Problem A - Conflict Between Patrilineal Clans

2018 SCUDEM Competition

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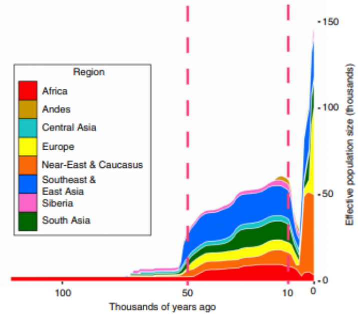
Problem A: Conflict Between Patrilineal Clans

There is evidence to suggest that the genetic variation in the Y chromosome decreased significantly 7,000 years ago because a large percentage of males died out due to conflict from warring patrilineal clans.

We model a scenario of multiple clans of different sizes warring over territory.
We track the ratio of males to females for all groups to see how the distribution of the Y chromosome changes throughout time.



Initial Conditions and Assumptions

- We are observing a square plot of land on Earth that contains each clan. The Earth is an infinite plane for simulation purposes.
- We have a given number of clans n; there will be no migration of clans.
- Each clan starts with some distinct initial population.
- Resources are evenly distributed among land and all land is equally viable for food. Each square unit of land supports a given number of people, c.
- Each clan starts at an initial location (x, y) in the square and does not move.

Initial Conditions and Assumptions - continued

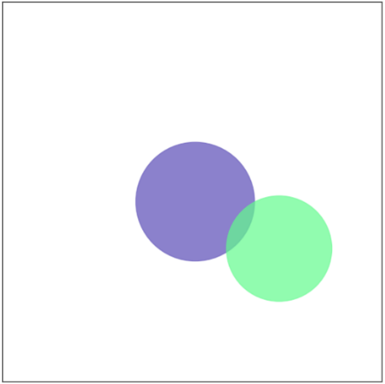


Figure: Example of the world with two clans

Model Definition: Gender

To model the change in gender for every clan it is useful to define the change in population for each clan as:

$$\frac{dP_i}{dt} = \frac{dM_i}{dt} + \frac{dF_i}{dt}$$

and the percent female as

$$s_i(t) = \frac{F_i(t)}{F_i(t) + M_i(t)} \in [0, 1]$$

Where M and F represent the number of males and females respectively.

Model Definition: Logistic Growth and Gender

The typical logistic function used to model population is:

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

We define our carrying capacity as $K_{net} = c\ d^2$ where c is the number of people supported by one square unit and d is the side length of the square plot.

$$r(s_i) = -s_i^3 \ln(s_i)$$

We can finalize the growth part of the model for both males and females as below since all births will be split between the genders:

$$\frac{dM_i}{dt}^+ = \frac{dF_i}{dt}^+ = \frac{1}{2} r(s_i) P_i \left(1 - \frac{P_i}{K_{net}}\right)$$

Model Definition: Clan Conflict

To measure a clan's internal discontent we can measure its distance from carrying capacity.

$$\Delta_i = K_{net} - P_i(t)$$

When two clans have sufficient contact they may have conflict based on their discontent:

$$A_{i,j} = \left(\lambda_{i,j} \frac{1}{\ln(\Delta_i \Delta_j)}\right)^p$$

$\lambda_{i,j}$ can have multiple definitions as the measure of contact. We define it as the overlapping area between two clan's territories, calculated with the radii and centers.

$$P_i = c(Area) = c\pi r_i^2 \;\Rightarrow\; r_i = \sqrt{\frac{P_i}{c\pi}}$$

ODE's and Parameter Outline

By summing up the conflict we now have a quantity for the decrease in rate or change for males and females.

$$\frac{dM_i}{dt} = \begin{cases} \frac{1}{2} r(s_i) P_i \left(1 - \frac{P_i}{K_{net}}\right) - g \sum_{j \neq i} A_{i,j}, & F_i > 0 \\ \eta M_i, & F_i \leq 0 \end{cases}$$

$$\frac{dF_i}{dt} = \begin{cases} \frac{1}{2} r(s_i) P_i \left(1 - \frac{P_i}{K_{net}}\right) - (1-g) \sum_{j \neq i} A_{i,j}, & M_i > 0 \\ \eta F_i, & M_i \leq 0 \end{cases}$$

- g the percent of deaths through conflict that are male
- $\eta \in [-1, 0]$ is a decaying factor for when births are no longer possible.

