - - - - - \frac{1}{L_m}$$

(3) Capacitors in ||al :-



$$I = I_1 + I_2$$

{ ||al current → divide
voltage → same }

$$I_1 = C_1\frac{d}{dt}V_{C_1}$$

$$I_2 = C_2\frac{d}{dt}V_{C_2}$$

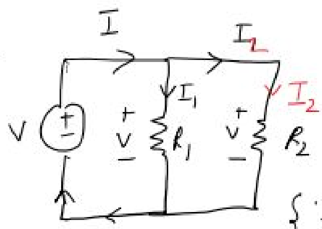
{ ||al $V_C = V_{C_1} = V_{C_2} = V$
voltage same }

$$C_{eq}\frac{dV}{dt} = C_1\frac{d}{dt}V_{C_1} + C_2\frac{d}{dt}V_{C_2}$$

$$C_{eq} = C_1 + C_2$$

$$C_{eq} = C_1 + C_2 + C_3 - - - - - + C_n$$

In ||al Current division Rule:-



$$I_1 = \frac{V}{R_1} = \frac{I \cdot R_{eq}}{R_1}$$

{ ∵ in ||al Voltage → same }

$$= \frac{I}{\cancel{R_1}} \cdot \left(\frac{\cancel{R_1} R_2}{R_1 + R_2}\right)$$

current through Resistance R_1

$$I_1 = I \cdot \frac{R_2}{(R_1 + R_2)}$$

→ overall current
→ Resistance in opposite branch.
→ Total Resistance

$$I_2 = I \cdot \frac{R_1}{R_1+R_2}$$ Current division Rule → In parallel Ckt

Imp

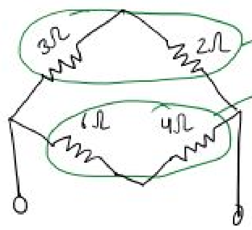
In Series Ckt → Voltage division Rule

$$V_1 = V \cdot \frac{R_1}{R_1+R_2} \quad \& \quad V_2 = V \cdot \frac{R_2}{R_1+R_2}$$

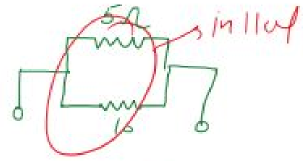
Imp * Current through series combination remains same & Voltage get Divided.

while Voltage remains same in parallel combination & current get divided.

* Q



$R_{eq} = ?$

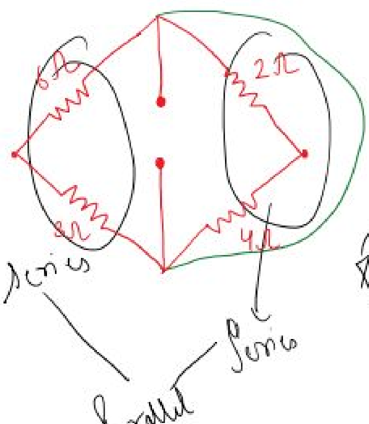


$3+2 = 5\Omega$

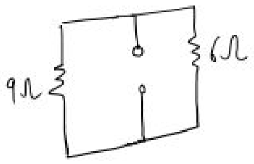
$6+4 = 10\Omega$

$$R_{eq} = 5 \,||\, 10 = \frac{R_1 R_2}{R_1+R_2} = \frac{50}{15} = 3.33\Omega$$

*



$R_{eq} = ?$



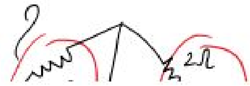
$R_{eq} = (6+3) \,||\, (2+4) \,||\, (0)$

$R_{eq} = 9 \,||\, 6 \,||\, 0$

$R_{eq} = 0$

*

R_{eq} or $R_{ab} = ?$



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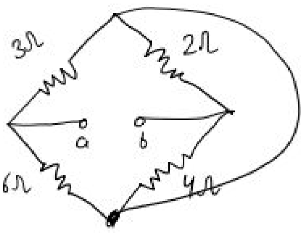
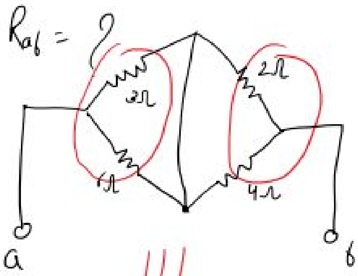


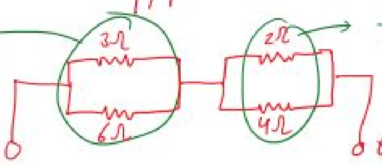
R_{eq} or R_{ab} = ?

$\equiv$



$\equiv$

$3 \| 6 = \dfrac{(3)(6)}{3+6} = \dfrac{18}{9} = 2$

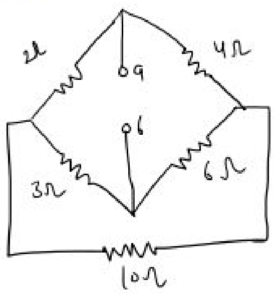


$\dfrac{2 \times 4}{2+4} = \dfrac{8}{6} = \dfrac{4}{3}$

$\equiv$

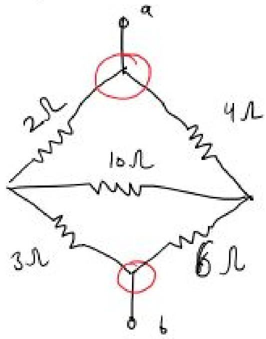
R_{eq} or $R_{ab} = 2 + \dfrac{4}{3} = \dfrac{6+4}{3} = \dfrac{10}{3}\Omega$

*

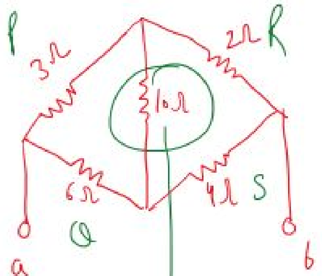


R_{ab} or R_{eq} = ?

$\equiv$



$\equiv$



$\longrightarrow$ Balanced Wheat Stone Bridge

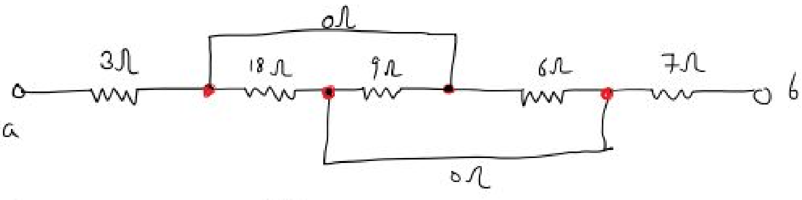
$\dfrac{P}{Q} = \dfrac{R}{S} \rightarrow \dfrac{3}{6} = \dfrac{2}{4}$

$\equiv$

$\longrightarrow i_{10\Omega} = 0 \longrightarrow$ O.C

$\longrightarrow R_{eq} = (3+2) \| (6+4)$
$= 5 \| 10 = \dfrac{5 \times 10}{5+10} = \dfrac{50}{15} = 3.33\Omega$

*



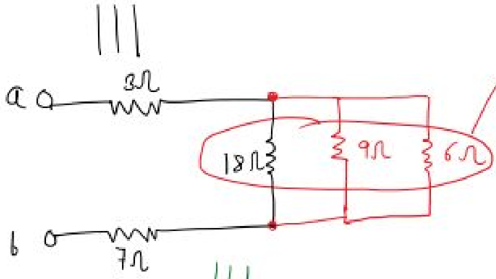
R_{ab} = ?

$\equiv$

3Ω

$6 \| 9 \| 18$

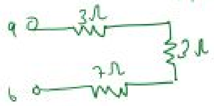
$R_{ab} = ?$



6 || 9 || 18

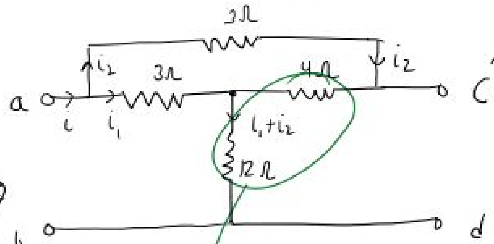
$\frac{6 \times 9}{6+9} = \frac{54}{15}$

$\frac{\frac{54}{15} \times 18}{\frac{54}{15} + 18} = 3\Omega$



$3 + 3 + 7 = 13\Omega$

Q *



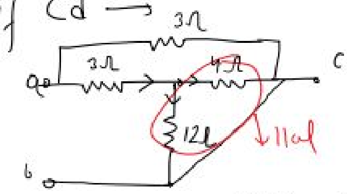
$12 || 4 = \frac{12 \times 4}{12+4} = 3\Omega$

$3 + 3 = 6\Omega$

$6 || 3 = \frac{6 \times 3}{6+3} = \frac{18}{9} = 2\Omega$

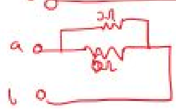
First Method

if Cd $\xrightarrow{S.C}$



$4 || 12 \longrightarrow \frac{4 \times 12}{4+12} = \frac{48}{16} = 3\Omega$

$3 + 3 = 6\Omega$



$3 || 6 = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\Omega$

Method of Analysis :-

→ Node Analysis

→ Mesh Analysis

(1) Mesh Analysis :-

→ Steady state Analysis

→ Transient Analysis

Steady state Analysis obtained from KVL

Transient Analysis obtained from KVL

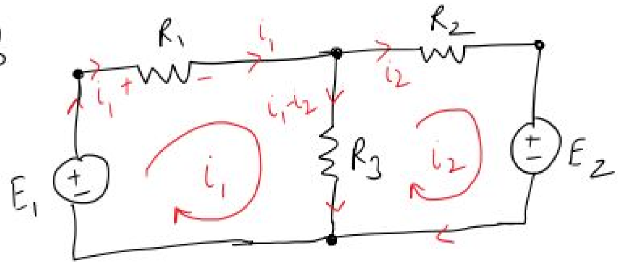
KVL mentioned in Branch Voltage

Branch Voltage required Branch current

Branch current described in term of loop current

We define mesh as a loop which does not contain any other loop with in it.

Case ①

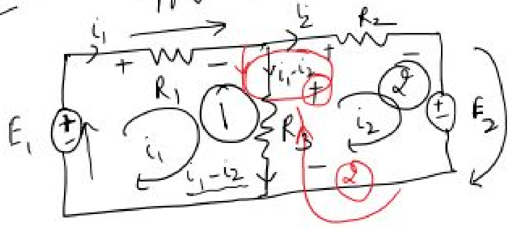


Step ① Select minimum no. of loops to solve the n/w.

$$l = b - n + 1 \qquad \text{Here } b = \text{no. of branches} = 5$$
$$n = \text{no. of nodes} = 4$$
$$l = 5 - 4 + 1 = 2$$

Step ② Express Branch current in terms of loop current & find branch voltage along with polarities.

Step ③ Apply KVL in each selected loop & solve the eq^n



Loop ① KVL

$$-i_1R_1 - R_3(i_1 - i_2) + E_1 = 0$$

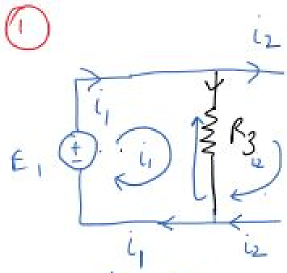
$$\boxed{-i_1(R_1 + R_3) + i_2R_3 = -E_1} \quad — ①$$

* Loop ② KVL

$$-i_2R_2 - E_2 - R_3(i_2 - i_1) = 0$$

$$-i_2R_2 - i_2R_3 + i_1R_3 = E_2$$

$$\boxed{-i_2(R_2 + R_3) + i_1R_3 = E_2} \quad — ②$$



Loop ②

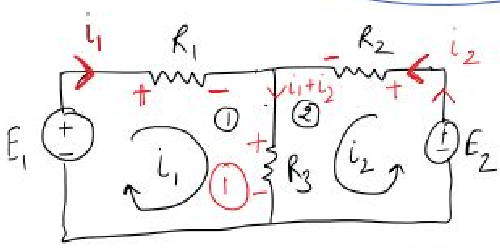
$$\begin{bmatrix} -(R_1 + R_3) & R_3 \\ R_3 & -(R_2 + R_3) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -E_1 \\ +E_2 \end{bmatrix}$$

Imp ① All Diagonal elements will have -ve sign.

② Matrix always satisfy the property of symmetry

$$R_{ij} = R_{ji}$$

Case ②



Apply KVL in loop ① → $-i_1R_1 - (i_1+i_2)R_3 + E_1 = 0$ —③

Apply KVL in loop ② → $-i_2R_2 - (i_1+i_2)R_3 + E_2 = 0$ —④

Rewrite eq ③ & ④

③ → $-i_1R_1 - i_1R_3 - i_2R_3 = -E_1$ —⑤

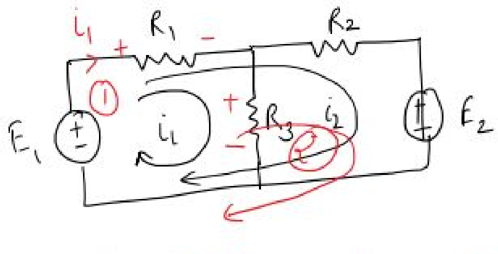
$$\boxed{-i_1(R_1+R_3) - i_2R_3 = -E_1} \quad \text{—(A)}$$

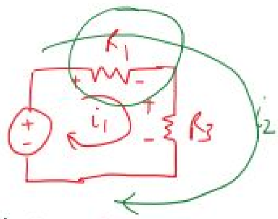
④ → $-i_2R_2 - i_1R_3 - i_2R_3 = -E_2$

$$\boxed{-i_1R_3 - i_2(R_2+R_3) = -E_2} \quad \text{—(B)}$$

$$\begin{bmatrix} -(R_1+R_3) & -R_3 \\ -R_3 & -(R_2+R_3) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -E_1 \\ -E_2 \end{bmatrix}$$

Case ③

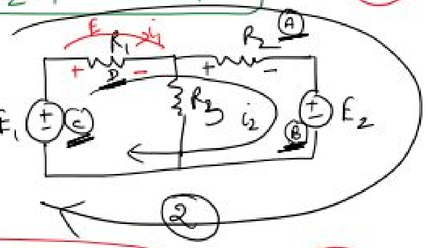




Apply KVL in loop ① → $-i_1R_1 - i_1R_3 - i_2R_1 + E_1 = 0$

Apply KVL in Loop ① → $-i_1R_1 - i_1R_3 - i_2R_1 + E_1 = 0$

$$\boxed{-i_1(R_1+R_3) - i_2R_1 = -E_1} \quad \text{— (7)}$$



Apply KVL in Loop ② :-

$$-i_2R_2 - E_2 + E_1 - i_1R_1 - i_2R_1 = 0$$

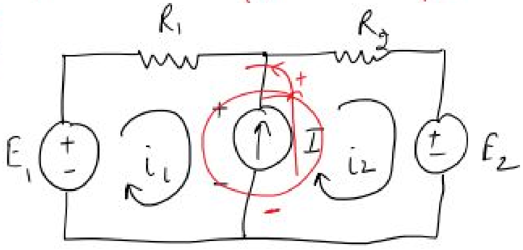
$$\boxed{-i_1R_1 - i_2(R_1+R_2) = -E_1+E_2} \quad \text{— (8)}$$

$$\begin{bmatrix} -(R_1+R_3) & -R_1 \\ -R_1 & -(R_1+R_2) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} -E_1 \\ -E_1+E_2 \end{bmatrix}$$

Imp * Common Resistance in both mesh

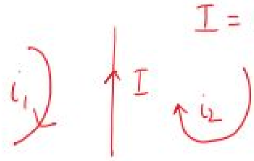
* Take -ve sign if loop current flow in same direction & take +ve sign if loop current flow in opposite dirⁿ

Case 4



Case: when ideal current source comes in one of the loop.

