Modeling Conflict between Patrilineal Clans

SCUDEM III Fall 2018 Executive Summary

I. Introduction

Given the hypotheses that around 7,000 years ago patrilineal clans formed causing a high death rate of males through conflict, we model the resulting distribution between males and females for a given number of clans[1].

II. Assumptions

To simplify the process of modeling, we made some reasonable simplifying assumptions based on typical human behaviour after the first agricultural revolution.

- Clans form cities that grow radially and do not move. There will be no migration of clans; there is no accounting for clans merging or forming.
- Resources (food, water, etc.) are evenly distributed among land, and all land is equally viable for farming.
- The Earth is an infinite plane, but we only model clans with centers in a specified square area.

III. Mathematical Model

A. Definition of a clan

Each clan will have a position (x, y) in the defined square and an initial population $P_{0,i}$ that is evenly split between males and females.

B. Population Growth

A main goal of the model is to show the dynamics of gender distribution for each clan going through conflict. We start by saying the total rate of change for a population is the sum of the rate of change of the males dM/dt and females dF/dt.

$$\frac{dP_i}{dt} = \frac{dM_i}{dt} + \frac{dF_i}{dt} \tag{1}$$

Looking at a typical differential equation for a population $P(t)$, $\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$ gives a population curve that follows a logistic function starting at initial population P_0 with an upper bound carrying capacity of K. Given that the observed land is square with side length d and each km^2 supports c people, we can define the carrying capacity of each clan as the total carrying capacity of the square as shown in (2).

$$K_{net} = c\, d^2 \tag{2}$$

The r in the differential equation represents some type of growth rate. We define growth rate as a function of gender ratio for each clan. We define $r : [0,1] \to \mathbb{R}$ in a manner that supports no growth with a completely single-gender population and peak rate where there more females than males. An easy to evaluate function that supports these goals is shown in (3) and (4).

$$r(s_i) = -s_i^3 \ln(s_i) \tag{3}$$

$$s_i(t) = \frac{F_i(t)}{P_i(t)} = \frac{F_i(t)}{F_i(t) + M_i(t)} \tag{4}$$

We can finalize the growth part of the model for both males and females as below since all births will be split between the genders:

$$\frac{dM_i}{dt}^+ = \frac{dF_i}{dt}^+ = \frac{1}{2} r(s_i)\, P_i(t) \left(1 - \frac{P_i(t)}{K_{net}}\right) \tag{5}$$

C. Population Decay

We will define a specific clan's discontent, or desire to expand, Δ_i as the difference between carrying capacity and current population.

$$\Delta_i = K_{net} - P_i(t) \tag{6}$$

When two clans i and j are in sufficient contact and there is ample individual discontent, conflict arises as given in where p is a possible tuning parameter for hostility between clans.

$$A_{i,j} = \left(\frac{\lambda_{i,j}}{\ln(\Delta_i \Delta_j)}\right)^p \tag{7}$$

To measure sufficient contact, $\lambda_{i,j}$ can have multiple definitions; we will evaluate it as shared area between clans i, j. We assumed earlier each clan maintains a circle proportional to population. With both radii and clan centers known, the overlapping area can be calculated. [3] The radii can be calculated as show in (8).

$$P_i = cA = c\pi r_i^2 \;\Rightarrow\; r_i = \sqrt{\frac{P_i}{c\pi}} \tag{8}$$

Finally, the total rate of change for death of males and females can be defined based on the hypotheses that males die at a much higher rate from conflict than females. We now have final versions of a differential equation where $g \in [0,1]$ represents the percent of deaths from conflict that are male. Each equation only needs to be evaluated if there are currently males and females to avoid these values becoming negative, and we will account for natural death with some factor $\eta \in [-1, 0)$ which will determine the rate of decay when reproduction is not possible; this represents the case where only one gender left in a clan (usually females). Thus, the differential equations defining the operation of our model are defined in (9) and (10).

$$\begin{aligned}\frac{dM_i}{dt} &= \begin{cases} \frac{dM_i}{dt}^+ + \frac{dM_i}{dt}^-, & F_i > 0 \\ \eta M_i, & F_i \le 0 \end{cases} \\ &= \begin{cases} \frac{1}{2} r(s_i) P_i \left(1 - \frac{P_i}{K_{net}}\right) - g \sum_{j \ne i} A_{i,j}, & F_i > 0 \\ \eta M_i, & F_i \le 0 \end{cases}\end{aligned} \tag{9}$$

$$\begin{aligned}\frac{dF_i}{dt} &= \begin{cases} \frac{dF_i}{dt}^+ + \frac{dF_i}{dt}^-, & M_i > 0 \\ \eta F_i, & M_i \le 0 \end{cases} \\ &= \begin{cases} \frac{1}{2} r(s_i) P_i \left(1 - \frac{P_i}{K_{net}}\right) - (1-g) \sum_{j \ne i} A_{i,j}, & M_i > 0 \\ \eta F_i, & M_i \le 0 \end{cases}\end{aligned} \tag{10}$$

IV. SIMULATIONS

To perform simulations of the model, the Forward Euler method [2] was used to approximate the solution. The time step dt is 0.01 and $t = 0$. The simulation was run until $t = 700$ to approximate 7000 years with each time step representing 10 years.