ODE's and Parameter Outline - Continued

- $g = .85$ the percent of deaths through conflict that are male
- $\eta \in [-1, 0], \eta = .7$ is a decaying factor for when the population growth is no longer possible.
- $c = 5$ the number of people supported by one square unit of land
- $d = 100$ the side length of the square plot
- $p = 3$ the global hostility level
- Each clan has a Cartesian coordinate (x, y), initial population, and initial gender ratio.

Results: Two Clans

Only one of the two clans are expected to survive unless two class has exactly same number of population and sex ratio, in which case two clans could oscillate and reach equilibrium.

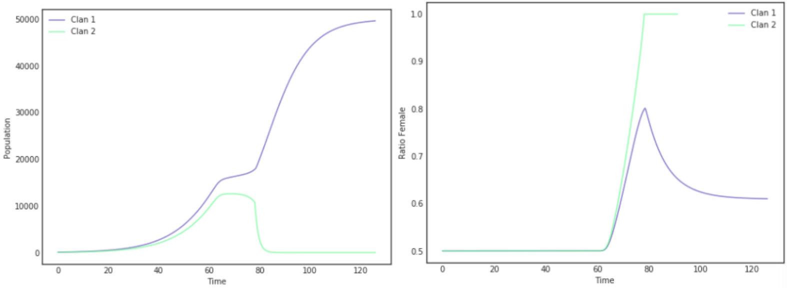


Figure: Population and gender ratio of clans by time when $n = 2$.

Results: Multiple Clans ($n = 8$)

As before, eventually only one clan takes over everyone and reaches the carrying capacity, i.e. stability.

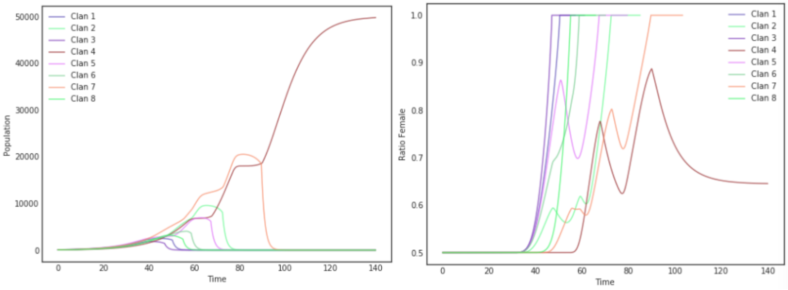


Figure: Population and gender ratio of clans by time when $n = 8$.

What is the role of human mobility in the model?

We assume that larger groups travel slower than smaller groups and clans are moving in x and y directions independently. Our earlier model assumed stationary clans but the ability to add movement is possible by defining:

$$\frac{dx_i}{dt} = \frac{dy_i}{dt} = \frac{\sqrt{\Delta_i}}{d} B = \frac{\sqrt{K_{net} - P_i}}{d} B$$

$$B \sim DiscreteUniform[-1, 1]$$

- As a group reaches carrying capacity, movement goes to zero.
- Movement is scaled based on the size of the environment.
- B is used to determine direction.

Supplemental Issue (Results)

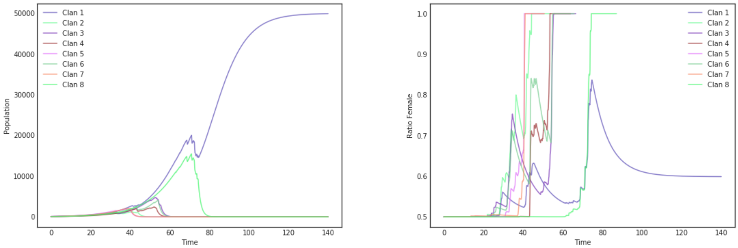


Figure: Population and gender ratio of clans by time when $n = 8$ movement is accounted for.

Conclusion

- Almost all simulations result in a single clan reaching carrying capacity and all other clans dying out.
  - An equilibrium with multiple clans occurs when there is no variation between them.
- Once the majority of conflict ends, the sex ratio of the remaining clan is mostly female, supporting the hypothesis.
- The clan that starts the largest does not always win due to the joint conflict with multiple clans.
- Adding movement causes clans to die out sooner, as they lose the stability of a single location. Adding movement may cause a different clan to come out on top than without.
- It is clear why patrilineal clans declined over time: conflict focused on one gender causes birth rates to slow too quickly.