$I = i_2 - i_1$

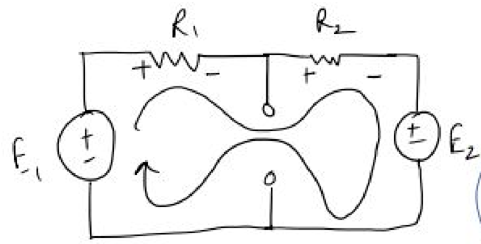


* Super Mesh Analysis :-

① Express loop current (i_1 & i_2) in terms of current source (I)

$$\boxed{I = i_2 - i_1}$$

② Replace current source by its internal resistance i.e (by open ckt) which results in super mesh Analysis



Supermesh

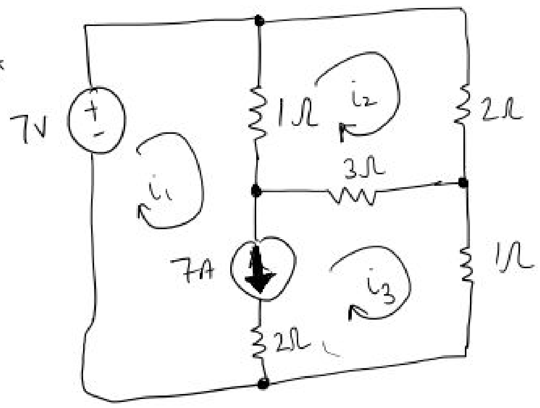
Apply KVL in supermesh by maintaining previous branch voltages.

$$\boxed{E_1 - i_1R_1 - i_2R_2 - E_2 = 0}$$

Alternative of 2^{nd} step

↳ Select a Loop which doesn't contain current source & apply KVL in ckt.

Q. *



i_1, i_2 & $i_3 = ?$

$l = b - n + 1$

$b = 6$

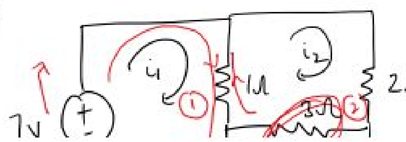
$n = 4$

$\therefore l = 6 - 4 + 1 = 3$

* 7A current source is connected along two meshes mesh ① & mesh ③ ∴ Along these two meshes a super mesh form.

$$\boxed{7 = i_1 - i_3} \quad -①$$

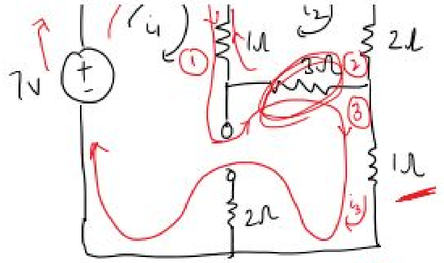
*



* Apply KVL in supermesh

$V = IR$

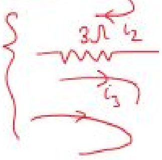
$\rightarrow \; 1(i_2 - i_1) - 3(i_2 - i_1) - 1(i_2) = 0$



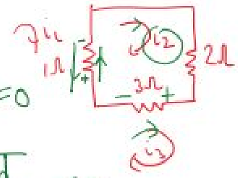
$V = IR$

$$7 - 1(i_1 - i_2) - 3(i_3 - i_2) - 1(i_3) = 0$$

$$\boxed{i_1 - 4i_2 + 4i_3 = 7} \quad \text{— (2)}$$



* Apply KVL in mesh (2)



$$-3(i_2 - i_3) - 1(i_2 - i_1) - 2i_2 = 0$$

$$\boxed{i_1 - 6i_2 + 3i_3 = 0} \quad \text{— (3)}$$

$$\boxed{7 = i_1 - i_3} \quad \text{— (1)}$$

Solve eq (1), (2) & (3)

(1) → $\boxed{i_1 = 7 + i_3}$ put in eq. (2) & (3)

in eq (2)

$$\cancel{7} + i_3 - 4i_2 + 4i_3 = \cancel{7}$$

$$-4i_2 + 5i_3 = 0 \quad \text{— (4)}$$

in eq (3) $\quad 7 + i_3 - 6i_2 + 3i_3 = 0$

$$\boxed{-6i_2 + 4i_3 = -7} \quad \text{— (5)}$$

Solve eq (4) & (5)

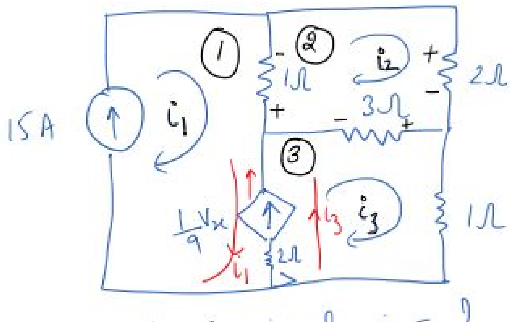
$i_2 = 2.5\,A$

$i_3 = 2\,A$

$i_1 = 7 + i_3 = 7 + (2)$

$i_1 = 9\,A$

Q *



Note

* Presence of two current sources in the 3 mesh ckt makes it necessary to apply KVL only once around mesh (2).

$i_1 = ?$ $\quad i_2 = ?$ $\quad i_3 = ?$

Solution:

Apply KVL in mesh (2)

Solution

Apply KVL in mesh ②

$$-1(i_2-i_1) - 2(i_2) - 3(i_2-i_3) = 0$$

Rewrite

$$\boxed{+i_1 - 6i_2 + 3i_3 = 0} \quad — ①$$

Loop ①

$$i_1 = 15A \quad — ②$$

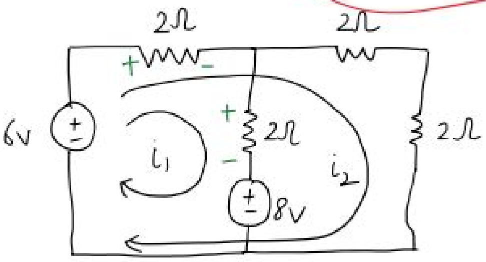
through current source →

$$\boxed{\frac{1}{9}V_x = (i_3 - i_1)} \quad — ③$$

* $\frac{1}{9}V_x = i_3 - 15$ $\therefore$ $i_3 = \frac{1}{9}V_x + 15$

Put value of i_1 & i_3 in eq ① = i_2

Question :-



find

a) i_1 & i_2

b) V_0

c) P_{8V} & P_{6V}

(S.P. / A.P.)

S.P. → Supply power

A.P. → Absorbing power.

Solⁿ Apply KVL in loop ①

$$+6 - 2(i_1+i_2) - 2i_1 - 8 = 0$$

Rewrite

$$\boxed{-4i_1 - 2i_2 = 2} \quad — ①$$

Apply KVL in loop ②

$$6 - 2(i_1+i_2) - 2i_2 - 2i_2 = 0$$

Rewrite

$$\boxed{-2i_1 - 6i_2 = -6} \quad — ②$$

In form of matrix

$$\boxed{\begin{bmatrix} -4 & -2 \\ -2 & -6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 2V \\ -6V \end{bmatrix}}$$

$-4i_1 - 2i_2 = 2$ $\quad \times 2 \longrightarrow$ $\cancel{-8i_1} - 4i_2 = 4$

$-2i_1 - 6i_2 = -6$ $\quad \times 4 \longrightarrow$ $\cancel{-8i_1} - 24i_2 = -24$

$+ \quad + \quad +$

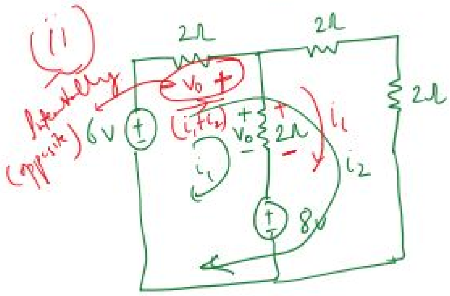
$20i_2 = 28 \rightarrow i_2 = \frac{28}{20} = \frac{14}{10} = \frac{7}{5} = 1.4A$

$\therefore$ $i_2 = 1.4A \rightarrow -4i_1 - 2(1.4) = 2$

$\therefore$ $-4i_1 = 2 + 2.8$ $\quad i_1 = \frac{-4.8}{4} = -1.2A$

$i_1 = -1.2A$ $\quad$ $i_2 = 1.4A$

(ii)



$V_0 = ?$ $\quad$ Ohm's Law

$V_0 = IR$ $\quad$ $R = 2\Omega$

$V_0 = 2(i_1) = V_0 = 2i_1$

+ve

$\boxed{V_0 = -2(i_1+i_2)}$

(iii) $P_{8V} = V \times I$

$= 8 \times (-1.2) = -9.6W \rightarrow$ Absorbing power

$P_{8V} = 9.6W$ supply power.

(iv) $P_{6V} = V \times I = 6(i_1+i_2)$

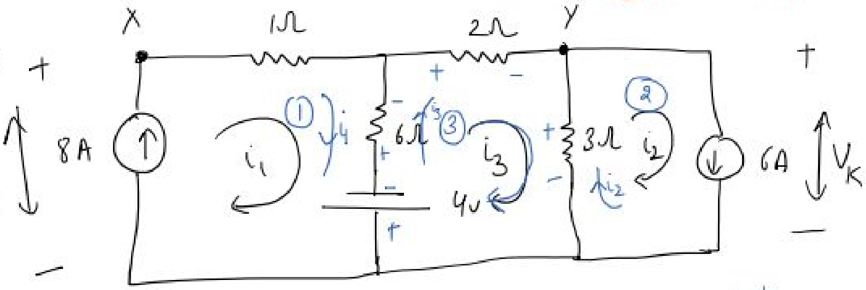
$= 6(-1.2+1.4)$

$= 6 \times (0.2) = 1.2W$ supply power.

Question ⊛

Find
a) i_1, i_2, i_3
b) V_{XY}
c) P_{4V}
d) P_{6A}
e) P_{8A}

??



Sol^n

Loop ① $\longrightarrow$ $i_1 = 8A$

Loop ② $\longrightarrow$ $i_2 = 6A$

Loop ③ Apply KVL

$-2i_3 - 3(i_3 - i_2) - 4 - 6(i_3 - i_1) = 0$

Rewrite

$-2i_3 - 3(i_3 - 6) - 4 - 6i_3 + 6(8) = 0$

$-2i_3 - 3i_3 + (18-4) - 6i_3 + 48 = 0$

$-11i_3 + 62 = 0$

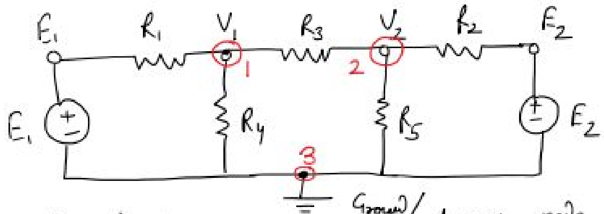
$i_3 = +\frac{62}{11} = 5.64A$

② Method of Analysis $\rightarrow$ ① Mesh Analysis

$\rightarrow$ ② Node Analysis

Node Analysis:

Node Analysis:



Case ① ⟶

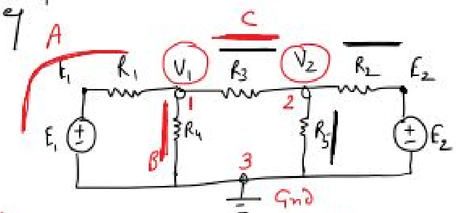
Step ① Determine minimum no. of nodes to solve the n/w

Ground/reference node or Datum node or Zero node

$$n = 3 \begin{cases} 1 \rightarrow \text{ref. node} \\ n-1 \rightarrow \text{non-ref. nodes} \ (2) \end{cases}$$

Step ② Express branch voltage interms of node voltage & also find branch currents along with dir.

Step ③ Apply KCL at each selected (Node 1 & Node 2) non-reference node & solve the eq^n



Apply KCL at node ①

(Assumption $V_1 > V_2$)

$$\underbrace{\frac{V_1 - E_1}{R_1}}_{A} + \underbrace{\frac{V_1 - 0}{R_4}}_{B} + \underbrace{\frac{V_1 - V_2}{R_3}}_{C} = 0$$

{ohm's law ⇒ $I = \frac{V}{R}$

Rewrite ⟶
$$\boxed{V_1\left[\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4}\right] - V_2\left[\frac{1}{R_3}\right] = \frac{E_1}{R_1}} \quad \text{—— ①}$$

Apply KCL at node ② (Assumption $V_2 > V_1$)

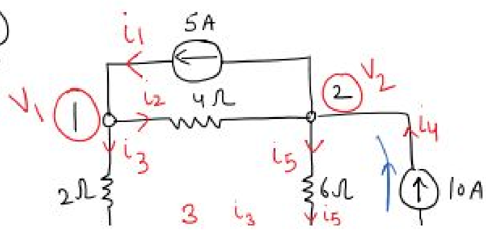
$$\frac{V_2 - V_1}{R_3} + \frac{V_2 - 0}{R_5} + \frac{V_2 - E_2}{R_2} = 0$$

Rewrite
$$\boxed{-V_1\left[\frac{1}{R_3}\right] + V_2\left[\frac{1}{R_3} + \frac{1}{R_5} + \frac{1}{R_2}\right] = +\frac{E_2}{R_2}} \quad \text{—— ②}$$

In Matrix form

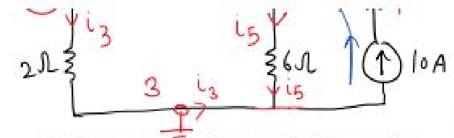
$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} E_1/R_1 \\ E_2/R_2 \end{bmatrix}$$

** Question ①



Calculate node voltages in the circuit shown.