A. Choosing Model Parameters

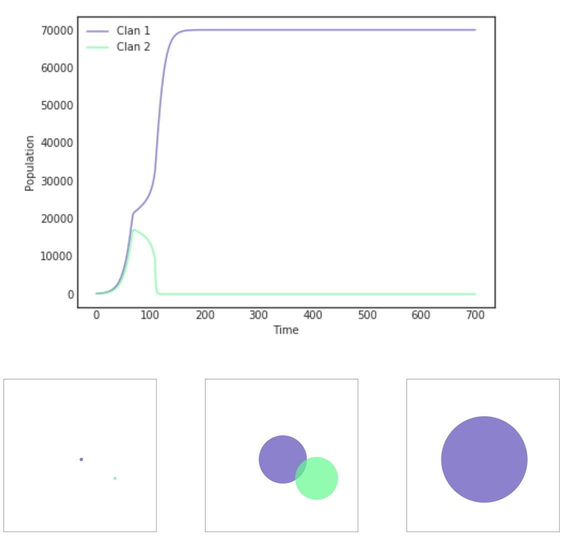
The initial conditions for simulating each individual clan was uniformly chosen by pseudo-randomly sampling over a range of values. A random seed was used to reproduce results. A complete list of parameter ranges and values is summarized in Table I.

Parameter	Symbol	Value/Range	Selected (if Global)
Dimension	d	100	100
Density	c	$(0,10)$	7
Death Disparity	g	$(0.5,0.9)$	0.58
Hostility	p	$(0,10)$	3.4
Natural Death	η	$(0,1)$	0.7
Clan Center	(x_i,y_i)	$(0,d)$	-
Initial Population	P_0	$(50,100)$	-

TABLE I
PARAMETERS FOR SIMULATIONS

B. Results

The model defined by the parameters in Table I was run first with 2 clans as the most basic case. This produced the results visualized in Fig. 1. The simulation was further run with 8 clans as well. The results for 8 clans are depicted in Fig. 2.

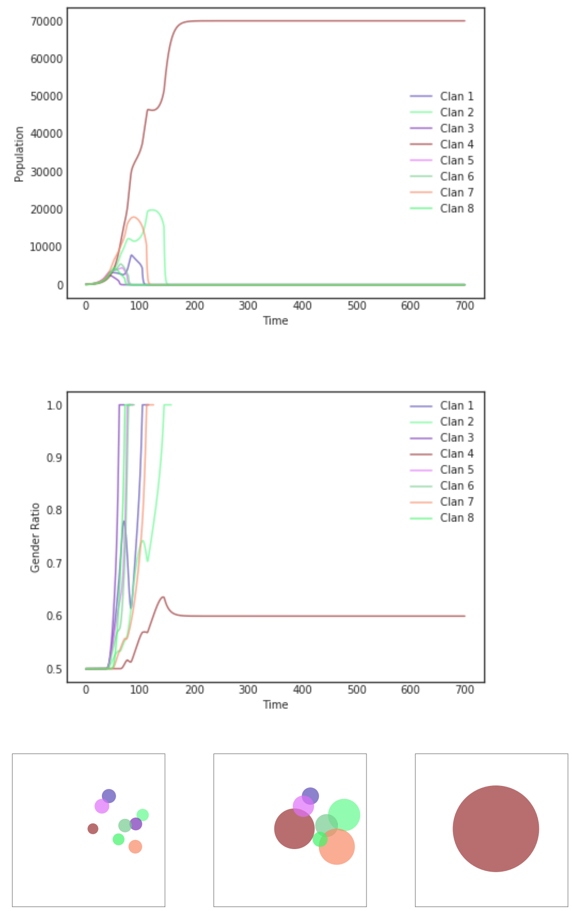


$t = 0$ $t = 100$ $t = 700$

Fig. 1. Running the Simulation with 2 Clans

V. CONCLUSIONS

The results indicate the starting distance of a clan and its neighboring clans as the main factor(s) directly affecting the amount of population decrease, or the conflict that a clan faces before reaching its carrying capacity. It is clear from Fig. 1 that the clan that is able to grow the largest the fastest tends to win conflicts versus other clans. Clan 4 has a smaller population than Clan 7 at $t = 50$, but because it doesn't have to compete for resources, it is able to eventually able to take over by $t = 200$. Equilibrium may be reached if all clans are equally spaced apart and have the same starting population, causing the patrilineal clans to maintain a small male population over time while only growing to a point that allows the birth rate to equal the death rate. Given that the world is not in uniform conditions, it is clear why patrilineal clans declined over time.



$t = 50$ $t = 100$ $t = 700$

Fig. 2. Running the Simulation with 8 Clans

Furthermore, The decline of patrilineal clans in a restricted region is inevitable. The conflict between clans in a region will always result in a winner given some type of variation or advantage in any clan. The plot of gender ratio versus time in Fig. 2 show this clearly. The clans that die out all reach a completely single-gender population when they die out, which is a direct consequence of patrilineal conflict in a closed region.

REFERENCES

[1] SIMIODE. Problem A - Conflict Between Patrilineal Clans. SCUDEM III 2018, October 2018.
[2] Eric W. Weisstein. Euler forward method. From MathWorld–A Wolfram Web Resource., 2001.
[3] Eric W. Weisstein. Circle-circle intersection. From MathWorld–A Wolfram Web Resource., 2002.