

Numbered Theorems, Definitions, Corollaries, and Lemmas in the Document

Theorem 1. (*Pythagorean Theorem*) *In a right-angled triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.*

$$a^2 + b^2 = c^2 \tag{1}$$

Definition 1. (*Prime Number*) *A prime number is a natural number greater than 1 that is not divisible by any number other than 1 and itself.*

- *Example: 2, 3, 5, and 7 are prime numbers.*

Corollary 1. (*Euclid's Corollary*) *There are infinitely many prime numbers.*

- *Proof: Assume there are finitely many primes. Let them be p_1, p_2, \dots, p_n . Consider the number $N = p_1 \cdot p_2 \cdots p_n + 1$. This number is not divisible by any of the primes p_1 through p_n . Therefore, there must be a prime factor not in the list, contradicting the assumption that there are only finitely many primes.*

Lemma 1. (*Basic Arithmetic Identity*) *For any real numbers a and b , we have:*

$$(a + b)^2 = a^2 + 2ab + b^2. \tag{2}$$