{ohm's law?



Sol[4] At Node 1, Apply KCL $\rightarrow$ $i_1 = i_2 + i_3$ — (A) {ohm's law $i = \frac{V}{R}$}

$5 = \frac{V_1 - V_2}{4} + \frac{V_1 - 0}{2}$

Rewrite $\rightarrow$ $\boxed{V_1 - V_2 + 2V_1 = 20}$ — (1) $\rightarrow$ $\boxed{3V_1 - V_2 = 20}$

At node (2), Apply KCL $\rightarrow$ $i_2 + i_4 = i_1 + i_5$ — (B)

$\left\{\frac{V_1 - V_2}{4}\right.$ $\cancel{\frac{V_2 - V_1}{4}} + 10 = 5 + \frac{V_2 - 0}{6}$

$\because V_1 \rightarrow$ higher potential (∵ the current direction)

$\boxed{5V_2 - 3V_1 = +60}$ — (2)

(1) $3V_1 - V_2 = 20$ — (1) $-3V_1 + 5V_2 = 60$ — (2)

Cramer's Rule:

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

$V_1 = \frac{\Delta_1}{\Delta}$

$V_2 = \frac{\Delta_2}{\Delta}$

(Δ) Determinant

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - (-3)(-1) = 15 - 3 = 12$$

replace

Calculate value of V_1 & V_2

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

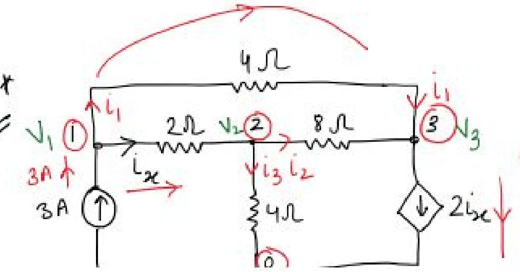
$$V_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{12} = \frac{100 - (-60)}{12}$$

$= 13.33$ volt

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{bmatrix} 3 & 20 \\ -3 & 60 \end{bmatrix}}{12} = \frac{180 - (-60)}{12}$$

$= \frac{240}{12} = 20$ volts

Q*

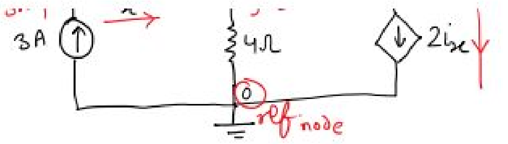


Sol[4]

At Node (1), Apply KCL

ohm's law

$\{ i = V$



At Node (1), Apply KCL

ohm's Law $\left\{ i = \frac{V}{R} \right.$

$$3 = i_1 + i_x$$

$$3 = \frac{V_1 - V_3}{4} + \frac{V_1 - V_2}{2}$$

Rewrite $\longrightarrow$ $\boxed{3V_1 - 2V_2 - V_3 = 12}$ — (A)

Apply KCL at node (2)

$$i_x = i_2 + i_3$$

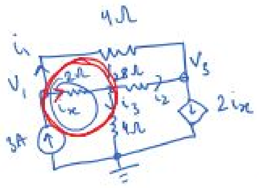
$$\frac{V_1 - V_2}{2} = \frac{V_2 - V_3}{8} + \frac{V_2 - 0}{4}$$

$\left\{ \text{ohm's Law } i = \frac{V}{R} \right\}$

$$\boxed{-4V_1 + 7V_2 - V_3 = 0}$$ — (B)

Apply KCL at node (3)

$$\underbrace{i_1 + i_2}_{\text{incoming current}} = \underbrace{2\,i_x}_{\text{outgoing current}}$$



$$\frac{V_1 - V_3}{4} + \frac{V_2 - V_3}{8} = 2\left(\frac{V_1 - V_2}{2}\right)$$

$\left\{ \text{ohm's law } i = \frac{V}{R},\ i_x = \frac{V_1 - V_2}{2} \right.$

Rewrite $\boxed{2V_1 - 3V_2 + V_3 = 0}$ — (C)

(A) $3V_1 - 2V_2 - V_3 = 12$

(B) $-4V_1 + 7V_2 - V_3 = 0$

(C) $2V_1 - 3V_2 + V_3 = 0$

* Cramer's Rule $V_1 = \frac{\Delta_1}{\Delta}$, $V_2 = \frac{\Delta_2}{\Delta}$, $V_3 = \frac{\Delta_3}{\Delta}$

$$\Delta = \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & +1 \end{vmatrix} \equiv \begin{vmatrix} 3 & -2 & -1 \\ -4 & 7 & -1 \\ 2 & -3 & +1 \\ 3 & -2 & -1 \\ -4 & 7 & -1 \end{vmatrix}$$

{we repeat first two rows & then crossmultiply} (−) (+)

$$\Delta = [21 - 12 + 4 + 14 - 9 - 8] = 10$$

$$V_1 = \frac{\Delta 1}{\Delta} = \frac{?}{10}$$

$$\Delta_1 = \begin{vmatrix} 12 & -2 & -1 \\ 0 & +7 & -1 \\ 0 & -3 & +1 \\ 12 & -2 & -1 \\ 0 & +7 & -1 \end{vmatrix}$$

(−) (+)

$$\Delta_1 = [84 + 0 + 0 - 0 - 36 - 0] = 48$$

$$V_1 = \frac{48}{10}$$

$V_1 = 4.8$ volts

$$V_2 = \frac{\Delta_2}{\Delta} = \frac{}{10}$$

$$\Delta_2 = \begin{vmatrix} 3 & 12 & -1 \\ -4 & 0 & -1 \\ 2 & 0 & 1 \\ 3 & 12 & -1 \\ -4 & 0 & -1 \end{vmatrix} = [0 + 0 + 24 + 0 + 0 + 48]$$

(−) (+) $= [+24]$

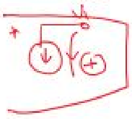
$$V_2 = \frac{24}{10} =$$ 24 volts

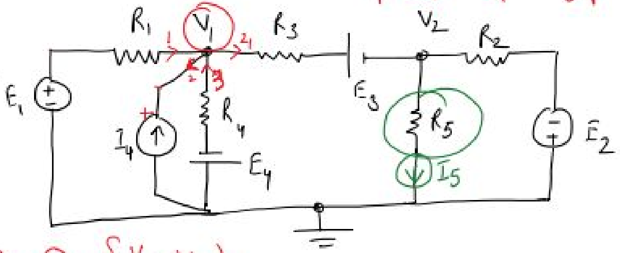
$$V_3 = \frac{\Delta_3}{\Delta} = \frac{-24}{10} = -2.4 \text{ volts}$$

$$\Delta_3 = \begin{vmatrix} 3 & -2 & 12 \\ -4 & 7 & 0 \\ 2 & -3 & 0 \end{vmatrix} = -24$$

$V_1 = 4.8\,V$, $V_2 = 2.4\,V$, $V_3 = -2.4\,V$

*Case 2





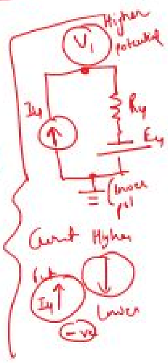
nodes → 3: ref node (3), non-ref node 2 (V_1, V_2)

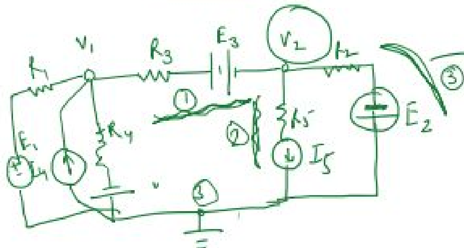
KCL at node ① $\{V_1 > V_2\}$

$$\underbrace{\frac{V_1 - E_1}{R_1}}_{1} - \underbrace{I_4}_{2} + \underbrace{\frac{V_1 + E_4}{R_4}}_{3} + \underbrace{\frac{V_1 - V_2 - E_3}{R_3}}_{4} = 0$$

Rewrite

$$\boxed{V_1\left[\frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_3}\right] - \frac{V_2}{R_3} = I_4 + \frac{E_1}{R_1} - \frac{E_4}{R_4} + \frac{E_3}{R_3}} \quad (1)$$





KCL at node 2 $(V_2 > V_1)$

$$\underbrace{\frac{V_2 + E_3 - V_1}{R_3}}_{1} + \underbrace{I_5}_{2} + \underbrace{\frac{V_2 + E_2}{R_2}}_{3} = 0$$

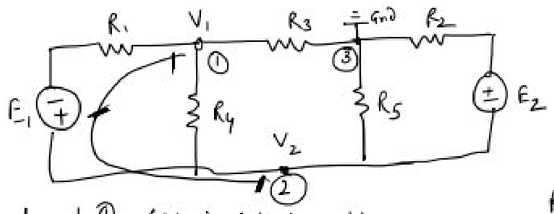
Rewrite

$$\boxed{-\frac{V_1}{R_3} + V_2\left(\frac{1}{R_3} + \frac{1}{R_2}\right) = -I_5 - \frac{E_3}{R_3} - \frac{E_2}{R_2}} \quad (2)$$

Matrix form

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_3} & -\frac{1}{R_3} \\ -\frac{1}{R_3} & -\left(\frac{1}{R_2} + \frac{1}{R_3}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \frac{E_1}{R_1} + I_4 - \frac{E_4}{R_4} + \frac{E_3}{R_3} \\ -I_5 - \frac{E_2}{R_2} - \frac{E_3}{R_3} \end{bmatrix}$$

*Case ③



① No. of nodes = 3: 2 non-ref, 1 ref.

Apply KCL at node ① (V_1 is at higher pot)

$$\frac{V_1 + E_1 - V_2}{R_1} + \frac{V_1 - V_2}{R_4} + \frac{V_1 - 0}{R_3} = 0$$

Rewrite eq.

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_4} + \frac{1}{R_3}\right) - V_2\left(\frac{1}{R_1} + \frac{1}{R_4}\right) = -\frac{E_1}{R_1}$$

$$\frac{V_1+E_1-V_2}{R_1} + \frac{V_1-V_2}{R_4} + \frac{V_1-0}{R_3} = 0$$

Rewrite

$$\begin{cases} V_1\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_3}\right) - V_2\left(\frac{1}{R_1}+\frac{1}{R_4}\right) = -\frac{E_1}{R_1} \\ -V_1\left(\frac{1}{R_1}+\frac{1}{R_4}\right) + V_2\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_5}+\frac{1}{R_2}\right) = \frac{E_1}{R_1}-\frac{E_2}{R_2} \end{cases}$$

Apply KCL at node ② ($V_2 \to$ higher pot.)

$$\frac{V_2-E_1-V_1}{R_1} + \frac{V_2-V_1}{R_4} + \frac{V_2-0}{R_5} + \frac{V_2+E_2}{R_2} = 0$$

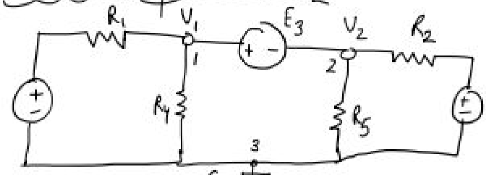
Matrix form

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_3} & -\left(\frac{1}{R_1}+\frac{1}{R_4}\right) \\ -\left(\frac{1}{R_1}+\frac{1}{R_4}\right) & +\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_5}+\frac{1}{R_2}\right) \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -\frac{E_1}{R_1} \\ \frac{E_1}{R_1} - \frac{E_2}{R_2} \end{bmatrix}$$

Case ③

* Case ④ Special Case

* Ideal voltage source b/w 2 non-ref nodes with zero internal resistance.



\+ Note:

Normal node analysis fails when ideal voltage source is connected b/w two non-ref. nodes. → sol ??

KCL at node ① (let $V_1 \to$ higher pot.)

$$\frac{V_1-E_1}{R_1} + \frac{V_1}{R_4} + \cancel{\frac{V_1-E_3-V_2}{0}} = 0$$

fails

method 1 (Supernode)

method 2 (Change ref. node) or Ref. node shifting method.

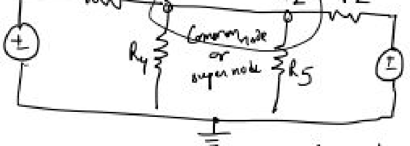
step① Express node voltage in term of voltage source (E_3)

$$\boxed{V_1 - V_2 = E_3} \quad \text{—①}$$

step② Replace volt. source (E_3) by its internal resistance.

* Int. resistance of ideal voltage source is zero

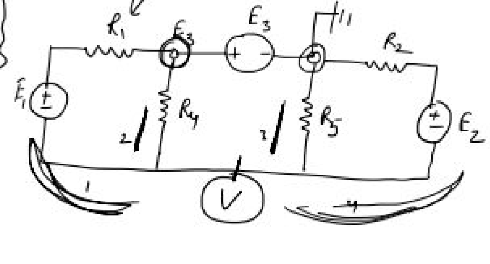
∴ to replace S.C which leads to supernode



Apply KCL at super node by maintaining the previous branch current

$$\boxed{\frac{V_1-E_1}{R_1} + \frac{V_1-0}{R_4} + \frac{V_2-0}{R_5} + \frac{V_2+E_2}{R_2} = 0} \quad ②$$

Solve eq. ① & ② $\xrightarrow{to}$ find V_1 & V_2



$$\underbrace{\frac{V+E_1-E_3}{R_1}}_{1} + \underbrace{\frac{V-E_3}{R_4}}_{2} + \underbrace{\frac{V-0}{R_5}}_{3} + \underbrace{\frac{V+E_2-0}{R_2}}_{4} = 0$$

$$\boxed{V\left[\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_5}+\frac{1}{R_2}\right] = \frac{-E_1+E_3}{R_1} + \frac{E_3}{R_4} - \frac{E_2}{R_2}} \quad ①$$

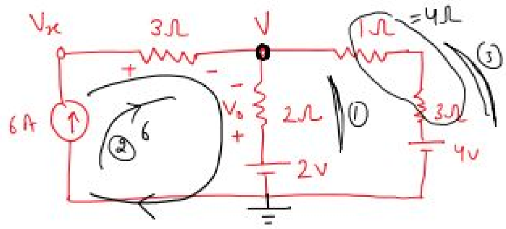
Here only 1 eq. is sufficient to solve the ckt.

