Answer 1.

The statement $(\exists m \in \mathcal{N})(\exists n \in \mathcal{N})(3m + 5n = 12)$ is false

Proof: The statement says that there are two natural numbers m and n such that

(3m+5n)=12

Since m, n are natural numbers m, n>=1

Let us take first few values of m and n as per the following table

| m | n | (3m+5n) |
|---|---|---|
| 1 | 1 | 8 |
| 1 | 2 | 13 |
| 2 | 1 | 11 |

For all other combinations of m and n (3m+5n) will be greater than 12 hence the statement is false.

Answer 2.

The statement is true

To prove that the sum of any five consecutive numbers is divisible by 5

Proof: Let n be any integer. Then n, n+1, n+2, n+3, n+4 will be any five consecutive numbers. Let s be the sum of these five numbers

i.e. s=(n)+(n+1)+(n+2)+(n+3)+(n+4)

or s= 5.n+5

or s= 5(n+1)

since the sum of these numbers s is a multiple of 5 it would be divisible by 5.

Hence the statement that the sum of any five consecutive number is divisible is proved to be true.

Answer 3.

The statement is true

To prove that for any integer n the number x = n^2+n+1 is an odd number

Proof:

Case 1. For n=0 the number x=1 which is odd

Case 2. If n is even. Then n^2 and n are both even and their sum is even. Adding 1 to that makes x an odd number.

Caes 3. If n is odd. Then n^2 is odd and n is odd and their sum is even. Adding 1 to that makes x an odd number.

Thus we see that in all possible cases the number is odd. Hence proved

Answer 4.

To prove that every odd natural number is of the form 4n+1 or 4n+3, where n is an integer.

Proof: The division theorem states that given integers a, b such that b<>0, there exist integers q, r such that

$$a = bq+r \text{ (division theorem)}$$

If we take q=4 and b=n then we get

$$a = 4n+r \text{ such that } 0<= r < 4$$

Since r is an integer we take all possible values of r

Case 1. If r = 0 or 2 then a = 4n or a = 4n+2. Or in other words a is even

Case 2. If r = 1 or 3 the a = 4n+1 or a = 4n+3. Which are odd numbers

Since this is true for any value of n, 4n+1 or 4n+3 represent any odd number. Hence proved.

Answer 5. To prove that for any integer n at least one of n, n+2 or n+4 is divisible by 3.

Proof: Using division theorem:

The division theorem states that given integers a, b such that b<>0, there exist integers q, r such that

$$a = bq+r \text{ (division theorem)}$$

Let q=3 and b=n

Then $a = 3n+r$ where $0<= r < 3$

Since r is an integer we get we get the following then we get the following three possibilities

$$a = 3n, a = 3n+1 \text{ and } a = 3n+2$$

If we substitute value of n as n+1 then we have three scenarios as

a = 3n+3, a = 3n+4, a = 3n+5

If n is substituted by n+2 then we have the following three scenarios

2 = 3n+6, a = 3n+7, a = 3n+11

In all the three we see that at least there is one value of a divisible by three.

This proves that for any integer n at least one of n, n+2 or n+4 is divisible by 3

Answer 6.

To prove that the only prime triplets are 3, 5, 7 (each separated by 2)

Proof: Proof by contradiction

Let us assume that n such that n>3 is prime and there exist a prime triplets as n, n+2 and n+4.

In answer to question number 5 we have proven that for any integer n one of n, n+2, n+4 is divisible by 3. (*excuse me for not reproducing the proof again)

Hence the assumption is wrong

Hence all the three cannot be prime and the only case is where n=3. Proved by contradiction.

Answer 7.

To prove that for any natural number

$$2 + 2^2 + 2^3 + \cdots \ldots \ldots \ldots + 2^n = 2^{n+1} - 2$$

Proof: By induction

Let us prove this for n = 1

In this case left hand side of the identity becomes $2^1 = 2$

And the right hand side of the identity becomes $2^{1+1} - 2 = 4 - 2$

= 2

Now let us prove this for any natural number n+1

Adding (n+1)th number on both the sides of the identity we get

$$2 + 2^2 + 2^3 + \cdots \ldots \ldots \ldots + 2^n + 2^{n+1} = 2^{n+1} - 2 + 2^{n+1}$$

$2 + 2^2 + 2^3 + \cdots \ldots \ldots \ldots + 2^n + 2^{n+1} = \ 2 * 2^{n+1} - 2$ (by combining the two $2^{n+1}$ together)

$2 + 2^2 + 2^3 + \cdots \ldots \ldots \ldots + 2^n + 2^{n+1} = \ 2^{n+2} - 2$ (by adding the exponents)

Let us call the right hand side as A

Now let us substitute n+1 in place of n in the original identity

Then the right hand side becomes

$$2^{n+1+1} - 2$$

$$2^{n+2} - 2$$

We find that this is same as A. Hence proved by induction that the identity

$$2 + 2^2 + 2^3 + \cdots \ldots \ldots \ldots + 2^n = 2^{n+1} - 2$$

Is true.