Note* There is no restriction in selection of ref. node i.e any

Solve eq. (1) & (2) to find V_1 & V_2

Note* There is no restriction in selection of ref. node i.e any node can be selected as ref. node.

Question (1)



Find (1) V_o (2) P_{6A} (3) P_{4V} = ??

Sol^n (1) Apply KCL

$$\underbrace{\frac{V-2}{2}}_{①} + \underbrace{(-6)}_{②} + \underbrace{\frac{V-4}{4}}_{③} = 0$$

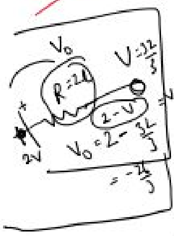
∵ current flowing from lower to higher (opposite)

Rewrite

$$V\left[\frac{1}{2}+\frac{1}{4}\right] = 6+1+1$$

$$V\left[\frac{3}{4}\right] = 8$$

$$V = \frac{32}{3}\text{ volts}$$

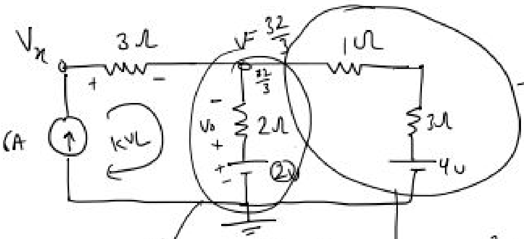


$$V_o = 2 - V = 2 - \frac{32}{3} = -\frac{26}{3}\,V$$

(ii) $P_{6A} = ?$

$$P_{6A} = V \cdot I = V_x(6)$$

Apply KVL in LHS loop

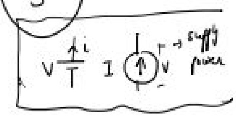


ohm's law $V = IR = (6)(3) = 18V$

$$V_x - 18 - \frac{26}{3} - 2 = 0$$

$$V_x = \cancel{\frac{34}{3}} \quad \frac{86}{3}\text{ volts}$$

$$P_{6A} = V_x \times 6 = \frac{86}{3} \times 6 = 172 \text{ supply power}$$



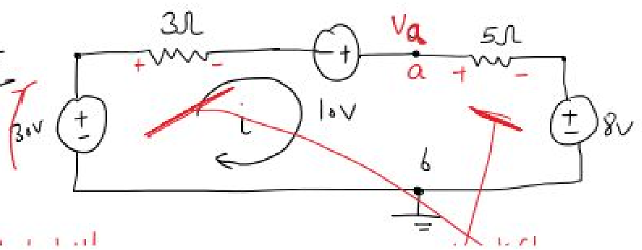
(iii) $P_{4V} = V \times I = 4 \times \left(\frac{V-4}{4}\right) = \frac{80}{3}$ watt → Absorbing power

$V \to 32/3$

{ ohm's law $I = \frac{V}{R}$ }

(iv) $P_{2V} = V \times I = 2 \times \left(\frac{V}{R}\right) = 2 \times \left(\frac{V-2}{2}\right) = 2 \times \left(\frac{32}{3} - 2\right) = \frac{26}{3}$ Absorbing power

* **Question:-**



Find $V_{ab} \to$??

$i \to ?$

$i \to ?$

Sol^n Apply KVL

$$30 - 3i + 10 - 5i - 8 = 0$$
$$40 - 8i - 8 = 0$$
$$32 - 8i = 0$$
$$\boxed{i = 4A}$$

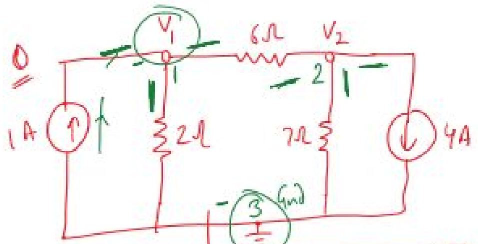
Apply KCL (At node a)

$$\frac{V_a - 8}{5} + \frac{V_a - 30 - 10}{3} = 0$$
$$V_a\left[\frac{1}{5} + \frac{1}{3}\right] = \frac{8}{5} + \frac{40}{3} \qquad \boxed{V_a = 28\ \text{volts}}$$
$$i = \frac{V}{R} = \frac{30 + 10 - 8}{8} = \frac{32}{8} = \boxed{4\ \text{Amp}}$$

$\cancel{V_a = V_a = 28 = 28\ \text{volts}}$

$\boxed{i = 4\ \text{amp}}$

$$4A = \frac{V_a - 8}{5}$$
$$V_a = 20 + 8 \Rightarrow \boxed{V_a = 28\ \text{volts}}$$



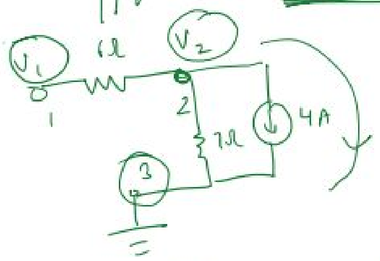
Sol^n Find V_1 & V_2 = ??

Total nodes → 3
→ 1 ref. (3)
2 nonref. (1, & 2)

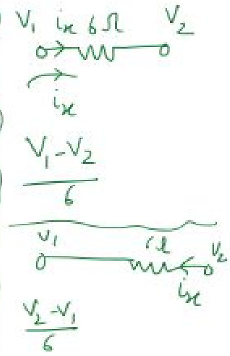
Apply KCL at ①

$$-1 + \frac{V_1 - 0}{2} + \frac{V_1 - V_2}{6} = 0$$
$$V_1\left[\frac{1}{2} + \frac{1}{6}\right] - V_2\left[\frac{1}{6}\right] = 1 \longrightarrow \frac{2}{3}V_1 - \frac{V_2}{6} = 1$$
$$\boxed{4V_1 - V_2 = 6} \quad ①$$

* Apply KCL at node 2



$$\frac{V_2 - V_1}{6} + \frac{V_2 - 0}{7} + 4 = 0$$
$$-\frac{V_1}{6} + V_2\left[\frac{1}{6} + \frac{1}{7}\right] + 4 = 0$$
$$\boxed{7V_1 - 13V_2 = 168} \quad ②$$



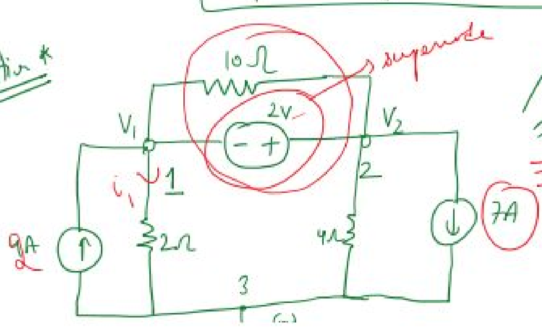
$$\frac{V_1 - V_2}{6}$$

$$\frac{V_2 - V_1}{6}$$

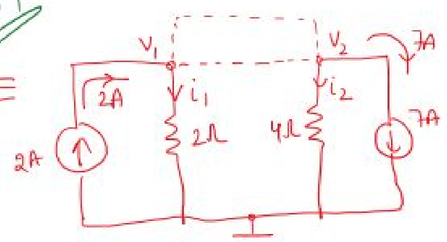
Solve eq ① & ②

$$\boxed{V_1 = -2V \ \& \ V_2 = -14V}$$

Question*



Sol^n



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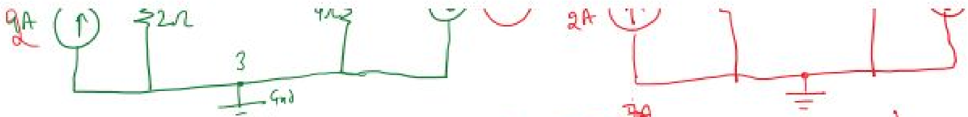
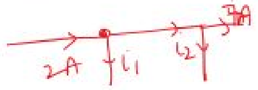


Supernode contains 2V source, node 1, node 2 & 10 Ω resistor.



Apply KCL at supernode

$$2 = i_1 + i_2 + 7$$

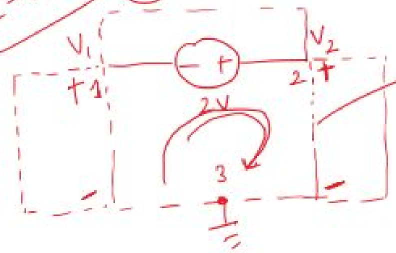
Rewrite

$$2 = \frac{V_1 - 0}{2} + \frac{V_2 - 0}{4} + 7$$

$2V_1 - V_2 = 20$ (1)

$V_2 = 20 - 2V_1$ (A)

*



KVL

$V_1 + 2 - V_2 = 0$

$V_2 = V_1 + 2$ (B)

$V_1 = -\frac{22}{3}$

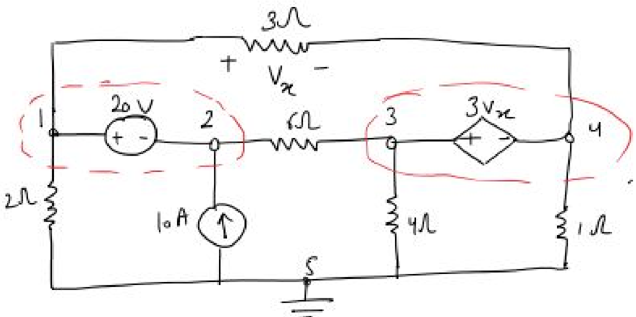
$V_1 = -7.333$ Volt

$V_2 = -7.333 + 2$

$V_2 = -5.333$ Volt

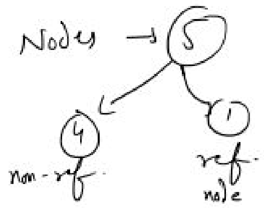
*

Question:



Find node voltages in the circuit.

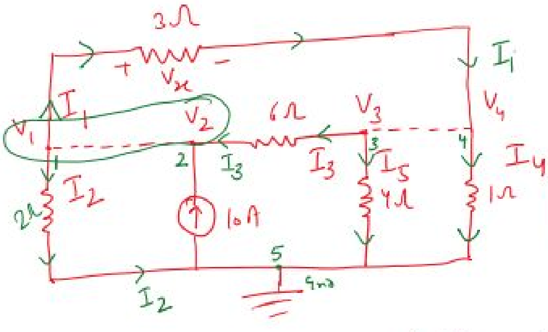
Nodes → 5

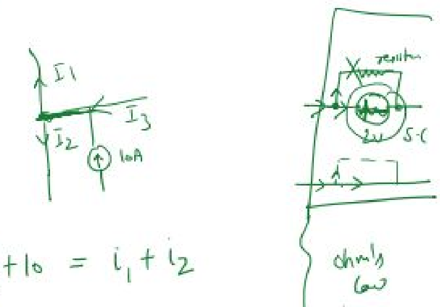


Sol^n

Node 1 & 2 $\xrightarrow{\text{form a}}$ Supernode

Node 3 & 4 $\xrightarrow{\text{form a}}$ Supernode. ∴ Apply KCL at Supernode 1-2.





$$i_3 + 10 = i_1 + i_2$$

Rewrite

$$\frac{V_3 - V_2}{} + 10 = V_1 - V_4 + V_1 - 0$$

ohm's law $i = \frac{V}{R}$

I_2

Rewrite $\frac{V_3-V_2}{6}+10 = \frac{V_1-V_4}{3}+\frac{V_1-0}{2}$ $\quad 6\Omega \quad i=\frac{V}{R}$

$$\boxed{5V_1+V_2-V_3-2V_4=60} \quad (1)$$

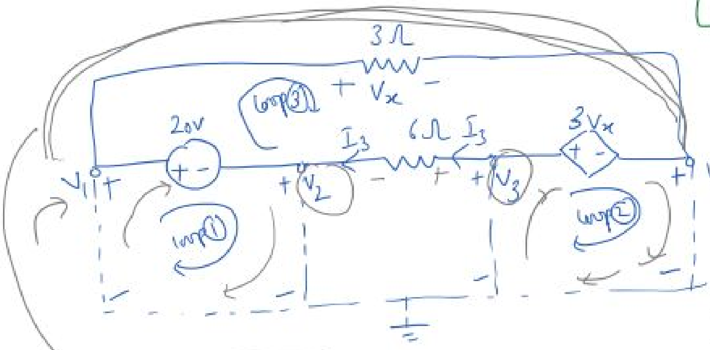
Apply KCL at supernode ③ & ④

$i_1 = i_3+i_4+i_5$

$$\frac{V_1-V_4}{3} = \frac{V_3-V_2}{6}+\frac{V_4-0}{1}+\frac{V_3-0}{4}$$

Rewrite

$$\boxed{4V_1+2V_2-5V_3-16V_4=0} \quad (2)$$



$$\boxed{V_x = V_1-V_4}$$

Apply KVL in loop ①

$V_1-20-V_2=0$

$$\boxed{V_1-V_2=20} \quad (A)$$

Apply KVL in loop ②

$V_3-3V_x-V_4=0$

$$\therefore \boxed{3V_1-V_3-2V_4=0} \quad (B)$$

Apply KVL in loop ③

$$20-V_x+3V_x-6I_3=0 \quad (C)$$

As we know $6I_3 = V_3-V_2$ — (D)

Rewrite

$6I_3 = V_3-V_2$
$V_x = V_1-V_4$

$20-V_1+V_4+3V_1-3V_4-V_3+V_2=0$

$$\boxed{2V_1+V_2-V_3-2V_4=-20} \quad (3)$$

from eq. (A) $V_2 = V_1-20$

$\therefore$

$5V_1+V_2-V_3-2V_4=60$ — (1)

$5V_1+V_1-20-V_3-2V_4=60$

$$\boxed{6V_1-V_3-2V_4=80} \quad (4)$$

$$4V_1+2V_2-5V_3-16V_4=0 \quad (2)$$

1
2
3
4
A
B
C
D
(5)

Solve eq ④ & ⑤

$$4V_1 + 2V_2 - 5V_3 - 16V_4 = 0 \quad (2)$$

$$4V_1 + 2V_1 - 40 - 5V_3 - 16V_4 = 0$$

$$\boxed{6V_1 - 5V_3 - 16V_4 = 40} \quad (5)$$

Solve eq (4) & (5)

$$\begin{bmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \end{bmatrix} \begin{bmatrix} V_1 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 80 \\ 40 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -2 \\ 6 & -1 & -2 \\ 6 & -5 & -16 \\ 3 & -1 & -2 \\ 6 & -1 & -2 \end{vmatrix} = 48 + 60 + \cancel{12} - \cancel{12} - 30 - 96 = 108 - 126$$

$$\boxed{\Delta = -18}$$

$$\Delta_1 = \begin{vmatrix} 0 & -1 & -2 \\ 80 & -1 & -2 \\ 40 & -5 & -16 \\ 0 & -1 & -2 \\ 80 & -1 & -2 \end{vmatrix} = 0 + 800 + \cancel{80} - \cancel{80} - 0 - 1280$$

$$\boxed{\Delta_1 = -480}$$

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{-480}{-18}$$

$\therefore$ $V_1 = +26.667$ volts

$$\Delta_3 = \begin{vmatrix} 3 & 0 & -2 \\ 6 & 80 & -2 \\ 6 & 40 & -16 \\ 3 & 0 & -2 \\ 6 & 80 & -2 \end{vmatrix} = -3120$$

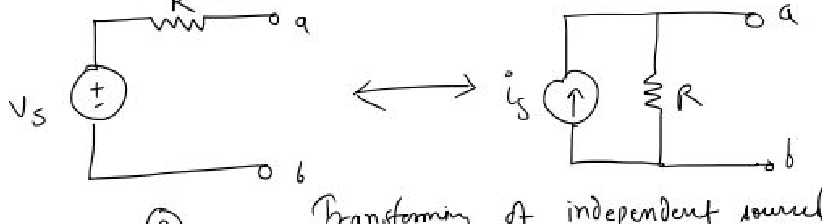
$$V_3 = \frac{\Delta_3}{\Delta} = \frac{-3120}{-18}$$

$V_3 = 173.333$ volt

||ly $V_4 = -46.667V$ $\quad \{V_2 = V_1 - 20$

Put value of V_1, V_3 & V_4 $\therefore$ $\rightarrow V_2 = 26.667 - 20 = 6.667$ volts

* Source Transformation: → Another tool for simplify the ckt.
↳ is a process of replacing a voltage source V_s in series with a resistor R by a current source i_s in parallel with a resistor R or vice-versa.



(a) Transforming of independent source (b)

* Both ckt (a) & (b) are equivalent provided they have

(a) Transforming of independent sources

* Both ckt (a) & (b) are equivalent, provided they have voltage & current relation at terminal a-b.

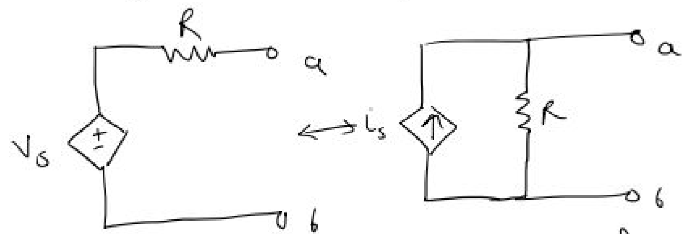
* if source are turned off $\xrightarrow{\therefore}$ eq. Resistance $\xrightarrow[\text{both ckt}]{\text{in}}$ 'R'.

* if terminal a-b short ckt $\xrightarrow[\text{current}]{\text{S.C}}$ $i_{sc} = \frac{V_s}{R}$ (a)

& $i_{sc} = i_s$ (b)

$$\boxed{\frac{V_s}{R} = i_s}$$

* Transformation of dependent source:



Transformation of dependent source.

Imp

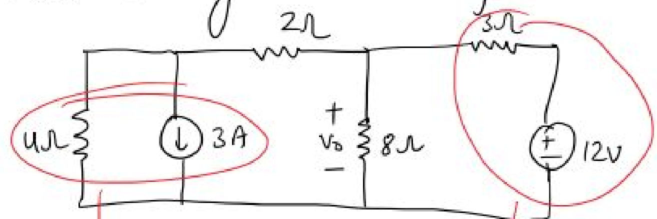
* Note in both figures arrow of current source is directed towards the positive terminal of voltage source.

* Source Transformation $\xrightarrow[\text{possible}]{\text{is not}}$ (i) when $R = 0$ {case of Ideal Voltage source}

(Practically $R \neq 0$ non-ideal Voltage source)

(ii) $\xrightarrow[\text{possible.}]{\text{is not}}$ when ideal current source with $R = \infty$, can't be replaced.

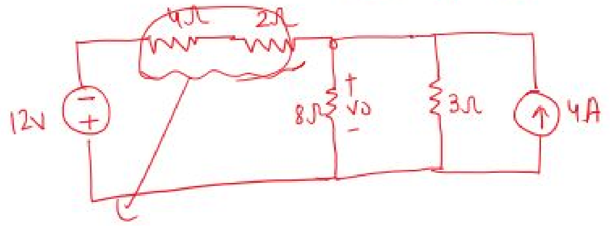
Q* Find V_o using source transformation.



Sol^n 4Ω & 3A & 3Ω & 12V

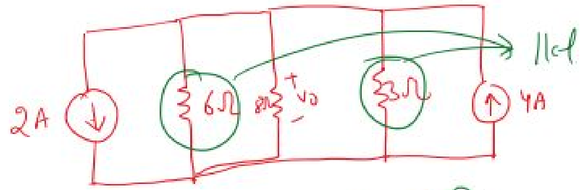
Sol: $4\Omega \parallel 3A$ $\quad$ $3\Omega \parallel 12V$

{ ohm's law }

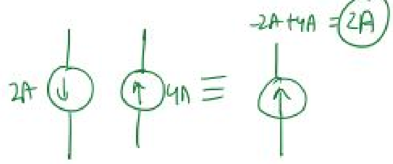


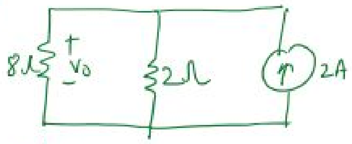
$4+2 = 6\Omega \parallel 12V$

{ ohm's law $V = IR$ }



$6 \parallel 3 \longrightarrow \frac{6\times 3}{6+3} = \frac{18}{9} = 2\Omega$





* Current divider Rule

Actual current source, opposite resistor

$i = 2 \times \frac{2\Omega}{8+2} = \frac{4}{10} = 0.4\,amp$

{ ohm's law $i = \frac{V}{R}$ }

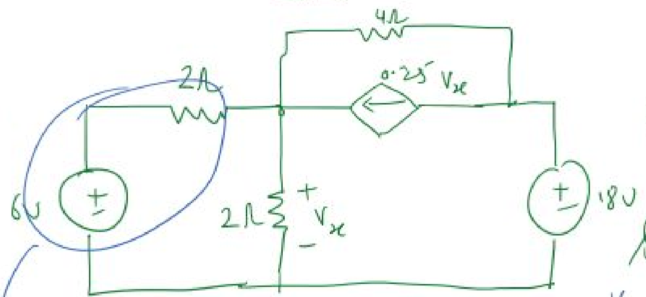
$V_o = IR = (0.4)(8) = 3.2$ volts

OR

* $(8\Omega \parallel 2\Omega) \rightarrow$ in ||el. $\rightarrow$ voltage same.

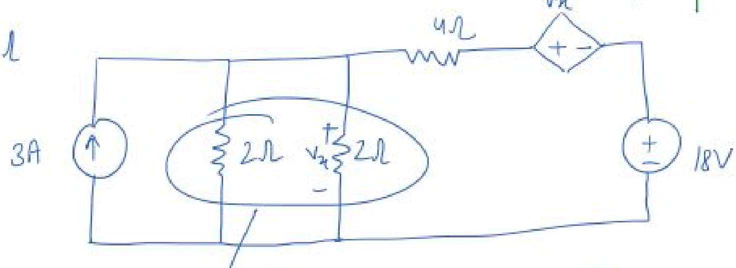
$V_o = \left(\frac{8\times 2}{8+2}\right)(2A) = \frac{16}{10}\times 2 = \frac{32}{10} = 3.2$ volts

Question



Find V_x using Source Transformation.

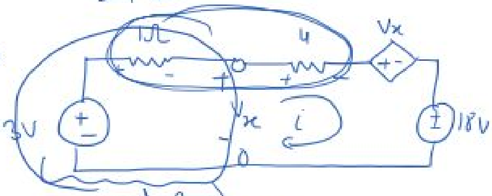
Sol $6V \parallel 2\Omega$



|| el $\quad \frac{2\times 2}{...} = \frac{4}{...} = 1\Omega$

$+ 11 cl$ $\qquad \frac{2\times 2}{2+2} = \frac{4}{4} = 1\Omega$

Now $1\Omega + 3A \rightarrow 11 cl \rightarrow$ Voltage



Apply KVL in loop

$$3 - 5i - V_x - 18 = 0$$

$$\cancel{3} - 5i - \cancel{3} + i - 18 = 0$$

$$-4i = 18 \quad i = 4.5 \text{ amp}$$

loop $\Rightarrow$ KVL

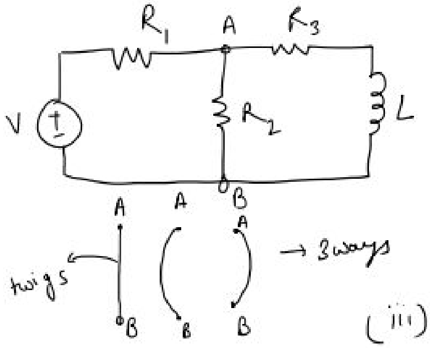
$$-V_x + 3 - 1i = 0$$

$$V_x = 3 - i$$

$$V_x = 3 - i$$
$$= 3 - (-4.5)$$
$$= 3 + 4.5 = 7.5 \text{ volts}$$

Graph Theory :-

→ **Tree** :- a set of branches which together connects all the nodes of graphs without forming a loop.



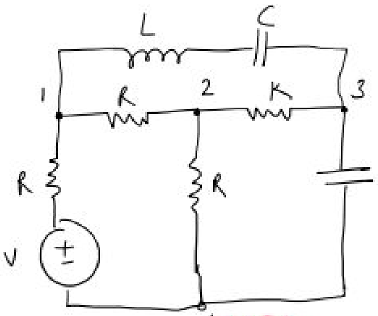
Prop. of trees :- ①

(i) All nodes of the graph must be present in the tree.

(ii) Branches of tree (twigs) must be equal to $n-1$. where n = no. of nodes.

(iii) Twigs of tree must be connected in such a manner that no close loop is formed.

Question :-



Graph → ?

Tree → ?

Co-tree → ? } link

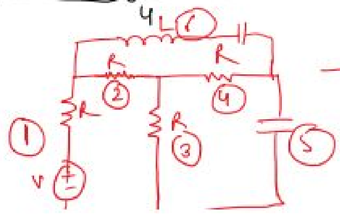
nodes →

branches →

Sl^n branches = 6

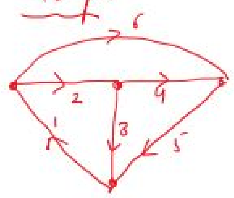
Nodes = n = 4

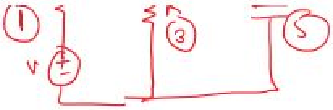
branches of tree (twigs) = $n-1$ = 3



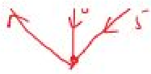
→ (branch) → Graph

Graph

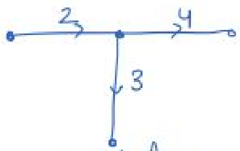




Graph



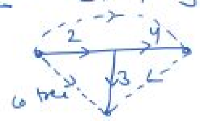
Tree



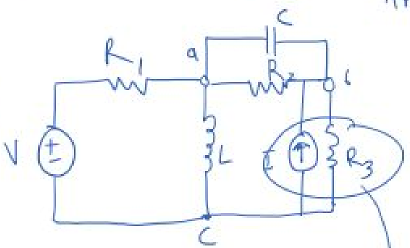
Tree links → [2, 3, 4]

Co-Tree link → [1, 5, 6]

Co-Tree → set of Remaining branches of a graph after forming tree is known as co-tree or sub graph



Q



Graph → ?

tree → ?

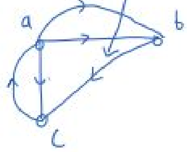
Co-Tree → ?

nodes → ?

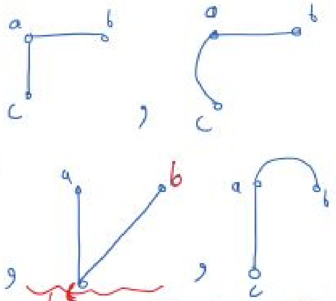
branches → ?

Sol Nodes → 3 (a, b, c)

Graph:



Trees :-



nodes → 3

twigs → n-1 = 2

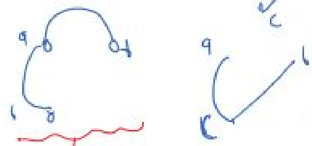
Co-Tree

cotree

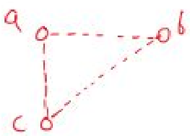
tree

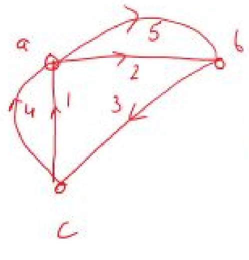
co-tree

correct

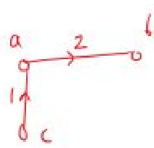


Co-tree

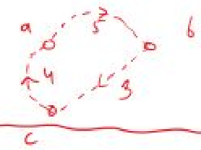




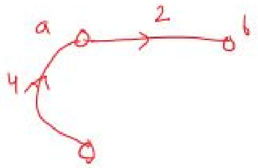
Tree link [1, 2]



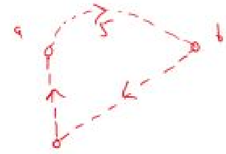
Co tree link [3, 4, 5]



Tree



Co-Tree



Imp Tree → Never forms a closed loop

* Incidence Matrix $[A_{ij}]$ → This matrix is drawn b/w nodes & branches.

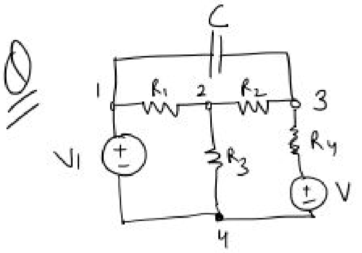
nodes ↓ \ branches →	b_1	b_2	b_3	- - -
n_1				
n_2				
⋮				

* 1 → 2 incoming, for node 2 ⇒ (−1)

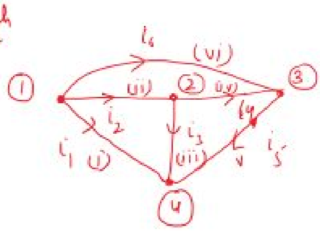
* ← 2 outgoing ⇒ (+1)

* 1   2 O.C ⇒ (0)

Q



Sol^n Graph



Incidence Matrix

nodes ↓ \ branches →	(i)	(ii)	(iii)	(iv)	(v)	(vi)
1	+1	+1	0	0	0	+1
2	0	−1	+1	+1	0	0
3	0	0	0	−1	+1	−1
4	−1	0	−1	0	−1	0

Prop of Incidence Matrix

(i) Sum of entries in any column is zero.

(ii) Determinant of incidence matrix of a linear loop → zero.

(iii) Rank of incidence matrix of connected graph → n−1

* Reduced Incidence Matrix:- $[A]_{n-1 \times b}$

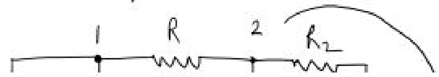
By eliminating one row from complete incidence matrix.

[A]

$$[A] = \begin{bmatrix} +1 & +1 & 0 & 0 & 0 & +1 \\ 0 & -1 & +1 & +1 & 0 & 0 \\ 0 & 0 & 0 & -1 & +1 & -1 \end{bmatrix}$$

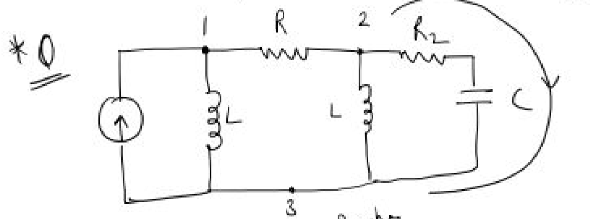
* Number of possible tree:- Det $\left[[A] [A^T] \right]$

*Q

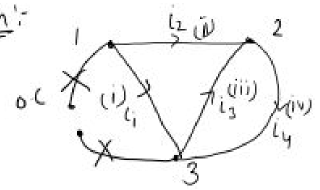


$\text{Det}\left[[A]\,[A']\right]$

*①



Graph:-



node ↓ \ Branches →	(i)	(ii)	(iii)	(iv)
n_1 1	+1	+1	0	0
2	0	−1	−1	+1
3	−1	0	+1	−1

$[A]_{n \times 6}$

Reduced incidence Matrix

$$\text{Reduce } [A]_{n-1 \times 6} = [A] = \begin{bmatrix} +1 & +1 & 0 & 0 \\ 0 & -1 & -1 & +1 \end{bmatrix}_{2\times4} \therefore [A^T] = \begin{bmatrix} +1 & 0 \\ +1 & -1 \\ 0 & -1 \\ 0 & +1 \end{bmatrix}_{4\times2}$$

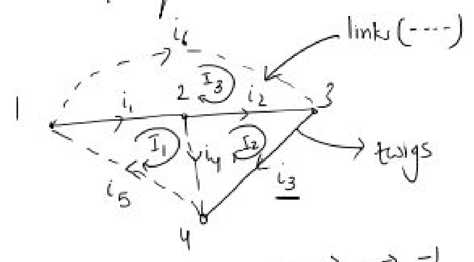
$$\text{No. of possible trees} = \text{Det}\left[[A]\,[A^T]\right]$$

$$= \text{Det}\begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}_{2\times2}$$

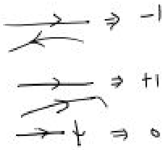
$$\text{Possible trees} = (2\times3) - (-1\times-1) = 6-1 = 5$$

* TIE-SET Matrix [B] → drawn b/w branch current & loop current

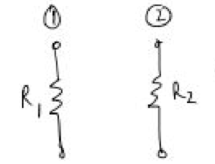
loop curr. ↓ \ B-curr →	i_1	i_2	i_3	i_4	i_5	i_6
I_1	+1	0	0	+1	+1	0
I_2	0	+1	+1	−1	0	0
I_3	−1	−1	0	0	0	+1



$$\text{Number of links} = 6 - 3 \rightarrow (n-1) \text{ nodes}$$
$$= 6 - 3 = 3$$

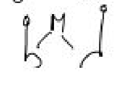


* Branch Impedance Matrix → $[Z]_b$ → Square matrix of branch by branch.

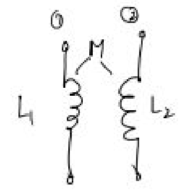


$Z_{11} = R_1$

$Z_{12} = 0$



$Z_{11} = L_1$



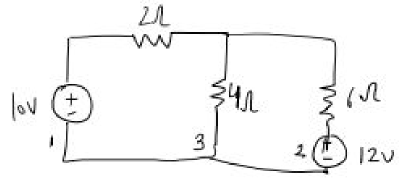
$Z_{11} = L_1$

$Z_{22} = L_2$

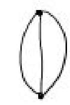
$Z_{12} = M$

* Impedence Matrix $[Z] = [B][Z_b][B^T]$

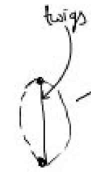
Q



Sol^n Graph: Branches → 3 nodes 2

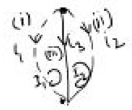


Tree →



→ links = b − T = 3 − 1 = 2

F-tree



Tie-set Matrix (B) →

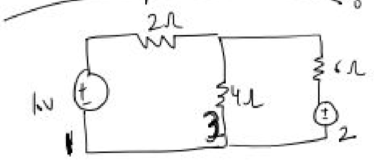
loop current ↓ / branch current →	i_1	i_2	i_3
I_1	−1	0	+1
I_2	0	+1	−1

$$[B] = \begin{bmatrix} -1 & 0 & +1 \\ 0 & +1 & -1 \end{bmatrix} \quad [B^T] = \begin{bmatrix} -1 & 0 \\ 0 & +1 \\ +1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix}$$

Branch Impedence Matrix : → $[Z_b] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3\times3}$

Z = R



$$[Z] = [B][Z_b][B^T]$$

$$= \begin{bmatrix} -1 & 0 & +1 \\ 0 & +1 & -1 \end{bmatrix}_{2\times3} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 4 \end{bmatrix}_{3\times3} \begin{bmatrix} -1 & 0 \\ 0 & +1 \\ +1 & -1 \end{bmatrix}_{3\times2}$$

$$[Z] = \begin{bmatrix} -1 & 0 & +1 \\ 0 & +1 & -1 \end{bmatrix}_{2\times3} \begin{bmatrix} -2 & 0 \\ 0 & 6 \\ 4 & -4 \end{bmatrix}_{3\times2} = \begin{bmatrix} 6 & -4 \\ -4 & 10 \end{bmatrix}_{2\times2}$$

* Equation of KVL : $[Z][I_e] = [B]\left[[V_s] - [Z_b][I_s]\right]$

4Ω 3Ω

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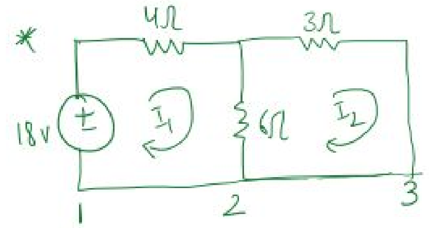
* Equation of KVL: $[Z][I_e] = [B]\left[[V_s] - [Z_b][I_s]\right]$

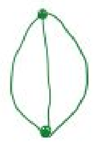
* 

Find I_1 & I_2 in the ckt using graph theory

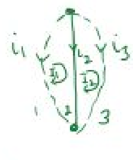
Node → 2

twigs = 1

Sol^n Graph:-



Tree:-



B = Tie set Matrix

Link curr ↓ / Branch curr →	i_1	i_2	i_3
I_1	-1	+1	0
I_2	0	-1	+1

$$B = \begin{bmatrix} -1 & +1 & 0 \\ 0 & -1 & +1 \end{bmatrix}_{2\times3}$$

$$B^T = \begin{bmatrix} -1 & 0 \\ +1 & -1 \\ 0 & +1 \end{bmatrix}_{3\times2}$$

Z_b = Branch Impedance Matrix

$$Z_b = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{bmatrix}_{3\times3}$$

$$Z = [B][Z_b][B^T] = \begin{bmatrix} -1 & +1 & 0 \\ 0 & -1 & +1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ +1 & -1 \\ 0 & +1 \end{bmatrix}$$

$$\boxed{Z = \begin{bmatrix} 10 & -6 \\ 6 & 9 \end{bmatrix}_{2\times2}} \quad -(1)$$

Equation of KVL

$$[Z][I_e] = [B]\left[[V_s] - \cancel{[Z_b][I_s]}\right]$$

→ source curr not availble ∴ (0)

$$I_e = \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} 10 & -6 \\ -6 & 9 \end{bmatrix} [I_e] = \begin{bmatrix} -1 & 1 & 0 \\ 0 & - & +1 \end{bmatrix}_{2\times3} \begin{bmatrix} 18V \\ 0 \\ 0 \end{bmatrix}_{3\times1}$$

$$\begin{bmatrix} 10 & -6 \\ -6 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -18 \\ 0 \end{bmatrix}$$

$$10I_1 - 6I_2 = -18$$
$$-6I_1 + 9I_2 = 0$$
→ I_1 & I_2

$$60 I_1 - 36 I_2 = -108$$
$$+60 I_1 + 90 I_2 = 0$$

$$+54 I_2 = -108$$

$$I_2 = \frac{+108}{54} = -2A$$

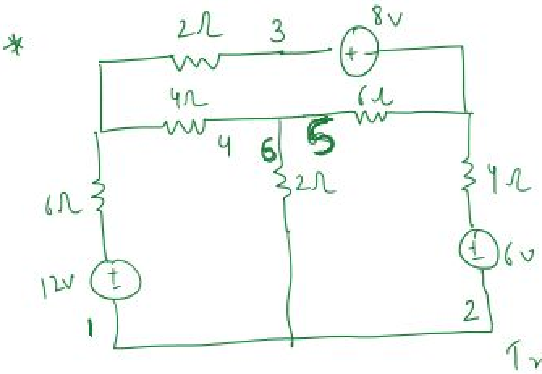
~~$I_2 = 4.028A$~~

$$10 I_1 - 6(-2) = -18$$
$$I_1 = \frac{-30}{10}$$
$$\boxed{I_1 = -3A}$$

$T(3)4 I_2 = -108$

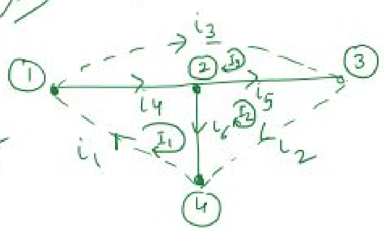
$\cancel{I_1 = +1.028A}$ | $I_1 = -3A$

$\cancel{60 I_1 - 36(2) = -108 \quad 60 I_1 = -108 + 72 \quad I_1 = -\frac{4}{6}}$

* 

Calculate I_1 & I_2 & I_3 = ?

Sol^n Nodes → 4   twigs = n − 1 = 3

Tree :- 

	i_1	i_2	i_3	i_4	i_5	i_6
I_1	1	0	0	1	0	1
I_2	0	+1	0	0	+1	−1
I_3	0	0	+1	−1	−1	0

$$B = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & +1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix}_{3\times6}$$

$$B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & +1 & -1 \\ +1 & -1 & 0 \end{bmatrix}_{6\times3} \qquad Z_b = \begin{bmatrix} 6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}_{6\times6}$$

$$[Z] = [B][Z_b][B^T]$$

$$[Z] = \begin{bmatrix} 12 & -2 & -4 \\ -2 & +12 & -6 \\ -4 & -6 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 12 & -2 & -4 \\ -2 & +12 & -6 \\ -4 & -6 & 12 \end{bmatrix}_{3\times3} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}_{3\times1} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 0 \end{bmatrix}_{3\times6} \begin{bmatrix} 12 \\ 6 \\ 8 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{6\times1}$$

$$\begin{bmatrix} 12 \\ 6 \\ 8 \end{bmatrix}_{3\times1}$$

$12 I_1 - 2 I_2 - 4 I_3 = 12$ — (1)

$-2 I_1 + 12 I_2 - 6 I_3 = 6$ — (2)

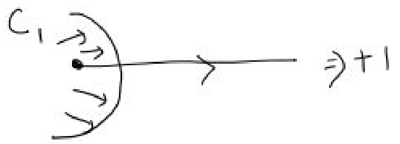
$-4 I_1 - 6 I_2 + 12 I_3 = 8$ — (3) I_1 & I_2 & I_3

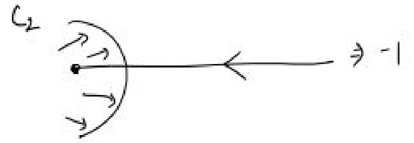
*

* Cut-set Matrix :- [Q] → Drawn b/w branch voltage & cut-set voltage.

* No. of cut-set = No. of twigs
* Cut-set never cut two twigs.

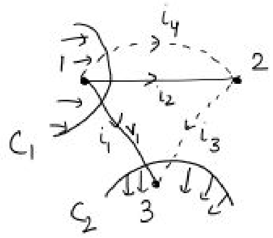
Cut-set voltage ↓ \ Branch voltage →	V_1	V_2	V_3	- - - - -
C_1				
C_2				
C_3				
⋮				





e.g.

Tree:



Branches = 4
↳ voltage V_1, V_2, V_3, V_4

Nodes = 3 = n

Twigs = n-1 = 3-1 = 2

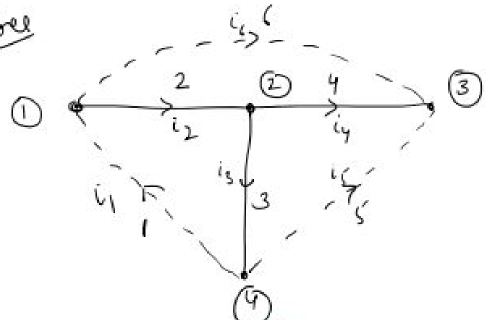
No. of cut-set = No. of twigs = 2

Cut-set Voltage ↓ \ Branch voltage →	V_1	V_2	V_3	V_4
C_1	+1	+1	0	+1
C_2	+1	0	+1	0

$$Q = \begin{bmatrix} +1 & +1 & 0 & +1 \\ +1 & 0 & +1 & 0 \end{bmatrix}$$

e.g. Tree

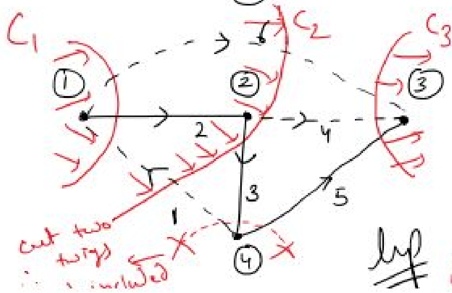
*



Branch = 6
nodes = 4

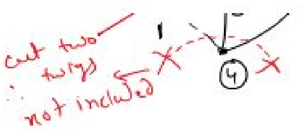
twigs = 3
↳ cut-set = 3

*



Branch = 6
nodes = 4
twigs = 3 = Cut-set

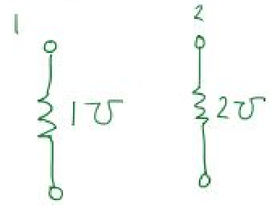
Imp Cut-set never cut two twigs



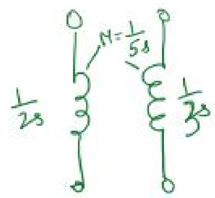
imp: Cut-set never cut two twigs

	V_1	V_2	V_3	V_4	V_5	V_6
C_1	-1	+1	0	0	0	+1
C_2	-1	0	+1	+1	0	+1
C_3	0	0	0	+1	+1	+1

* Branch Admittance Matrix :- (y_b)



$$y_b = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}_{2\times 2} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$



$$y_b = \begin{bmatrix} \frac{1}{2s} & \frac{1}{5s} \\ \frac{1}{5s} & \frac{1}{3s} \end{bmatrix}$$

* Admittance Matrix :- [Y]

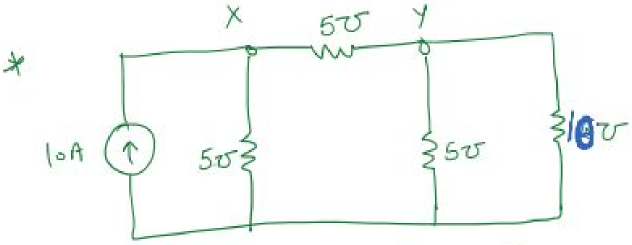
$$[Y] = \underbrace{[Q]}_{\text{cut-set}}\ \underbrace{[y_b]}_{\text{Branch Admittance Matrix}}\ \underbrace{[Q^T]}_{\text{cut-set Transpose}}$$

* Equation of KCL :-

$$\text{KVL} \rightarrow [Z][I_l] = [B][V_s] - [Z_b][I_s]$$

$$\text{KCL} \rightarrow \underbrace{[Y]}_{\text{Admittance}}\ \underbrace{[V_t]}_{\text{terminal voltage}} = \underbrace{[Q]}_{\text{cut-set Matrix}} \left[\underbrace{[I_s]}_{\text{source current matrix}} - \underbrace{[y_b]}_{\text{Branch Admittance}} \underbrace{[V_s]}_{\text{source voltage Matrix}} \right]$$

Admittance Matrix — terminal voltage — cut-set Matrix — Branch Admittance

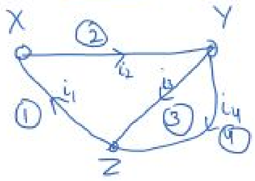
* 

Find voltage at node X & Y using KCL Graph theory.

Sol^n

Graph

Nodes ⇒ 3

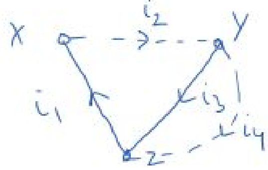


Branches

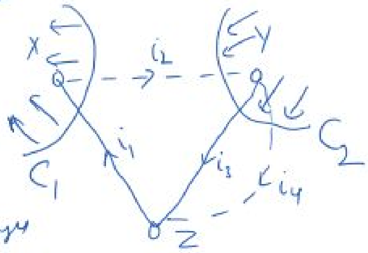
* current source
↳ ||al resistance
∴ it will be considered one branch

Tree

Nodes = 3 → twigs = 2 → cut-set = 2



Cut-set



$$Q = \begin{bmatrix} +1 & -1 & 0 & 0 \\ 0 & -1 & +1 & +1 \end{bmatrix}_{2\times 4}$$

Branch voltages →, cut-set voltage ↓

	V_1	V_2	V_3	V_4
C_1	+1	−1	0	0
C_2	0	−1	+1	+1

$[Q] = $ ∴

$$[Q^T] = \begin{bmatrix} 1 & 0 \\ -1 & -1 \\ 0 & +1 \\ 0 & +1 \end{bmatrix}$$

∵ 4 branches

$$[Y] = [Q]\,[y_b]\,[Q^T]$$

$$= \begin{bmatrix} +1 & -1 & 0 & 0 \\ 0 & -1 & +1 & +1 \end{bmatrix}_{2\times4} \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 10 \end{bmatrix}_{4\times4} \begin{bmatrix} +1 & 0 \\ -1 & -1 \\ 0 & +1 \\ 0 & +1 \end{bmatrix}_{4\times2}$$

$$[Y] = \begin{bmatrix} 10 & 5 \\ 5 & 20 \end{bmatrix}_{2\times2}$$

$$[Y][V_t] = [Q]\left([I_s] + [y_b][V_s]\right)$$

$\times 0$ ∵ no voltage source

$$\begin{bmatrix} 5 & 20 \end{bmatrix}_{2\times2}$$

$$[I_s] = \begin{bmatrix} -10 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{4\times1} \qquad [V_s] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{4\times1}$$

$$[Y][V_t] = [B][I_s] - \underbrace{[Y_s][V_s]}_{\times 0}$$

∵ no voltage source

$$\begin{bmatrix} 10 & 5 \\ 5 & 20 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}_{2\times4} \begin{bmatrix} -10 \\ 0 \\ 0 \\ 0 \end{bmatrix}_{4\times1}$$

OR

$$\begin{bmatrix} 10 & 5 \\ 5 & 20 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix}_{2\times1} \begin{bmatrix} & \\ & \end{bmatrix}_{2\times2}$$

$$\begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \end{bmatrix} \begin{bmatrix} 20 & -5 \\ -5 & -10 \end{bmatrix}$$

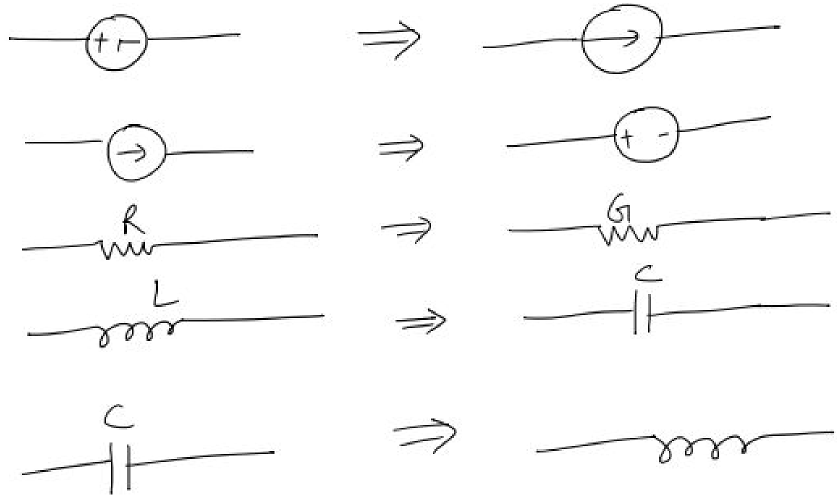
$10V_x + 5V_y = -10$ ①

$5V_x + 20V_y = 0$ ②

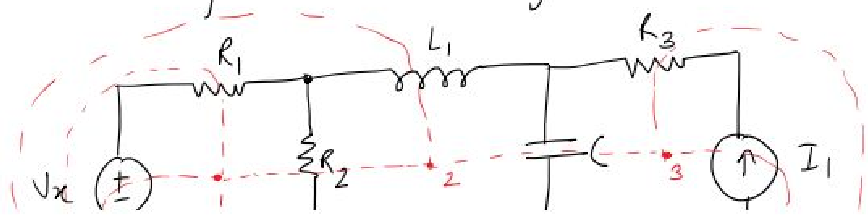
V_x & V_y

$$V_x = -8/7$$
$$V_y = 2/7$$

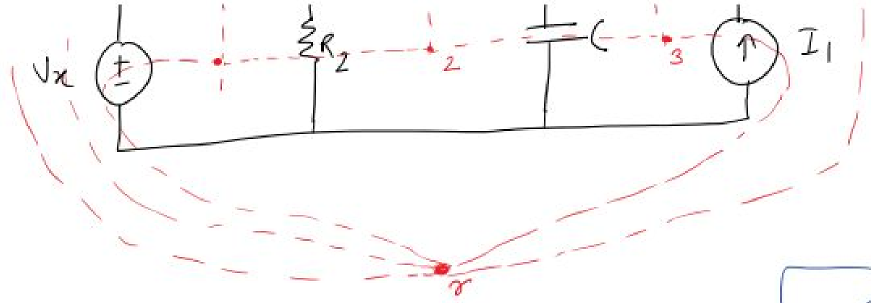
* Concept of Duality

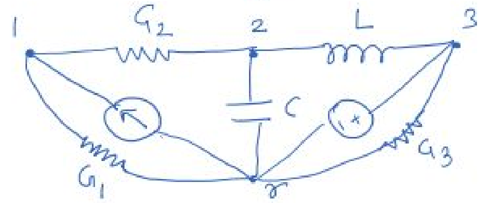


Q Dual ckt for the Given ckt by:-



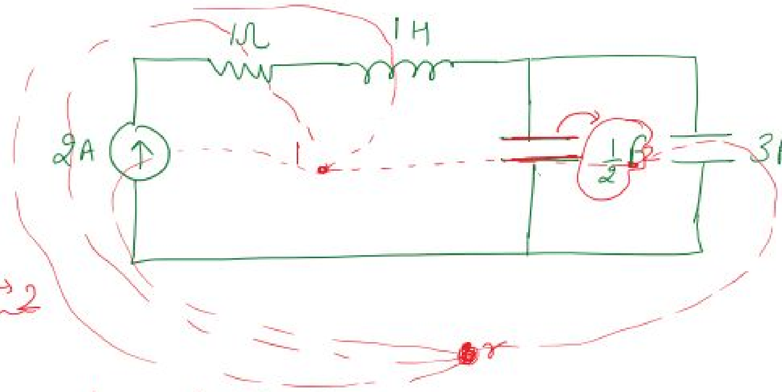
Sol^n

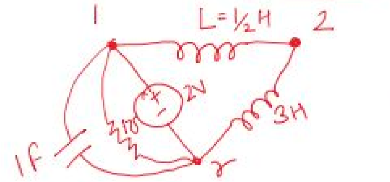


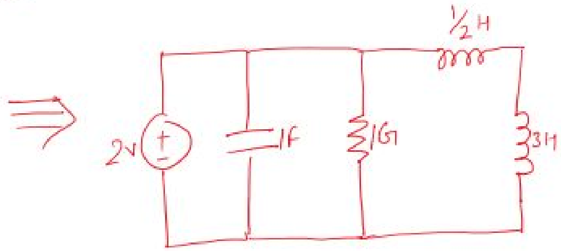


* Q Draw dual network of the given ckt.

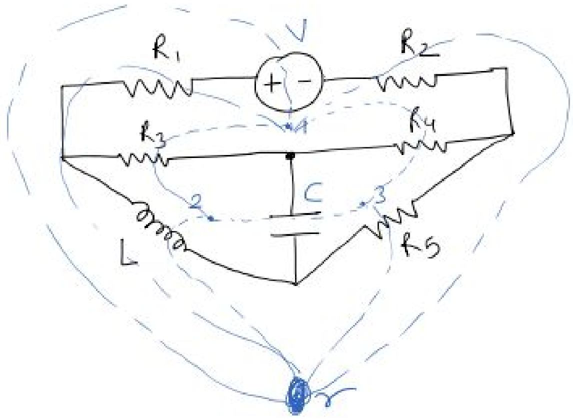
Nodes → 2



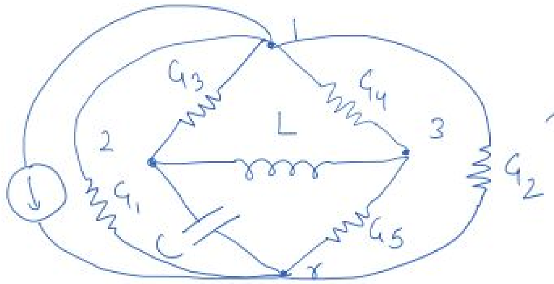




* Q Draw Dual N/ω



Sol^n Nodes → 4 → 1-ref.
3 non-ref.



→ Dual n/ω.

Q Draw oriented Graph from following matrix ??

Sol cut-set Matrix

cut-set matrix ↓ / branch voltage →	1	2	3	4	5	6	7
	twigs				links		
C_1	1	0	0	0	-1	0	0
C_2	0	1	0	0	1	0	1
C_3	0	0	1	0	0	1	1
C_4	0	0	0	1	0	1	0

Identity Matrix

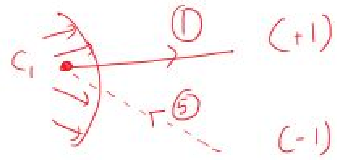
No of cut-set = 4

No of twig = 4

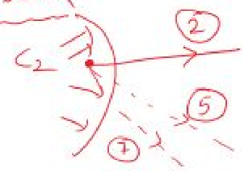
No of nodes → n = t+1 = 4+1 = 5

No of branches → 7

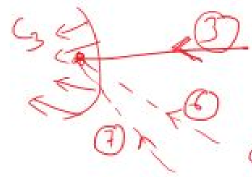
Cut-set 1



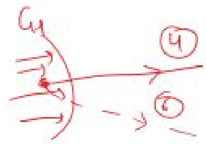
Cut-set 2

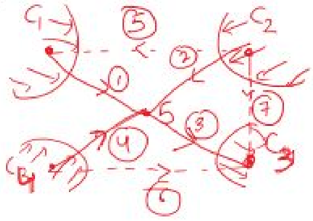


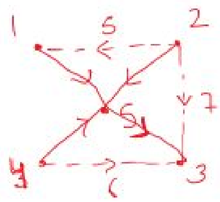
Cut-set 3



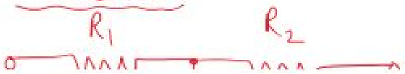
Cut-set 4





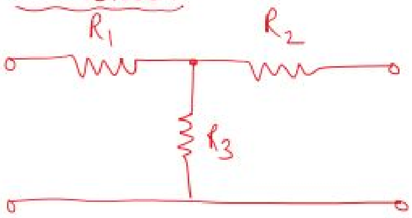


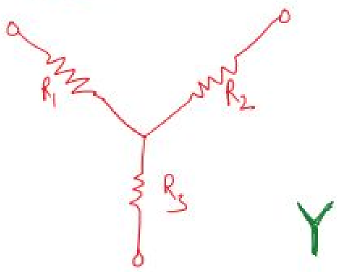
T-network



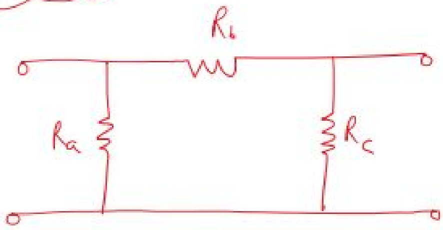
Star

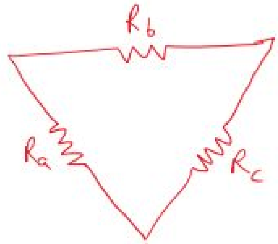
T - network





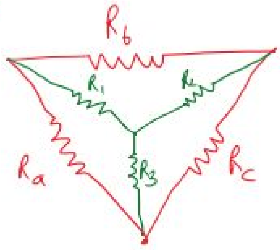
* π - N/ω





Delta

* Delta to star



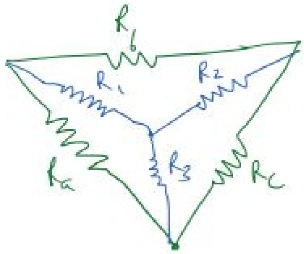
Delta to star

$$R_1 = \frac{R_a R_b}{R_a + R_b + R_c}$$

$$R_2 = \frac{R_b R_c}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_c}{R_a + R_b + R_c}$$

Star to Delta



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

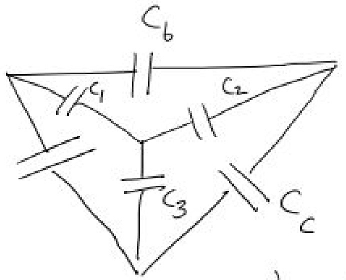
$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \quad , \quad R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

* Procedure of transformation either from Delta to star

or star to Delta $\xrightarrow{\text{for}}$ resistors & inductors

Impedances $\xrightarrow{\text{is}}$ same.

* if →



(Y) Star to Eq. Delta (Δ)

$$\frac{1}{C_a} = \frac{\frac{1}{C_1C_2} + \frac{1}{C_2C_3} + \frac{1}{C_3C_1}}{1/C_2}$$

$$\frac{1}{C_b} = \frac{\frac{1}{C_1C_2} + \frac{1}{C_2C_3} + \frac{1}{C_3C_1}}{1/C_3}$$

$$\frac{1}{C_c} = \frac{\frac{1}{C_1C_2} + \frac{1}{C_2C_3} + \frac{1}{C_3C_1}}{1/C_1}$$

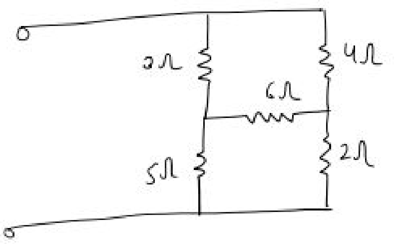
Delta to Eq. Star

$$\frac{1}{C_1} = \frac{\frac{1}{C_a}\cdot\frac{1}{C_b}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

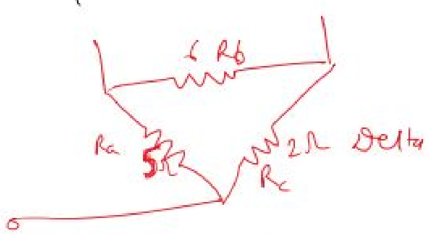
$$\frac{1}{C_2} = \frac{\frac{1}{C_b}\cdot\frac{1}{C_c}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

$$\frac{1}{C_3} = \frac{\frac{1}{C_a}\cdot\frac{1}{C_c}}{\frac{1}{C_a} + \frac{1}{C_b} + \frac{1}{C_c}}$$

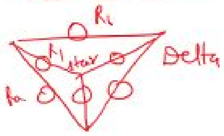
*Q



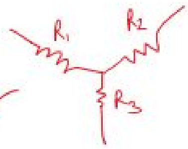
$R_{eq} = ??$



Sol. Delta to Star :→



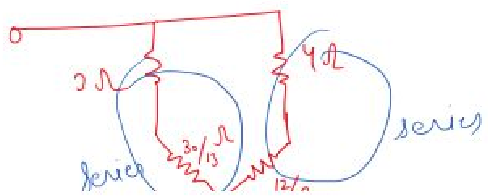
Star

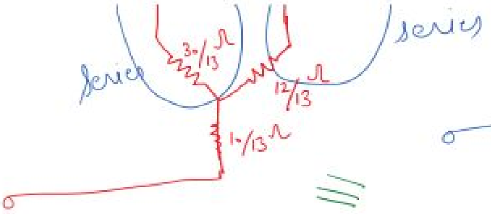


$$R_1 = \frac{6\cdot5}{6+5+2} = \frac{30}{13}$$

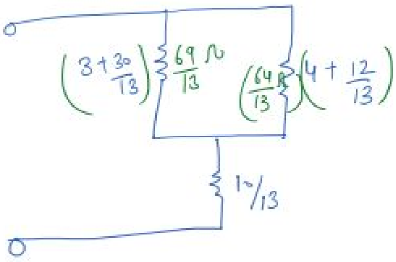
$$R_2 = \frac{6\cdot2}{6+5+2} = \frac{12}{13}$$

& $$R_3 = \frac{(5)(2)}{6+5+2} = \frac{10}{13}$$





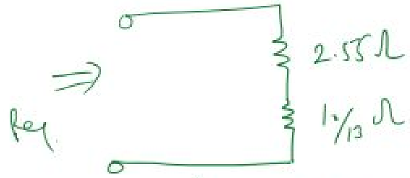
$\equiv$



$\equiv$

$\frac{69}{13} \parallel \frac{64}{13}$

$$\frac{\frac{69}{13}\cdot\frac{64}{13}}{\frac{69}{13}+\frac{64}{13}} = 2.55\,\Omega$$

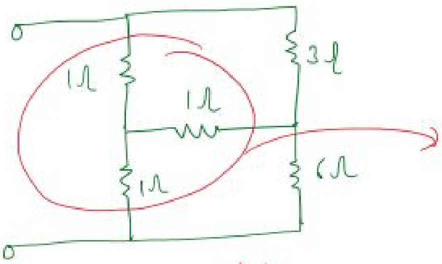


$R_{eq} = 2.55 + \frac{10}{13}$

$R_{eq} = 3.319\,\Omega$

*Q

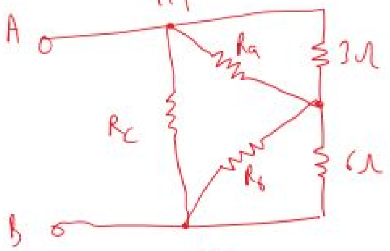
$R_{eq} = ??$



Star to Delta

Sol

$\equiv$



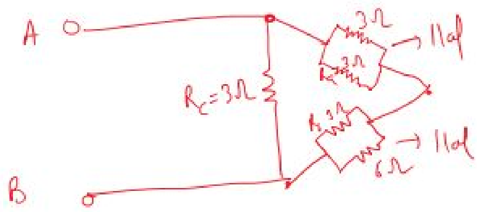
$$R_a = \frac{(1)(1)+(1)(1)+(1)(1)}{1}$$

$R_a = 3\Omega$

$\text{||ly } R_b = 3\Omega$

$R_c = 3\Omega$

$\equiv$

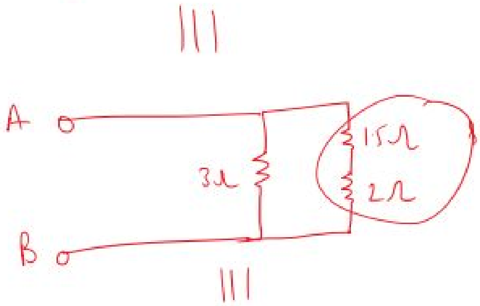


$3\|3 = \frac{3\times3}{3+3} = \frac{9}{6}$

$= \frac{3}{2} = 1.5\Omega$

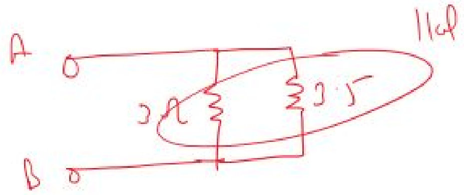
$3\|6 = \frac{3\times6}{3+6} = \frac{18}{9}$

$\equiv$



$3 \parallel 6 = \frac{3 \times 6}{3+6} = \frac{18}{9} = 2\Omega$

$\Rightarrow 1.5 + 2 = 3.5\Omega$



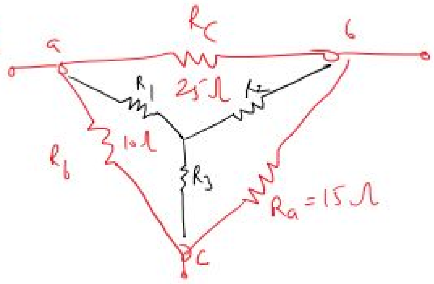
$$\frac{3 \times 3.5}{3 + 3.5} = \frac{10.5}{6.5}$$

$$R_{eq} = 1.61\ \Omega$$

* When Resistance of Equal Value transformed from star to delta, Resistors increased by 3 times.

* When Capacitors of equal Value transformed from star to delta, capacitance decreased by 3 times

* Question



Convert Δ n/w to eq Y n/w

1) $R_1 = 5\Omega$ b) $R_2 = 7.5\Omega$ c) $R_3 = 3\Omega$

2) $R_1 = 7.5\Omega$ b) $R_2 = 5\Omega$ $R_3 = 3\Omega$

3) $R_1 = 3\Omega$, $R_2 = 7.5\Omega$, $R_3 = 5\Omega$

Soln Delta to star

$$R_1 = \frac{(25)(10)}{25+10+15} = \frac{250}{50} = 5\Omega$$

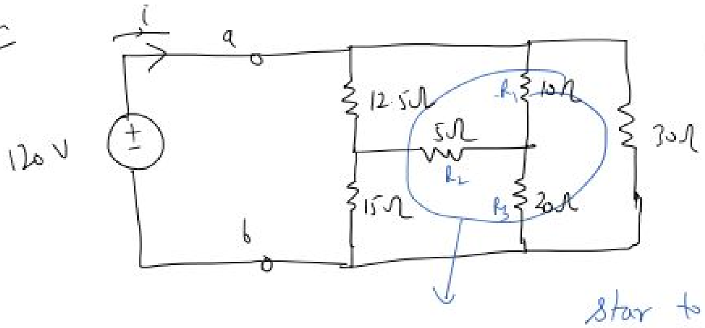
$$R_2 = \frac{(25)(15)}{50} = \frac{375}{50} = 7.5\Omega$$

$$R_3 = \frac{(15)(10)}{50} = \frac{150}{50} = 3\Omega$$

* Question

Obtain eq Resistance

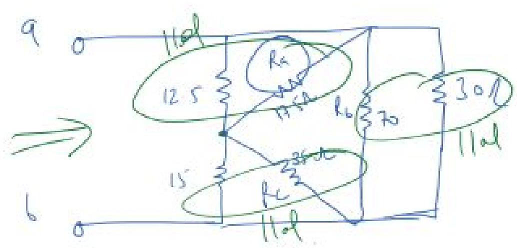
* Question



Obtain eq. Resistance R_{ab} & use it to find current i.

Sol^n

Star to Delta



$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$R_a = \frac{50 + 100 + 200}{20}$$

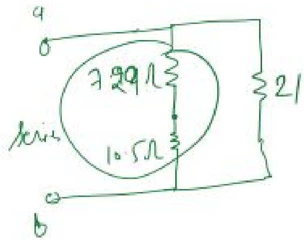
$$R_a = 350/20 = 17.5\,\Omega$$

$R_b = \frac{350}{5} = 70\,\Omega$ Ilty $R_c = \frac{350}{10} = 35\,\Omega$

→ $12.5 \,||\, 17.5 = 7.29\,\Omega$

→ $15 \,||\, 35 = 10.5\,\Omega$

→ $70 \,||\, 30 = 21\,\Omega$



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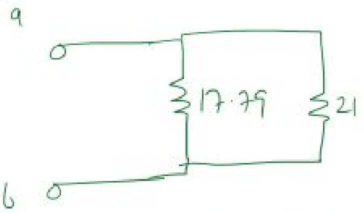

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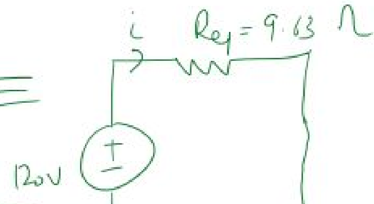

Ohm's law $i = \frac{V}{R} = \frac{120}{9.63} = 12.46\,A$